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# Application of the Real-Time Hilbert Huang Transform to a Noise Perturbed Buck Converter

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*Abstract:* - Extraneous noise signals can greatly downgrade the performance of a buck converter. In this paper, we investigate the use of an adaptive notch filter to dampen oscillations caused by perturbations of the input voltage. Typically, time-frequency analysis is carried out using traditional techniques such as the Fourier Transform or the Short-Term Fourier Transform. However, both of these methods require the signal to be stationary in its entirety or to be broken down into the sum of several stationary signals. Thus, these techniques are limited in their application. We propose using the Hilbert-Huang Transform to analyse in the time-frequency domain and identify the frequency of the non-stationary input. This can be used to tune the adaptive notch filter and the paper outlines a method for real-time application of the Hilbert Huang Transform.

*Key-Words:* - dc-dc power converters, empirical mode decomposition, Hilbert-Huang transform.

## 1 Introduction

As technology sizes shrink and functional density increases, supply voltage noise has become more significant in recent years [1]. The degradation of signals can come from a wide variety of sources; coupling from adjacent signals, baseband and RF oscillators, reflection from impedance disturbances, *IR* voltage drops to just name a few [2]–[4]. If switch mode power supplies (SMPS) are not protected against these intrusions, they can cause unwanted oscillations (ringing), intermittency or even chaotic operation [5]. Previous work on the effect of input perturbations on the buck converter [5]–[7] has been limited to sinusoidal, triangular and saw-tooth waveforms. Furthermore, the research has been analytical in the prediction of the effect of the noise rather than designing methods for its reduction or elimination. In this work, we propose the inclusion of an adaptive notch filter to reduce the effects of noise perturbing the input voltage. The filter is tuned using the Hilbert-Huang Transform (HHT).

Fourier analysis is one of the most popular tools for analysis in the time-frequency domain. It allows the user to describe any signal as the sum of sinusoidal functions [8]. However, because each sinusoidal function must be time-independent, Fourier analysis is only suited to stationary signals i.e. the signal being studied must have a frequency content that does not change with time [9]. To combat this, the short-term Fourier transform (STFT) was developed. The STFT divides a signal into a series of sections. Each section contains a stationary signal. This enables Fourier analysis to be carried out on each respective section. However, issues arise with the selection of the window size. For example, very small window sizes only capture high-frequency components and omit the low frequency elements. Very large window sizes will increase the range of frequencies that can be identified but also increase the risk of a section containing a non-stationary signal which leads to inaccurate results [10]. Another popular method is the use of the Wigner-Ville distribution [9]. It is similar to the STFT but is advantageous in that it does not have to sacrifice resolution for a range of identifiable frequencies. However, the distribution includes the possibilities of non-physical harmonics and negative amplitudes [11]. Thus, the real frequencies must be picked out from the non-physical components. An alternative tool proposed by Norden Huang, is the HHT [12]. This consists of two components; Empirical Mode Decomposition (EMD) and the Hilbert Transform (HT). EMD is an adaptive iterative algorithm which breaks a signal down into a series of Intrinsic Mode Functions (IMFs) whose HT gives the instantaneous frequency values [13]. These IMFs have time-varying amplitudes and frequencies unlike in Fourier analysis where the signal is the sum of sine waves. The HT is a spectral analysis tool that describes nonstationary data locally.

We will first present an overview of the HHT and describe the buck converter model. We will then introduce an adaptive notch filter and show how the HHT can be used to tune the filter to reduce the effect of the noise.

# 2 Buck Converter Model

The buck converter is considered in Fig.1 (A). The purpose of the buck converter is to convert an input voltage, $V_{IN}$, to a smaller output voltage, $v_o$, where $v_o = dV_{IN}$ and $d$ is the duty cycle of switch $SW_1$. $SW_1$ is closed for a time $dT$ and open for the remainder of the switching period i.e. $(1\text{-}d)T$, where $T$ is the period of the ramp signal. $SW_2$ operates complimentary to switch $SW_1$. The open loop buck converter is described by the following equations:

$$\frac{d}{dt}\begin{bmatrix} v_o(t) \\ i(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} v_o(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V_{IN} \quad (1)$$

Fig. 1 (B) illustrates the control loop used in this work. The output voltage is measured against a reference voltage to give the error signal. This passes through the controller where the proportional control law is applied. The output of the controller is compared to a ramp signal. This can mathematically be described by the switching surface $h(x,t)$.

$$h(x,t) = K_P(V_{REF} - v_o) - V_L - (V_U - V_L)t \bmod T \quad (2)$$

When $h(x,t) > 0$, $SW_1$ is closed. When $h(x,t) < 0$, $SW_1$ is open.

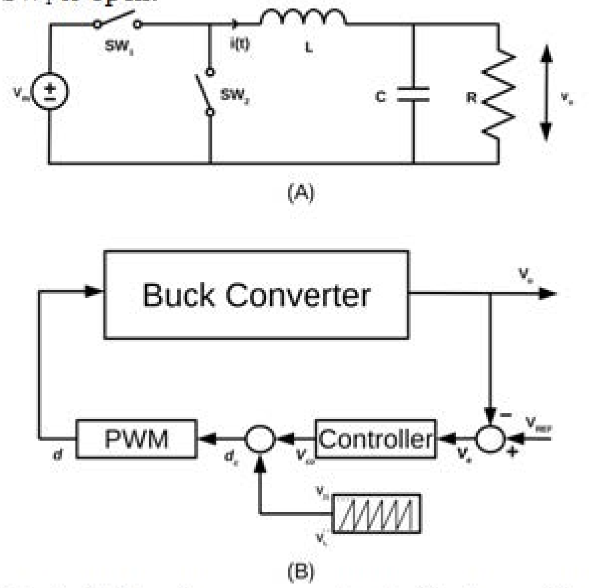


Fig. 1: (A) Buck converter circuit (B) Control loop

# 3 Hilbert-Huang Transform

The HHT is a method to decompose a signal into a series of IMFs whose instantaneous frequency is obtained using the HT. The signals are typically nonstationary and nonlinear.

## 3.1 Empirical-Mode Decomposition

EMD decomposes any multicomponent signal into a set of nearly mono-component signals termed IMFs. Once the signal is decomposed, the instantaneous frequency of the IMFs can be found using the HT. However, there are conditions to define a physically meaningful instantaneous frequency [14]:

1. In the data set, the number of extrema (maximum and minimum points) and the number of zero crossings must either be equal or differ by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima must be zero at every point.

The steps to find the IMFs are [12]:

1. Identify all the local maxima in the input signal and connect them with a cubic spline line. This is termed the upper envelope.
2. Repeat step 1 with the local minima. This is termed the lower envelope.
3. Determine the mean of both of these envelopes, $m_1(t)$.
4. The mean is then subtracted from the original signal. The result is the first component, $d_1(t)$.

In ideal circumstances, after performing step 4, the result should be the aforementioned IMF. However, during the process, overshoots and undershoots may exist which leads to new extrema [14]. The overshoots or undershoots can shift and cause distortion of the existing extrema. Furthermore, the envelope mean may be different to the true nonlinear signal mean which may cause $d_1(t)$ to be asymmetric. To avoid this, Huang repeats the steps as many times as are required in order to reduce the extracted signal to an IMF. Therefore, the final step, step 5 is to treat $d_1(t)$ as the signal and repeat steps 1-4. This is known as the sifting process. The sifting process is thus:

$$d_{1(k-1)} - m_{1(k-1)} = d_{1k} \quad (3)$$

This sifting process is repeated until $d_{1k}(t)$ becomes an IMF. Thus, the sifting process must be terminated by some stopping criterion. This is achieved by limiting the size of the standard deviation which is found from two consecutive sifting results.

$$\sigma = \sum_{t=0}^{N} \left[ \frac{| d_{1(k-1)}(t) - d_{1k}(t) |^2}{d^2_{1(k-1)}(t)} \right] \quad (4)$$

According to Huang, a standard deviation of 0.2-0.3 for the sifting process is a rigorous limit [12]. Thus, the first IMF, $c_1$, is extracted from the signal. The steps are then repeated several times to remove all IMFs. Thus, the original signal can now be written as:

$$x(t) = \sum_{i=1}^{n} c_i + r_n \quad (5)$$

where $c_i$ is the i$^{th}$ IMF and $r_n$ is the final residue and $n$ is the number of IMFs generated by the HHT. Once all of the IMFs have been extracted, the Hilbert transform can be applied to find the instantaneous frequency of each of the IMFs.

### 3.2 Hilbert Transform

Once all of the IMFs have been found, the HT is then used to find the instantaneous frequency of each.

$$H\left[c(t)\right] = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{c(\tau)}{t-\tau} d\tau \quad (6)$$

Using this definition, *c(t)* and *H*[*c(t)*] form a complex conjugate pair, which defines an analytical signal *z(t)*. Applying this to the $j^{th}$ IMF:

$$z_j(t) = c_j(t) + iH\left[c_j(t)\right] \quad (7)$$

This can be expressed

$$z_j(t) = a_j(t) e^{i\theta_j(t)} \quad (8)$$

where the amplitude $a_j(t)$ and the phase $\Theta_j(t)$ are defined by:

$$a_j(t) = \sqrt{c_j^{\,2}(t) + H^2\left[c_j(t)\right]} \quad (9)$$

$$\theta_j(t) = \tan^{-1}\left( \frac{H\left[c_j(t)\right]}{c_j(t)} \right) \quad (10)$$

Thus, the instantaneous frequency $\omega_j(t)$ is given by:

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} \quad (11)$$

After obtaining all of the IMFs, the HT can be applied to each IMF and the instantaneous frequency calculated using (9), (10) and (11). The original signal can now be expressed as:

$$x(t) = \mathrm{Re}\sum_{j=1}^{n} a_j(t) e^{i\int \omega_j(t) dt} \quad (12)$$

where the final residue $r_n(t)$ is omitted. (12) enables us to find the instantaneous frequency of each of the IMF signals which is used to tune the notch filter.

## 4 Injection of Noise

The inductor and capacitor in Fig. 1 (A) act as a low pass filter with a cut-off frequency at $\omega_C = 1/\sqrt{LC}$. However, as shown in [15], if $|\omega_S - \omega_N| < \omega_C$ the low pass filter will not remove the noise. Instead, it is shifted to a lower frequency which the authors in [15] term a beat frequency, where $\omega_{S,N}$ are the switching and noise frequency respectively. In this work, we propose filtering the output voltage with a notch filter, as shown in Fig. 2, to reduce the effect of the input perturbation. This changes the order of the differential equations from 2 to 4. The new circuit diagram can be seen in Fig. 2 and equations used to model the system are given in (13):

$$\dot{x} = \begin{bmatrix} 0 & \frac{1}{C_F} & 0 & 0 \\ -\frac{1}{L_F} & -\frac{R_F}{L_F} & \frac{1}{L_F} & 0 \\ 0 & -\frac{1}{C} & -\frac{1}{RC} & \frac{1}{C} \\ 0 & 0 & -\frac{1}{L} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\delta}{L} \end{bmatrix} V_{IN} \quad (13)$$

where $x = \begin{bmatrix} v_{CF} & i_{LF} & v_O & i_L \end{bmatrix}^T$, $R_F$, $L_F$ and $C_F$ are the filter resistor, inductor and capacitor values, $\delta = 1$ when $SW_1$ is closed and $\delta = 0$ when $SW_1$ is open. Without a filter and under normal operating conditions and with the following circuit parameters, T = 400 μs, $V_{IN}$ = 22 V, L = 20 mH, C = 47 μF, R = 22 Ω, $V_{REF}$ = 11.3 V, $V_L$ = 3.8 V, $V_U$ = 8.2 V and $K_P$ = 8.4, the system operates with a stable period-1 orbit. Now consider a chirp-like interference signal $v_N$, shown in Figure 3 (A), chosen due to its varying frequency nature. The interference signal perturbs the input voltage for several switching cycles.

$$v_N = e^{2\pi j\left(-\frac{(t_1 - t)^2}{k}\right)} \quad (14)$$

where $k = 2t_{sw}^{\,2}$. Thus, the new input voltage is given by:

$$V_{IN}^{\,*} = V_{IN}(1 + \alpha v_N \lambda) \quad (15)$$

where α is the strength of the chirp-like signal and λ = 1 when the chirp-like signal is perturbing the input voltage and λ = 0 when it is not. Fig. 3 (B) shows the effect that the chirp-like signal has on the steady state response of the buck converter. For the first 0.012 (s) λ = 0 and the system operates with a stable period-1 orbit. At t = 0.012 (s), the chirp-like signal perturbs the input voltage which causes the output voltage to lose stability and oscillate at a different frequency to the ripple voltage and with a much larger amplitude. At t = 0.02 (s), the chirp-like signal no longer perturbs the input voltage.

However, the system does not return to a stable period-1 orbit immediately but takes 0.0616 (s) for the effect of the input voltage perturbation to be fully eradicated. We will now investigate the use of tuning an adaptive notch filter with the HHT to dampen oscillations caused by the unwanted noise source.

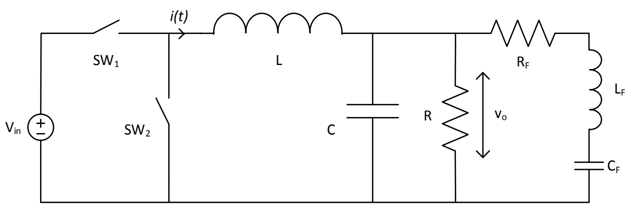


Fig. 2: Buck converter circuit model inclusive of notch filter

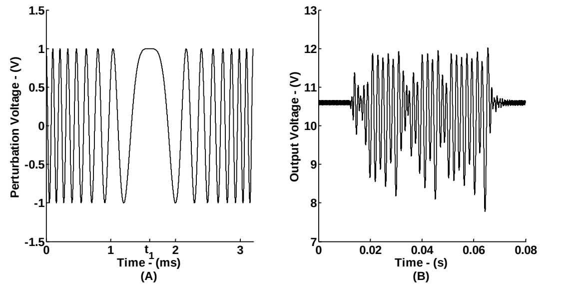


Fig. 3: (A) Chirp-like signal (B) Response of buck converter to chirp-like signal after 80 ms. Chirp-like signal perturbs input voltage for 8 ms

## 5 Application of the Hilbert Transform

The algorithm presented above for the EMD process is for off-line calculations that use an entire measured data-set. Therefore, the algorithm was modified to process the data in real time using a sliding window. At the end of each switching cycle, the input voltage from the previous N-switching cycles undergoes EMD, where N = 10 gives the best results for the given system. In order to determine the IMF signal that has the largest effect on the system, the variance of each of the IMF signals is found and the HT is applied to the signal with the highest standard deviation from the mean. This gives the instantaneous frequency of the dominant IMF. This is used to adapt or update the new cut-off frequency for the notch filter.

Fig. 4 shows the output of the EMD algorithm at t = 0.018 ( s). The output voltage is broken into 3 IMFs and the residue function. The algorithm calculates the standard deviation for each IMF to determine the dominant function. In this instance, $\sigma_1 = 0.3404$, $\sigma_2 = 0.1016$ and $\sigma_3 = 0.0490$. Thus, the instantaneous frequency of $c_1$, 1224 Hz, is used as the cut-off frequency, $f_c$, of the adaptive notch filter for the next switching period. Table 1 shows how the cut-off frequency and variance of the dominant IMF vary over 4 s witching periods. At each switching point, $f_c$ and σ change from the previous value. Furthermore, the table demonstrates that the 1st IMF is not necessarily the dominant one.

Table 1: Standard deviation of dominant IMF and cut-off frequency of adaptive notch filter over four switching periods

| Time (s) | 0.0152 | 0.0156 | 0.016 | 0.0164 |
|---|---|---|---|---|
| $f_c$ (Hz) | 498 | 461 | 459 | 615 |
| $\sigma_N$ | 0.3803 | 0.4214 | 0.3287 | 0.3787 |
| N | 1 | 1 | 2 | 2 |

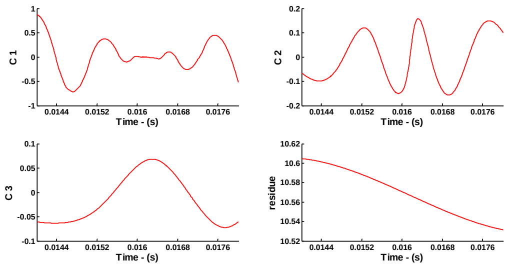


Fig. 4: IMFs and residue function obtained by real-time EMD algorithm.

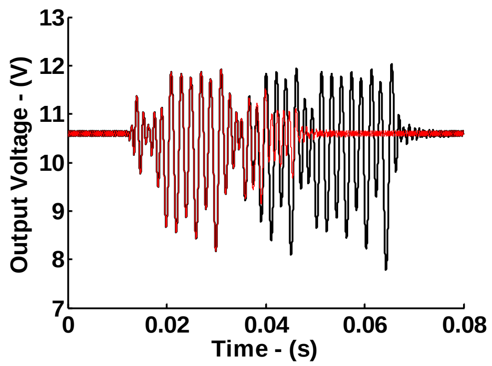


Fig. 5: Response of buck converter to perturbed input voltage with a notch filter (red) and without a notch filter (black)

Figure 5 shows the effect that the perturbing chirp-like signal has on the steady state response of the buck converter when the output is filtered with an adaptive notch filter (red). To ease comparison between the inclusion and exclusion of the notch filter, the response without the filter is also plotted (black). It is clear from Fig. 5 that the inclusion of the notch filter greatly reduces the effect of the chirp-like signal perturbation. The notch filter eliminates oscillations in 0.0392 (s). This is 0.0224 (s) faster than when the notch filter is not used.

## 6 Conclusion

Due to the switching action of the buck converter, high frequency disturbances at the input are not always removed by the low-pass filter. A chirp-like disturbance signal was shown to cause oscillations on the output voltage. The system took 0.0616 (s) to return to period-1 behaviour. An adaptive notch filter was then considered. The HHT is a method used to decompose a signal into a series of IMFs whose instantaneous frequencies can be obtained using the HT. The traditional algorithm was modified so that data could be analysed in real-time. The output voltage was broken into a series of IMFs and the variance of each IMF was determined. This enabled the identification of the dominant frequency and thus, the frequency used to tune the notch filter. The adaptive notch filter was shown to dampen oscillations 0.0224 (s) faster than when no filter was used.

## 7 Acknowledgments

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*References:*

[1] M. H. Chowdhury, Noise separation in analog integrated circuits using blind source separation, *IEEE International Conference on Electro/Information Technology*, 2007, pp.16-20.

[2] H. H. Chen and D. D. Ling, Power Supply Noise Analysis Methodology For Deep-submicron Vlsi Chip Design, *Proceedings of the 34th Design Automation Conference*, 1997, pp. 638-643.

[3] Noise Management in Portable RF Systems, [Online]. Available: http://www.ti.com/lit/ml/slyp091/slyp091.pdf.

[4] P. Deivasundari, G. Uma, and K. Murali, Chaotic Dynamics of Voltage-Mode Controlled Buck Converter with Periodic Interference Signals, *International Journal of Bifurcation and Chaos*, Vol. 23, No. 6, 2013, pp. 1-12.

[5] S. C. Wong, C. K. Tse, and K. C. Tam, Intermittent chaotic operation in switching power converters, *International Journal of Bifurcation and Chaos*, Vol. 14, No. 8, 2004, pp. 2971–2978.

[6] C. K. Tse, F. C. M. Lau, and S. S. Qiu, Intermittent chaos in switching power supplies due to unintended coupling of spurious signals, *IEEE 34th Annual Conference on Power Electronics Specialist*, Vol. 2, 2003 , pp. 642–647.

[7] F. W. Byron and R. W. Fuller, *Mathematics of Classical and Quantum Physics*, Courier Corporation, 2012.

[8] B. L. Barnhart, The Hilbert-Huang Transform: theory, applications, development, PhD. Dissertation, University of Iowa, 2011.

[9] E. O. Brigham, *The Fast Fourier Transform and Its Applications*, Prentice Hall, 1988.

[10] L. Cohen, *Time-frequency Analysis*, Prentice Hall, 1995.

[11] N. E. Huang et al., The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, *Proceedings of the Royal Society of of London A,* Vol. 454, No. 1971, 1988, pp. 903-995.

[12] P. Flandrin, G. Rilling, and P. Goncalves, Empirical Mode Decomposition as a Filter Bank, *IEEE Signal Processing Letters*, Vol. 11, No. 2, 2004, pp. 112-114.

[13] Z. K. Peng, P. W. Tse, and F. L. Chu, An improved Hilbert–Huang transform and its application in vibration signal analysis, *Journal of Sound and Vibration*, Vol. 286, No. 1-2, 2005, pp. 187-205.

[14] M. Zhang, W. Zhang, and Z. Zhang, Study of High Frequency Input Interference for Buck Converter, *Progress in Electrocmagnetics Research Symposium*, 2010, pp.1126-1131.