Minimizing Product Cost Crashing using Graph Network System

NESHAR BASUMATARY, ABDUR ROHMAN, SURAJIT KR. NATH*
[1]Department of Mathematical Sciences,
Bodoland University, Kokrajhar, 783370,
INDIA

*Corresponding Author

Abstract: - Graph is mathematical representation which can use in any real life situation where a relationship is present between the objects/elements. In this article, we try to find minimizing product cost crashing using graph network system. The minimizing product cost crashing is a system that used by businesses to reduce the expenses related with the manufacturing unit and product processes. Here we, Analyzed the various cost reduction strategies and their impact on product development cycles and identified efficient method for lowering production expenses while maintaining quality standards. The outcome of this article contributes to enhancing cost management practices and improving overall profitability in the industry.

Key-Words: - Graph theory, Graph Network, Optimization, Crashing, Critical Path, Slope.

Received: August 24, 2023. Revised: November 17, 2023. Accepted: December 13, 2023. Published: December 31, 2023.

1 Introduction

Minimizing product cost is the system that identifies and reduces expenses related to a running business. The main focus of minimizing product cost is to lower of the overall costs of a business without compromising quality and negative impact in the area of the company. A graph Network system, [1], is involved in optimizing the transformation of goods from one location to another while taking into account factors such as the cost of transformation, the distance traveled, and the availability of transformation routes.

Crashing, [2], on the other hand, is a technique when it appears, an additional costs related to crashing are viewed against a minimum possible benefit to complete a project within a short period. It helps to speed up the timeline of a project through additional resources. Crashing is a one way process to compress the rest of the path and to make up for delays in the beginning.

In some situations, [3], minimizing product cost and crashing may be related. For example, if a company is scheduled on a project it may need to allocate its resources to certain activities to speed up its completion and meet the deadline however this may result in additional costs that need to be factored into the overall product cost.

In mathematics, graph theory, [4], is a branch of mathematics that deals with the study of graphs, which are mathematical structures used to model pairwise relationships between any two objects. A graph is determined as a mathematical structure that represents a particular function by connecting a set of points. It involves analyzing properties and characteristics of graphs such as connectivity, paths, cycles, and graph coloring, to solve in various fields including computer science, operations research, social networks, etc. In mathematics, all these networks are called graphs. By using the graph theory we can find a critical path. The concept of minimizing product cost crashing is various strategies that can be employed to achieve this objective.

2 Preliminary

Definitions, [4], [5], [6], [7]

Graph: Graph theory is the study of points and lines. It is a pictorial representation that represents the Mathematical truth. Graph theory is the study of the relationship between the vertices(nodes) and edges(lines). Formally, a graph is denoted as a pair G = (V, E) where V represents the finite set of vertices and E represents the finite set of edges.

Subgraph: A graph of G is a subgraph having all of its points and lines in G.

Clearly from Figure 1 and Figure 2, we see that Figure 1 is a example of Graph and Figure 2 is a example of subgraph.

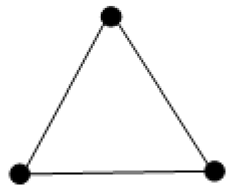

Fig. 1: Graph

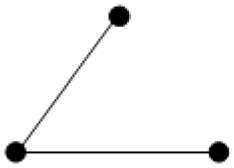

Fig. 2: Subgraph

Degree of vertex: The degree of a vertex in an undirected graph is the number of links incident with it, with the exception that a loop at a vertex contributes twice to the degree of that vertex. It is denoted by deg(v), where v is the vertex of the graph.
A vertex whose degree is zero is called an isolated vertex.
Clearly, we see that Figure 3 is a example of degree of vertices.

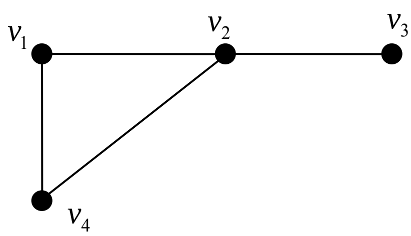

Fig. 3: Example of degree of vertices

In this figure the degree of the vertices are $deg(v_1) = 2$, $deg(v_2) = 3$, $deg(v_3) = 1$, $deg(v_4) = 2$.
Directed and undirected graph: If in a graph G(V, E) each edge of a graph G has a direction, then G is called a directed graph.
If each edge of graph G has no direction, then graph G is called an undirected graph.
Example: Clearly, Figure 4 is a example of undirected graph and Figure 5 is a example of directed graph.

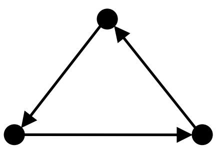

Fig. 4: Undirected graph

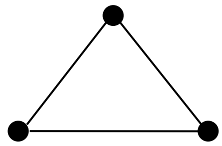

Fig. 5: Directed graph

Weighted graph: A weighted graph is a graph in which each line is given a numerical weight. A weighted graph is, therefore, a special type of labeled graph in which the labels are numbers usually taken to be positive.
Example: Here Figure 6 is a example of weighted graph.

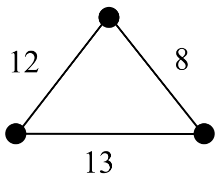

Fig. 6: Example of Weighted Graph

Walk: A walk of graph G is an alternating sequence of points and lines beginning and ending with points, in which each line is independent with the two points immediately preceding and following it.
Trail: A trail is a walk with no repeated edge.
Path: A path in a graph is an infinite or finite sequence of edges that connect a sequence of vertices which by most definitions are all distinct from one another.
Cycle: A trail of a non-zero length from a vertex V to itself in a graph is called a cycle except for the beginning and the ending vertices that are both equal to V there are no repeated vertices in the walk.
In Figure 7, we clearly mention walk, trail, path, cycle.

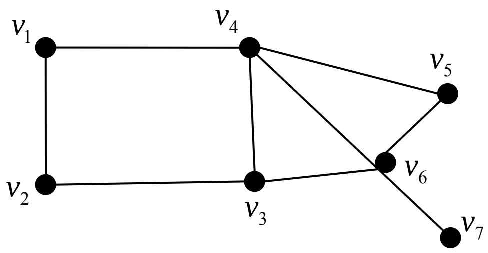

Fig. 7: Example of Cycle

$v_1\ v_2\ v_4\ v_3\ v_2\ v_1\ v_4$ (walk)
$v_1\ v_2\ v_4\ v_3\ v_6\ v_4\ v_1$ (walk) (trail)
$v_4\ v_3\ v_2\ v_4\ v_6\ v_7$ (walk) (trail)
$v_1\ v_2\ v_3\ v_6\ v_5\ v_4$ (walk) (trail) (path)
$v_1\ v_2\ v_3\ v_6\ v_5\ v_4\ v_1$ (walk) (trail) (cycle)

Critical path: The critical path is the sequence of tasks that determine the project's shortest duration and must be completed on time for the project to be completed successfully.

Transportation: Transportation is crucial for connecting people and goods or services from one place to another. It can include various modes like cars, trains, planes, buses, bicycles, and walking.

Slope: In mathematics, the slope refers to the measure of the steepness of a line it represents the ratio of vertical change (rise) to horizontal change (run)between two points on the line.

3 Minimizing Product Cost Crashing using Graph Network

Minimizing product cost crashing means reducing the project completion time by adding extra resources to it. The project may crash by reducing the normal completion time of critical activities is called the crashing of activities. This can be obtained by increasing resources to perform.

Many companies suffer from challenges and difficulties due to complex projects. Because of dependency on popular ways to plan schedule and control the project development, control of time, cost and good performance also need to be completed on time with good quality within the allocated budget.

For this research paper, we have given an example below in Table 1 about finding the product cost and crashing of a network with two critical path in Figure 8, Figure 9, Figure 10 and Figure 11 and we find Crash limit and Slope in Table 2, Table 3 and Table 4.

Table 1. Product Cost

Activity	Normal time(week)	Crash time(weeks)	Normal cost(Rs)	Crash cost(Rs)	Cost Slope
1-2	7	4	700	850	50
1-3	5	3	500	700	100
1-4	8	5	600	1200	200
2-5	9	7	800	1250	225
3-5	5	3	700	1000	150
3-6	6	5	1100	1300	200
4-6	7	5	1200	1400	125
5-7	2	1	400	500	100
6-7	3	2	500	850	350
			Total = 6500		

Let the indirect cost per week is Rs.200

$$Slope = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}} = \frac{850-700}{7-4} = \frac{150}{3} = 50$$

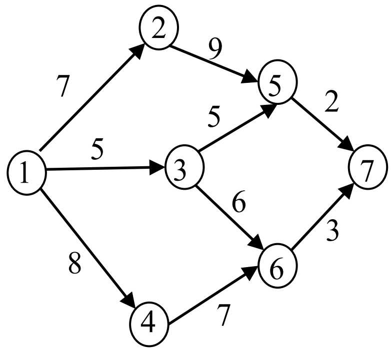

Fig. 8: Graph of Critical Path

Critical Path:

$$1 \to 2 \to 5 \to 7 = 7 + 9 + 2 = 18$$
$$1 \to 3 \to 5 \to 7 = 5 + 5 + 2 = 12$$
$$1 \to 3 \to 6 \to 7 = 5 + 6 + 3 = 14$$
$$1 \to 4 \to 6 \to 7 = 8 + 7 + 3 = 18$$

Normal project completion time = 18 weeks

Critical path:

$1 \to 2 \to 5 \to 7$ and $1 \to 4 \to 6 \to 7$

$$\text{Total Direct Normal cost} = 6{,}500$$
$$\text{Indirect cost}(200 \times 18) = \frac{3{,}600}{10{,}100}$$

Crash limit and Slope:

Table 2. Crash limit and Slope

Critical path	Critical Activity	Crash Limit	Cost Slope
$1 \to 2 \to 5 \to 7$	$1 \to 2$	3	50
	$2 \to 5$	2	225
	$5 \to 7$	1	100
$1 \to 4 \to 6 \to 7$	$1 \to 4$	3	200
	$4 \to 6$	2	125
	$6 \to 7$	1	350

(i). <u>Iteration</u>:

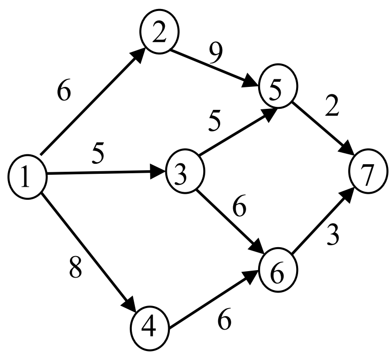

Fig. 9: Graph of Critical Path

Critical Path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 = 6 + 9 + 2 = 17$$
$$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 = 5 + 5 + 2 = 12$$
$$1 \rightarrow 3 \rightarrow 6 \rightarrow 7 = 5 + 6 + 3 = 14$$
$$1 \rightarrow 4 \rightarrow 6 \rightarrow 7 = 8 + 6 + 3 = 17$$

Project Completion time = 17 weeks
Critical path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \text{ and } 1 \rightarrow 4 \rightarrow 6 \rightarrow 7$$

Total Cost = pre. Total cost + direct cost(cost slope) – indirect cost
=10,100 +[50+125]- 200
=10,075

Crash limit and Slope:

Table 3. Crash limit and slope

Critical path	Critical Activity	Crash Limit	Cost Slope
1 → 2 → 5 → 7	1 → 2	2	50
	2 → 5	2	225
	5 → 7	1	100
1 → 4 → 6 → 7	1 → 4	3	200
	4 → 6	1	125
	6 → 7	1	350

(ii). <u>Iteration</u>:

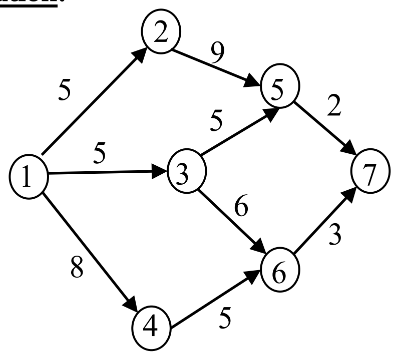

Fig. 10: Graph of Critical path

Critical Path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 = 5 + 9 + 2 = 16$$
$$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 = 5 + 5 + 2 = 12$$
$$1 \rightarrow 3 \rightarrow 6 \rightarrow 7 = 5 + 6 + 3 = 14$$
$$1 \rightarrow 4 \rightarrow 6 \rightarrow 7 = 8 + 5 + 3 = 16$$

Project Completion time = 16 weeks
Critical path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \text{ and } 1 \rightarrow 4 \rightarrow 6 \rightarrow 7$$

Total Cost = pre. Total cost + direct cost(cost slope) – indirect cost
=10,075 +[50+125]- 200
=10,050

Crash limit and Slope:

Table 4. Crash limit and slope

Critical path	Critical Activity	Crash Limit	Cost Slope
1 → 2 → 5 → 7	1 → 2	1	50
	2 → 5	2	225
	5 → 7	1	100
1 → 4 → 6 → 7	1 → 4	3	200
	4 → 6	0	125
	6 → 7	1	350

(iii). <u>Iteration</u>:

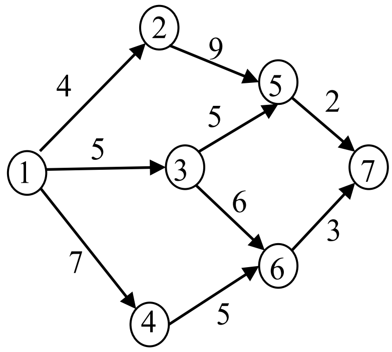

Fig. 11: Graph of critical path

Critical Path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 = 4 + 9 + 2 = 15$$
$$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 = 5 + 5 + 2 = 12$$
$$1 \rightarrow 3 \rightarrow 6 \rightarrow 7 = 5 + 6 + 3 = 14$$
$$1 \rightarrow 4 \rightarrow 6 \rightarrow 7 = 7 + 5 + 3 = 15$$

Project Completion time = 16 weeks
Critical path:

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \text{ and } 1 \rightarrow 4 \rightarrow 6 \rightarrow 7$$

Total Cost = pre. Total cost + direct cost(cost slope) – indirect cost
=10,050+[50+125]- 200
=10,100

<u>Final Result</u>:
Since the total cost of this iteration **(iii)** is more than that of the previous iteration, stop the procedure and treat the solution of the previous iteration **(ii)** as the best solution for implementation. The final crashed project completion time is 16 weeks. Corresponding critical paths are-

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \text{ and } 1 \rightarrow 4 \rightarrow 6 \rightarrow 7$$

4 Conclusion

Minimizing product cost involves optimizing the cost of producing and delivering goods or services to customers while crashing is a technique used to reduce the duration of critical activities in a project schedule to meet a deadline minimizing product cost could involve optimizing the transportation of goods from one location to another. While taking into account factors such as the cost of transportation distance traveled and available transportation routes.

If a company is behind schedule on a project it may need to allocate more resources to certain activities to speed up their completion and meet the deadline which could result in additional costs that need to be factored into the overall product costs.

Companies need to find a balance between minimizing product costs and crashing to meet project deadlines. Careful planning analysis and decision-making are essential to achieve the desired result and minimize cost without compromising project timelines or quality

References:

[1] Petrina C. Feng, "Impact of Graphs and Network in Minimizing Project and Product cost", *The International Journal of Business Management and Technology*, vol.2, no. 1, pp. 01-6, 2018.

[2] M.N. Islam, B. Rana, S. Rafique, T. Aziza, "Crashing Project Time with Least Cost: A Linear Programming Approach", *Journal of Business Research*, vol. 6, 2004.

[3] C.L. Karmaker and P. Halder, "Scheduling Project Crashing Time Using Linear Programming Approach: Case Study", *International Journal of Research in Industrial Engineering*, 2017.

[4] D.B.West, "*Introduction to Graph Theory*", Pearson, 2000.

[5] R. Likaj, A. Shala, M. Mehmetaj, P. Hyseni, X. Bajrami, "Application of Graph theory to find the optimal paths for the transportation problem," *IFAC Proceedings* Vol., vol.46, Issue 8, pp. 235-240, 2013.

[6] R. Rehab and R.I. Carr, "time-cost trade-off among related activities",*.J.construct.Eng. manage.*,1977.

[7] G. Ambrus, P.Hajnal, "*The Slope parameter of Graphs*", Acta Scientiarum Mathematicarum, 2006.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The author has no conflict of interest to declare.