Aspects of Symmetry in Petri Nets

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Abstract: - Symmetry is a fundamental mathematical property applicable to the description of various shapes both geometrical and representational. Symmetry is central to understanding the nature of various objects. It can be used as a simplifying principle when structures are created. Petri nets are widely covered formalisms, useful for modeling different types of computer systems or computer configurations. Different forms of Petri nets exist along with several forms of representation. Petri nets are useful for i) deterministic and ii) non-deterministic modeling. The aspect of symmetry in Petri nets requires in-depth treatment that is often overlooked. Symmetry is a fundamental property found in Petri nets. This work tries to briefly touch on these properties and explain them with simple examples. Hopefully, readers will be inspired to carry out more work in this direction.

Key-Words: - Computer Science**,** Graph Theory, Mathematics, Matrices, Petri Nets, Software Models, Structural Representation, Symmetry.

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1 Introduction

In everyday language symmetry expresses the principles of balanced proportions and a sense of harmony, [1], [2], [3], [4], [5], [6]. Symmetry deals with things like coherence, orchestration, consonance and respect for proportions. The very opposite of symmetry is asymmetry which is synonymous with disharmony in nature. Asymmetry causes dissonance, disproportions, disunity, and imbalances, [1], [2], [3], [4]. In the case of two-dimensional shapes geometric symmetry at the most basic level implies that a dividing line can be drawn through the object generating two new shapes that are mirror images or reflections of one another. This is known as mirror symmetry, similarly to reflecting an object in a mirror. Symmetry should be central to describing graphical objects. This is even more so for structures like Petri nets, which are types of bipartite digraphs, [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19].

For symmetry in Petri nets, I give the following top-level classification. i) Geometrical Symmetry, ii) Matrix Symmetry, and iii) Operational or executional symmetry. Geometrical symmetry can be further subdivided into Euclidean, Reflectional, Point Reflectional, Rotational, Translational, Glide Reflectional, Helical, Double rotational, Shape etc. However, in this work, the reference will be kept to basic mirror and rotational symmetry. Matrix symmetry refers to symmetry in the basic representational matrices for Petri nets which are the input, output, and incidence matrices. These can be used for different classification of Petri net properties, including symmetry. Matrix analysis of Petri nets is very useful for limited-sized Petri nets. Operational or executional symmetry can be derived from the marking graph of the Petri net or some other form of representation.

Petri nets have well over three decades of coverage, apart from extensive uses to model different system types, [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. It is sometimes argued that their composition and representation are complex and incomplete. There are several ways to represent Petri nets, however, the main two ways are i) graphically and ii) mathematically. Graphical representation implies having a graph structure to which many aspects of geometric symmetry are fully applicable. The mathematical representation gives other interesting forms of understanding. Much of the available work on symmetry in Petri nets is not exclusive, [7], [8], [9], [12], [14], [15] but focuses on other unrelated issues, barely touching on the full possibilities of symmetry issues involved in Petri nets. Symmetry has been used for understanding sequential and parallel composition. However these topics are not exclusively of Petri nets and can be found in other graph based structures.

The topic of symmetry in Petri nets is so vast that it cannot be pinned down to a few topics only like in [7], [8,] [9], [10], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]. This work will try to summarize and classify these, however each of the topics and sub-topics that will be referred to can be used for in-depth research exclusively in their own right. Volumes can be written on just a single aspect. Some parts of these topics can prove to be quite intuitive whilst other parts would be rather abstract. For Petri nets just at a glance there can be other explorable symmetry classifications, like i) total or ii) partial symmetry. According to Plato, symmetry should lie at the core or center of nature. It is the author's own opinion that the aspect of symmetry in Petri nets, requires an in depth treatment, that has often been overlooked. It is this fundamental property that will be dealt with here.

2 Related Works

In [7], symmetries of Petri nets are described, these are given for the reachability graph, structural analysis, whilst symmetries are compared with place/transition invariant computation. Algorithms to compute these symmetries are described, the main idea of using symmetries here is to reduce the complexities of Petri nets and to solve issues like deadlock, etc. In [7], high level abstractional representational notations have been introduced. The possible types and classifications of symmetries that are given from the Petri net theory point of view. This differs from the normal classical approach because in [7] the start off is from the mathematical aspects of symmetry and not Petri net representation. No concrete practical examples of symmetry are given in [7]. Compared with the classic Petri net theory, there is the issue that several other forms of symmetry are easily observable at a glance. These are not considered. It is possible to find many problems with the approach presented in [7], e.g. even though symmetries are mentioned, there is no specific explanation of what these are in relation to Petri nets, symmetries are only seen as useful for devising and creating algorithms for Petri net analysis. This is just one aspect of Petri nets.

The symmetries of a system [6] are used for understanding the state space of the Petri nets, [8]. The idea of using symmetries is to reduce the state space of large Petri net structures. This would help with the state explosion problem. An abstraction technique called the state class graph (SCG) is used for symmetry reduction for timed Petri nets. It is evident that real symmetry is not the starting point for this work. The limitation of this work is to timed Petri nets, which is just one of the many classes of Petri nets and the symmetry is used as a technique only for simplification purposes.

In [9], it is shown that there are categories of Petri nets with symmetry. The concept of symmetry in Petri net unfolding is presented. The Petri net is unfolded to create an occurrence net. This is a very exclusive use of symmetry. The work in [9], concludes by explaining that an implicit symmetry on the unfolding of a generic Petri net does exist. The topics addressed in [9], do not directly deal with symmetry in Petri nets but explains the concepts of symmetry, from the view of the unfolding process in the net. There are several ways to include symmetry in Petri and these are explained. However, the main aspect of the work in [9], is not just about symmetry.

The examined literature previously presented, uses concepts of symmetry at the definition level of the Petri nets. It is implied that the concepts of symmetry presented in these works use high level abstraction along with algebraic notations.

Petri nets are useful for describing supervisory control systems such as those used in manufacturing, [10]. The concept of symmetry is presented as being useful to solve key issues in the field of control synthesis. However the work in [10], is rather vague how to actually and practically use symmetry, even though it is referred to. In [10], there is no real explanation of what symmetry is and then again what type of symmetry is to be used. On the other hand the work in [10], shows the practical importance of symmetry when setting up Petri net models for workstations to be controlled by a main actor which is the supervisor actor, represented by the supervisor Petri net structure. The authors in [10], discuss the reduction of forbidden states in the Petri net and how to improve the layout and design of the models. The concepts of symmetry are briefly mentioned and however they can be greatly expanded upon.

Petri nets can also be abstracted to monoids, [11]. Here sets of operations can be used for transition composition. The work in [10], does not deal directly with symmetry, however the fact that new morphisms can be defined to represent Petri nets and their respective structures is indicative that Petri nets do have many symmetry properties. The construction of new algebraic representations for Petri nets presented in [10], is useful for creating new theories. In [11], Petri nets are treated as monoids. Even though the title of this paper does not explicitly mention symmetry the models given in this work exhibit some symmetrical properties.

In [12], a Petri net model with a repeated sub-graph structure is given. The paper in part shows how a repeated symmetrical sub-graph can be removed, simplifying the net. This idea is very good. The only issue is that this paper is restricted to one type of symmetry used for reduction.

The work in [13], is an important dissertation. It does explain symmetry in Petri nets from a particular perspective. The idea is restrictive to Petri net main properties. There is much more to symmetry that has not been explored. Even different views to symmetry can be used. The work in [14], focuses on algorithms used to analyze symmetry in the Petri net. There is no focus on drawing symmetry. In [15], again the work focuses on timed continuous Petri nets. These are just a single class of Petri nets.

There are many examples of Petri net uses. This paper just contains some, [16], [17], [18], [19], [20], [21], [22]. Petri nets are used in information technology, hardware, software modeling etc. These papers describe some substantial uses and possibilities of Petri nets. E.g. in [20], Petri nets are used for modeling the reliability and availability of an electrical power system.

From the literature there is firsthand evidence that the starting point of symmetry in Petri nets should be the net structure itself. This has not been done. The types of symmetry presented in these publications is not real symmetry in the mathematical sense.

This differs greatly from what is being presented in this paper, which is symmetry from a mathematical prerogative. The idea of this paper is to examine directly the structure of the net and not deviate using other forms of symmetry that do not have a direct relationship to the Petri nets.

3 Problem Formulation

The issues of symmetry and its possible uses in Petri nets is non-trivial. There is a problem where to start using symmetry for Petri nets. This is directly related to several types of symmetry found in mathematics and group theory and how to classify these forms. The research papers, [7], [8], [9], [10], [11], [12], [13], [14], [15], clearly create additional problems by considering singular or particular aspects. To simplify the problem this work will be restricted to symmetry from purely a mathematical perspective. So in this part classification of symmetry will refer to symmetry classification in mathematics.

Shall the focus be on rotational symmetry, incidence or input, output matrix symmetry? At the simplest level of definition a geometrical object can be called symmetrical if operations that are carried out on it will leave the original structure unchanged. The big problem is to devise correct ways of classification and structure.

Classification issues relative to symmetry and applicable to Petri nets are the first part of the problem. Petri nets are widely covered formalisms that are useful for modeling different types of complete systems, [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]. Their composition, abstraction, and rules have various different types of interpretation and representation. Petri nets are representable in two main ways i) graphically and ii) mathematically using things like equations, matrices, etc. It is evident that matrices used to represent Petri nets exhibit many features including several aspects of symmetry. A simple classification is to divide Petri nets into i) Petri net matrix symmetry and ii) Diagrammatic or drawing symmetry. The word diagrammatic symmetry is used to represent all possible forms of geometrical symmetry that would be possible for the Petri net drawing. Figure 1 indicates this classification.

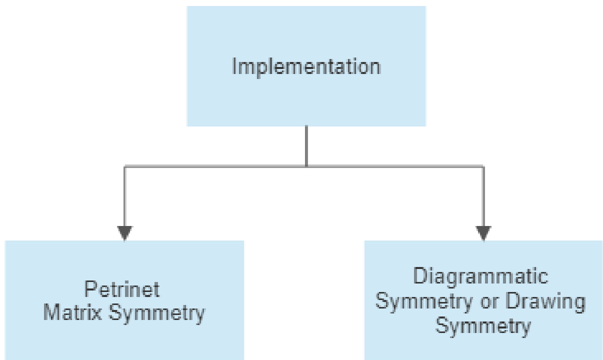

Fig. 1: Simplified Classification of Petri Net Symmetry

4 Solutions

In classifying symmetry of the matrices in ordinary Petri nets, the following basic types are possible. i) total symmetry, ii) partial symmetry. These can be established for the i) incidence matrix, ii) input matrix and iii) output matrix. It is also possible to find symmetry for the marking graph which can be represented either i) as a graph or ii) using matrices. In this case the symmetry would be in the firing matrix.

Here several examples will be presented, but readers of this work should note that indeed many other examples are possible.

4.1 Input Matrix Symmetry

This is one of the simplest forms of symmetry in the Petri net. It is quite easy to understand. A Petri net has three types of matrices that describe its structure. These are input, output and incidence matrices. Many operations can be performed on these matrices. For the example in this section, matrix symmetry is considered from the point of view of the input matrix generated for Figure 2.
For this to be possible the input matrix of the Petri net must be square. I.e. the number of places = number of transitions. In this case transposing the matrix should leave it unchanged.
E.g. consider the following input matrix

$$I = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

for the Petri net depicted in Figure 2.

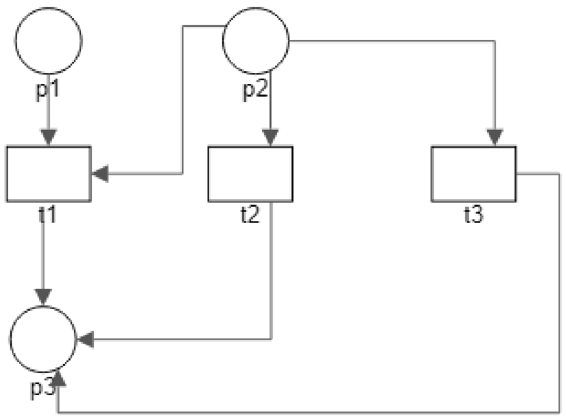

Fig. 2: Petri net no input matrix symmetry

Clearly the transposition of this matrix yields a different matrix.

$$I^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Only one row and one column remain the same thus there is no proper symmetry in this part. Although it could be argued that there is some form of partial symmetry. However this is beyond the scope of this paper. Now a second figure, Figure 3 is given. An input matrix is defined below.

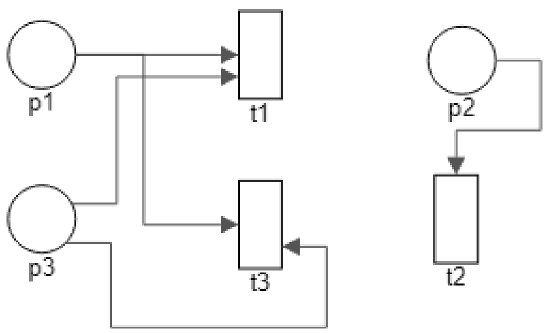

Fig. 3: Petri net with input matrix symmetry

Consider the following input matrix for Figure 3.

$$I = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Transposing this matrix leaves its values unchanged.

$$I^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

In this case there is perfect symmetry where,

$$I = I^T$$

4.2 Output Matrix Symmetry

Output matrix symmetry is similar to the input matrix symmetry, but in this case the output matrix for the Petri net is considered.
The following

$$O = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

is the output matrix for the Petri net depicted in Figure 4.

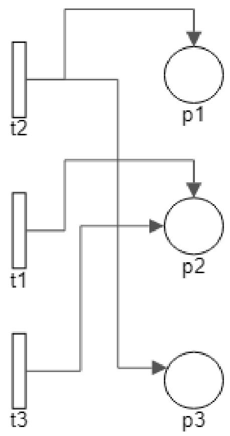

Fig. 4: Petri net with output matrix symmetry

Transposing this matrix does not change the values as shown below. i.e. $0 = 0^{\mathrm{T}}$.

$$0^{T} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

4.3 Incidence Matrix Symmetry

When the square incidence matrix is transposed, i.e. the places are transposed with the transitions, the resultant net is left completely and structurally unchanged. This is why the author of this paper is calling this total matrix symmetry. This can be addressed as perfect symmetry too. The resultant net is completely reversible too. It could be possible to design the incidence matrix and the Petri net in such a way as to give perfect symmetry.

The net in Figure 5 has the incidence matrix A.

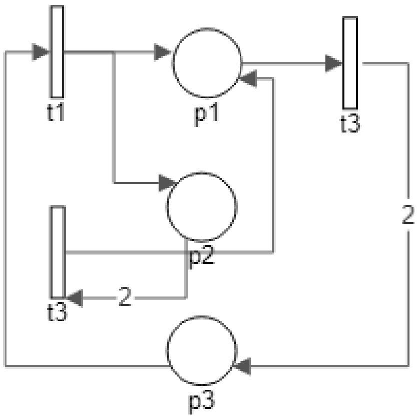

Fig. 5: Petri net with symmetry in incidence matrix

$$\mathrm{A} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Transposing this matrix A^{T} leaves it unchanged. Due to the simple design of this net it is possible to draw it in several other ways. This is indicated and shown in Figure 6.

$$\mathrm{A}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

It should be clearly noted that for these forms to be obtained, the Petri net structures need to follow certain rules. E.g. The incidence matrix for the Petri net has to be a square matrix. This places a restriction on the types of Petri net structures that can be drawn. The general conditions to allow this would mean that the nP=nT (no of places = no of transitions). However having an identical number of places and transitions does not guarantee that these types of matrix symmetries that are described do exist.

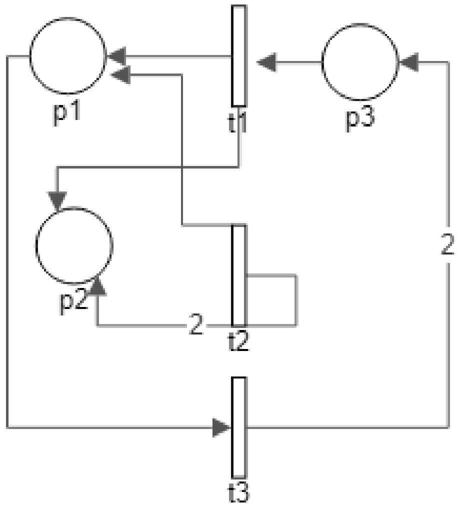

Fig. 6: Redrawn Petri net identical to Figure 5

4.4 Other Non-Matrix Forms of Symmetry

There are other forms of symmetry that can be applied to Petri net structures. These forms are visual in nature and apply to the drawing of the net. They are simpler to apply and interesting. The practical examples given below show this. Due to the size of this paper only some toy examples have been presented. There are several forms of mirror symmetry and drawing symmetry. The mathematical formulae for these types of symmetry are not explained in this paper. The concern is more with the idea and possibility of uses.

4.5 Executional Graph Symmetry

Figure 7 is an execution graph or marking graph of a typical Petri net. Figure 7 is used to explain mirror symmetry.

For this type of symmetry it does not matter if the net has an equal number of transitions at all. The execution graph is relatively left unchanged from the structural point of view if some of the entries in the graph are put on the opposite side. This graph is symmetric with respect to a line if reflecting the graph over that line leaves the graph structurally unchanged. This is indicative of execution symmetry. I.e. it implies that there are alternate execution paths, but the final outcome is unchanged. In the example in Figure 7, the marking graph can be rotated 180° leaving it essentially unchanged. I.e. both sides can be flipped. There is much more to this than meets the eye.

4.6 Mirror Symmetry

So much can be written and explained about mirror symmetry. This type of symmetry is found in different types of shapes. Mirror symmetry is described in 4.5. A definition is not presented in section 4.6. Rotational or line symmetry can be considered although these are not necessarily connected to mirror symmetry. The graph in Figure 8 is replicated from Figure 7 and has no node and edge labeling. This graph is another example of mirror symmetry that can be found in the Petri net execution graph.

4.7 Drawing Symmetry

Drawing Symmetry in this work refers to symmetry that is possible in the drawings of the Petri net.
Drawing symmetry is in short a reference to mirror images or images that are rotated in a certain form to create symmetry. They are being called drawing symmetry here to simplify and generalize the idea. The possibilities of creating symmetry using this approach are so vast that it is possible to write several publications just to deal with this aspect only. On the other hand, this work just briefly touches upon these properties. Drawing symmetry can imply geometrical or rotational symmetry.

This type of symmetry is different from matrix symmetry. Consider the diagram in Figure 9, a mid-line can be drawn vertically exactly in the middle dividing it into two perfectly symmetrical halves.

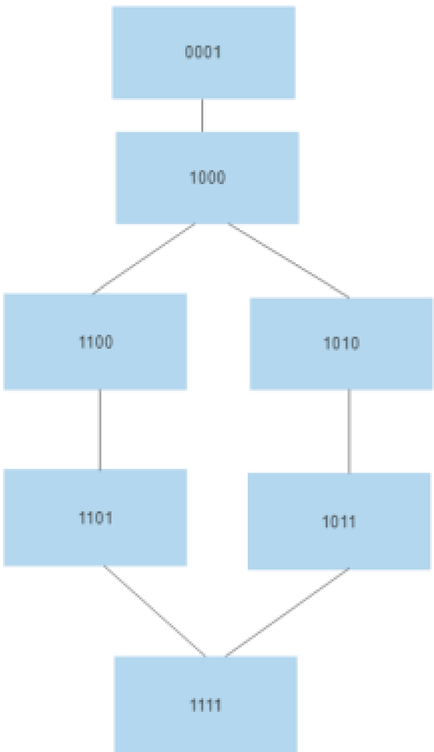

Fig. 7: Mirror Symmetry in Execution Graph

5 Results and Observations

i) In the majority of ordinary Petri net structures, it is difficult to find symmetry in the Petri net matrices unless the net is restricted. I.e. the number of places = the number of transitions (Table 1). However other forms of symmetry like mirror symmetry are still possible and do not depend at all on the structural matrices (Table 2).

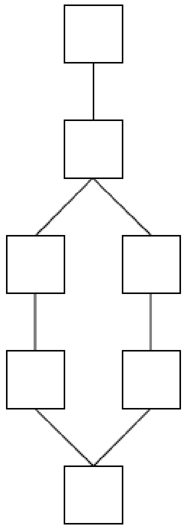

Fig. 8: Replicated Graph for Figure 7

ii) It is possible for the net to have symmetry in all or some of its matrices. However, this might not be evident in the drawing or representation of the net. iii) Diagrammatic symmetry or drawing symmetry does not necessarily depend on the structural matrices of the Petri net. I.e. it is independent of the matrices. iv) The principles of symmetry apply both to the drawing and the structure of the net using matrices.

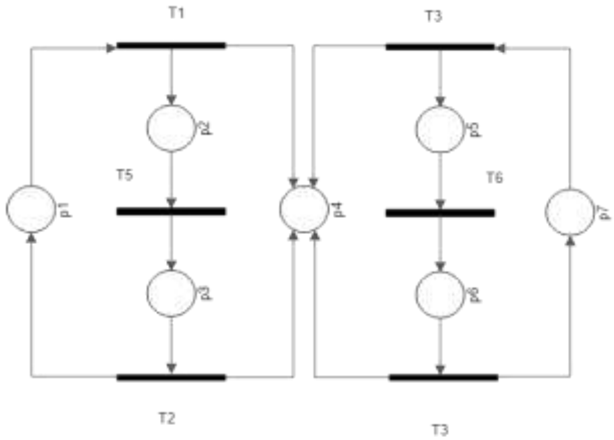

Fig. 9: Petri net model for Drawing Symmetry

v) It is possible to find many different classes of diagrammatic or drawing symmetry. E.g. rotational symmetry, plane symmetry, mirror symmetry, etc. Even something like rotational symmetry can be further classified into the order of 1,2,..n. This part on its own would require detailed further investigations and studies beyond the scope of this paper. vi) Operational symmetry or execution symmetry is observable from the marking graph or the firing matrix. Even this part requires in-depth treatment. This finding is very interesting because it shows that certain fundamental structures of the net are preserved even if the firing order can be slightly varied. These principles are not exclusive only to the Petri net marking graphs but are found in other graph structures too.

Ordinary Petri net structures share the principle of duality and preserve it concerning their input, output, and incidence matrices.

Creating the Petri net diagrams for showing the input, output, and incidence matrix symmetry has been a selective process that is limited to certain structures only. It follows that most naturally occurring models will not exhibit these forms of symmetry by default. From the experiments in this paper, it is obvious that if a Petri net has the symmetry property in the incidence matrix then the input and output matrices will also exhibit similar properties. But does this imply that there is a repeatable pattern? Possibly the inverse will also be possible. If both the input and output matrices of the Petri net have symmetry then the resultant incidence matrix will have symmetry too. This property could be useful to reduce the size of the Petri net model. Another feature is that a large Petri net model can be decomposed into smaller parts or sub-nets that can exhibit symmetry individually.

An important finding of this study is that the Petri net shape-related symmetry does not depend at all on the matrix symmetry. The shape-related symmetry is shown in Table 2 and this list is by no means exhaustive. So this is an important property where further work can be carried out.

Table 1. Petri Net Matrix Symmetry

	incidence	Input	Output
nP=nT	Possible	Possible	Possible
nP≠nT	No	No	No

Geometrical or rotational symmetry is different from matrix symmetry, however, there could be some form of overlap where matrix symmetry co-exists with geometrical symmetry.

The composition of the net identified through symmetry will have profound implications on other things like parallel, concurrent, and even sequential processing. So what comes first? Is it the composition of the net or symmetry?

The results and observations also show promising ideas on how to use symmetry for visualization of the nets.

Table 2. Petri Net Shape-Related Symmetry

Shape Related Symmetry	Possible
Mirror Symmetry	Yes
Drawing Symmetry	Yes
Rotational Symmetry	Yes
Reflectional Symmetry	Yes
Point Symmetry	Yes

There are forms of symmetry that can be identified through perception and not mathematically. In [2], [5], [6] there are several examples of this. I.e. you can have two triangles that look visually symmetrical to one another. Symmetry would simplify design and repeatability. It could be possible to create a recursive form of design in principle. Symmetry in Petri nets could be used for pattern and complexity identification. A software system described via Petri nets could be decomposed into manageable components. When creating a software or hardware system asymmetrical components would be identifiable. The structures could be decomposed into symmetrical components. This would facilitate their understanding. For having a telecommunications network modeled using Petri nets the components of the network, i.e. the subnets can be checked for symmetry. This could provide valuable insight into the fundamental design properties of the system. Images or graphs represent abstractions of systems. Some representations are universally better than others. They can represent a pattern that possibly repeats itself indefinitely even in other types of systems or solutions. A case in point is the producer/consumer pattern. Then is it possible to find such types of patterns through symmetry? Rotational symmetry might not obey algebraic commutative rules like A.B = B.A. Then clockwise rotations that are not shown in this work can be considered. E.g. A clockwise rotation of 90^0 does not necessarily give the same result as an anticlockwise rotation of the same order. The results that will be obtained depend on the net structure.

Another possibility is the fact that a net can be reversed. I.e. the net could be inverted. This again needs more exploration and experimentation.

System complexity would benefit from understanding different forms of symmetry. Symmetry has a profound effect on the thought processes of persons. So this is bound to have an effect when describing computer systems.

6 Conclusion

This theoretical work about symmetry in Petri nets just touches on this aspect. This work presents a very short concise summary of the types of symmetry. On each of the aspects that have been presented volumes of research can be done. E.g. if just mirror symmetry is considered it is possible to delve into greater depths and come up with many other works.

Symmetry is very difficult to define. It considers how an object or entity may be moved or transformed but the object or entity remains the same after the transformation even though it might look completely different from the observer's point of view.

Symmetry is one of the most important principles that is used in physics. Symmetry governs patterns in the real world.

As for the usefulness of this work in the real world, the Petri net models that exhibit symmetry properties are useful for representing systems. These models could be used to present different viewpoints and the fact that they are symmetrical could mean that their properties will be used for verification and analysis.

Key principles are often neglected when designing software and software systems. The neatness and diagram layout are visualization tools. Good models exhibit good drawing characteristics and layouts. Symmetry in the Petri nets focuses on this aspect.

Some questions are important. Is it possible to use symmetry instead of formal methods to describe computer systems via Petri net structures? This would require the creation of some notation for representing this as the current notations given in literature suffer from various drawbacks. Symmetrical systems should exhibit well-balanced properties. Symmetry could be used to create reduced and restricted forms of Petri nets. Possibly non-symmetrical Petri nets would have some more difficulty to interpret. However, to understand and check these conjectures more theoretical and experimental work has to be undertaken.

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