The Analysis of Wireless Relay Communication System in the Presence of Nakagami-m Fading

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Abstract: - The wireless relay communication system with three sections operating over Nakagami-m multipath fading channel is determined in this work. The outage probability of proposed relay system is calculated for two cases. In the first case, the outage probability is defined as probability that signal envelope falls below the specified threshold at any section. In the second case, the outage probability is defined as probability that output signal envelope is lower than predetermined threshold. For the first case, the outage probability is calculated by using cumulative distribution function of minimum of three Nakagami-m random variables. For the second case, the outage probability is evaluated by using the cumulative distribution function of product of three Nakagami-m random variables. Numerical expressions for the outage probability of wireless relay communication system are presented graphically and the influence of Nakagami-m parameter from each section on the outage probability is estimated. Then, the probability density functions and cumulative distribution functions of the ratios of product of two Nakagami-m random variables and Nakagami-m random variable and the ratio of Nakagami-m random variable and product of two Nakagami-m random variables are derived.

Key-Words: - Nakagami-m short term fading; relay communication system; outage probability

1 Introduction

The outage probability, as one of the most important wireless communication systems performance, is determaned in [1] for wireless relay communication system with three sections in the presence of Nakagami-m short term fading. In this paper, an extended analysis of Nakagami-m multipath fading influence on desired signal and interference is done.

Refractions, reflections, diffractions and scatterings cause multipath propagation resulting in degradation of the outage performance of wireless relay communication radio system [2], [3]. Certain numbers of channel models exist to describe the statistics of the amplitude and phase of multipath fading signals. Nakagami-m distribution has some advantages versus the other models, such as that this is a generalized distribution which can model different fading environments. It has greater flexibility and accuracy in matching some experimental data than the Rayleigh, lognormal or Rice distributions, and also, Rayleigh and one-sided Gaussian distribution are special cases of Nakagami-m model. So the Nakagami-m channel model is of more general applicability in practical fading channels [4] - [9].

Nakagami-m fading modelling in the frequency domain is investigated in [7]. For frequency-selective Nakagami-m fading channels, the magnitudes of the channel frequency responses to be Nakagami-m distributed random variables with fading and mean power parameters as explicit functions of the fading and mean power parameters of the channel impulse responses are shown. Based on this model, the bit error rate (BER) performance of an orthogonal frequency-division multiplexing system with receive diversity over correlated Nakagami-m fading channels is analytically evaluated.

BER of band-limited binary phase-shift keying in a fading and cochannel interference (CCI) environment is derived for the case of perfect coherent detection in [8]. The assumed fading-and-interference model is general and of interest for microcellular systems. The model allows both desired signal and interfering signals to experience arbitrary amounts of fading severity.

In first part of this paper, the wireless relay system with three sections is considered. In sections, Nakagami-m channel is present. This channel can be denoted as Nakagami- Nakagami- Nakagami channel. It has three parameters which are denoted with m_1, m_2 and m_3. Also, Nakagami- Nakagami-Nakagami relay channel is general channel and several channels can be derived from this channel. For m_1=1, Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Nakagami- Nakagami channel; for m_1=1 and m_2=1, Nakagami- Nakagami-Nakagami channel becomes Rayleigh - Rayleigh - Nakagami channel, and for m_1=1, m_2=1 and m_3=1, Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Rayleigh – Rayleigh channel. For m_1=0.5, Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian- Nakagami- Nakagami channel; for m_1=0.5 and m_2=0.5, Nakagami- Nakagami-Nakagami channel becomes One sided Gaussian-One sided Gaussian-Nakagami channel, and for m_1=1/2, m_2=1/2 and m_3=1/2, Nakagami- Nakagami-Nakagami channel becomes One sided Gaussian-One sided Gaussian- One sided Gaussian relay channel. Also, for m_1 goes to infinity, Nakagami-Nakagami- Nakagami relay channel becomes no fading- Nakagami- Nakagami channel, for m_1 goes to infinity and m_2 goes to infinity, Nakagami-Nakagami- Nakagami relay channel becomes no fading- no fading - Nakagami relay channel, and for m_1 goes to infinity, m_2 goes to infinity and m_3 goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - no fading channel.

The wireless relay system with two sections in the presence of κ-μ and η-μ multipath fading is processed in [10]. The wireless relay communication mobile radio system with two sections, subjected to κ-μ short term fading is considered in [11]. The outage probability is derived and parameters influence is analyzed.

In this work, the outage probability of wireless communication relay radio mobile system with three sections operating over Nakagami multipath fading channel is considered. For relay system, the outage probability can be defined at two manners. In the first case, the outage probability is defined as probability that signal envelope at any section falls below the specified threshold. For this definition, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

At the second manner, the outage probability is defined as probability that signal envelope at output of relay mobile radio system is lower than specified threshold. Signal envelope at the output of relay system with three sections can be written as product of three Nakagami random variables. Therefore, the outage probability by the second definition can be evaluated by using cumulative distribution function of product of three Nakagami random variables.

In this paper, probability density function and cumulative distribution function of minimum of three Nakagami random variables and product of three Nakagami random variables are calculated. The joint probability density function of minimum of three Nakagami random variables can be calculated and used for calculation the level crossing rate of minimum of three Nakagami random processes. After that, the ratio of product of two Nakagami-m random variables and Nakagami-m random variable is derived and PDF and CDF of this ratio. Also, the ratio of Nakagami-m random variable and product of two Nakagami-m random variables is determined. The CDF are graphically presented and parameters influence is observed.

2 Statistics of Minimum of Three Nakagami Random Variables

Random variables x_1, x_2 and x_3 follow Nakagami-*m* distribution:

$$p_{x_1}(x_1) = \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x_1^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x_1^2}\ ,\ x_1 \geq 0\ , \quad (1)$$

$$p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} x_2^{2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\ ,\ x_2 \geq 0\ , \quad (2)$$

$$p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)}\left(\frac{m_3}{\Omega_3}\right)^{m_1} x_3^{2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}\ ,\ x_3 \geq 0\ , \quad (3)$$

where Γ(.) is the (complete) gamma function [12].

Cumulative distribution functions (CDF) of x_1, x_2 and x_3 are:

$$F_{x_1}(x_1) = \frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x_1^2\right),\ x_1 \geq 0\ , \quad (4)$$

$$F_{x_2}(x_2) = \frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x_2^2\right),\ x_2 \geq 0\ , \quad (5)$$

$$F_{x_3}(x_3) = \frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x_3^2\right), \quad x_3 \geq 0, \tag{6}$$

Minimum of x_1, x_2 and x_3 is:

$$x = \min(x_1, x_2, x_3) \tag{7}$$

Probability density function (PDF) of x is:

$$p_x(x) = p_{x_1}(x)F_{x_2}(x)F_{x_3}(x) +$$

$$+p_{x_2}(x)F_{x_1}(x)F_{x_3}(x) + p_{x_3}(x)F_{x_1}(x)F_{x_2}(x) =$$

$$= \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x_1^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x^2} \frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x^2\right)\cdot$$

$$\cdot\frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x^2\right) + \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} x^{2m_2-1} e^{-\frac{m_2}{\Omega_2}x^2}\cdot$$

$$\cdot\frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x^2\right)\cdot\frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x^2\right) +$$

$$+\frac{2}{\Gamma(m_3)}\left(\frac{m_3}{\Omega_3}\right)^{m_1} x^{2m_3-1} e^{-\frac{m_3}{\Omega_3}x^2} \cdot\frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x^2\right)\cdot$$

$$\cdot\frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x^2\right), \; x \geq 0 \tag{8}$$

Cumulative distribution function of minimum of three Nakagami-*m* random variables is:

$$F_x(x) = \int_0^x dt \; p_x(t) =$$

$$F_x(x) = \left(1 - \left(1 - F_{x_1}(x)\right)\cdot\left(1 - F_{x_2}(x)\right)\cdot\left(1 - F_{x_3}(x)\right)\right) =$$

$$= 1 - \left(1 - \frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x^2\right)\right)\cdot\left(1 - \frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x^2\right)\right)\cdot$$

$$\left(1 - \frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x^2\right)\right), \; x \geq 0 \tag{9}$$

In previous expressions, parameter m_1 is severity parameter of Nakagami-m fading in the first section, m_2 is severity parameter of Nakagami-m fading in the second section and m_2 is the severity parameter of Nakagami-m fading in the third section. The Ω_1 is signal envelope average power in the first section, Ω_2 is signal envelope average power in the second section and Ω_3 is signal envelope average power in the third section.

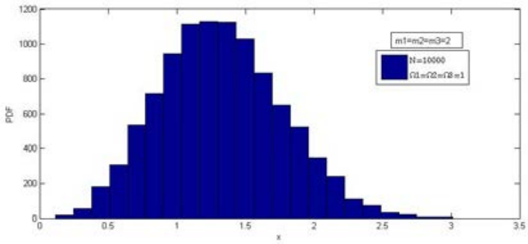

Fig.1. PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.

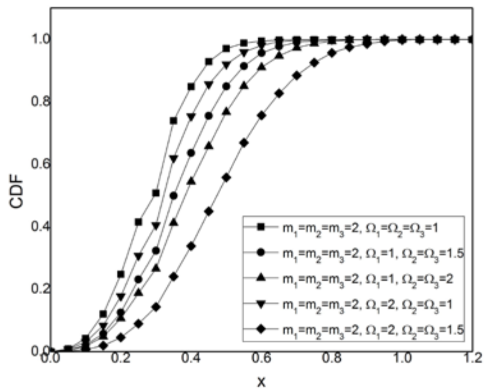

Fig. 2. The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.

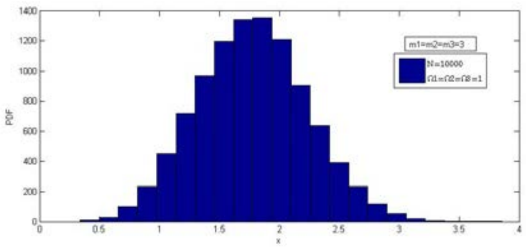

Fig. 3. PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

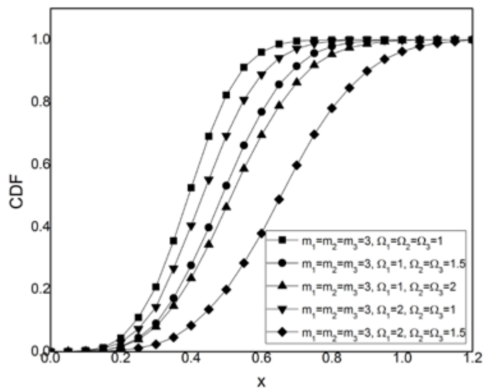

Fig. 4. The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

Probability density functions of x are shown in Figs. 1. and 3 versus of minimum of three Nakagami-m random variables. Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 1. and $m_1 = m_2 = m_3 = 3$ in Fig. 2. Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures.

In Figs. 2 and 4, the outage probability in terms of minimum of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections. The outage probability decreases when severity Nakagami parameter m_1 in the first section increases, severity Nakagami parameter m_2 in the second section increases, and severity Nakagami parameter m_3 in the third section increases. The influence of severity Nakagami parameter in the first section on the outage probability is the highest for higher values of severity Nakagami parameters in the second section and in the third section. The outage performance is better as signal envelope average power in the first section increases, when, also, signal envelope average power in the second section increases and when signal envelope average power in the third section increases. Signal envelope average power in the first section has higher influence on the outage probability when severity Nakagami parameters in the first, in the second and in the third section have higher values.

3 Statistics of Product of Three Nakagami Random Variables

The statistics of three random variables are very important for performance analyzing of wireless mobile communication radio systems. Sum of three random variables can be used for calculation the outage probability of wireless system with equal gain combining (EGC) receiver with three inputs operating over short term fading channel. EGC receiver is used for reduction multipath fading effects on system performance [3]. The statistics of the ratio of more random variables can be used in performance analysis of wireless communication systems operating over short term fading channel in the presence of cochannel interference [13]. In cellular radio interference limited environment, the ratio of signal envelope and interference envelope is important system performance. The statistics of product of two or more random variables have application in performance analysis of wireless relay communication systems with more sections [14]. Under determined conditions, signal envelope at output of relay system can be expressed as product of signal envelope at each section [14].

Product of three Nakagami-m random variables is:

$$x = x_1 \cdot x_2 \cdot x_3 \, , \; x_1 = \frac{x}{x_2 \cdot x_3} \tag{10}$$

Conditional probability density function of x is:

$$p_x\left(x / x_2 x_3\right) = \left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x}{x_2 x_3}\right) \tag{11}$$

where:

$$\frac{dx_1}{dx} = \frac{1}{x_2 x_3} \, . \tag{12}$$

After substituting and averaging, probability density function of x becomes:

$$p_x(x) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1}\left(\frac{x}{x_2 x_3}\right) p_{x_2}(x_2) \, p_{x_3}(x_3) =$$

$$= \frac{2}{\Gamma(m_1)} \frac{2}{\Gamma(m_2)} \frac{2}{\Gamma(m_3)} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_3}{\Omega_3}\right)^{m_3} \cdot$$

$$x^{2m_1-1} \int_0^\infty dx_2 \;\, x_2^{-1-2m_1+1+2m_2-1} e^{-\frac{m_2}{\Omega_2} x_2^2} \cdot$$

$$\cdot \int_0^\infty dx_3 \;\, x_3^{-1-2m_1+1+2m_3-1} \cdot e^{-\frac{m_1}{\Omega_1}\frac{x^2}{x_2^2 x_3^2} - \frac{m_3}{\Omega_3} x_3^2} =$$

$$= \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_3}{\Omega_3}\right)^{m_3} \cdot x^{2m_1-1} \cdot$$

$$\cdot \int_0^\infty dx_2 \;\, x_2^{2m_2-2m_1-1} e^{-\frac{m_2}{\Omega_2} x_2^2} \cdot$$

$$\cdot \left(\frac{m_1 x^2 \Omega_3}{\Omega_1 m_3 x_2^2}\right)^{m_3-m_1} \cdot K_{2m_3-2m_1}\left(2\sqrt{\frac{m_1 x^2 m_3}{\Omega_1 \Omega_3 x_2^2}}\right) =$$

$$= \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_3}{\Omega_3}\right)^{m_3} \cdot$$

$$x^{2m_1-1+2m_3-2m_1} \cdot \left(\frac{m_1 \Omega_3}{\Omega_1 m_3}\right)^{m_3-m_1}$$

$$\cdot \int_0^\infty dx_2 \;\, x_2^{2m_2-2m_3-1} \, e^{-\frac{m_2}{\Omega_2} x_2^2} \cdot K_{2m_3-2m_1}\left(2\sqrt{\frac{m_1 x^2 m_3}{\Omega_1 \Omega_3 x_2^2}}\right) \tag{13}$$

where $K_n(x)$ denotes the modified Bessel function of the second kind [15, eq. (3.471.9)].

Cumulative distribution function of x is:

$$F_x(x)=\int_0^x dt\ p_x(t)=$$

$$=\int_0^x dt\ \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\cdot$$

$$\cdot t^{2m_1-1}\int_0^\infty dx_2\ x_2^{2m_2-2m_1-1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot\int_0^\infty dx_3\ x_3^{2m_3-2m_1-1}\cdot e^{-\frac{m_1}{\Omega_1}\frac{t^2}{x_2^2x_3^2}-\frac{m_3}{\Omega_3}x_3^2}=$$

$$=\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\cdot$$

$$\cdot\int_0^\infty dx_2\ x_2^{2m_2-2m_1-1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot\int_0^\infty dx_3\ x_3^{2m_3-2m_1-1}\cdot e^{-\frac{m_3}{\Omega_3}x_3^2}\cdot$$

$$\cdot\int_0^x dt\ t^{2m_1-1}e^{-\frac{m_1}{\Omega_1}\frac{t^2}{x_2^2x_3^2}} \qquad (14)$$

The integral I_1 is:

$$I_1=\int_0^\infty dx_2\ x_2^{2m_2-2m_1-1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot\int_0^\infty dx_3\ x_3^{2m_3-2m_1-1}\ e^{-\frac{m_3}{\Omega_3}x_3^2}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_1}{m_1}\right)^{m_1}\cdot x_2^{2m_1}\cdot x_3^{2m_1}\gamma\left(m_1,\frac{x^2}{x_2^2x_3^2}\frac{m_1}{\Omega_1}\right)=$$

$$=\frac{1}{m_1}\frac{x^{2m_1}}{x_2^{2m_1}x_3^{2m_1}}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\cdot\sum_{j_1=0}^\infty\frac{1}{(m_1+1)(j_1)}\frac{m_1^{j_1}}{\Omega_1^{j_1}}\frac{x^{2j_1}}{x_2^{2j_1}x_3^{2j_1}}e^{-\frac{m_1x^2}{\Omega_1x_2^2x_3^2}}=$$

$$\int_0^\infty dx_2\ x_2^{2m_2-2m_1-1-2j_1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot\frac{1}{2}\int_0^\infty dx_3\ x_3^{2m_3-2m_1-1-2j_1}\ e^{-\frac{m_1}{\Omega_1}\frac{x^2}{x_2^2x_3^2}-\frac{m_3}{\Omega_3}x_3^2}=$$

$$=\frac{1}{m_1}x^{2m_1}\cdot\sum_{j_1=0}^\infty\frac{1}{(m_1+1)(j_1)}\frac{m_1^{j_1}x^{2j_1}}{\Omega_1^{j_1}}$$

$$\int_0^\infty dx_2\ x_2^{2m_2-2m_1-1-2j_1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot\left(\frac{m_1x^2\Omega_3}{\Omega_1x_2^2m_3}\right)^{m_3-m_1-j_1}\cdot K_{2m_3-2m_1-2j_1}\left(2\sqrt{\frac{m_1x^2m_3}{\Omega_1\Omega_3x_2^2}}\right) \qquad (15)$$

By substituting the last expression in the expression (14), we obtain for cumulative distribution function of x:

$$F_x(x)=\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\cdot$$

$$\cdot\frac{1}{m_1}x^{2m_1}\cdot\sum_{j_1=0}^\infty\frac{1}{(m_1+1)(j_1)}\frac{m_1^{j_1}x^{2j_1}}{\Omega_1^{j_1}}\cdot$$

$$\cdot\int_0^\infty dx_2\ x_2^{2m_2-2m_3-1}\ e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot\left(\frac{m_1x^2\Omega_3}{\Omega_1m_3}\right)^{m_3-m_1-j_1}\cdot K_{2m_3-2m_1-2j_1}\left(2\sqrt{\frac{m_1m_3x^2}{\Omega_1\Omega_2x_2^2}}\right) \qquad (16)$$

Probability density functions of x are shown in Figs. 5. and 7 versus of product of three Nakagami-m random variables. Severity parameters of Nakagami-m fading are $m_1=m_2=m_3=2$ in Fig. 5. and $m_1=m_2=m_3=3$ in Fig. 7. Signal envelope average powers are $\Omega_1=\Omega_2=\Omega_3=1$ in both figures.

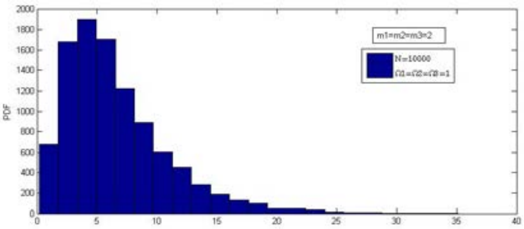

Fig. 5. PDF of product of three Nakagami-m random variables for $m_1=m_2=m_3=2$.

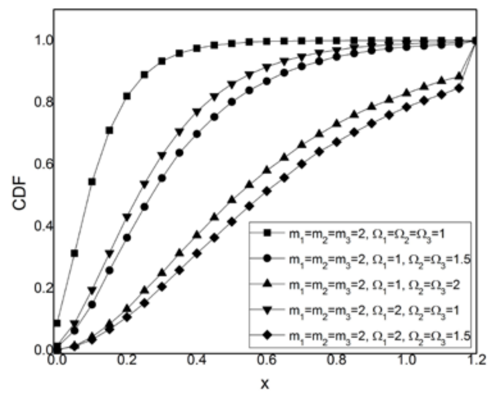

Fig. 6. The outage probability of product of three Nakagami-m random variables for $m_1=m_2=m_3=2$.

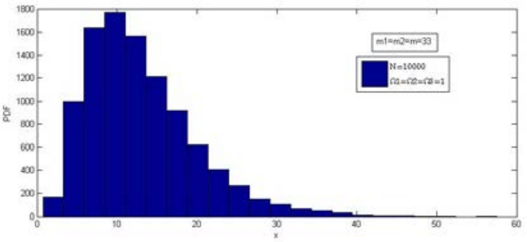

Fig. 7. PDF of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

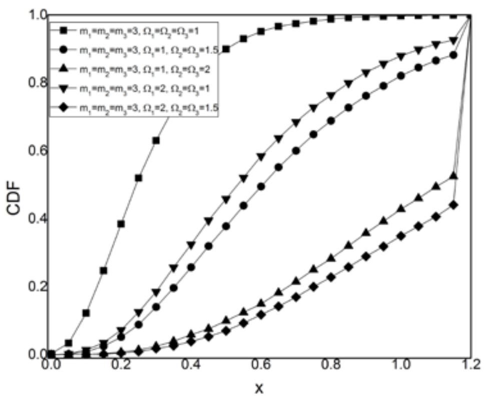

Fig. 8. The outage probability of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

In Figs. 6 and 8, the outage probability depending of product of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections. The outage probability decreases when severity Nakagami parameter m_1 in the first section increases, severity Nakagami parameter m_2 in the second section increases, and severity Nakagami parameter m_3 in the third section increases.

4 The Ratio of Product of Two Nakagami-m Random Variables and Nakagami-m Random Variable

The distribution of the ratio of random variables is important in statistical analysis in a number of different fields of science, and also in analysis of wireless communication systems in the presence of fading [16]. The random variable in nominator represents the desired signal envelope whiles the random variable in denominator presents the interference signal envelope. Because desired signal in wireless communication systems can be subjected to cochannel interference (CCI) beside of fading and shadowing. In this composite fading-shadowing environment, the signal envelope can be modeled by product of two random variables [17]. Also, when two fading affect together at the combiner inputs, the equivalent envelope is equal to the product of two random variables.

The ratio of product of two Nakagami-m random variables and Nakagami-m random variable is:

$$x = \frac{x_1 x_2}{x_3}, \; x_1 = \frac{x x_3}{x_2}, \tag{17}$$

where random variables x_1, x_2 and x_3 follow Nakagami-m distribution:

$$p_{x_1}(x_1) = \frac{2}{\Gamma(m_1)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} x_1^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x_1^2}, \quad x_1 \ge 0$$

$$p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)} \cdot \left(\frac{m_2}{\Omega_2}\right)^{m_1} x_2^{2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2}, \quad x_2 \ge 0$$

$$p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)} \cdot \left(\frac{m_3}{\Omega_3}\right)^{m_1} x_3^{2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}, \quad x_3 \ge 0 \cdot \tag{18}$$

Conditional probability density function of x is:

$$p_x(x / x_2, x_3) = \left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x x_3}{x_2}\right) \tag{19}$$

where

$$\frac{dx_1}{dx} = \frac{x_3}{x_2}. \tag{20}$$

After substituting and averaging, the expression for probability density function $p_x(x)$ is:

$$p_x(x) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{x_3}{x_2} p_{x_1}\left(\frac{x x_3}{x_2}\right) p_{x_2}(x_2) p_{x_3}(x_3) =$$

$$= \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$x^{2m_1-1} \int_0^\infty dx_2 x_2^{-1-2m_1+1+2m_2-1} \int_0^\infty dx_3 x_3^{1+2m_1-1+2m_3-1}$$

$$\cdot e^{-\frac{m_1}{\Omega_1}\frac{x^2 x_3^2}{x_2^2} - \frac{m_2}{\Omega_2}x_2^2 - \frac{m_3}{\Omega_3}x_3^2} \tag{21}$$

The two-fold integral is:

$$J = \int_0^\infty dx_2 x_2^{-2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot$$

$$\int_0^\infty dx_3 x_3^{2m_1+2m_3-1} e^{-x_3^2\left(\frac{m_1 x^2 \Omega_3 + m_3 \Omega_1 x_2^2}{\Omega_1 \Omega_3 x_2^2}\right)} =$$

$$=\frac{1}{2}\left(\Omega_1\Omega_3\right)^{m_1+m_3}\cdot\Gamma\left(m_1+m_3\right)$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-1+2m_1+2m_3} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot\frac{1}{\left(m_1\Omega_3 x^2+m_3\Omega_1 x_2^2\right)^{m_2+m_3}} \qquad (22)$$

We make replacement:

$$\frac{m_3\Omega_1 x_2^2}{m_1\Omega_3 x^2}=y,\ \ x_2^2=\frac{m_1\Omega_3 x^2}{m_3\Omega_1}y,$$

$$x_2 dx_2=\frac{m_1\Omega_3 x^2}{2m_3\Omega_1}dy \qquad (23)$$

and then, from (22) and (23), it is:

$$J=\frac{1}{2}\left(\Omega_1\Omega_3\right)^{m_1+m_3}\cdot\Gamma\left(m_1+m_3\right)$$

$$\cdot\frac{1}{\left(m_1\Omega_3 x^2\right)^{m_1+m_3}}\frac{m_1\Omega_3 x^2}{m_3\Omega_1}\frac{1}{2}\left(\frac{m_1\Omega_3}{\Omega_1 m_3}\right)^{m_2+m_3-1}$$

$$\int_0^\infty dy\ y^{m_2+m_3-1} e^{-\frac{m_2}{\Omega_2}\frac{m_1\Omega_3 x^2}{m_3\Omega_1}y}\cdot\frac{1}{(1+x)^{m_1+m_3}}=$$

$$=\frac{1}{2}\left(\Omega_1\Omega_3\right)^{m_1+m_3}\cdot\Gamma\left(m_1+m_3\right)$$

$$\cdot\frac{1}{\left(m_1\Omega_3\right)^{m_1+m_3}}x^{-2m_1-2m_3}\frac{1}{2}\frac{\left(m_1\Omega_3\right)^{m_2+m_3}}{\left(\Omega_1 m_3\right)^{m_2+m_3}}$$

$$\Gamma\left(m_2+m_3\right)$$

$$U\left(m_2+m_3, m_2-m_1+1, \frac{m_2 m_1\Omega_3 x^2}{\Omega_2 m_3\Omega_1}\right) \qquad (24)$$

where $U(a, b, z)$ is Tricomi's (confluent hypergeometric) function [18].

Cumulative distribution function of x is:

$$F_x(x)=\int_0^x dt\ p_x(x)=$$

$$=\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot\int_0^\infty dx_3 x_3^{-2m_1+2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}$$

$$\int_0^x dt\, t^{2m_1-1} e^{-\frac{m_1}{\Omega_1}\frac{t^2 x_3^2}{x_2^2}}=$$

$$=\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot\int_0^\infty dx_3 x_3^{-2m_1+2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}$$

$$\frac{1}{2}\left(\frac{\Omega_1}{m_1}\right)^{m_{11}}\frac{x_2^{2m_1}}{x_3^{2m_1}}\cdot\gamma\left(m_1, \frac{m_1}{\Omega_1}\frac{x^2 x_3^2}{x_2^2}\right)=$$

$$=\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\frac{1}{2}\left(\frac{\Omega_1}{m_1}\right)^{m_{11}}\frac{1}{m_1}\left(\frac{m_1}{\Omega_1}\right)^{m_1}x^{2m_1}\sum_{j=0}^{\infty}\frac{1}{(m_1+1)(j_1)}\frac{m_1^{j_1}}{\Omega_1^{j_1}}x^{2j_1}\cdot$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-2j_1-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\int_0^\infty dx_3 x_3^{-2m_1+2m_3+2j_1-1} e^{-\frac{m_3}{\Omega_3}x_3^2} e^{-\frac{m_1}{\Omega_1}\frac{t^2 x_3^2}{x_2^2}}=$$

$$=\frac{4}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot\frac{1}{m_1}x^{2m_1}\sum_{j=0}^{\infty}\frac{1}{(m_1+1)(j_1)}\frac{m_1^{j_1}}{\Omega_1^{j_1}}x^{2j_1}\cdot$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-2j_1-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\frac{1}{2}\left(\Omega_1\Omega_2\right)^{m_1+m_3}\cdot\Gamma\left(m_1+m_3\right)$$

$$\cdot\frac{1}{2}\left(\Omega_1\Omega_3\right)^{-m_1+m_3+j_1}x_2^{-2m_1+2m_3+2j_1}\cdot\Gamma\left(m_1+m_3+j_1\right)$$

$$\cdot\frac{1}{\left(m_3\Omega_1 x_2^2+m_1\Omega_3 x^2\right)^{-m_1+m_3+j_1}} \qquad (25)$$

The integral J_1 is [19]:

$$J_1=\int_0^\infty dx_2 x_2^{-4m_1+2m_2+2m_3-1} e^{-\frac{m_2}{\Omega_2}x_2^2}\cdot$$

$$\cdot \frac{1}{\left(m_3\Omega_1 x_2^2 + m_1\Omega_3 x^2\right)^{-m_1+m_3+j_1}} \tag{26}$$

The new substitution is:

$$\frac{m_3\Omega_1 x_2^2}{m_1\Omega_3 x^2} = t, \;\; x_2^2 = \frac{m_1\Omega_3 x^2}{m_3\Omega_1} t,$$

$$x_2 dx_2 = \frac{m_1\Omega_3 x^2}{2m_3\Omega_1} dt \tag{27}$$

After substituting, the integral J_1 becomes:

$$J_1 = \frac{1}{2}\left(\frac{m_1\Omega_3 x^2}{m_3\Omega_1}\right)^{-2m_1+m_2+m_3} \frac{1}{\left(m_1\Omega_3 x^2\right)^{-m_1+m_3+j_1}}$$

$$\int_0^\infty dt\, t^{-2m_1+m_2+m_3-1} e^{-\frac{m_2}{\Omega_2}\frac{m_1\Omega_3 x^2}{m_3\Omega_1}t}$$

$$\cdot \frac{1}{(1+x)^{-m_1+m_3+j_1}} \tag{28}$$

It can be solved by using the formula:

$$\int_0^\infty dz\, z^{a-1} e^{-\alpha z} \frac{1}{(1+z)^n} = \Gamma(a) U(a, a+1-n, \alpha). \tag{29}$$

The previous integral becomes:

$$J_1 = \frac{1}{2}\left(\frac{m_1\Omega_3 x^2}{m_3\Omega_1}\right)^{-2m_1+m_2+m_3} \frac{1}{\left(m_1\Omega_3 x^2\right)^{-m_1+m_3+j_1}}$$

$$\cdot \Gamma(-2m_1+m_2+m_3)$$

$$\cdot U\left(-2m_1+m_2+m_3, -2m_1+m_2+m_3+1+m_1-m_3-j_1, \frac{m_2 m_1 \Omega_3 x^2}{\Omega_2 m_3 \Omega_1}\right) \tag{30}$$

After putting obtained solution in (25), the final expression for cumulative distribution function is obtained. Then, the outage probability can be calculated by using this cumulative distribution function.

5 The Ratio of Nakagami-m Random Variable and Product of Two Nakagami-m Random Variables

The ratio of Nakagami-m random variable and product of two Nakagami-m random variables is:

$$x = \frac{x_1}{x_2 x_3}, \;\; x_1 = x x_2 x_3, \tag{31}$$

where random variables x_1, x_2 and x_3 have Nakagami-m distribution given by (18).

Conditional probability density function of x is:

$$p_x(x / x_2, x_3) = \left|\frac{dx_1}{dx}\right| p_{x_1}(x x_2 x_3) \tag{32}$$

where

$$\frac{dx_1}{dx} = x_2 x_3 \tag{33}$$

After substituting and averaging, the expression for probability density function $p_x(x)$ becomes:

$$p_x(x) = \int_0^\infty dx_2 \int_0^\infty dx_3\, x x_3\, p_{x_1}(x x_2 x_3) p_{x_2}(x_2) p_{x_3}(x_3) =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$x^{2m_1-1} \int_0^\infty dx_2\, x_2^{1+2m_1-1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \int_0^\infty dx_3\, x_3^{1+2m_1-1+2m_3-1}$$

$$\cdot e^{-\frac{m_1}{\Omega_1}x^2 x_2^2 x_3^2 - \frac{m_3}{\Omega_3}x_3^2} =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$x^{2m_1-1} \int_0^\infty dx_2\, x_2^{2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2}$$

$$\frac{1}{2}\left(\Omega_1\Omega_2\right)^{m_1+m_3} \cdot \frac{1}{\left(m_1\Omega_3 x_2^2 x^2 + m_3\Omega_1\right)^{m_1+m_3}} \tag{34}$$

The substitution is introduced:

$$\frac{m_1\Omega_3 x^2 x_2^2}{m_3\Omega_1} = t, \;\; x_2^2 = \frac{m_3\Omega_1}{m_1\Omega_3 x^2} t,$$

$$x_2 dx_2 = \frac{1}{2}\frac{m_3\Omega_1}{m_1\Omega_3 x^2} dt \tag{35}$$

If we put now (35) into (34), $p_x(x)$ becomes:

$$p_x(x) = \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot \frac{1}{2}\left(\frac{m_3\Omega_1}{m_1\Omega_3 x^2}\right)^{m_1+m_3} \frac{1}{\left(m_3\Omega_1\right)^{m_1+m_3}}\,.$$

$$\int_0^\infty dt\, t^{m_1+m_2-1} e^{-\frac{m_2}{\Omega_2}\frac{m_3\Omega_1}{m_1\Omega_3 x^2}t} \cdot \frac{1}{(1+t)^{m_1+m_3}} \quad (36)$$

By using the formula [19]:

$$\int_0^\infty dz\, z^{a-1} e^{-\alpha z} \frac{1}{(1+z)^n} = \Gamma(a) U(a, a+1-n, \alpha) \quad (37)$$

the expression for $p_x(x)$ from (36) is:

$$p_x(x) = \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot \frac{1}{2} \frac{(m_3\Omega_1)^{m_2-m_3}}{(m_1\Omega_3 x^2)^{m_1+m_3}} \cdot \Gamma(m_1+m_2)$$

$$U\left(m_1+m_2, m_1+m_2+1-m_1-m_3, \frac{m_2}{\Omega_2}\frac{m_3\Omega_1}{m_1\Omega_3 x^2}\right) \quad (38)$$

Cumulative distribution function of x is:

$$F_x(x) = \int_0^x dt\, p_x(t) =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot \int_0^\infty dx_3 x_3^{2m_1+2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}$$

$$\int_0^x dt\, t^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x_2^2 x_3^2 t^2} =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\int_0^\infty dx_2 x_2^{-2m_1+2m_2-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot \int_0^\infty dx_3 x_3^{2m_1+2m_3-1} e^{-\frac{m_3}{\Omega_3}x_3^2}$$

$$\frac{1}{2}\left(\frac{\Omega_1}{m_1}\right)^{m_{11}} \frac{1}{x_2^{2m_1} x_3^{2m_1}} \cdot \gamma\left(m_1, \frac{m_1}{\Omega_1} x^2 x_2^2 x_3^2\right) =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\frac{1}{2}\left(\frac{\Omega_1}{m_1}\right)^{m_{11}} \frac{1}{m_1}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x^{2m_1} \sum_{j_1=0}^\infty \frac{1}{(m_1+1)(j_1)} \frac{m_1^{j_1}}{\Omega_1^{j_1}} x^{2j_1} \cdot$$

$$\int_0^\infty dx_2 x_2^{2m_1+2m_2+2j_1-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot$$

$$\int_0^\infty dx_3 x_3^{2m_1+2m_3+2j_1-1} e^{-\frac{m_3}{\Omega_3}x_3^2 - \frac{m_1}{\Omega_1}x^2 x_2^2 x_3^2} =$$

$$= \frac{4}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot \frac{1}{m_1} x^{2m_1} \sum_{j_1=0}^\infty \frac{1}{(m_1+1)(j_1)} \frac{m_1^{j_1}}{\Omega_1^{j_1}} x^{2j_1} \cdot$$

$$\int_0^\infty dx_2 x_2^{2m_1+2m_2-2j_1-1} e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot \frac{1}{2}(\Omega_1\Omega_2)^{m_{11}+m_3+j_1}$$

$$\cdot \frac{1}{(m_3\Omega_1 + m_1\Omega_3 x_2^2 x^2)^{m_1+m_3+j_1}} \Gamma(m_1+m_3+j_1) =$$

$$= \frac{2}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot \frac{1}{m_1} x^{2m_1} \sum_{j_1=0}^\infty \frac{1}{(m_1+1)(j_1)} \frac{m_1^{j_1}}{\Omega_1^{j_1}} x^{2j_1} \cdot$$

$$(\Omega_1\Omega_2)^{m_{11}+m_3+j_1} \Gamma(m_1+m_3+j_1)$$

$$\cdot \frac{1}{2} \frac{1}{(m_3\Omega_1)^{m_1+m_3+j_1}} \left(\frac{m_3\Omega_1}{m_1\Omega_3 x^2}\right)^{m_1+m_3+j_1}$$

$$\int_0^\infty dt\, t^{m_1+m_2+j_1-1} e^{-\frac{m_2}{\Omega_2}\frac{m_3\Omega_1}{m_1\Omega_3 x^2}t} \cdot \frac{1}{(1+t)^{m_1+m_3+j_1}} =$$

$$= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \cdot \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_2}{\Omega_3}\right)^{m_3}$$

$$\cdot \frac{1}{m_1} x^{2m_1} \sum_{j_1=0}^\infty \frac{1}{(m_1+1)(j_1)} \left(\frac{m_1}{\Omega_1}\right)^{j_1} x^{2j_1} \cdot$$

$$(\Omega_1\Omega_2)^{m_{11}+m_3+j_1} \Gamma(m_1+m_3+j_1)$$

$$\cdot \frac{1}{(m_3\Omega_1)^{m_1+m_3+j_1}} \left(\frac{m_3\Omega_1}{m_1\Omega_3 x^2}\right)^{m_1+m_3+j_1}$$

$$\Gamma(m_1+m_3+j_1)$$

$$U\left(m_1+m_2+j_1, m_1+m_2+j_1-m_1-m_3-j_1, \frac{m_2}{\Omega_2}\frac{m_3\Omega_1}{m_1\Omega_3 x^2}\right) \tag{39}$$

In the next few figures, the outage probability (P_{out}) of the ratio of three Nakagami-m random variables and Nakagami-m random variable is presented. It is visible from Fig. 9 that the outage probability decreases with increasing of the signal power Ω_i, i=1,2,3.

From Fig. 10, one can see that outage probability has smaller values for the smaller values of the interference power Ω_4. The system performance is better for smaller values of the outage probability.

In Fig. 11, different combinations of Nakagami-m parameter and signal and interference envelope powers are presented.

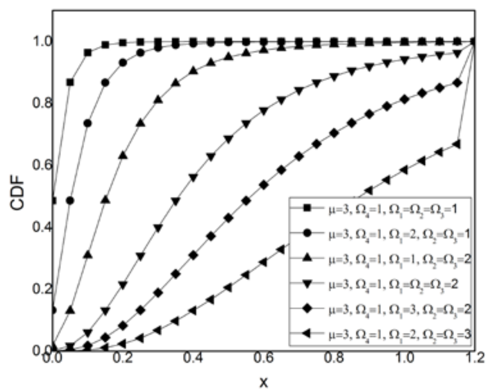

Fig. 9. The outage probability of the ratio of product of three Nakagami-m random variables and Nakagami-m random variable

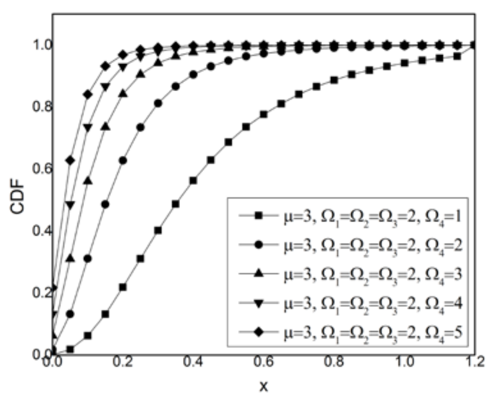

Fig. 10. The outage probability of the ratio of product of three Nakagami-m random variables and Nakagami-m random variable

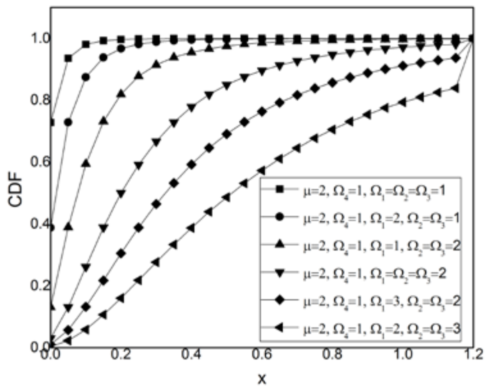

Fig. 11. The outage probability of the ratio of product of three Nakagami-m random variables and Nakagami-m random variable

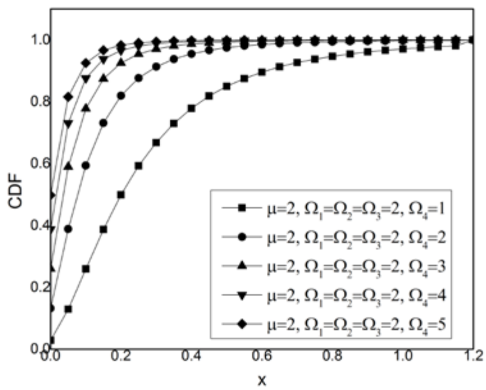

Fig. 12. The outage probability of the ratio of product of three Nakagami-m random variables and Nakagami-m random variable

From last Fig. 12, it is possible to see that P_{out} increases with decreasing of Nakagami fading parameter signed by μ in these figures. This means that system performance start to be worse. The results from this paper are helpfull to wireless relay system designers to choose optimal system parameters.

6 Conclusion

In this paper, wireless mobile relay radio communication system with three sections operating over Nakagami small scale fading channel is considered**.** Nakagami- Nakagami- Nakagami relay channel is defined. For proposed relay system, the outage probability is determined.

The outage probability can be defined at two manners. To the first case, the outage probability is defined as probability that signal envelope at any section falls below the specified threshold. For this case, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

To the second manner, the outage probability is defined as probability that signal envelope at the output of wireless relay communication system with three sections is lower than the predetermined threshold. Signal envelope at the output of relay system with three sections can be written as product of signal envelopes at sections. Therefore, the outage probability for the second case can be derived from cumulative distribution function of product of three Nakagami random variables.

In this work, probability density functions and cumulative distribution functions of minimum of three Nakagami random variables and product of three Nakagami random variables are evaluated. Cumulative distribution function of minimum of three Nakagami random variables is derived in the closed form. Cumulative distribution function of product of three Nakagami random variables is obtained as expression with one integral. For the both cases, the outage probability decreases when severity parameters of Nakagami fading increase at any sections.

On the second part of this work, the distributions of the ratio of product of two Nakagami-m random variables and Nakagami-m random variable and of the ratio of Nakagami-m random variable and product of two Nakagami-m random variables are derived. These distributions of the ratios of random variables are important in statistical analysis of wireless communication systems in the presence of fading and cochannel interference (CCI). In such composite fading-shadowing environment, the signal envelope can be modelled by product of random variables. Also, when two or more fading affect together at the combiner inputs, the equivalent envelope is equal to the product of random variables.

The probability density functions and cumulative distribution functions are determined and the outage probability based on CDF. The numerical results are shown to point out the influence of different parameters.

These results are useful for designing of wireless mobile relay radio communication system with more sections in the presence of fading.

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