Adaptive Neural Networks Based Robust Output Feedback Control for Nonlinear System

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Abstract: - The performance of the control system is reduced by uncertain nonlinearities behaviors, which can be enhanced by implementing an adaptive approach represented by the robust output-feedback control and artificial neural network, which is proposed in this paper and utilized for identification and control of a nonlinear system. The Cart Pole System (CPS) is treated as a multi-body dynamical system, and the nonlinear swing-up problem is handled by designing an adaptive neural network which trained using a modified conventional controller called Linear Quadratic Optimal State Estimator with Integral Control (LQOSEIC). In this paper, the nonlinear system CPS stabilized utilizing robust output feedback control called LQOSEIC, this controller allows a linearized system to act as a model reference for the original nonlinear system, but they are only valid for a limited range of operations and will fail if the plant characteristics are unknown or uncertainty. An adaptable neural network is used to overcome this challenge., in which the adaptive neuro controller is trained offline using LQOSEIC to get the initial weights of neurons for layers network, after finished the training the LQOSEIC will be replaced by adaptive neural control. The real advantage of a neuro-controller is its ability to update online depending on the error signal. The neuro-controller demonstrates that when any disturbance or uncertainty arises in a non-linear system, neural networks characterized by online learning compensate for the effect of unpredictable conditions. The suggested adaptive neural network improves control performance and ensures the closed-loop control system's robust stability. Finally, numerical simulations are used to demonstrate the efficacy of the proposed controllers.

Key-Words: - Adaptive Neural Network, Integral Control, LQOSEIC, Nonlinear System.

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1 Introduction

The cart pole system (CPS), commonly known as an inverted pendulum on a cart. CPS is a well-known nonlinear control issue. It's a non-minimum phase system, unstable and under actuated. The task of balancing a pole on a moving cart is a common benchmark problem for evaluating various control algorithms. Despite the fact that the CPS is a well-studied control issue, its identification and control remain a hot issue. The nonlinearities behavior of real plants is extremely difficult to represent analytically. The approximation linear model can be used to analyze their behavior in some instances, although it is only valid for small nonlinearities. Non-linear systems, on the other hand, can be stabilized using a modified conventional controller. These controllers allow a linearized system to perform as a model reference for a non-linear system, but they are only valid for a limited range of operations and will fail if the plant's characteristics are unknown or vary. One of the goals of this paper is to show the different techniques that may be used to construct a nonlinear control system, such as Adaptive Neural Networks (ANN) based robust output feedback control. One of the most crucial characteristics of neural networks is their ability to adapt. Artificial neural networks that adapt to changing surroundings are known as adaptive artificial neural networks. Neural networks have been used effectively in a wide range of applications, including nonlinear system identification and control. Different control techniques such as classical, optimal, and intelligent for control of CPS were addressed in [1-4]. The following is a description of how the paper is structured: Description Modeling of CPS in section two, Control Design Approach is available in section three, Neural networks in Process Modeling and Control shown in section four, and the Conclusion is provided in section five.

2 Description model of the CPS

The CPS is an open-loop system that is compulsively non-linear, single input multi output (SIMO), the system input is control voltage, while the system outputs are cart position and angle. Because of its strong non-linearity and lack of stability, it's a good way to test prototype controllers. As a result, traditional linear approaches are unable to represent and regulate the nonlinear system (CPS). The unstable system's output data does not provide adequate information about the system. Before identification, feedback controllers are created to stabilize the system [5]. Within the confines of a one-dimensional track, the cart is free to move. The pole can move parallel to the track in the vertical plane. A force F, parallel to the track, can be applied by the controller to the cart. The pole has a mass of mp and a length of 2l, whereas the cart has a mass of mc. The position of the cart represented by x (m) and the angle formed by the pole and the vertical is θ (rad), cart velocity $\dot{x}$ (m/s) and angle velocity $\dot{\theta}$ (rad/sec). The friction coefficient between the cart and the track is γ, and there is friction in the articulation linking the pole and the cart, resulting in a torque of α and the gravitational acceleration is g. The system dynamical equations must be derived before the plant model can be constructed in Simulink [6].

$$\ddot{\theta} = \frac{g\sin\theta + \cos\theta\,\{N_a + \gamma\, g\, sgn(N_c\dot{x})\} - \frac{\alpha}{l\, m_p}}{l\left\{\frac{4}{3} - \frac{m_p\cos\theta}{m_p + m_c}[\cos\theta - \gamma\, sgn(N_c\dot{x})]\right\}} \qquad (1)$$

$$\ddot{x} = \frac{F + l\, m_p(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta) - \gamma\, N_c\, sgn(N_c\dot{x})}{m_p + m_c} \qquad (2)$$

Where

$$N_a = \frac{-F - m_p l\dot{\theta}^2[\sin\theta + \gamma\, sgn\,(N_c\dot{x})\cos\theta]}{m_p + m_c}$$

$$N_c = (m_p + m_c)g - m_p l(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$$

3 Control Design Approach

The study and design of control systems are focused on three basic goals: achieving stability, investigate the desired transient response, and reducing steady-state errors. In addition, there are several types of performance indexes such as minimum time problem and regulator problem [7].

3.1 Modified Conventional Control Design

Creating a state feedback controller usually has one major disadvantage: it generates a significant steady-state inaccuracy. As a result, Integral Control (IC) or Reference Input Signal (RIS) is used to correct for this problem, removing the steady-state response inaccuracy. Link the LQR with the IC to configure LQRIC and connect the Eigen-assignment (ES) with the RIS to configure ESRIS. Also, some state variables may not be available or may be too costly to measure. If the state variables are not available due to system design or computation, it is possible to approximate cases using the observer. Taking into account that the system is controllable and observable. The observer can be designed using two different methods: Place Estimation (PE) and Optimal State Estimation (OSE). The basic idea now is to create an observer-based controller basis by linking ESRIS with PE to form ESRISPE and similarly linking LQRIC with OSE to create LQRICOSE. The purpose of these controllers is to make the linearized system act as a model reference system for the nonlinear system as shown in Fig. 1.

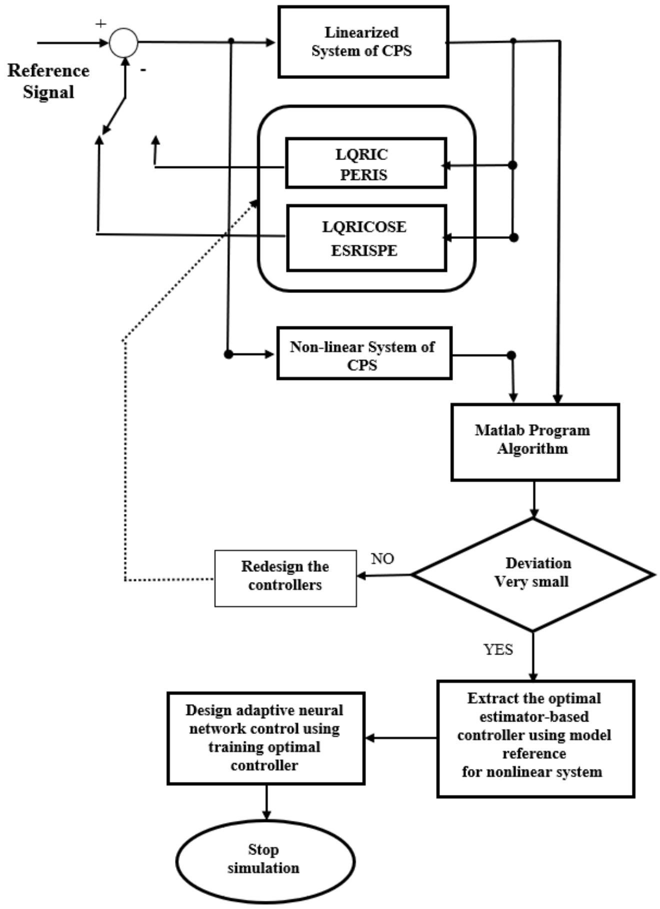


Fig. 1. Design optimal controller using model reference

The state space model of linearized CPS is

$$\dot{x} = Ax + Bu \qquad (3)$$

The error e is the new state with integral control

$$\dot{X}_\iota = \dot{e} = r - y = r - Cx \qquad (4)$$

Where y is the output system and r are desired reference.

$$\begin{bmatrix}\dot{x}\\ \dot{X}_i\end{bmatrix} = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x\\ X_i\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u + \begin{bmatrix}0\\ 1\end{bmatrix}r$$
$$\begin{bmatrix}\dot{x}\\ \dot{X}_i\end{bmatrix} = A_a\begin{bmatrix}x\\ X_i\end{bmatrix} + B_a \mathrm{u} + \begin{bmatrix}0\\ 1\end{bmatrix}r \qquad (5)$$
$$\mathrm{y} = [C \quad 0]\begin{bmatrix}x\\ X_i\end{bmatrix} = C_a\begin{bmatrix}x\\ X_i\end{bmatrix} \qquad (6)$$
Where
$A_a = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}$, $B_a = \begin{bmatrix}B\\ 0\end{bmatrix}$ and $C_a = [C \quad 0]$
The system is completely state controllable and the state feedback control (SFC) can be written as:
$$u = -[K \quad -K_i]\begin{bmatrix}x\\ X_i\end{bmatrix} = -K_a X_a \qquad (7)$$
Where
$K_a = [K \quad -K_i]$ and $X_a = \begin{bmatrix}x\\ X_i\end{bmatrix}$
$\mathrm{K_a}$: gains of LQR with integral control (LQRIC).
K: gains of LQR.
Ki: Integral gain.
The LQR approach is based on the minimization of a quadratic cost function J, which is defined as
$$J = \int_0^{\infty}(x^T Q x + u^T R u)dt \qquad (8)$$
Where Q is a symmetric positive semi-definite matrix and R is a symmetric positive definite matrix [8].
The closed-loop state equation with the state feedback control u(t) is
$$\begin{bmatrix}\dot{x}\\ \dot{X}_i\end{bmatrix} = A_a\begin{bmatrix}x\\ X_i\end{bmatrix} - B_a K_a\begin{bmatrix}x\\ X_i\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}r$$
$$\begin{bmatrix}\dot{x}\\ \dot{X}_i\end{bmatrix} = (A_a - B_a K_a)\begin{bmatrix}x\\ X_i\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}r \qquad (9)$$
The gain matrix K_a must be found such that the solutions to the equation,
$$\det(\lambda I - (A_a - B_a K_a))$$
$$det\left(\lambda I - \begin{bmatrix}A - BK & BK_i\\ -C & 0\end{bmatrix}\right) = 0$$

The primary goal of state feedback control is to stabilize a linearized system so that all closed-loop eigenvalues are in the complex plane's left side. Now we will design Eigen-assignment with reference input signal (ESRIS).
The control signal is
$$\mathrm{u} = -K_e \mathrm{x} + \mathrm{K_{RIS}} \cdot \mathrm{r} \qquad (10)$$
As a result of which the closed-loop system, provided by:
$$\dot{x} = (\mathrm{A} - \mathrm{B}K_e)\,\mathrm{x} + \mathrm{B}\,\mathrm{K_{RIS}}.\,\mathrm{r} \qquad (11)$$
Where
K_e: gains of ES.
KRIS: Feed forward scaling factor (Reference input signal)
The steady state solution, x_{ss}, for x is
$$x(\infty) = x_{ss} = Constant$$
$$x_{ss} = [V_1 \;\; V_2 \;\; V_3 \; \ldots\ldots V_{\mathrm{n}}]^T = \mathrm{V}$$
Also, At the staedy state $\dot{x}_{ss} = 0$
$$0 = (\mathrm{A} - \mathrm{B.K_e})\,\mathrm{V} + \mathrm{KRIS} \cdot \mathrm{B.\,r}$$
Solving V yields.
$$\mathrm{V} = -(\mathrm{A} - \mathrm{B.K_e})^{-1}\,\mathrm{K_{RIS}}.\,\mathrm{B.r} \qquad (12)$$
The steady state output
$$\mathrm{V_{ss}} = \mathrm{CV} = \mathrm{C}(-(A - B.\mathrm{K_e})^{-1}\,\mathrm{K_{RIS}}.\,\mathrm{B.\,r}) \qquad (13)$$
The steady state error for a reference input r as a final outcome is
$$e(\infty) = r_{ss} - y_{ss}$$
$$0 = -[1 + \mathrm{C}(A - B.\mathrm{K_e})^{-1}\,\mathrm{K}_{RIS} \cdot \mathrm{B}]$$
Since K_{RIS} is a scalar, we can easily solve to show the value of feed forward scaling factor to get ess = 0 is,
$$\mathrm{K}_{RIS} = \frac{-1}{C.(A-BK)^{-1}B.} \qquad (14)$$
Some state variables may not be available at all in other applications, or measuring them is too expensive. It is feasible to estimate the states if the state variables are unavailable due to system design or cost. The oobserver dynamic
$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \qquad (15)$$
and
$$\hat{y} = C\hat{x} \qquad (16)$$
Introduce estimation error
$$\hat{e} = x - \hat{x}$$
and
$$\dot{\hat{e}} = \dot{x} - \dot{\hat{x}}$$
$$\dot{\hat{e}} = (A - LC)(x - \dot{x}) = (A - LC)\hat{e} \qquad (17)$$

The controllers and estimators gain for LQRICOSE and ESRISPE can be determined using the Matlab program and Fig. 2 present the simulink model of LQRICOSE and ESRISPE. We constructed complex Simulink model of nonlinear system and linearized system with LQRICOSE and ESRISPE controllers. In this design, the linearized system can be used as a model reference for the non-linear system [9]. Figure 3 and Fig. 4 demonstrate the results of LQRICOSE and ESRISPE controllers on the behavior of linearized and actual nonlinear plant outputs of CPS. Table 1 present the numerical values of time specifications (rise time Tr, settling time Ts, and overshot OS %) and steady state error (e_{ss}). From the simulation result, it can be determined that the LQRICOSE performs better. In addition, the difference between actual states and estimated states using observers-based controllers (LQRICOSE and ESRISPE) for the linearized and nonlinear system is presented in Fig. 5 and Fig. 6 respectively. Through the results obtained, it is clear that the LQRICOSE gives accurate results with very small errors for linearized and nonlinear systems. The optimal estimator-based controller is LQRICOSE.

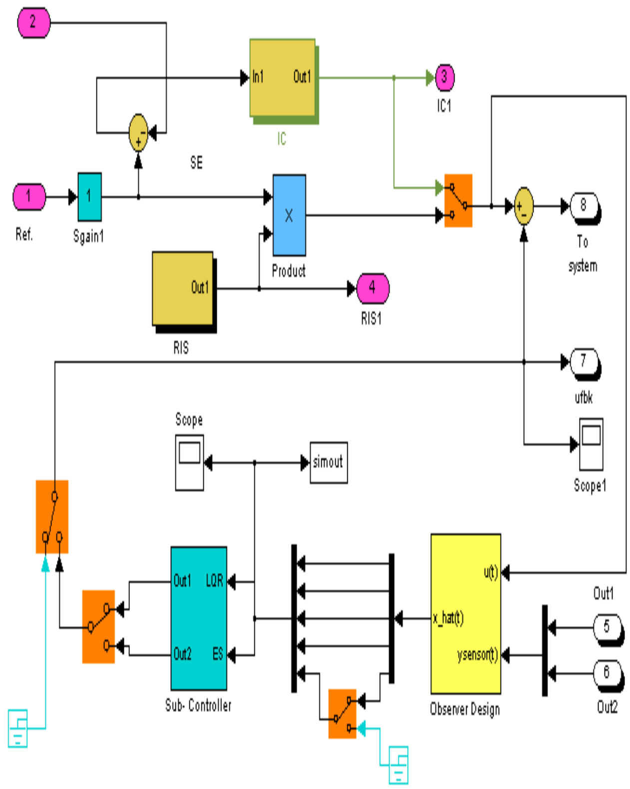


Fig. 2. Simulink model of LQRICOSE and ESRISPE

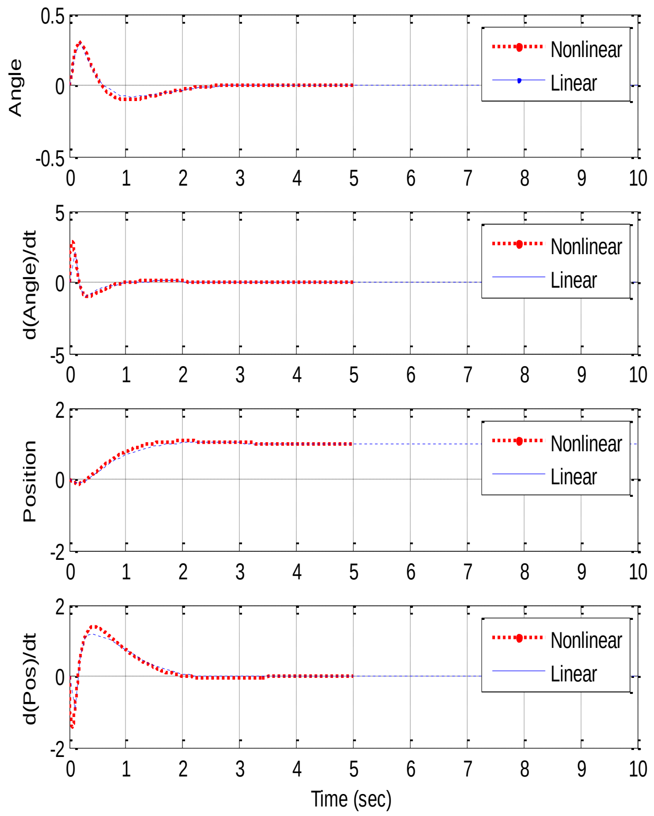


Fig. 3. Simulation states using LQRICOSE

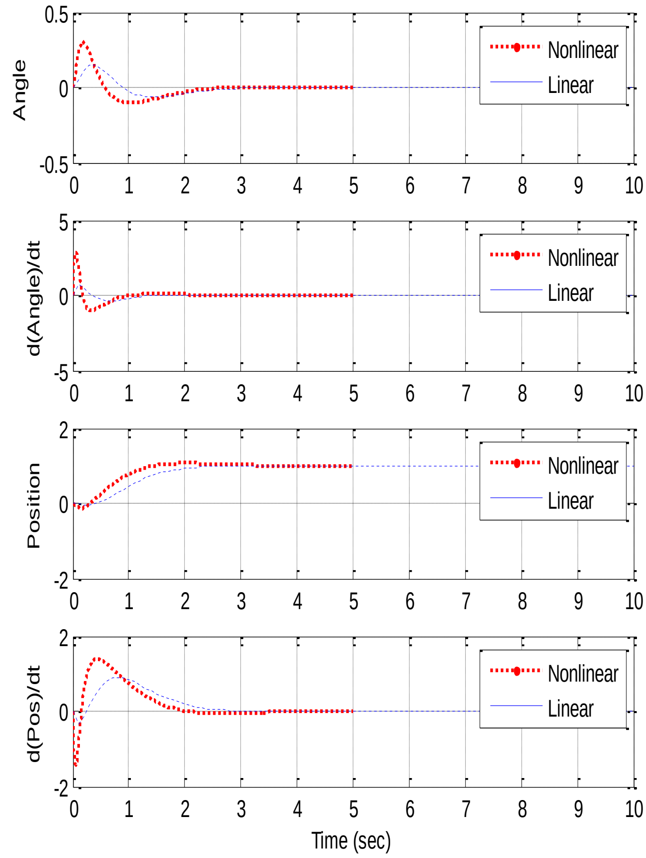


Fig. 4. Simulation states using ESRISPE

Table 1. Specification of linearized and non-linear System

Linearized System				
Controller	***LQRICOSE***		***ESRISPE***	
Specification	x (m)	θ (rad)	x (m)	θ (rad)
T_r	1	0.7	1.5	0.8
T_s	2	2	2	3
OS %	0	25	0	10
e_{ss}	0	0	0	0
Non-linear System				
Controller	***LQRICOSE***		***ESRISPE***	
Specification	x (m)	θ (rad)	x (m)	θ (rad)
T_r	1.1	0.78	1.25	0.6
T_s	2.1	2.3	4	4
OS %	1	30	8	30
e_{ss}	0	0	0	0

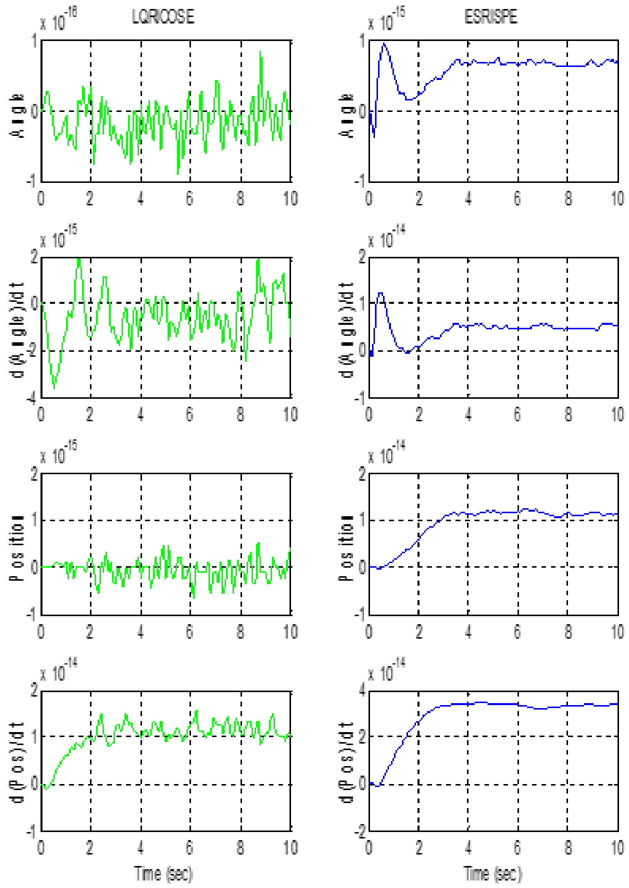


Fig. 5. Eestimated error states for linearized system

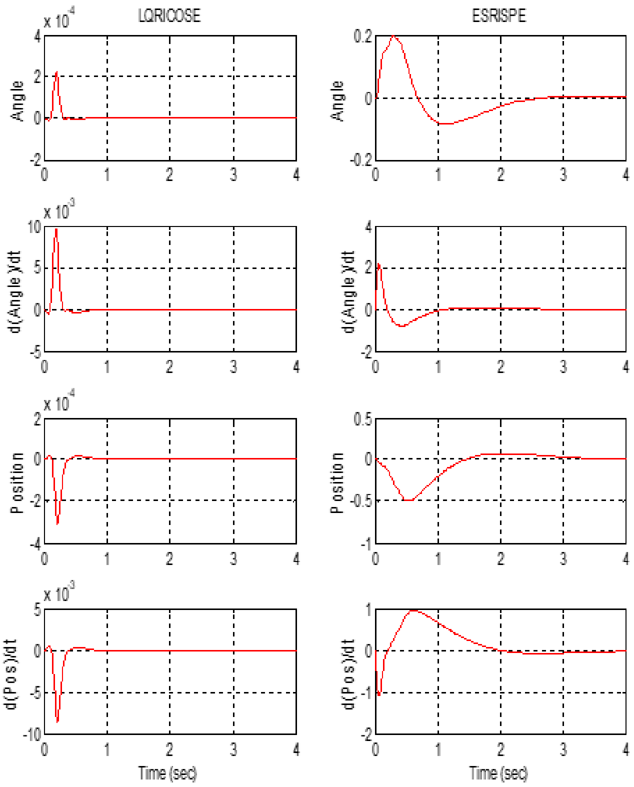


Fig. 6. Eestimated error states for nonlinear system

Despite the fact that the preceding results are good while the system parameters remain constant, Fig. 7 illustrates the implications of parameter changes with a noise signal is added. It is evident from this diagram that the controller has chosen (LQRICOSE) is unable to operate the system to an appropriate degree. The acquired controller has the disadvantage of failing if there is any ambiguity or change in the plant's parameters. This issue will be addressed in the next design stage.

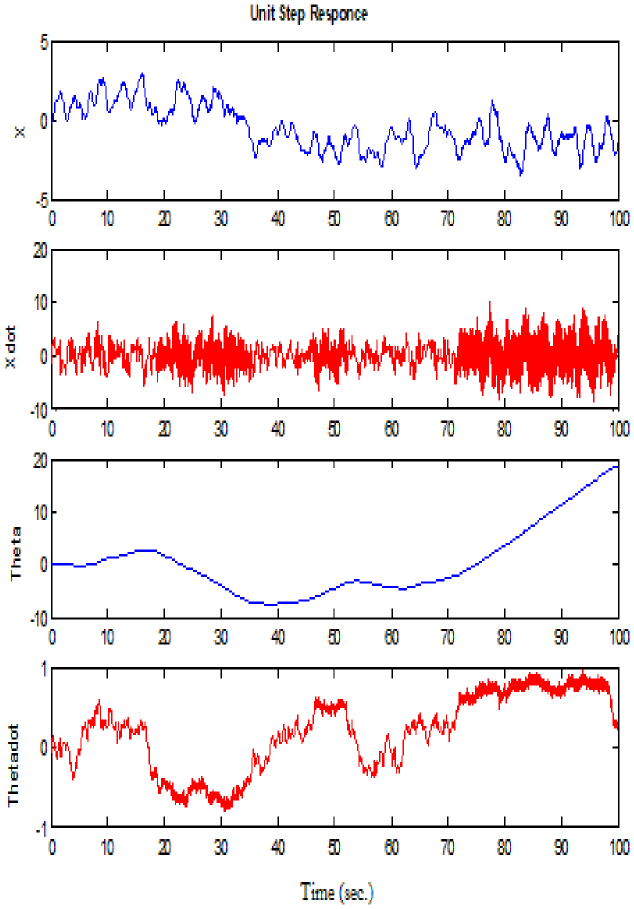
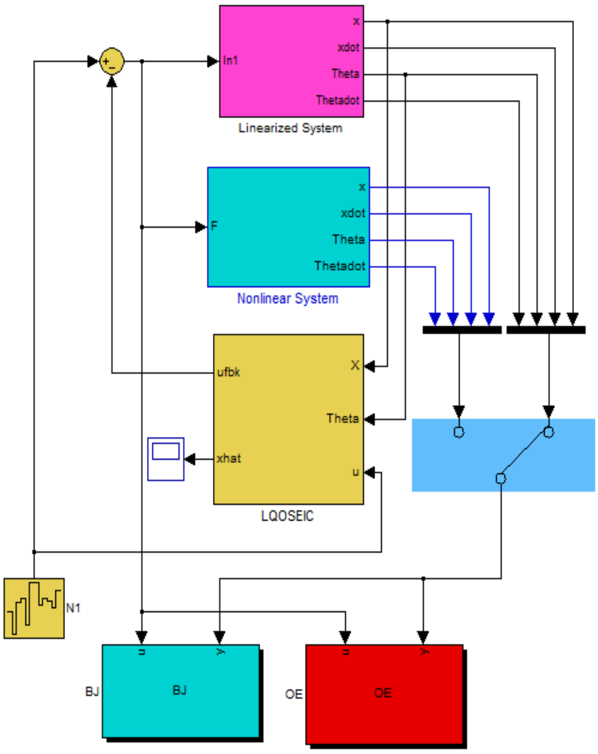


Fig. 7. Effects disturbance and varying parameters on the non-linear system controlled using LQRICOSE

3.2 Non-linear Identification Using Linear Techniques

The model dynamics of linear systems is the most essential work in control systems, but when dealing with non-linear systems, obtaining the model becomes a highly difficult problem that may be solved using system identification techniques. Figure 8 depicts a linearized system and nonlinear system with a feedback controller LQRICOSE with linear identification techniques. The non-linear model of CPS was identified using linear approaches such as ARX, ARMAX, Output Error (OE), and Box-Jenkins (BJ) Model. The nonlinear system's input and output signals are sent to ARX, ARMAX, BJ, and OE which create the estimated models. In the case of the CPS system, both the position and the angle can be measured.



Figur 8. Simulated models using the system identification toolbox

The simulation results presented in Fig. 9 to Fig. 12 show that the linear identification methods have acceptable results in the case of a linearized system, but because the inaccuracy in the estimated nonlinear model is quite large. Linear identification will not be able to develop a decent model for a non-linear system of CPS. As a result, the artificial neural network methodology will be examined in the next section.

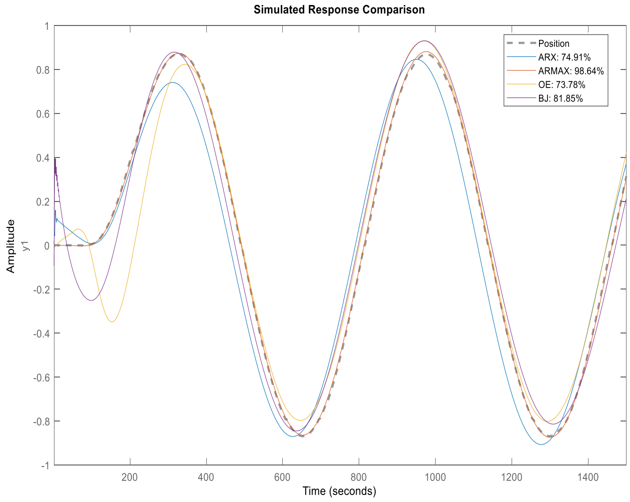


Fig. 9. Estimated position state of linearized

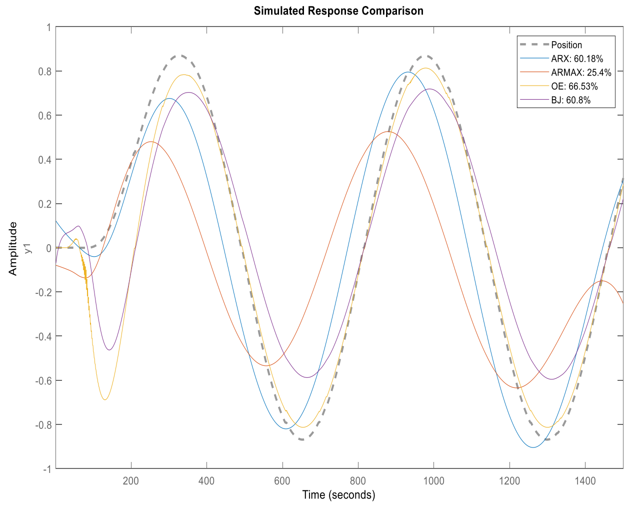


Fig. 10. Estimated position state of nonlinear

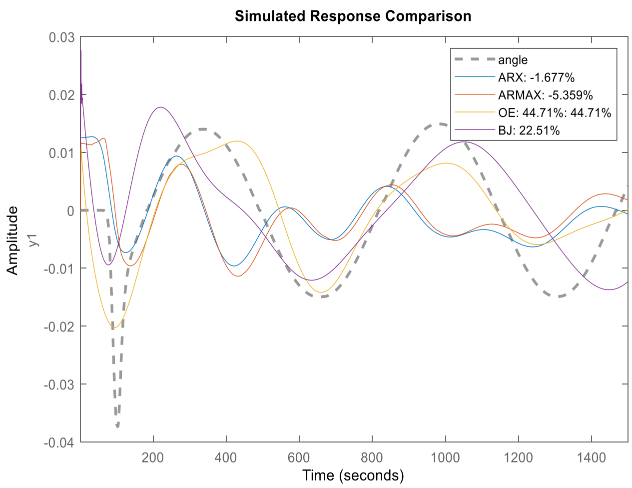


Fig. 11. Estimated angle state of linearized system

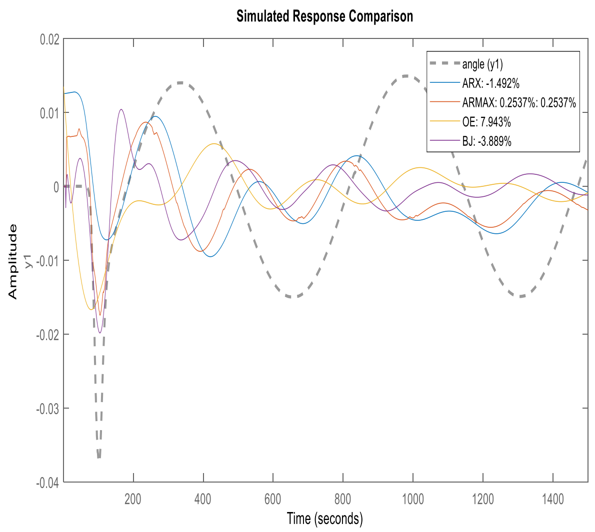


Fig. 12. Estimated angle state of nonlinear system

4 Neural networks in Process Modeling and Control

Control of non-linear systems is a prominent application field for neural networks (NNs), which have been used to identify and control dynamic systems with great success. The fact that this control approach does not use a mathematical model of the system gives it a benefit. Input-output relationships are used instead. When employing neural networks for control, there are usually two-step procedure: The system's identification and control design. A NNs model of the plant must be constructed at the system identification step. The controller is designed or trained using the developed model [10],[11].

4.1 Non-linear Identification Using Neural Networks

There are many ways of recognizing the non-linear model using neural networks. Feedforward (FF) neural network structure is the most popular approach of neural network identification. Both the process and the NN model receive the same input during training, and then the actual and neuro model outputs are compared, with the error signal being used to update the NN weights and biases. The nonlinear model of CPS gives goal values for the learner of the neural network model. The Simulink model with feedback control is utilized to give a set of targets for the network to learn, as shown in Fig. 13. Using Matlab it is possible to training and developing multi-layer perceptrons. The inaccuracy between the network and the output plant is considerable at the start of the training. The Mean Square Error (MSE) is a useful indicator of the model's correctness. The MSE decreases as the number of epochs grows. It is feasible to establish the right number of epochs for training the position and angle states by looking at the training diagram in Fig. 14 and Fig. 15. To compare the dynamics, the output from the model and process is shown. More complicated functions can be mimicked by increasing the number of hidden layer neurons where the training Epochs = 300 and Learning rate = 0.001. Increasing the number of hidden neurons improves the MSE between the neuro model and the process, according to presented Table 2.

Table 2. Results of the training feedforward networks

Neurons	Angle (MSE)	Position (MSE)
1	1×10^{-7}	3×10^{-4}
5	2×10^{-7}	4×10^{-4}
20	7×10^{-8}	6×10^{-5}
50	8×10^{-8}	7×10^{-5}
100	9×10^{-9}	8×10^{-6}

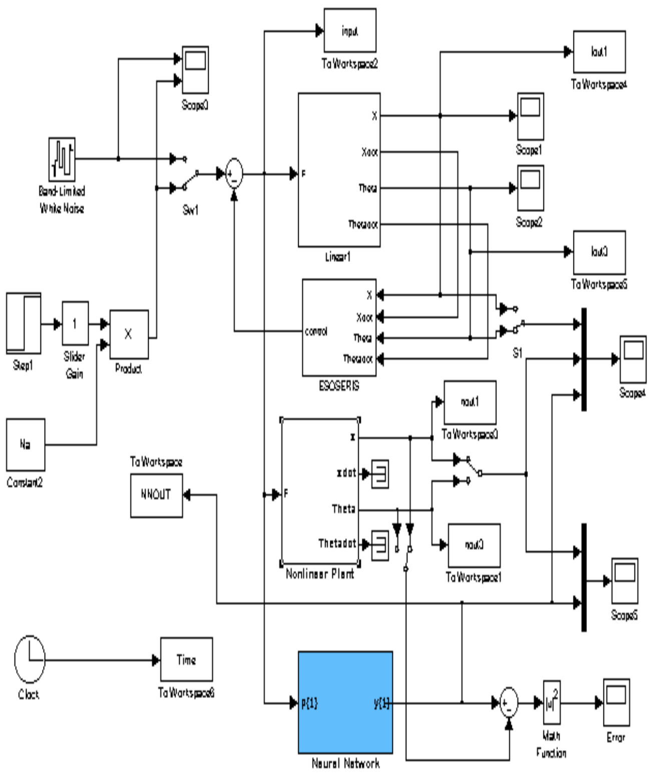


Fig. 13. Simulink is used to evaluate the actual and neuro model quality.

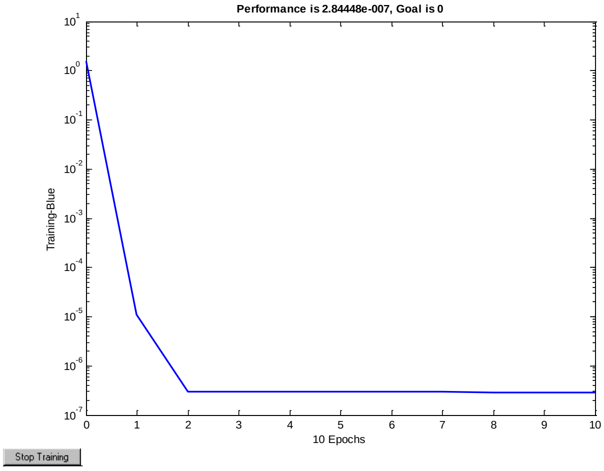


Fig. 14. Training angle state

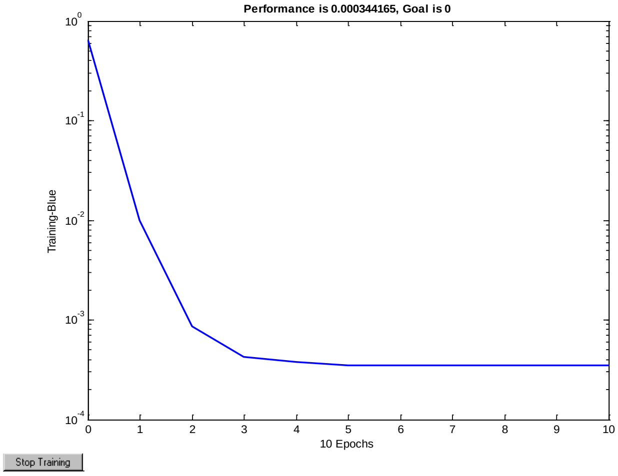
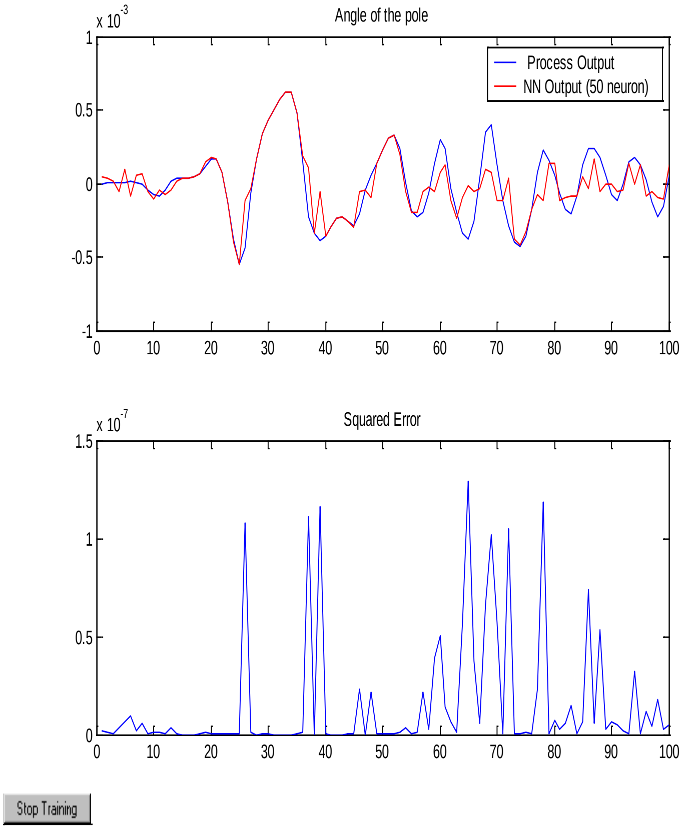
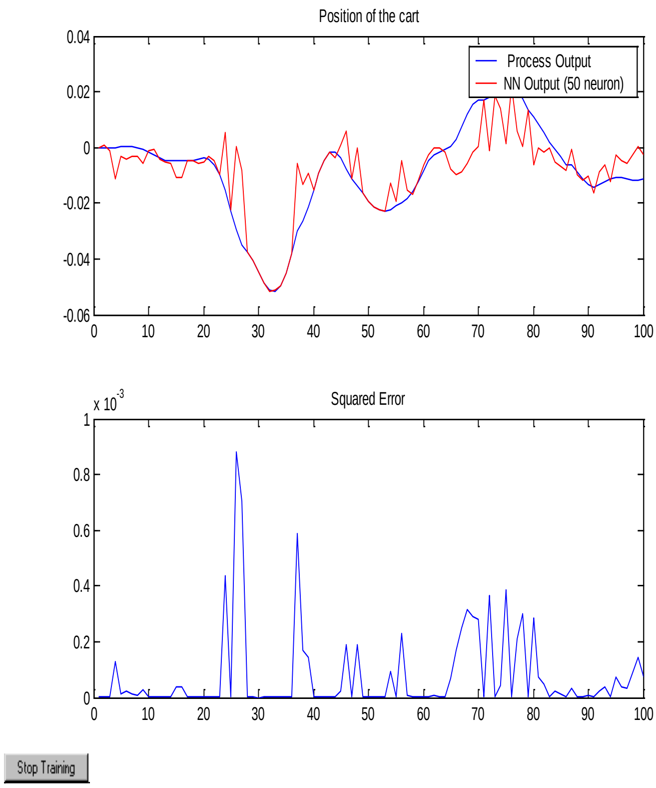


Fig. 15. Training position state

The process outputs are displayed against the neural model outputs in Fig. 16. When the number of hidden layer neurons is increased, the MSE drops dramatically, and the neural model accurately predicts the target. As a result, the feedforward networks can accurately mimic the process. FF networks will be used to simulate a multi-output system as presented in Fig. 17, The neural network's quality is assessed by contrasting the neural network's outputs with the process's outputs. The process and the neural model will both get the same input, but the neural network will have four targets to train instead of just one. At each time period, the neural network is trained by showing the four objectives concurrently. Different sizes of FF networks (10 and 50 neurons) will model the multi-output process. The simulation result in multi-output case indicates that increasing the number of hidden neurons will be improving the MSE between the neuro outputs model and process. The optimal FF neural network model multi output when hidden layer contain 100 neurons as illustrated in Fig. 18.



(a) Angle state



(b) Position state

Fig. 16. FF network, 1 hidden layer, 50 neurons

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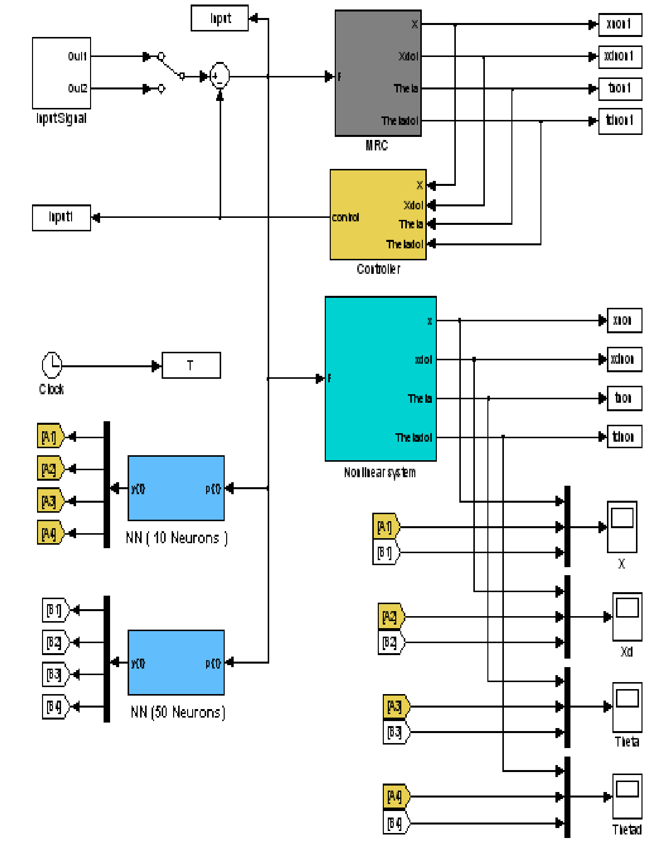

Fig. 17. Multi-output FF network, 2 hidden layer, [10 – 50] neurons

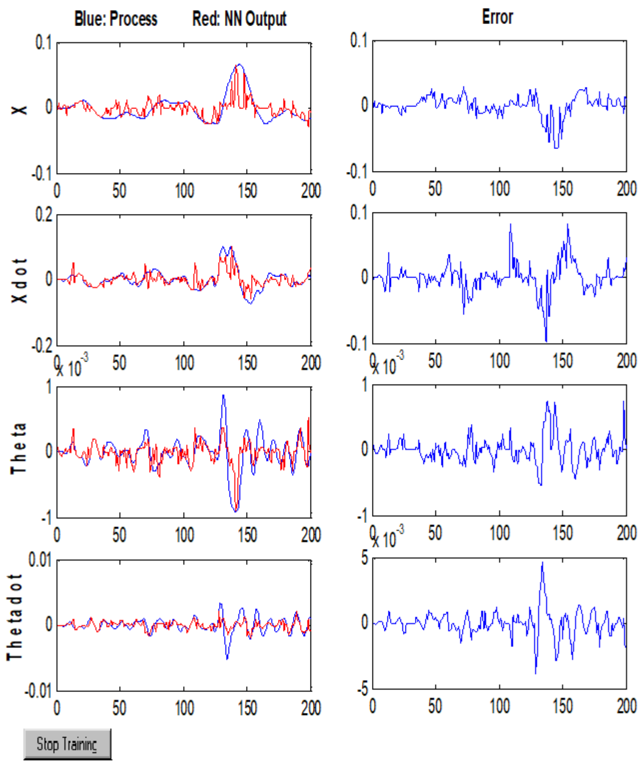

Fig. 18. Optimal FF neural network model multi output

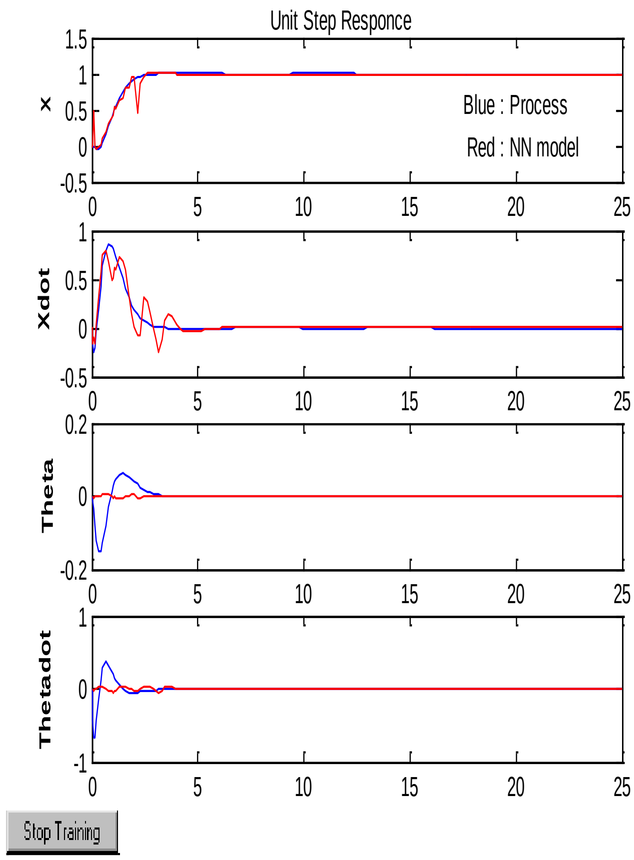

Fig. 19. Unit step response of FF neural network model

4.2 Adaptive Neural network in control

In this part, adaptive neural control for CPS system is provided, and the closed-loop system's stability characteristics have been shown. An existing controller is required to construct a supervised neural controller [10]. A feedback controller (LQRICOSE) has already been created, and this controller might be utilized as a reference for neural network (neuro controller). The neural controller will be created in the same way as the identification techniques were. Fig. 20 shows LQRICOSE controller and neuro controller, where the target of the neuro controller is the output from the original controller. The weights and biases are set when the training is completed and a Simulink model structure of neuro controller is generated, and instead of the existing controller, the network is placed in the feedback loop. The type of adaptive network to be utilized is the Adaline, which gradually adjusts the weights and biases of a network during training to reduce the error e(t). The ADALINE (ADAptive LInear NEuron) networks are similar to the perceptron utilized in the identification section, but its transfer function is linear instead of hard limiting. Figure 21 depicts the ADALINE's weight values as the training proceeds. The error between the LQRICOSE controller and the adaptive neural controller is shown in Fig. 22, and as the error decreases, the network weights converge to their ultimate values.

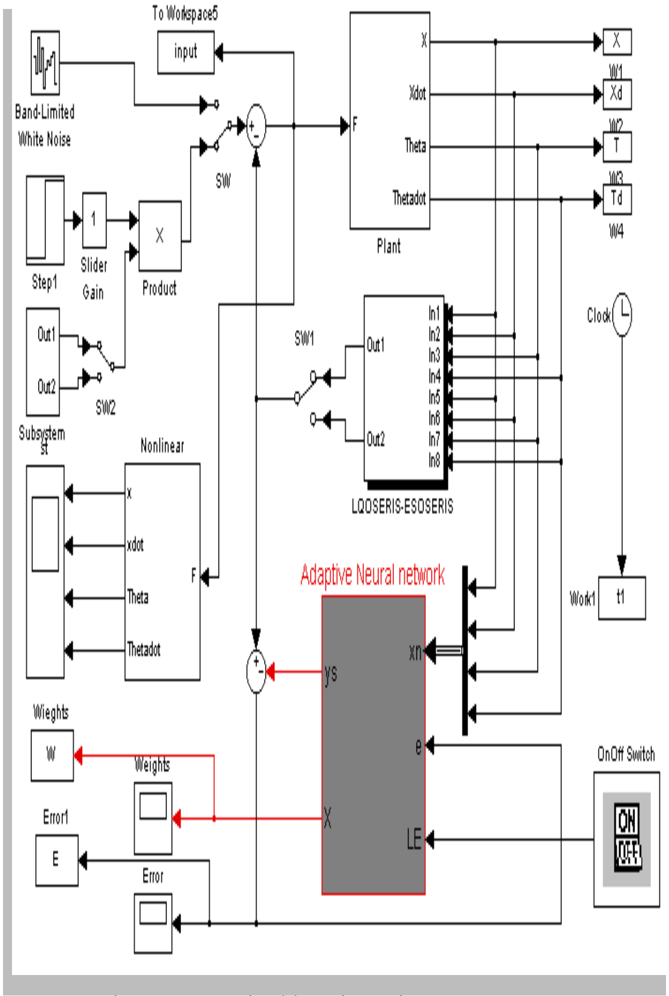

Fig. 20. Supervised learning using LQRICOSE

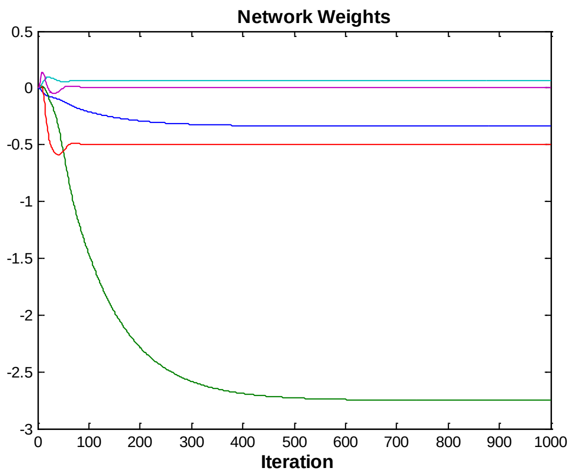

Fig. 21. Plot of the neuro controller weights

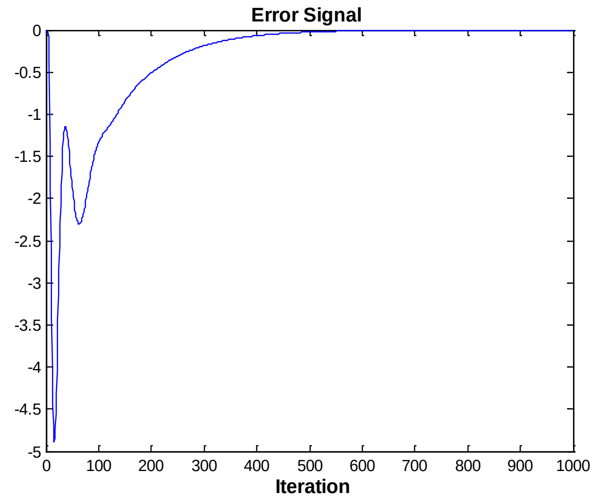

Fig. 22. Plot of the error signal

We compare the result of system output using the original controller and the neural controller in Fig. 23. The difference in error between the neuro controller and the original controller is approximately 10^{-7}, indicating that the neuro controller is a close match to the LQRICOSE controller. The main benefit of the adaptive neuro controller is that it has the

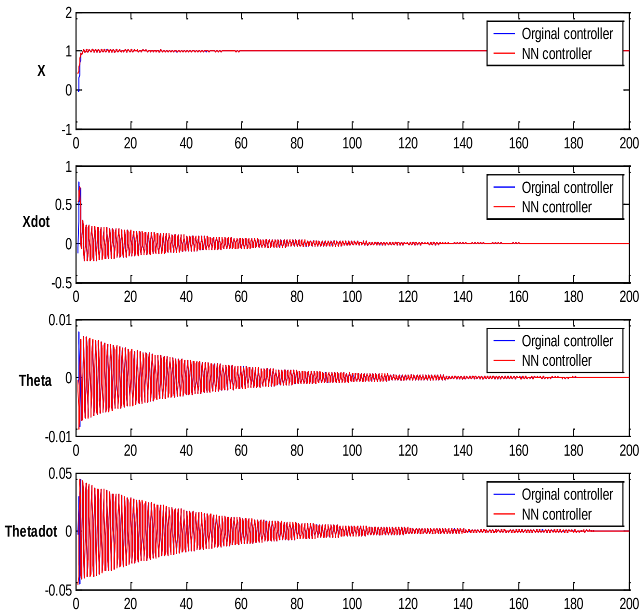

Fig. 23. Response of the non-linear system to unit step

The Simulink set-up for the adaptive neural controller is shown in Fig. 24, where the previously learned weights are now utilized as startup weights for the ADALINE controller. The input error signal in an ADALINE network is equal to the desired output minus the actual output. The ADALINE receives this error signal and adjusts the weights online. This enhances the networks performance. The previous neural controller developed shows that a NN can be trained offline using is LQRICOSE controller as a trainer. The Adaptive neural controller can be then placed online where it will continuously update its weights. The potential of adaptive neural controller to cancel disturbances that arise during operation is one of its advantages. When the controller is LQRICOSE and a noise signal is added to the setpoint while varying the parameters of a non-linear system, the system becomes unstable; however, we will now test the system's response by varying the system's parameters and adding multiple types of disturbances to the setpoint when the system controlled using adaptive neural networks. The adaptive neuro controller will cancel the effect of any disturbance for different references as shown in Fig. 25 to Fig. 27. It can be seen that the neuro controller has ability to predict the desired values with high accuracy.

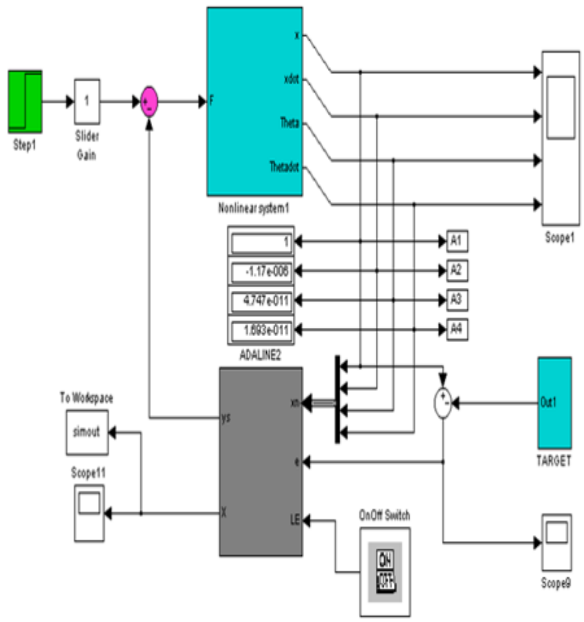

Fig. 24. Online training adaptive controller

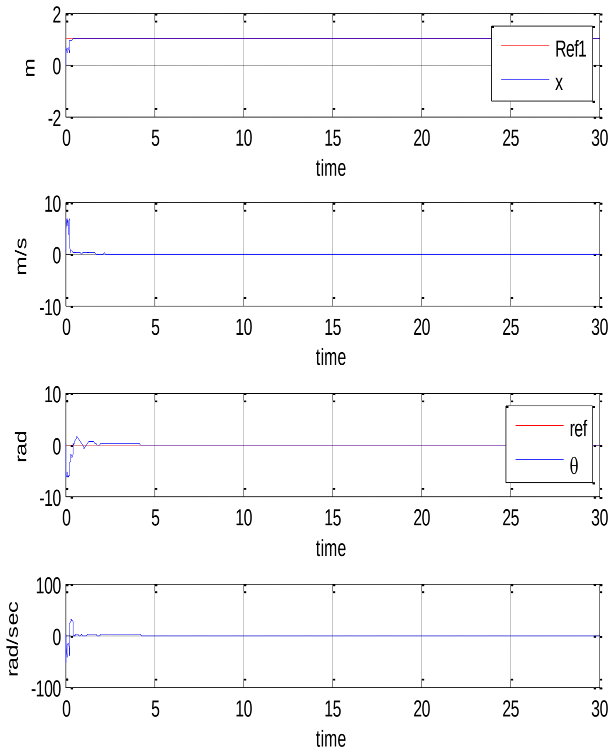

Fig. 25. Response to unit step reference with disturbance and modifying the nonlinear system's parameters

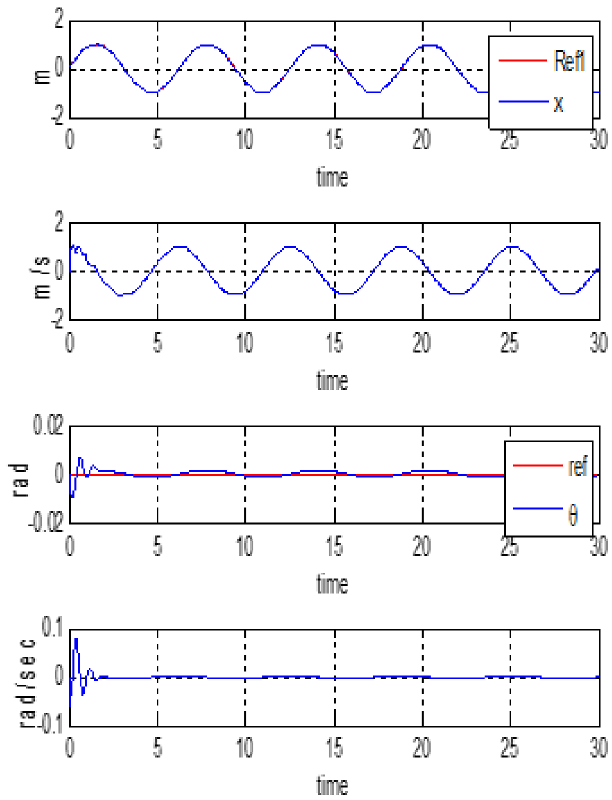

Fig. 26. Response to sinewave reference with disturbance and modifying the nonlinear system's parameters

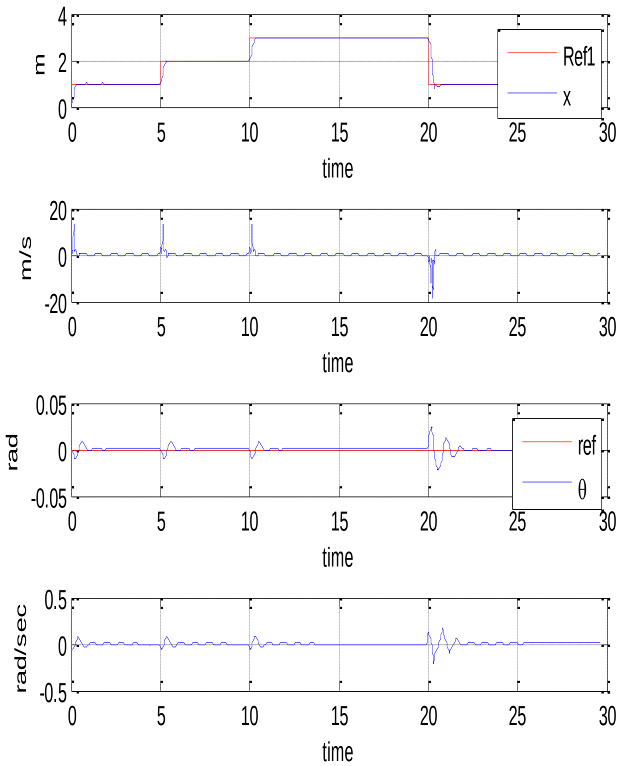

Fig. 27. Response to multi step reference with disturbance and modifying the nonlinear system's parameters

5 Conclusion

The stability and tracking performance behavior of the nonlinear CPS system for reference trajectories has been studied and improved using a robust feedback control and an adaptive neural network, with uncertainties taken into account in the control design methods. The linear identification models like ARX, ARMAX, OE and BJ models were applied to estimate the non-linear system (CPS), which is found inadequate in modeling the non-linear system. A variety of hidden layer neurons were used to create feedforward neural networks. The feedforward networks accurately represented the nonlinear system, with a very low MSE between the process and the neuron model. The LQRICOSE was able to stabilize the non-linear system, but it failed when there was any uncertainty. When a disturbance is introduced to the process and the plant's parameters are changed during simulation, the LQRICOSE loses control of the non-linear system. The problem is handled using an adaptive neural network, in which the neuro controller is trained offline using an existing controller LQRICOSE to obtain the initial weights, and then replaced to regulate the non-linear system. The adaptive neuro controller has the benefit of being able to correct for a disturbance or any type of uncertainty that arises during operation.

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