Modeling and Performance Analysis of Dynamic Interconnected Production Processes

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Abstract: - This article comprehensively examines the issues of modeling and evaluation of dynamically interacting production processes. In the modern production environment, areas of work require the operational consideration of numerous and sometimes conflicting factors. These factors encompass intricate parallel and sequential interactions among system components, the unstable and ambiguous nature of dynamic processes, as well as the extensive range of their state spaces. In such a complex environment, the construction of adequate mathematical models for decision-making and effective process management is of particular importance. At the same time, the significant complexity and uncertainty of the tasks involved, along with the influence of the human element, which largely shapes the quality and standards of modern solutions, are crucial factors in the modeling process.The article proposes appropriate methods and approaches for the assessment of dynamic production systems, taking into account these features, analyzes the relationships between processes, and provides practical recommendations for increasing the efficiency of the system. The proposed approach allows for the optimization of the decision-making process in the production environment and more efficient use of resources.

Key-Words: modeling, Petri net, production, fuzzy, dynamic, incident matrix.

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1 Introduction

Given the systemic requirements for the unity of methodological approaches, a comprehensive study of inherently fuzzy processes is essential at all stages of the product life cycle to ensure the creation of systems for product competition in a marketplace. The study and modeling of domain processes is typically associated with the following actions: description of system elements, its structure, main subsystems, functions, material and information flows; construction of the state space and possible state transitions; compilation of functional interaction diagrams; analysis of cause-and-effect relationships, dynamics, and interaction mechanisms; algorithmization of decision-making; formalization and modeling of data and knowledge based on special formalized domain objects obtained at the preliminary design stages. Properties of inherent fuzzy nature are largely characteristic of all stages of the life cycle of products, technologies, and production facilities. The work at these stages is carried out under conditions of fuzzy, uncertain, and resource constraints, including time constraints. This, in turn, leads to fuzzy representation of processes, which must be taken into account in practical implementations.

Traditional methods for constructing decision-making systems have mainly concentrated on deterministic or probabilistic processes, which makes them inadequate for handling situations with highly uncertain and vague state spaces. Models grounded in fuzzy set theory often do not explicitly capture parallelism or dynamic interactions, nor do they fully incorporate the wide range of parameters and domain-specific characteristics. Addressing these challenges effectively calls for an integrated systems perspective that relies on developing models, techniques, and algorithms using advanced mathematical frameworks, modeling instruments, and computational intelligence approaches. Accordingly, this study explores the analysis of dynamic, interacting process models represented as production systems through the use of fuzzy PN.

A key characteristic of fuzzy models [1], [2] is their ability to offer a flexible approach for handling heterogeneous, dynamically interacting processes that represent both data and knowledge within an inherently uncertain state space of the analyzed objects. These dynamic interactions are expressed using both quantitative and linguistic variables. In this context, fuzzy models are primarily aimed at representing system components through linguistic descriptions (fuzzy sets); describing system characteristics in a linguistic format; and representing and processing data under uncertainty.

2 Problem Formulation

Fuzzy models built on rule-based principles provide a transparent and efficient way to represent dynamically interacting processes, expressing both data and knowledge through "IF…THEN" rules. In such rules, the "IF" segment is referred to as the premise, while the "THEN" segment represents the outcome or action. In general, a production can be written in the form:

$$Q\ ;\ P\ ;\ A \rightarrow B;\ n, \qquad (1)$$

where k denotes the identifier of the rule, Q specifies its domain of application, $A \rightarrow B$ forms the central logical component, P determines the conditions under which the core is applicable, and n describes the resulting state after execution. In practical implementations, component A often has a complex structure that may include predicates and logical operators such as NOT, AND, OR, and their variations, [3], [4], [5].

A production system typically consists of three main parts: a rule base, a global database, and an inference mechanism. The rule base serves as a storage unit for knowledge expressed in the form of IF.…THEN rules. The global database contains current data describing inputs and the system's state. Although database structures may vary across systems, they generally consist of data items with associated names, attributes, and values. The inference engine is responsible for deriving conclusions by applying rules from the rule base to the data stored in the database, [6], [7], [8], [9].

3 Problem Solution

Consider a production rule represented as:
IF A1 and A2 and … and An THEN B (2)

This indicates that if all conditions from A1 through An hold true, then B is also true, or equivalently, once all these conditions are satisfied, action B is executed.

In Boolean logic, this can be expressed as: B = TRUE( )

$$B = TRUE(\text{ A1 AND A2 AND … AND An}) = TRUE \qquad (3)$$

Similar to (2), (3), one can demonstrate the validity of the corresponding solutions for production rules containing the operation NOT- OR and their derivative. Expression (2), (3) in fuzzy representation is defined as follow:

$$IF\ \ \overline{A_1}\ \text{ AND }\ \mu_{\overline{A_1}}(k)\ \text{ İS }\ \overline{A_2}\ \ is\ \mu_{\overline{A_2}}(k)\ \text{ AND .. AND }\ \overline{A_n}\ \text{ İS }\ \mu_{\overline{A_n}}(k)$$

$$THEN \qquad B \quad \text{AND} \quad \mu_{\overline{B}}(k)$$

$$\begin{aligned}\overline{B} = TRUE| & [\,(\overline{A_1} and\ \overline{A_2}\ and\ ...\ and \overline{A_n} = TRUE] \\ & and\ [\,\mu_{\overline{A1}}(K1) \geq \mu_{\overline{A_1}}(k1\ \ )^*\,]\ and\ \ [\ \mu_{\overline{A2}}(K1) \geq\ \mu_{\overline{A_2}}(k\ \ 1)^*\ ]\ \ and \\ & ...and\ [\ \mu_{\overline{A_n}}(k1\ \ ) \geq\ \mu_{\overline{A_n}}(k1\ \ )^*\,] and [\,\mu_{\overline{B}}(k1\ \ ) \geq \mu_{\overline{B}}(k1\ \ )^*]\,,\end{aligned}$$

where
$\mu_{\overline{A1}}(k1\ \ )^*, \mu_{\overline{A\ \ 2}}(k1\ \ )^*, ...,\mu_{\overline{AN}}(k1\ \ )^*, \mu_{\overline{B}}(k1\ \ )^*$

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- the permissible value of the corresponding and membership functions, [10], [11].

Membership functions represent a collection of mechanisms that specify the conditions and actions within a given domain and are mapped onto the places and transitions of a fuzzy PN.

Considering the above, the production model for dynamically interacting parallel processes can be formulated using a fuzzy PN structure as follows:

$N = (P, T, I, O, \mu)$, $p=\{p_1, p_2, ..., p_n\}$, non-empty set of positions; $T=\{t1,\ t2, ...,\ tm\}$, $m>0$ - finite non-empty set of transition; I: $I{:}P{\times}T\text{-}(0,1,...)$, $O{:}T{\times}P\text{-}\ (0,1,..)$ respectively, the functions of input and output incidences; in the display $\mu{:}P$-[0,1] assign to each positions pi a vector of distribution of degrees of membership of tokens to the positions $\mu(pi)$.

Here is your text rewritten in English with different wording while keeping the same meaning:

$$\exists\, tj \in T,\ (\,/I(\,pi)\,/\ \geq 1)\ AND\ (\,/O(\,pi)\,/\geq 1)$$

A certain execution condition of the process includes at least one input and one output process:

$$\exists\, pi \in P,\ (\,/I\,(ti)/\geq 1)\ AND\ (\,/O(\ t_j)/\geq 1)$$

The process is performed under multiple input conditions exceeding one and multiple output conditions also exceeding one:

$$\exists t_j \in T, [\,|I(p_i)| > 1\,] \text{ and } [\,|O(p_i)| > 1\,]$$

A specific execution condition involves several input processes greater than one and several output processes greater than one:

$$\exists p_i \in P, [\,|I(t_j)| > 1\,] \text{ and } [\,|O(t_j)| > 1\,]$$

When addressing practical tasks, it is often convenient to represent the structure of a fuzzy control system in matrix form, [12]. The elements of the input, output, and incidence matrices are defined as follows:

$$f_{ij} = \begin{cases} 1, & if\ pi \in I(tj); \\ 0, & if\ pi \notin O(tj); \end{cases}$$

$$h_{ij} = \begin{cases} 1, & if\ pi \in O(tj); \\ 0, & if\ pi \notin O(tj); \end{cases}$$

$$d_{ij} = \begin{cases} -1, & if\ pi \in I(tj)\ and\ pi \notin O(tj); \\ 1, & if\ pi \notin I(tj)\ and\ pi \in O(tj); \\ 0, & if\ pi \notin I(tj)\ and\ pi \notin O(tj); \end{cases}$$

The rules governing the firing of transitions and the evolution of states in a fuzzy PN are defined as follows [2]: if, within the membership degree distribution vectors for each input place $p_i \in P$, there exists a nonzero component whose count is equal to or exceeds the number of arcs connecting that place to a transition $t_j \in T$, then the transition t_j becomes enabled and fires. Once the transition fires, tokens are redistributed among the places. The number of tokens present in the places determines the current state of the network.

The transition t_j when marking f_i allowed, subject to the following conditions: all $f_{ij} \neq 0$, are selected f_i, for $\dot{I}=1,n$; for each fixed i there must be $\exists \mu_{ik} \neq 0$, at $K = \overline{f_{ij}, k\dot{I}}$, where k_i is the length of the distribution vector of degrees of membership in the i position.

4 Conclusion

Once the transition t_j fires, the evolution of the network state, the redistribution of tokens among the places, and the resulting new marking are determined according to the algorithm, [13], [14], [15]:

1) Formation of the distribution vector of the degrees of membership of each input position after the triggering of the transitions tj:

a) calculation of the zero component of the membership degree distribution vectors:

$$\mu'_{i0} = \bigvee_{\alpha=0}^{f_{ij}} \mu_{i\alpha},$$

where $\vee$- maximum operation;

b) calculation of the remaining component of the membership degree distribution vectors:

$$\mu_{i\beta} = \mu_{i,\beta} + f_{ij}$$
$$\beta = 1,2,\ldots,k_i - f_{ij};$$

c) when the transition t_j is launched, the dimension of the distribution vectors of the degrees of membership of each input position decreases according to arcs:

$$ki = ki - fij$$

2) Formation of the distribution vectors of the degrees of membership of each output position after the triggering of the transitions $r=(r_0,r_1,\ldots,r_{hjk})$:

a) selection of all $h_{jk} \neq 0$, at $z = \overline{1,n}$;

b) calculation of vector r:

$$r_{h_{jz}} = \bigwedge_i \bigvee_{\alpha=f_{ij}}^{k_i-1} \mu_{i\alpha}$$

for all fixed i, with $f_{ij} \neq 0$, Where $\wedge$- minimum operation;

c) calculation of the zero component:

$$r_0 = 1 - r_{h_{jz}}$$

d) calculation of the remaining components: $ri=0$, $\dot{I} = \overline{1, H_{jk} - 1}$.

3. Formation of the Gram matrix:

$$G(\mu z, r^T) = (\mu_{z0}\mu_{z1}\ldots\mu_{zk_z}) \circ \begin{pmatrix} r_0 \\ r_1 \\ \cdot \\ \cdot \\ \cdot \\ r_{h_{jz}} \end{pmatrix} = \begin{vmatrix} \mu_{z0} \wedge r_0 \mu_{z0} \wedge r_1 \ldots \mu_{z0} \wedge r_{h\ jz} \\ \mu_{z_1} \wedge r_0 \mu_{z1} \wedge r_1 \ldots \mu_{z1} \wedge r_{h\ jk} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \mu_{zk_z} \wedge r_0 \mu_{zk} \wedge r_1 \ldots \mu_{zk} \wedge r_{h\ jk} \end{vmatrix}$$

where

$$g_{\ell k} = \mu_{z\ell} \wedge rk, \quad L = \overline{0, kZ}, \quad K = \overline{0, h_{jz}}.$$

4. Calculation Gram matrix $C(G(\mu Z, r^T))$:

$$C(G(\mu_z, r^T)) = \begin{vmatrix} (\mu_{z0} \wedge r_0) \\ (\mu_{z1} \wedge r_0) \vee (\mu_{z0} \wedge r_1) \\ \ldots\ \ldots\ \ldots\ \ldots \\ (\mu_{zk_z} \wedge r_0) \vee (\mu_{zk_{z-1}} \wedge r_1) \vee \ldots \vee (\mu_{z-h_{jz}} \wedge r_{h_{jz}}) \\ \ldots\ \ldots\ \ldots \\ (\mu_{zk_z} \wedge r_{h_{jz}}) \end{vmatrix}$$

The elements of this vector are calculated using the formula:

$$c_\ell = \vee_{\underline{k+i=l}} (\mu_{zk} \wedge r_i),$$

at $k = \overline{0, k_z}$, $i = \overline{1, h_{jz}}$;

the dimension of the vector of membership degrees distribution increases according to the number h_{jz}: *hjz* : *kz=kz+hjz ;*

5. Formation of a vector $\mu'z = (\mu'z_0\ \mu'z_1 \dots \mu'k_z)$ distribution of degrees of membership of the output position p_z:

$$\mu'_{zk} = c_k,\ k = \overline{0, k_z}.$$

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Contribution of individual authors in the creation of a scientific article (Ghostwriting Policy)

- SHALALA JAFAROVA MEHDI- setting the topic and general guidance and building a model on the topic.
- GUNEL KAMALOVA SULEYMAN - conducting a computer experiment based on the developed model and obtaining the result.

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