Simulating the Soft Power Effect in Two-Dimensional Models of A. Lotka – V. Volterra

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Abstract: - The most famous models of A. Lotka and V. Volterra, "predator-prey" and "competition," are considered in relation to social systems. The peculiarity of the social systems that distinguishes them from the biological ones is a quick reaction to the current situation. It turns out that the straightforward struggle of preys against predators is ineffective, whereas the methods of "soft power" are able to get rid of the predators completely. The article considers the competition equations as mathematical model of cross-cultural interaction. The study of the model reveals the possibility of a paradoxical situation when one of the cultures positively treats the other, though this other one is actually harmful to it. Soft power, in this case, masks a negative attitude, presenting it as a friendly one. Conversely, in some cases, a harmless culture may be mistakenly perceived as a very negative one because of the "brutality" of some of its manifestations.

Key-Words: Soft Power, Double Standards, Simulation, Predator – Prey, Competition

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1 Introduction

Soft power – they say that this term was introduced in 1990 by Joseph Nye of Harvard University, [1], for the first time, but something similar can be found even in ancient times – for example, in Lao Tzu's Tao Te Ching [2]. It is possible to say that cultural values, capable of inducing others to want what is wanted by the soft power operator, are the cornerstone of this concept.

2 Predator – Prey Model

Take a look at the most famous model of A. Lotka and V. Volterra, – "perdator-prey," [3].

$$\begin{aligned} \frac{dN}{dt} &= \alpha N\left(1-\frac{M}{\overline{M}}\right), \\ \frac{dM}{dt} &= \beta M\left(\frac{N}{\underline{N}}-1\right). \end{aligned} \tag{1}$$

Here N is the population size of preys, M is the population size of predators, α, β are the Malthusian factors, $\underline{N}$ is the minimum number of preys necessary to feed the predators, $\overline{M}$ – is the maximum number of predators that the prey population can withstand.

Note that the essence of the model (1) was described, including the oscillatory nature of its solutions (although in natural language), by L.N. Tolstoy in [4], almost 40 years before A. Lotka and V. Volterra.

2.1 Elements of Macro Analysis

We transform our equations (1):

$$\frac{dN}{dt} = \frac{\alpha N(\overline{M}-M)}{\overline{M}}, \frac{dM}{dt} = \frac{\beta M(N-\underline{N})}{\underline{N}}.$$

Next, – divide the first equation by the second:

$$\frac{dN}{dM} = \frac{\alpha N\underline{N}(\overline{M}-M)}{\beta M\overline{M}(N-\underline{N})}. \tag{2}$$

Let us separate the variables:

$$\frac{\overline{M}-M}{\beta M\overline{M}}dM + \frac{\underline{N}-N}{\alpha N\underline{N}}dN = 0,$$

or,

$$\frac{dM}{\beta M} - \frac{dM}{\beta\overline{M}} + \frac{dN}{\alpha N} - \frac{dN}{\alpha\underline{N}} = 0.$$

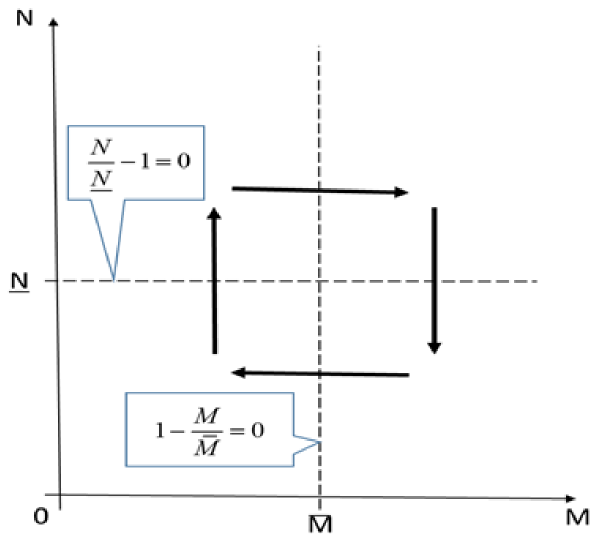

Figure 1: The Phase Plane in N, M coordinates.

Integrate both parts – is the next step.

$$\frac{1}{\beta}\left(\ln M - \frac{M}{\overline{M}}\right) + \frac{1}{\alpha}\left(\ln N - \frac{N}{\underline{N}}\right) = C. \quad (3)$$

Consider equality (3) as the function of N and M.

$$C(N, M) = \frac{1}{\alpha}\left(\ln N - \frac{N}{\underline{N}}\right) + \frac{1}{\beta}\left(\ln M - \frac{M}{\overline{M}}\right).$$

Let's call the function $C(N, M)$ the potential of the predator-prey system, and the functions

$$C(M) = \frac{1}{\beta}\left(\ln M - \frac{M}{\overline{M}}\right),$$

$$C(N) = \frac{1}{\alpha}\left(\ln N - \frac{N}{\underline{N}}\right),$$

potentials of predator and prey populations, respectively. This definition of potential is good because it has the same functional form for both populations. Then $C(N, M) = C(N) + C(M)$.

Finding a gradient $C(N, M)$ at the point (N, M). This is the vector

$$\left(\frac{1}{\alpha N} - \frac{1}{\alpha \underline{N}}, \frac{1}{\beta M} - \frac{1}{\beta \overline{M}}\right).$$

The appearance of the gradient suggests that the function $C(N, M)$ is strictly concave and reaches a strict global maximum at the point $\left(\underline{N}, \overline{M}\right)$.

Indeed, the partial derivative of the function $C(N, M)$ by N is

$$\frac{\partial C(N, M)}{\partial N} = \frac{1}{\alpha}\left(\frac{1}{N} - \frac{1}{\underline{N}}\right).$$

We see that if $N < \underline{N}$, the derivative is strictly positive, at $N = \underline{N}$, it is zero, and if $N > \underline{N}$, it is strictly negative.

Similarly, the partial derivative of the function $C(N, M)$ by M is

$$\frac{\partial C(N, M)}{\partial M} = \frac{1}{\beta}\left(\frac{1}{M} - \frac{1}{\overline{M}}\right).$$

Again, we see that if $M < \overline{M}$, the derivative is strictly positive, at $M = \overline{M}$, it is zero, and if $M > \overline{M}$, it is strictly negative. Since for any solution $N(t), M(t)$ of the predator-prey system, the following is true: $(N(t), M(t)) = const$, the plane $C(N, M) = C$, and the surface

$$C(N, M) = \frac{1}{\alpha}\left(\ln N - \frac{N}{\underline{N}}\right) + \frac{1}{\beta}\left(\ln M - \frac{M}{\overline{M}}\right),$$

intersecting if $const \leq C(\underline{N}, \overline{M})$, this gives us the trajectory of the system on the phase plane.

Moreover, from the above analysis of derivatives, it follows that if $N(t), M(t)$ and $\tilde{N}(t), \tilde{M}(t)$ are solutions of our predator-prey system with different initial conditions, $(N(t), M(t)) = C$, $\left(\tilde{N}(t), \tilde{M}(t)\right) = \tilde{C}$ and $C < \tilde{C}$, then the phase trajectory $\tilde{N}(t), \tilde{M}(t)$ lies strictly within the phase trajectory $N(t), M(t)$. The point $\left(\underline{N}, \overline{M}\right)$ is the global maximum point of the function; thus, (N, M) lies within the phase path of any solution. The example is shown in Fig. 2 below.

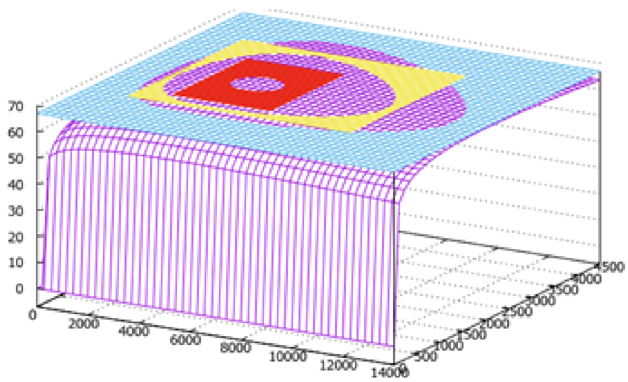

Figure 2: Level Planes $C(N, M) = const$.

Thus, we can say that our invariant

$$\frac{1}{\alpha}\left(\ln N - \frac{N}{\underline{N}}\right) + \frac{1}{\beta}\left(\ln M - \frac{M}{\overline{M}}\right) = C,$$

is the degree of proximity (topological, not metrical, as the belonging to the phase trajectory $C(N, M) = const$) of the phase trajectory to the equilibrium point – the center $\left(\underline{N}, \overline{M}\right)$. On any solution $N(t), M(t)$, this degree of proximity is preserved. Phase trajectories with a larger value of $C\left(N(t), M(t)\right)$ are inside the phase trajectories with a smaller value, and the stationary point $\left(\underline{N}, \overline{M}\right)$ is inside all the trajectories.

The above causes associations between the potential of the system and entropy. First, because it is a measure of proximity to equilibrium – the stationary point of the system, where the maximum potential $\left(\underline{N}, \overline{M}\right)$ is reached. Secondly, the essential role of the logarithm in its definition.

2.2 Simulation Experiment with the Classic Model

The Fig. 3 below shows examples of the model plots with different initial conditions, built in the AnyLogic system, [5].

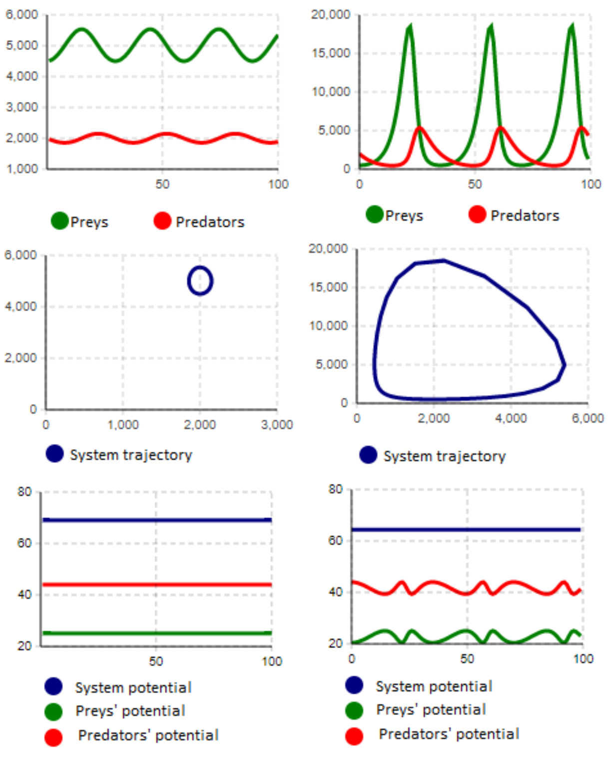

Figure 3: Simulation of the Predator-Prey Model.

We see that the potential of the system is preserved on the trajectory, although part of the potential of preys can go to predators, and vice versa (just as when a massive point moves around the center of gravity, part of its potential energy can go into kinetics, and vice versa but the total energy is preserved, and its magnitude determines the type of trajectory). If the phase trajectory of the system passes near the stationary point, the potentials of the populations can be considered approximately unchanged (their variances are of the highest order of smallness compared to the trajectory deviations from the stationary point).

Further, the higher the potential of the system, the closer its trajectory to the stationary point (in Fig. 3, $\underline{N} = 5000$, $\overline{M} = 2000$).

The graph also shows that the frequency of oscillations remains constant only in the small neighborhood of the center $\left(\underline{N}, \overline{M}\right)$ and with a decrease in the potential of the system, it also decreases. It might also be noted that in the small neighborhood of the center, the dynamics of populations are almost sines and cosines; phase trajectories are circles, althoufg "in large" they differ obviously.

2.3 Predator Control: Soft Power vs Fight

Suppose the preys resented their role as predator fodder in the model and decided to fight against oppression by the latter. The first thing that comes to mind, and what the oppressed peoples did throughout the 20th century, is to kill predators, or at least significantly reduce their number. As the history of the 20th century shows, it is most likely not be possible to eliminate everyone, but the reduction of the population size, as we will see later, does not solve the problem.

Suppose that there were about 1,000 preys and about 2,000 predators then the number of the latter was reduced to 300 at a time. The system simply jumped to another, more external trajectory, with higher peaks for both predators and preys. In the second example there were about 5,000 preys and more than 6,000 predators, and reduced the number of the latter was 1,500 that is more than four times less. The trajectory jumped to the internal trajectory with small oscillation amplitudes. Fundamentally, nothing has changed in the model.

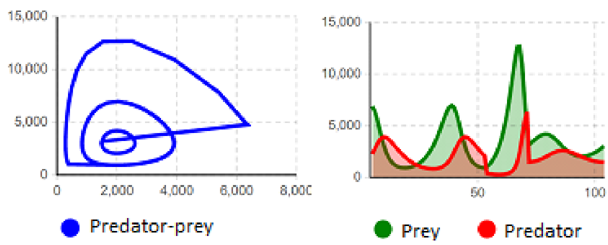

Figure 4: Instant Elimination of Part of the Predators.

What should preys do to improve their position in the system if the elimination of some predators does not have an effect?

Let's try to bring competition to the dynamics of the prey population. Add the competition summand with the environmental capacity N^* to the equation of the dynamics of preys in (1):

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{N^*} - \frac{M}{\overline{M}}\right).$$

The center $(\underline{N}, \overline{M})$ (see Fig. 1) turns into a stable focus and shifts left (see Fig. 5) to the point

$$\left(\underline{N}, \overline{M}(1 - \frac{\underline{N}}{N^*})\right).$$

As soon as the ratio $N^* \leq \underline{N}$ begins to be fulfilled, predators have no place in the system they die out (see Fig. 5 and Fig. 6).

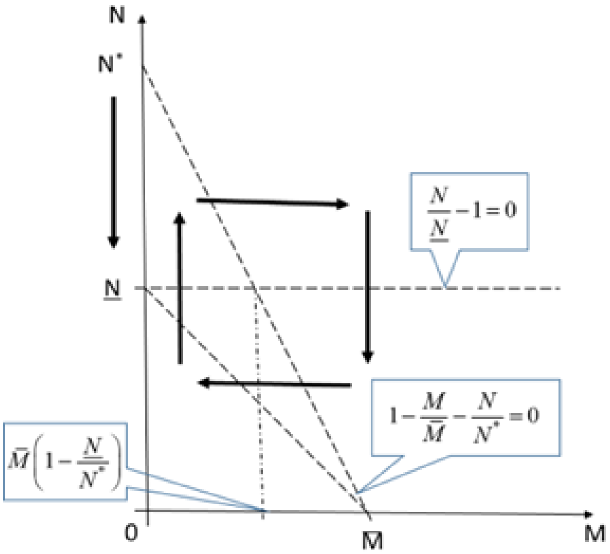

Figure 5: Competition among the Preys.

Note that the condition for the extinction of predators is that the entire resource is mastered in the process of preys' competition

$$\frac{\underline{N}}{N^*} \geq 1.$$

At the same time, preys do not lose anything: their number still fluctuates in the area $\underline{N}$, and after the extinction of predators, it is possible to weaken the competition – and increase N^*.

Note, that certain religions, as well as thinkers such as L.N. Tolstoy, [4], M.K. Gandhi, [6], advocate for non-resistance to evil trough violence, as an alternative to the retribution of "an eye for an eye", and this thesis often seems to us incomprehensible, impractical, idealistic, etc. Our simulations (Fig. 4) shows, that direct fighting with the predators is not fruitful for preys. Nevertheless, we see that preys can quite realistically get rid of predators without any violence, exclusively by the self-organization – by increasing internal competition, i.e. through the self-government, self-support, self-improvement.

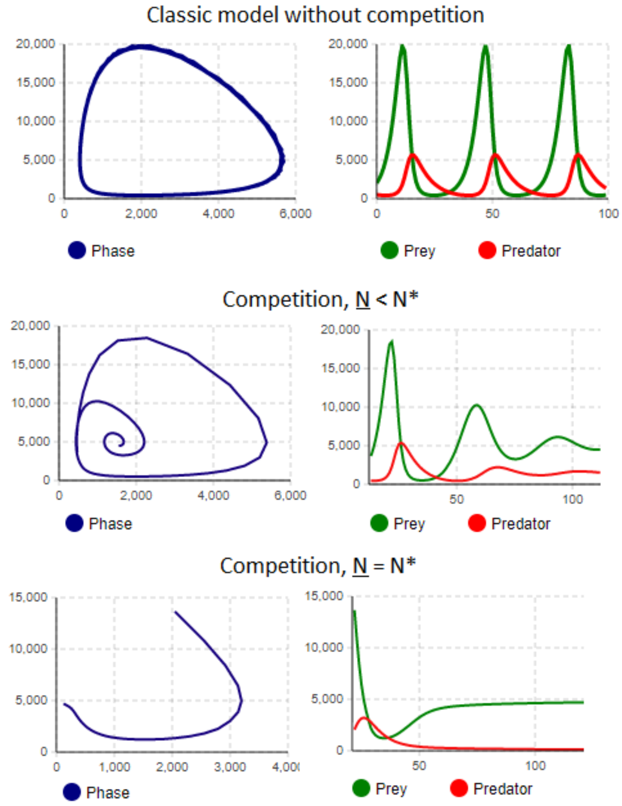

Figure 6: Preys' Competition, as a Soft Power.

If you look at the competition in the predator-prey model through the eyes of a reasonable predator, then, on the contrary, in order to maintain the status quo in the system, predators should avoid competition among preys in every possible way. For example, isolate them from each other and feed them (such as chickens or cows at a farm).

At the same time, in such a system, the average number of preys may be much greater than it could be in nature without reasonable predators. In this case, the question arises: if we consider the average population to be the criterion for its success, then is the presence of reasonable predators an evil, or, on the contrary, a good for a prey population in such a system?

3 Soft Power in a Competition Model

Now let us consider the next famous model of A. Lotka and V. Volterra: the "competition," [3],

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{N^*} - m\frac{M}{M^*}\right), \\ \frac{dM}{dt} = \beta M \left(1 - n\frac{N}{N^*} - \frac{M}{M^*}\right). \tag{4}$$

Here N and M – populations sizes of competitors, α, β are Malthusian factors, N^* and M^* are environmental capacities that is the maximal numbers of each type of competitor, for which the system resource is steel enough, n and m are double standard factors, they show how many times the competition with aliens differs from the one with compatriots.

Double standards are characterized by different applications of the principles, laws, rules, and estimates to the same actions of various subjects, depending on the degree of loyalty of these subjects to the estimator or other reasons of benefit to him. We will see that the double standards are effective control mechanisms in competitive systems [7].

It is interesting to consider system (4) not in the biological domain, as in [3], but in the social one, [8], where system (4) becomes the constraints of the differential game – the capability of social systems to change behavior in a short time in response to the current situation – turns the dynamic system (4) into a positional differential game, where the double standard factors n and m become the controls of players. That is why double standards are so popular in interstate relations.

We shall distinguish the following ranges of these double standard factors:

- Supertolerance, if $-\infty < n, m < 0$, $nm < 1$.
- Tolerance, if $0 \leq n, m < 1$.
- Equal treatment (no double standards), if n or m equals to 1.
- Intolerance, if $1 < n, m < \infty$.

It occurs, [7], that if the double standard factors are less than one (tolerance), the cultures are friendly – they can exist together. If the double standard coefficient of a culture is greater than one (intolerance), this culture constitutes a real danger to another, and may force it out of the system.

Now let us look at the situation, for example, from the position of the culture N representative. First, the value $\frac{N}{N^*}$ is well-known to him because it is a way of attitude to compatriots in the culture N – the manner of correct behavior that has been taught since childhood. Secondly, the value $m\frac{M}{M^*}$ is also known; it is the competitive pressure of the culture M, which the representatives of the culture N directly observe and feel because they are under this pressure. Most likely, these values are not equal $\frac{N}{N^*} \neq m\frac{M}{M^*}$, as the cultures are really different.

Further, it is quite natural to assume that if $\frac{N}{N^*} > m\frac{M}{M^*}$, then the culture M is pleasant to the representative of the culture N – usually it is pleasant to anybody when the pressure upon him weakens. Perhaps he assesses this situation approximately like: “Ah, what darlings, these well-mannered people of M – not as my rough compatriots!” On the contrary, if $\frac{N}{N^*} < m\frac{M}{M^*}$, then the representative of N does not like the M population; very few people like the pressure stronger than usual. Most likely, he will think, “Well, and how savage these M are! It is quite impossible to live near them! They are not able to behave at all!”

Actually, both first and second estimations can be deeply wrong. In the system (4), nothing depends upon the comparison between the values $\frac{N}{N^*}$ and $m\frac{M}{M^*}$, as well as the comparison between $\frac{M}{M^*}$ and $n\frac{N}{N^*}$. The behavior of the system (4) depends only upon the combination of ranges of double standard factors n and m, [7].

For example, if $\frac{N}{N^*} >> m\frac{M}{M^*}$, but at the same time $m > 1$, the situation can be dangerous for the culture N; it can disappear completely after a time because of the neighborhood with the “lovely and well-mannered” people, especially if it puts $n \leq 1$, having been under the illusion of M culture friendship due to the first inequality.

On the contrary, if $\frac{N}{N^*} < m\frac{M}{M^*}$ and even $\frac{N}{N^*} << m\frac{M}{M^*}$, but $m < 1$, there is no danger for the culture N to disappear near the culture M. The rival culture may be unpleasant since the competitive pressure is greater due to it, but it is by no means fatal since there is no danger of disappearing from such a neighborhood. Moreover, if $n > 1$, then the culture N forces out the alien culture trough time.

However, if the system (4) becomes a differential game, the double standard factors n and m are not observed directly. For the representative of the culture N to define m, it is necessary to compare given him in feelings $m\frac{M}{M^*}$ with $\frac{M}{M^*}$, but the last value, as a rule, is unknown to him: studying foreign cultures is a destiny of the rather narrow circle of specialists.

The only true measure of the culture is this culture itself, not any other one. The competitive pressure of a foreign culture is to be compared with its own internal competition, but by no means with the internal competition of the native culture.

4 Conclusion

The soft power (competition among preys) is able to solve the problem of preys in the "predator-prey" model, but the armed struggle of preys against the predators cannot. This brings to mind the works by L.N. Tolstoy and M.K. Gandhi about non-resistance to evil trough violence.

It is also interesting to look at the model through the eyes of a predator. It turns out that it is necessary to put the preys in cages like chickens or in stalls like cows at a farm and feed them to their fullest to exclude the competition. Does this remind you of something (like the chickencoop of cities with universal basic income)?

The elementary "competition" model teaches us that it is incorrect to measure one culture by the gauge of another; such a measurement is not valid because it is not informative. The only true yardstick for the culture is this culture itself, but by no means any other cultures.

In the authors' subjective view, this paradox illustrates why Russian cutting through a "window to Europe" was not too successful during the last 300 years. Because here $m > 1$, though, $\frac{N}{N^*} >> m\frac{M}{M^*}$, was fulfilled, which was attractive but not healthy for the N population in terms of our competition model. Slavs once lived in Europe, but little of them remained. At the same time, Russians survived under the Horde Yoke, and under the Ottoman Empire, Southern Slavs did ($m < 1$), though very unpleasant memories about these historical periods remained in the folklore of the survivors ($\frac{N}{N^*} << m\frac{M}{M^*}$).

These simplest models of A. Lotka and V. Volterra, which laid the foundation of mathematical biology at the beginning of the twentieth century, being transferred to the domain of social systems, cannot claim the accuracy of numerical prediction, of course, because of their primitiveness.

However, they draw the researcher's attention to the following qualitative questions about the social systems' dynamics:

1. Are the appeals of L.N. Tolstoy and M.K. Gandhi to the non-resistance as naive as they seem at first glance? The twentieth century was one of social "preys" armed struggle against the social "predators". Was the result good enough? Maybe improving sustainable development would be better?

2. Do our politicians, who face problems of cross-cultural interaction, understand well with whom to be friends and on whom to keep an eye with great wariness? The model of the soft power effect in a competition system shows that it is very easy to make a wrong choice even in a simple system of two differential equations with two variables!

It can be argued that social systems are much more complicated than simple two-dimensional differential equations. For instance, they are comprised of agents who have their own behaviors and goals. However, it turns out that the effects of the soft power and double standards described here also occur in the agent analogues of systems (1) and (4), implemented by cellular automata, [9].

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