System Engineering based on Remarkable Geometric Properties of Space

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Abstract: - In this paper, we regard designing systems based on remarkable geometric properties of space, namely valuable rotational symmetry and asymmetry harmonious, using schematic and diagrammatic presentations of the systems. Moreover, the relationships are a way to comprehend original information to serve as a source of research and designing the systems. The objective of the future methodology is the advanced study of spatial geometric harmony as profiting information for expansion fundamental and applied researches for optimal solutions of technological problems in systems engineering. These systems engineering designs make it possible to improve the quality indices of devices or systems concerning performance reliability, code immunity, and the other operating indices of the systems. As examples, both up to 25% errors of lengths correcting code and high-speed self-error-correcting vector data code formed under a toroidal coordinate system are presented.

Key-Words: - symmetry and asymmetry harmonious relationships, generative rotational symmetry, Ideal Ring Bundle, error-correcting cyclic code, vector monolithic coding system, manifold.

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1 Introduction

The General Relativity Einstein`s theory describes the Universe as a four-dimensional manifold topology, [1], [2]. This research relates to a better understanding of combinatorial properties of two- and multidimensional geometric structures for finding optimal solutions in system engineering, using schematic and diagrammatic presentations of the systems. There are innovative theoretical investigations, using schematic modeling advanced engineering systems based on combinatorial configurations such as projective planes [3], cyclic groups in extensions of the Galois field [4], difference sets [5], and the other combinatorial constructions, [6], [7].

2 Problem Formulation

The statement problem is to increase technical variabilities in the system with an incomplete set of elements and a number of bonds, using novel designs based on combinatorial geometric properties of space. Research involves configuring the circuits and system engineering related to finding the optimal placement of structural elements in spatially or temporally distributed systems, including the appropriate combinatorial constructions for configuring innovative engineering devices, schemes and systems in spatially closed topologically systems.

3 Problem Solution

3.1 Symmetry and Asymmetry: Harmonious Relationships

Symmetry is known form the foundation of the geometrical construction of the Universe, [8]. We refer to the process engineering for optimal solutions to technological problems, using harmonious penetration asymmetry into rotational symmetry, and curvature structure of general relativity, [9]. A complementary ensemble of 1-fold (black line) and 2-fold (red lines) cyclic asymmetries joined into 3-fold (S=3) rotational symmetry illustrates Figure 1.

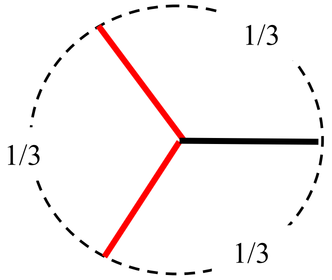

Fig. 1: Complementary ensemble of 1-fold (black line) and 2-fold (red lines) cyclic asymmetries joined into 3-fold (S=3) rotational symmetry

Note, the chart (Figure 1) backgrounds the ring-like models of the ideal ring protractor with two (n=2) marks (red lines) placed into a dial as the angular distance relation 1:2 (Figure 2).

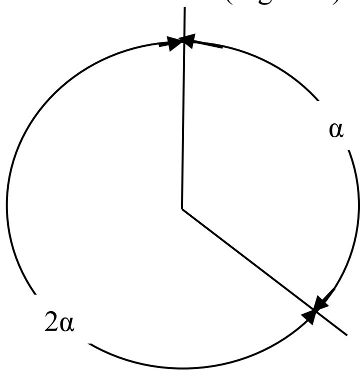

Fig. 2: Chart of ideal ring protractor with two (n=2) marks

It's evident, that two S–1 = 2 angular distances between marks (α_{min}=120° and α_{max}=240°) in the protractor represent the perfect cyclic relationship of integers. There are a lot of the underlying ideal protractors generated from the rotational symmetry of numerous orders. An example of 7-fold (S=7) rotational symmetry is given in Figure 3. We call this peculiarities "Generative Rotational Symmetry " (GRS).

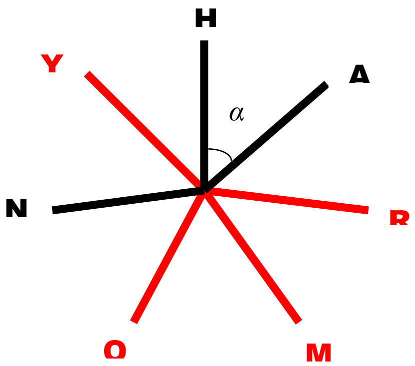

Fig. 3: Generative 7-fold (S=7) rotational symmetry

If we allow go round seven (S=7) lines of the 7-fold rotational symmetry, moving clockwise reference points **H→A→R→M→O→N→Y** (Figure 3), we can obtain a set of angular distances [α, 6α] between distinct pairs of three (n_1=3) black and four (n_2=4) red lines, α = 360°/S =360°/7(Table 1).

Table 1. Angular distances [α, 6α] between distinct pairs, moving clockwise reference points **H→A→R→M→O→N→Y** of three (n_1=3) black and four (n_2=4) red lines, α = 360°/S =360°/7

Angle	Starting point	Final point
α	**H**	**A**
2α	**N**	**H**
3α	**N**	**A**
4α	**A**	**N**
5α	**H**	**N**
6α	**A**	**H**
Angle	Starting point	Final point
α	**R**	**M**
2α	**R**	**O**
3α	**M**	**Y**
4α	**R**	**Y**
5α	**O**	**R**
6α	**M**	**R**
Angle	Starting point	Final point
α	**M**	**O**
2α	**O**	**Y**
3α	**Y**	**R**
4α	**Y**	**M**
5α	**Y**	**O**
6α	**O**	**M**

Hence, the ring scale reading system based on 7-fold (S=7) rotational symmetry allows on partition of two-dimensional space perfectly for a minimum number of intersections relative to the reading point by spatial interval α = 360°/7 exactly once(R_1=1) and/or twice (R_2=2) by the same interval. Easy to see, that the 7-fold rotational symmetry creates an intelligent system as an ensemble of two complementary numerical non-uniform cyclic structures {1, 4, 2}, and {1, 1, 2, 3}, followed by **H→A→N→H**, and **R→M→O→Y→R** cyclic sequences.
Parameters S, n_1, n_2, R_1, R_2 of GRSs for 3 ≤ S ≤ 31 are tabulated (Table 2).

Table 2. Parameters S, n_1, n_2, R_1, R_2 of GRSs for $3 \leq S \leq 31$

Parameters of the GRSs				
S	n_1	n_2	R_1	R_2
3	1	2	1	1
7	3	4	1	2
11	5	6	2	3
13	4	9	1	6
15	7	8	3	4
19	9	10	4	5
21	5	16	1	12
23	11	12	5	6
31	6	15	1	7

In Table 2, we observe, that all S-fold GRSs are completed from complementary pairs of ideal protractors. Note, anyone of the given GRSs consists of the protractors, each of them takes an even-fold (n_1), and odd-fold (n_2) number of marks. The set of all angular intervals $[\alpha, 360^o]$ occurs exactly R_1 (R_2) times by step $360^o/S$. In the underlying schematic demonstrations of rotational symmetry and asymmetry harmonious relation phenomenon it's perfectly well understood remarkable geometric properties of a space for comprehending original information to serve as a source for research and designing high-performance systems engineering.

3.2 Rotational Symmetry and Asymmetry Relationships

From Table 2 and the underlying relationships follow

$$S = n\,(n - 1)/R + 1, \qquad (1)$$

where S is the order of rotational symmetry, n - number of asymmetrically diverged beams from, and R - number times of enumerate the set of commensurable angular distances between these lines. Rotational symmetry-asymmetry relationships, provide, essentially, new information about the remarkable properties of GRSs as one more approval hypothesis on the existing world-wide harmony of the Universe, [8].

4 System Engineering based on Generative Rotational Symmetry

4.1 Ideal Ring Bundles

Generally, IRBs are cyclic sequences of ordered- chain sub-sequences of the sequence. The modular sums of consecutive sub-sequences of an Ideal Ring Bundle enumerate nodal points of a manifold coordinate system exactly R-times. In a particular case, the sums of one-dimensional IRB enumerate the set of integers from 1 to $S-1$ exactly R-times.

For example, the IRB {1,4,2} formed by 7-fold (S=7) rotational symmetry as three (n_1=3) black beams (Figure 3) enumerate the set of integers [1,6] exactly once (R_1=1):

1, 2, 3=2+1, 4, 5=1+4, 6=4+2.

At the same time the IRB {1, 1, 2, 3} follows from four (n_2=4) red beams - exactly twice (R_2=2):

1, 1; 2, 2=1+1; 3, 3=1+2; 4=3+1, 4=1+1+2; 5=2+3, 5=3+1+1; 6=1+2+3, 6=2+3+1.

Therefore, we comprehend this harmonious symmetry -asymmetry of order seven which generates this creative ensemble.

Ideal Ring Bundles (IRBs) can be used for optimal solutions to wide classes of technological problems, using the applicability of one- and multidimensional IRBs

4.2 IRBs Error Correcting Cyclic Codes

Designs of IRBs error correcting cyclic codes make it possible to correct up to 25% of the code combination lengths. For synthesis of the codes can be used formula

$$x_j - 1 \equiv \sum_{i=1}^{j} k_i \;(mod\, S),\; j=1, 2,\ldots, n \qquad (2)$$

x_j – is numerical position of symbols "1" in an initial code combination of the code combination length S, where k_j is i–th term of an IRB.

An example of error-correcting cyclic code based on the IRB {1,4,2,1,2,1} with parameters S = 11, n=6, R=3 is below (Table 3).

Table 3. Error correcting cyclic code based on the IRB {1,4,2,1,2,1} with parameters S =11, n=6, R=3

№	Code combinations										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	1	0	0	0	1	0	1	1	0	1	1
2	1	1	0	0	0	1	0	1	1	0	1
3	1	1	1	0	0	0	1	0	1	1	0
4	0	1	1	1	0	0	0	1	0	1	1
5	1	0	1	1	1	0	0	0	1	0	1
6	1	1	0	1	1	1	0	0	0	1	0
7	0	1	1	0	1	1	1	0	0	0	1
8	1	0	1	1	0	1	1	1	0	0	0
9	0	1	0	1	1	0	1	1	1	0	0
10	0	0	1	0	1	1	0	1	1	1	0
11	0	0	0	1	0	1	1	0	1	1	1

The first code combination 10001011011 is obtained by formula (2), while the rest combinations of the code are fulfilled by its cyclic shifting.

A number t_{cor} of corrected errors for optimal cyclic IRB-code to be depending n [10]:

$$t_{cor} \leq (n/2 - 1) \tag{3}$$

We proceed from (3) that cyclic code (Table 3) allows correct up to 2 errors.

The code size can be doubled if Table 3 is added by the next table with opposite values of all its binary characters (Table 4).

Table 4. Opposite values of binary characters of Table 3 for doubled of the code size

№	Code combinations										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	0	1	1	1	0	1	0	0	1	0	0
2	0	0	1	1	1	0	1	0	0	1	0
3	0	0	0	1	1	1	0	1	0	0	1
4	1	0	0	0	1	1	1	0	1	0	0
5	0	1	0	0	0	1	1	1	0	1	0
6	0	0	1	0	0	0	1	1	1	0	1
7	1	0	0	1	0	0	0	1	1	1	0
8	0	1	0	0	1	0	0	0	1	1	1
9	1	0	1	0	0	1	0	0	0	1	1
10	1	1	0	1	0	0	1	0	0	0	1
11	1	1	1	0	1	0	0	1	0	0	0

Table of quality indexing the error- correcting cyclic code based on the IRBs in ascending order of its parameters n and S is below (Table 5).

Table 5. Quality indexing the error- correcting cyclic code based on the IRBs

n	t_{cor}	S	Q×100%	P
6	2	11	18	22
8	3	15	20	30
16	7	31	22,6	62
32	15	63	23,8	126
64	31	127	24,4	254

Table 5 evidents improving quality indexes t_{cor}, $Q = t_{cor}/S$, and P of the error correcting codes based on the IRBs approaching Q =25% errors of the code length asymptotically if $S \to \infty$.

4.3 Two-dimensional Ideal Ring Bundles

Definition. An n-stage cyclic sequence of connected 2-stage sub-sequences of the sequence, for which the set of all two-modular (t=2) vector-sums (mod m_1, mod m_2) form two-dimensional grid $m_1 \times m_2$ over a torus surface is named two-dimensional Ideal Ring Bundle (2-D IRB).

For example, two-dimensional four-stage (n=4) cyclic sequence {(1,1), (0,1), (2,2), (2,1)}, where all vector-sums of connected sub-sequences form two-dimensional cyclic grid 3×4, taking modulo m_1 = 3 for the first component of the vector-sums, and modulo m_2 = 4 for the second ones, depicted in a chart (Figure 4).

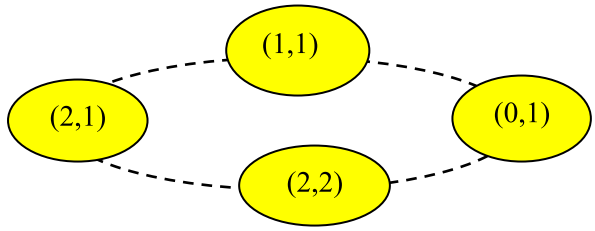

Fig. 4: A chart of two-dimensional (t=2) 4-stage (n =4) IRB {(1,1), (0,1), (2,2), (2,1)}

There are all two-dimensional vector-sums of the connected 2-D vectors of the IRB {(1,1), (0,1), (2,2), (2,1)}:

$$\left.\begin{aligned}
&(1,1)+(0,1)\equiv(1,2)\\
&(0,1)+(2,2)\equiv(2,3)\\
&(2,2)+(2,1)\equiv(1,3)\\
&(2,1)+(1,1)\equiv(0,2)\\
&(1,1)+(0,1)+(2,2)\equiv(0,0)\\
&(0,1)+(2,2)+(2,1)\equiv(1,0)\\
&(2,1)+(1,1)+(2,2)\equiv(2,0)\\
&(1,1)+(0,1)+(2,1)\equiv(0,3)
\end{aligned}\right\}\bmod 3, \bmod 4 \tag{4}$$

The set of two-dimensional vector-sums together with vectors (1,1), (0,1), (2,2), (2,1) of the IRB complete set of two-modular vector sums as follows:

(0, 0)	(0, 1)	(0, 2)	(0, 3)
(1, 0)	(1, 1)	(1, 2)	(1, 3)
(2, 0)	(2, 1)	(2, 2)	(2, 3)

A vector ring diagram of the 2-D IRB {(1,1), (0,1), (2,2), (2,1)} depicted as colored graph (Figure 5).

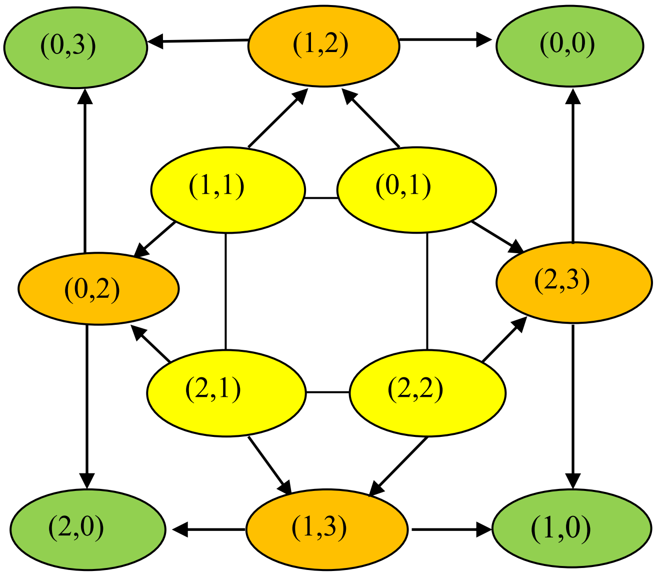

Fig. 5: Vector ring diagram of the 2-D IRB{(1,1), (0,1), (2,2), (2,1)}

The vector ring diagram (Figure 5) demonstrates the interconnection of all two-modular vector-sums from (0,0) to (2,3) inclusive formed on the IRB {(1,1), (0,1), (2,2), (2,1)}.

4.4 Optimum Vector Monolithic Coding Systems

An arbitrary vector monolithic coding system unlike of ordinary ones are that all allowed code words consist of no more than a single solid block both of connected symbols “1”, and block “0” in the words. The optimum vector monolithic coding systems are formed on t-dimensional IRBs with informative parameters n, S that makes it possible to offer code size $n^2 - n$ in the minimised basis of the system. Additionally, the set of all t-dimensional vectors covers node points of manifold reference system by t-axes taking t- modular (m_1, m_2, …, m_i,…, m_n) manifold coordinate system, allowing on binary encoding of vector signals arranged as no more two sequences with the same characters [10].

Optimum monolithic 2-D vector code based on the Ideal Ring Bundle {(1,1), (2,3), (0,3), (3,3), (1,3)} with parameters S=21, n=5, m_1=4, m_2=5, R=1 is given in Tabl.6.

Table 6. Optimum monolithic 2-D vector code based on the Ideal Ring Bundle {(1,1), (2,3), (0,3), (3,3), (1,3)}

Vector	Digit weights				
	(1,1)	(2,3)	(0,3)	(3,3)	(1,3)
(0,0)	1	1	1	0	1
(0,1)	0	0	0	1	1
(0,2)	1	1	0	0	1
(0,3)	0	0	1	0	0
(0,4)	0	0	1	1	1
(1,0)	1	0	1	1	1
(1,1)	1	0	0	0	0
(1,2)	1	0	0	1	1
(1,3)	0	0	0	0	1
(1,4)	0	1	1	1	0
(2,0)	1	1	1	1	0
(2,1)	0	1	1	0	0
(2,2)	0	1	1	1	1
(2,3)	0	1	0	0	0
(2,4)	1	0	0	0	1
(3,0)	1	1	0	1	1
(3,1)	0	0	1	1	0
(3,2)	1	1	1	0	0
(3,3)	0	0	1	0	0
(3,4)	1	1	0	0	0

Table 6 contains S –1 = 20 two-dimensional 5-digit (n=5) vector code combinations in a binary coding system of 2-D (t =2) vector digit weights {(1,1), (2,3), (0,3), (3,3), (1,3)}as the basis of the toroid coordinate grid of sizes $m_1 \times m_2 = 4 \times 5$, $m_1 = 4$, $m_2 = 5$.

Optimum monolithic 3-D (t=3) vector coding system formed under the IRB {(1,1,2), (0,2,2), (1,0,3), (1,1,1), (0,1,0), (0,2,3)} with informative parameters S=31, n=6, m_1=2, m_2=3, m_3=5, and R=1:

(0,0,0)⇒{(0,2,3)+(1,1,2)+(0,2,2)+(1,0,3)+(0,1,0)}
(0,0,1) ⇒{(0,2,2) + (1,0,3) + (1,1,1)}
(0,0,2) ⇒{(1,1,2) + (0,2,2) + (1,0,3)}
(0,0,3) ⇒{(0,2,3) + (0,1,0)}
(0,0,4)⇒{(0,2,2)+(1,0,3)+(1,1,1)+(0,1,0)+(0,2,3)}
(0,1,1) ⇒ {(0,2,2)+ (1,0,3) + (1,1,1) + (0,1,0)}
(0,1,2) ⇒ {(1,0,3)+ (1,1,1) + (0,1,0) + (0,2,3)}
(0,1,3)⇒{(1,1,1)+(0,1,0)+(0,2,3)+(1,1,2)+(0,2,2)}
(0,1,4) ⇒ {(0,1,3) + (1,1,1)}
(0,2,0) ⇒ {(0,2,3)+ (1,1,2) + (0,2,2) + (1,0,3)}
(0,2,1) ⇒ {(1,1,1)+ (0,1,0) + (0,2,3) + (1,1,2)}
…………………………………………………………
Finally,
(1,2,4)⇒{(0,2,3)+(1,1,2)+(1,1,1)+(1,0,3)+ (0,1,0)}

The set of S –1=30 these three-dimensional vectors form optimum coding system 2×3×5 in the minimized basis of manifold coordinate system formed by three (t=3) annular axes m_1=2, m_2=3, m_3=5 with origin of the coordinates in common point (0,0,0).

5 Discussion

Schematic presentations of remarkable geometric properties of space are given in Figure 1, Figure 2 and Figure 3. Complementary ensemble of 1-fold and 2-fold cyclic asymmetries joined into 3-fold (S=3) symmetry illustrates Figure 1. Table 1 reveals the essence of the generative rotational symmetry (GRS) as the "ideal" protractors, each of them takes even-fold, and odd-fold number of marks. Table 2 opens new information about geometric relationships of optimal distributed structural elements (events) as part of the "whole" and logical interpretation of the phenomenon. The 7-fold rotational symmetry (Figure 3) creates an intelligent system as an ensemble of two complementary numerical non-uniform cyclic structures {1, 2, 4}, and {1, 1, 2, 3} as the Fibonacci numbers, [11]. In Table 3, Table 4 and Table 5 we can observe improving quality indexes of the optimized *error-correcting* codes based on the IRBs approaching 25% errors of the code length asymptotically if $S \to \infty$. Unlike familiar coding systems, the monolithic codes provide faster self-correcting vector data signals for the transmission of multidimensional information by noise communication channels, as well as the processing of the signals under a manifold coordinate system. An example of an optimum monolithic 2-D vector code of size $m_1 \times m_2 = 20$, m_1=4, m_2=5 illustrates in Table 1. The second example demonstrates an optimum monolithic 3-D vector coding system of code size $m_1 \times m_2 \times m_3$= 30, where m_1=2, m_2=3, m_3=5.

6 Conclusion

System engineering based on remarkable geometric properties of space provides novel techniques for improving the quality indices of engineering devices and systems, using Ideal Ring Bundles prospected from generative rotational symmetry. The study of the generative rotational symmetry (GRS) peculiarity allows expanding the underlying combinatorial procedures for finding the optimal placement of structural elements in spatially or temporally distributed systems of numerous engineering devices under development. Ideal Ring Bundles (IRBs) can be used for optimal solutions to of wide classes of technological problems, using the applicability of one- and multidimensional IRBs. It's just what the generative rotational symmetry and asymmetry provide mutual penetration of existing eternal world intelligence of the Universe.

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Conflict of Interest

The authors have no conflicts of interest to declare.