Preventive Maintenance and Replacement Model for Mechanically Repairable Systems with Linearly Increasing Hazard Rate

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Abstract: - Most machines fail due to lack of appropriate preventive maintenance (PM) and replacement schedule, and this failure leads to higher cost of repair maintenance, distortion of production schedule, elongated downtime period and reduced productivity. These could however be avoided by the utilization of optimal PM and replacement models suited for the specific kind of system. It is on this premise that this work develops an optimal PM and replacement model for mechanically repairable systems with linearly increasing hazard rate which failure distribution of the system is characterized by the Rayleigh distribution. The failure times of a Rolls Royce dredging machine was used as real-time data to obtain the PM and replacement schedule for the machine at respective cost ratios. The results showed that the model provided an effective maintenance schedule for the machine and ensures optimal performance.

Key-Words: - preventive maintenance, replacement model, linearly increasing hazard rate, Rayleigh distribution, dredging machine, repairable systems

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1 Introduction

Preventive maintenance (PM) is an effective method of enhancing the condition of a machine's functionality. It aids in minimizing cost of maintenance and unexpected failure of machines. A PM policy outlines the scheduling requirements for PM activities. It may be periodic, which calls for machine maintenance at integer multiples of a predetermined period or sequential which keeps the system running at a series of intervals that may have different lengths. Both of these PM plans share the same presumption that the machine will only need minimal maintenance if it breaks down in between PM actions. When a machine breaks down, minimal repair just gets it back to working condition (as-good-as old); it does not get the machine healthier overall. In other words, minor repairs have no impact on the machine's age or hazard rate. Authors in [1] and [2] defined Preventive maintenance (PM) as a set of activities to be performed before system failures, aimed at keeping the system in a good working state and reducing its operational expenditure. A sequential PM policy with failure rate threshold for lease items with Weibull lifetime distribution was developed by [3] and applied it to leased equipment. The work showed that any product failure that occurs during the term of the lease is fixed with minimum repairs, and if a minimal repair takes longer than expected, the lessee may be charged a fee. Furthermore, additional PM actions were carried out in an effort to reduce product failures. The best threshold value and accompanying maintenance degrees were determined using this maintenance scheme and a mathematical model of the predicted total cost in order to reduce cost.

In 2009, [4] reviewed maintenance policies with emphasis on replacement, imperfect PM, and inspection policies in a broader context. The three common models of block replacement, simple replacement, and periodic replacement with minimum maintenance were changed into finite replacement models. Subsequently, an ideal preventive maintenance and replacement plan for a system was determined using a novel mathematical model by [5] with three potential actions; keeping the system up to date, replacing it, and doing nothing for each of the discrete, equally spaced intervals that made up the maintenance planning horizon, noting that every choice has a price and it influences the system's failure pattern. In a later development, [6] integrated dynamic programming with the branch and bound method to find the best PM plan for a sequence of repairable and maintainable system which components have increasing rate of failure. In order to meet the system's objectives, the best choices for each component during each period were examined. These were all based on the general increasing hazard rate condition of mechanically repairable systems. A repairable machine is a machine in which damaged parts can be serviced or readjusted so as to get the system into proper working condition. In this type of machine, a failure to one part of the machine does not necessary mean a failure to the entire machine. Also, damaged parts are not completely replaced regularly.

In 1880, [7] proposed the Rayleigh distribution to solve a problem in the field of acoustics. Since then, a lot of researches in various branches of science and technology has been done in relation to this distribution. The generalized Rayleigh distribution was put forth by [8], and various estimating techniques have been used to determine its parameters. Authors in [9] investigated the estimation of the Rayleigh distribution's parameter in the presence of various censoring sampling strategies, including type-I, type-II, and progressive type-II censored sampling. The Weibull distribution with scale parameter 2 is a particular instance of the Rayleigh distribution which is of interest in this work. When the shape parameter is adjusted to 1, the Rayleigh distribution changes into the chi square distribution with two degrees of freedom. The failure rate or hazard function of the Rayleigh distribution, according to [10], is a crucial property since it rises over time. This suggests that when failure time is dispersed in accordance with the Rayleigh model, excessive aging/piece occurs. The Rayleigh distribution's hazard rate increases linearly over time. It has several uses, including reliability analysis, clinical investigations, life testing experiments and applied statistics. It is frequently used to simulate the behaviour of systems with rising failure rates. The two-parameter Rayleigh distribution provides a simple but nevertheless useful model for the analysis of lifetimes, especially when investigating reliability of technical equipment.

Several replacement maintenance models and policies abounds in the literature. For instance, [11] proposed that the first and last triggering event approaches for replacement with minimal repairs of whichever occurs last should be used in optimizations for policy consideration when replacement times could be scheduled at a planned time, T of operation and at a number, N of minimal repairs to compare with the traditional approach of whichever occurs first. The long-run average cost rates was minimized by [12] to estimate the best scheduled replacement instants. In order to examine the effects of such variations, cost-rate minimizing models were created, presuming that the real PM time and the scheduled PM time varied from one another in a probabilistic manner. Also, [13] reviewed general maintenance policies under the key areas of maintenance: holistic review, concept planning, development planning and optimization planning. Furthermore, a non-periodic preventive maintenance schedule for repairable systems using failure rate threshold was developed by [14].

Over time, many combined PM and replacement models have been proposed to improve system maintenance. Of particular interest is [15] who proposed a PM and replacement schedule based on the age and hazard models. The result showed that the age model outperformed the hazard model. In a later development, [16] formulated a hybridized PM model on the assumption that PM is imperfect by combining the age reduction model and the hazard rate adjustment model of [15] for improved decision. Also, [17] developed a geometric imperfect preventive maintenance and replacement model for ageing repairable systems with higher degrees of deterioration. The model has three phases: the average life span, beyond the average life span, and beyond the initial replacement age of the system. The model was a generalization of [16] to produce a PM and replacement timeline for ageing mechanically repairable systems at various phases of deterioration.

It is however noted that these models were developed for the general case of repairable systems with increasing hazard rate mostly characterized by the Weibull failure function. In this work, we consider a special class of repairable systems with linear increasing hazard rate and therefore propose an optimal PM and replacement model for this class of system which were not considered in previous

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works. The Rayleigh distribution is utilized as the failure distribution characterizing the failure rate of this class of system, [10]. The proposed model is implemented on the Roll Royce dredging machine to obtain optimal PM and replacement schedule.

2 Problem Formulation

The Rayleigh distribution is the proposed failure distribution for this work because of it has the property of linear increasing hazard rate (LIHR). It is derived from the Weibull distribution when the shape parameter is 2. The 2-parameter Weibull distribution is given by;

$$f(t)=\alpha\beta^{\alpha}t^{\alpha-1}\,e^{-(\beta t)^{\alpha}}\ ;\ t>0,\quad \alpha>0,\ \beta>0$$

The hazard and cumulative hazard functions are respectively given as:

$$h(t)=\alpha\beta^{\alpha}t^{\alpha-1}\quad and\quad H(t)=\beta^{\alpha}t^{\alpha}$$

For $\alpha=2$, we have the Rayleigh distribution as follows;

$$f(t)=2\beta^{2}t\ \ e^{-(\beta t)^{2}}\ ;\ t>0,\ \ \beta>0$$

And its respective hazard and cumulative hazard functions are:

$$h(t)=2\beta^{2}t\ \ and\ \ H(t)=\beta^{2}t^{2}$$

2.1 Formulation of imperfect preventive and replacement model for repairable systems with linearly increasing hazard rate (LIHR)

In [16] a hybrid model which is a combination of the hazard rate adjustment model and the age reduction model of [15] was formulated. The model is given by; $\lambda(t_1+x)=dh(ut_1+x)$ where d is the hazard rate adjustment factor, u is the age reduction factor and *x* is the operating time before the next PM; $d\geq 1;\ 0\leq u\leq 1;\ x\in(0,t_2-t_1),$ $h(t)$ is the failure rate function for $t\in(0,t_1)$.

The PM activity at time, t_1 generates a new failure rate function, λ(t) for $t\in(t_2,t_1)$ with $dh(x)$ as the failure rate function in the subsequent PM interval which solely depends only on *h(x)* and the associated PM activity. In other words, *λ(t)* is dependent on both *h(t)* for $t\in(0,t_1)$ and *u*, the magnitude of the PM activity in time t_1.

2.2 The average cost of running the system per unit time

The associated cost model to PM and replacement model is often used to evaluate the performance of the repairable system and also to determine expected time for safe and appropriate maintenance. The aim is to minimize the expected cost of maintenance. Hence, the expected cost rate model is;

$$C=C(y_1,y_2,\ldots y_N)=\frac{Q_r+(N-1)Q_p+Q_m\sum_{k=1}^{N}D_k\left[H(y_k)-H(u_{k-1}y_{k-1})\right]}{\sum_{k=1}^{N-1}(1-u_k)y_k+y_N}\qquad(1)$$

where Q_r, Q_p *and* Q_m are respectively the cost of replacement maintenance, preventive maintenance and minimal repair of the machine, $D_k=\prod_{i=1}^{k-1}d_i$, $0=u_0<u_1<u_2<\ldots<1,$ D_k and $H(y_k)$ are the product of the hazard rate adjustment factor and the cumulative hazard function occurring within the interval (t_{k-1},t_k), which is between the time of $(k-1)^{th}$ *PM* and the *kth* PM respectively and $u_{k-1}y_{k-1}$ is the effective age of the system right after $(k-1)^{th}$ PM.

3 Problem Solution

3.1 Minimizing expected cost per unit time

To generate optimal PM and replacement plan for mechanically repairable systems with linearly increasing hazard rate, we shall determine optimal PM intervals by finding the optimal values of y_k (k =1,2,3,…,N) and at replacement point, N as decision variables to minimize the expected cost rate in (1); see [18], [15] and [16]. Let $C(y_1,y_2,\ldots y_N)=C$; In order to minimized the cost function, we take the partial derivative of (1) with respect to y_k and equate the obtained derivative to zero as follows;

$$\frac{\partial C}{\partial y_k}=\frac{\left[y_N+\sum_{k=1}^{N-1}(1-u_k)y_k\right]Q_m\left[D_kh(y_k)-u_kD_{k+1}h(u_ky_k)\right]-\left[Q_r+(N-1)Q_p+Q_m\sum_{k=1}^{N}D_k\left[H(y_k)-H(u_{k-1}y_{k-1})\right]\right](1-u_k)}{\left[y_N+\sum_{k=1}^{N-1}(1-u_k)y_k\right]^2}=0$$

$$=C_m\left[D_kh(y_k)-u_kD_{k+1}h(u_ky_k)\right]-C\left[(1-u_k)\right]=0$$

$$\Rightarrow Q_m\left[D_kh(y_k)-u_kD_{k+1}h(u_ky_k)\right]=C\left[(1-u_k)\right]\qquad(2)$$

$k = 1,2,3,\ldots, N-1$

Where; $h(y_k)$ is the hazard function and $H(y_k)$ is the cumulative hazard function, $h(u_k y_k)$ is the adjusted hazard function of the machine after k^{th} PM where $k = 1,2,3,\ldots, N-1$.

Similarly, at replacement point, N;

$$\frac{\partial C}{\partial y_N} = \frac{Q_m\left[D_N h(y_N) - D_N P_{k+1} h(u_N y_N)\right] - C}{\left[y_N + \sum_{k=1}^{N-1}(1-u_k)y_k\right]} = 0$$

$$= Q_m\left[D_N h(y_N) - r_N D_{k+1} h(u_N y_N)\right] - C = 0$$

$$\Rightarrow Q_m\left[D_N h(y_N) - u_N D_{k+1} h(u_N y_N)\right] = C \quad (3)$$

$$k = 1,2,3,\ldots, N-1$$

$D_{k+1} = 0$ since replacement occurs at the k^{th} PM.

$$C = Q_m\left[D_N h(y_N)\right] \quad (4)$$

By substituting (4) into (2) we have;

$$Q_m\left[D_k h(y_k) - u_k D_{k+1} h(u_{k-1} y_{k-1})\right] = Q_m\left[D_N h(y_N)\right]\left[(1-u_k)\right]$$

$$\Rightarrow \left[D_k h(y_k) - u_k D_{k+1} h(u_{k-1} y_{k-1})\right] = \left[D_N h(y_N)\right]\left[(1-u_k)\right] \quad \forall\ k = 1,2,3,\ldots, N-1 \quad (5)$$

where $d_k = \frac{6k+1}{2k+1}$ is the hazard rate adjustment factor and $u_k = \frac{k}{2k+1}$ is the age improvement factor, [16]

Also, from (1);

$$C\left[\sum_{k=1}^{N-1}(1-u_k)y_k + y_N\right] = Q_r + (N-1)Q_p + Q_m \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right]$$

$$\Rightarrow C\left[\sum_{k=1}^{N-1}(1-u_k)y_k + y_N\right] - Q_m \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = Q_r + (N-1)Q_p \quad (6)$$

Substituting (4) into (6), we obtain;

$$Q_m\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1}(1-u_k)y_k + y_N\right] - Q_m \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = Q_r + (N-1)Q_p$$

$$\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1}(1-u_k)y_k + y_N\right] - \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = \frac{Q_r + (N-1)Q_p}{Q_m} \quad (7)$$

3.2 Algorithm for generating PM and Replacement Schedule

Based on the preceding results, the following computational algorithm would be used;

Step1: solve for y_k as a function of y_N

Step2: Substitute y_k into (7)

Step3: Choose N to minimize $P_N h(y_N)$

Step4: Obtain y_k from the expression in step 1

Step5: obtain $x_k = y_k - u_{k-1} y_{k-1}$, $k = 1,2,3,\ldots, N$

The input parameters are the cost Q_r, Q_p and Q_m with ratios $\frac{Q_r}{Q_p}$ *and* $\frac{Q_m}{Q_p}$, the Weibull parameters are α and β, and the adjustment factors are d_k and u_k.

3.3 Implementation of the optimal PM and replacement algorithm

Step 1: Substituting the Rayleigh hazard function in (5), we have;

$$\left[D_k 2\beta^2(y_k) - u_k D_{k+1} 2\beta^2(u_k y_k)\right] = \left[D_N 2\beta^2(y_N)\right]\left[(1-u_k)\right]$$

$$(y_k)\left[D_k - D_{k+1} u_k^2\right] = D_N (y_N)(1-u_k)$$

$$y_k = \left[\frac{D_N(1-u_k)}{\left[D_k - D_{k+1} u_k^2\right]}\right] y_N \quad (8)$$

Step 2:

From (7) we have;

$$\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1}(1-u_k)y_k + y_N\right] - \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = \frac{Q_r + (N-1)Q_p}{Q_m}$$

By substituting Equation (8) into (7) we obtain;

$$\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1}(1-u_k)\left[\frac{D_N(1-u_k)}{\left[D_k - D_{k+1} u_k^2\right]}\right](y_N) + y_N\right] - \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = \frac{Q_r + (N-1)Q_p}{Q_m}$$

$$\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1}\left[\frac{(1-u_k)^2}{\left[D_k - D_{k+1} u_k^2\right]}\right] D_N(y_N) + y_N\right] - \sum_{k=1}^{N} D_k\left[H(y_k) - H(u_{k-1} y_{k-1})\right] = \frac{Q_r + (N-1)Q_p}{Q_m}$$

let $\omega_k = \left[\frac{(1-u_k)^2}{\left[D_k - D_{k+1} r_k^2\right]}\right]$

$$\left[D_N h(y_N)\right]\left[\sum_{k=1}^{N-1} \omega_k P_N(y_N) + y_N\right] - \sum_{k=1}^{N} D_k\left[H(y_k)\right] = \frac{Q_r + (N-1)Q_p}{Q_m}$$

Substituting the Rayleigh hazard and cumulative hazard functions, we have;

$$\left[D_N\alpha\beta^2(y_N)\right]\left[D_N\sum_{k=1}^{N-1}\omega_k(y_N)+y_N\right]-\sum_{k=1}^{N}D_k\left[\beta^2(y_k)^2\right]=\frac{Q_r+(N-1)Q_p}{Q_m}$$

$$\left[D_N 2(y_N)(y_k)\right]\left[D_N\sum_{k=1}^{N-1}\omega_k+1\right]-\sum_{k=1}^{N}D_k\left[(y_k)^2\right]=\frac{Q_r+(N-1)Q_p}{\beta^2 Q_m}$$

Note:

$$\sum_{k=1}^{N}D_k\left[(y_k)^2\right]=\sum_{k=1}^{N-1}D_N\left[(y_k)^2\right]+D_N(y_N)^2$$

Hence,

$$\left[D_N 2(y_N)(y_k)\right]\left[D_N\sum_{k=1}^{N-1}\omega_k+1\right]-\left[\sum_{k=1}^{N-1}D_N\left[(y_k)^2\right]+D_N(y_N)^2\right]=\frac{Q_r+(N-1)Q_p}{\beta^2 Q_m}$$

$$\therefore y_N=\left[\frac{\left[Q_r+(N-1)Q_p\right]}{Q_m\left\{\left[D_N^2\sum_{k=1}^{N-1}\omega_k+D_N\right]\right\}}\right]^{1/2}\times\frac{1}{\beta} \qquad (9)$$

Step 3: To obtain optimal N, we seek optimal number N^* which minimizes $D_N h(y_N)$

Let $B(N)=D_N h(y_N)=D_N\alpha\beta^2(y_N)$

$$B(N)=D_N 2\beta^2\left\{\left[\frac{Q_r+(N-1)Q_p}{Q_m\left\{\left[D_N^2\sum_{k=1}^{N-1}\omega_k+D_N\right]\right\}}\right]^{1/2}\times\frac{1}{\beta}\right\}$$

$$=\frac{D_N 2\beta\left[Q_r+(N-1)Q_p\right]^2}{\left[Q_m\right]^2\left\{\left[D_N^2\sum_{k=1}^{N-1}\omega_k+D_N\right]\right\}^2}$$

Let $A=\frac{2\beta}{\left[Q_m\right]^2}$

$$=\frac{A\left[Q_r+(N-1)Q_p\right]^2}{\left[D_N^{-1}+\sum_{k=1}^{N-1}\omega_k\right]^2}$$

A necessary condition for the existence of a finite N^* which minimizes B(N) is that N^* satisfies the inequalities;

$B(N+1)\geq B(N)$ *and* $B(N)<B(N-1)$. This follows;

$$\frac{A\left[Q_r+(N)Q_p\right]^2}{\left[D_{N+1}^{-1}+\sum_{k=1}^{N}\omega_k\right]^2}\geq\frac{A\left[Q_r+(N-1)Q_p\right]^2}{\left[D_N^{-1}+\sum_{k=1}^{N-1}\omega_k\right]^2}$$

$$=\frac{-(N-1)\left[D_{N+1}^{-1}-D_N^{-1}+\omega_N\right]}{\left[D_{N+1}^{-1}-D_N^{-1}+\omega_N\right]}+\frac{D_N^{-1}+\sum_{k=1}^{N-1}\omega_k}{\left[D_{N+1}^{-1}-D_N^{-1}+\omega_N\right]}\geq\frac{Q_r}{Q_p}$$

$$B(N^*)=\frac{D_N^{-1}+\sum_{k=1}^{N-1}\omega_k}{\left[D_{N+1}^{-1}-D_N^{-1/2-1}+\omega_N\right]}-(N-1)\geq\frac{Q_r}{Q_p}$$

(10)

Where

$$\omega_k=\left[\frac{(1-u_k)^2}{\left[D_k-D_{k+1}u_k^2\right]}\right] \quad \text{and} \quad D_k=\prod_{i=1}^{k-1}d_i,\ \forall\ k=1,2,3,...,N$$

Similarly, $B(N^*-1)<\frac{Q_r}{Q_p}$

3.4 Application of the proposed model

The inter failure times of Rolls Royce - RB211 Engine Dredging machine was studied and found to follow a Rayleigh distribution with rank 1 and scale parameter $\beta=2.5$ with the help of Easyfit (5.6) software. Rolls Royce is a dredging machine that is used to suck out accumulated sediment from the bottom or banks of bodies of water, rivers, lakes or streams. See Fig.1 in the appendix.

Recall:

$d_k=\frac{6k+1}{2k+1}$ is the hazard rate adjustment factor and $u_k=\frac{k}{2k+1}$ is the age improvement factor.

$D_k=\prod_{i=1}^{k-1}d_i,\ \forall\ k=1,2,3,...,N$ is the cumulative hazard rate adjustment factor

The optimal N, cost ratios $\frac{Q_r}{Q_p}$ and $\frac{Q_m}{Q_p}$ were obtained from equation (10) as shown in Table 1. Also, the effective age y_k and optimal preventive maintenance and replacement schedule x_k was obtained for the dredging machine from (8) and Step 5 of the algorithm respectively.

Table 1: Optimal PM and replacement Schedule for Rolls Royce Dredging Machine with LIHR ('0000)

N^*	1	3	5	7	9	11	13
Q_r / Q_p	2	5	10	20	30	50	80
x_1	0.2502	0.4225	0.5594	0.7911	0.9689	1.2508	1.5822
x_2		0.2305	0.2356	0.3194	0.3856	0.5742	0.6179
x_3		0.2315	0.1899	0.2525	0.3023	0.4918	0.4796
x_4			0.1567	0.2047	0.2434	0.3065	0.3821
x_5			0.2405	0.1809	0.2140	0.2680	0.3331
x_6				0.2004	0.1906	0.2375	0.2942
x_7				0.2111	0.1708	0.2118	0.2615
x_8					0.1542	0.1904	0.2344
x_9					0.1812	0.1718	0.2110
x_{10}						0.1557	0.1906
x_{11}						0.1955	0.1728
x_{12}							0.1550
x_{13}							0.2000

3.5 Discussion of results

Table 1 shows the optimal number, N^* of PM and replacement at the last point in row 1. The cost ratios are contained in row 2 while rows $x_1 \ldots x_{13}$ are the operating times of the machine under different cost ratios. The decreasing pattern of the operating times in columns 1 to 13 except the last one which is the replacement point shows shorter operating times before next PM. This calls for frequent PM due to usage and aging which is in line with the result obtained in the works by [15], [16], [19] and [17]. For instance, under the cost ratio of 20,000 in the first column of Table 1, replacement should be carried out on the machine after about 2,502hours of operation. If the company chooses to continue with the use of the machine, then it moves to the next column with a higher cost ratio of 50,000. In this column, the first PM is carried out on the machine after about 4,225hours of operation, the next PM is carried out after about 2,305hours of operation, being the second cycle and finally replacement is recommended in the third cycle after about 2,315hours of operation. If the operator still chooses to continue with the use of the machine, it moves to the next column with next higher cost ratio and so on.

4 Conclusion

A PM and replacement model has been developed in this work for a special class of mechanically repairable systems with linearly increasing hazard rate (LIHR) which failure rate is characterized by the Rayleigh distribution. The proposed model is shown to provide optimal PM and replacement schedule for this class of systems which were not provided for in earlier models. This model was applied to the Rolls Royce dredging machine which failure times was found to follow the Rayleigh distribution with scale parameter 2.5 to obtain optimal PM and replacement schedule. The machine has a linearly increasing hazard rate (LIHR), which means the machine deteriorates linearly with time. It is found that for a system with LIHR, PM is carried out more often at different costs levels which guarantees safe operation and of course, conforms to earlier results by [15], [16] and others. The frequent PM schedule obtained in this work will reduce the effective age and downtime of the machine as well as avoiding unplanned failures thereby increasing the uptime of a machine.

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Appendix

Figure 1: Rolls Royce dredging machine connected to a suction pipe