Optimization of Encoding Design Based on the Spatial Geometry Remarkable Properties

VOLODYMYR RIZNYK
Lviv Polytechnic National University
Lviv, S. Bandery str. 12, 79013
UKRAINE

Abstract: This paper involves techniques for optimization of encoding design based on the remarkable geometric property of ring symmetry which contains two complementary asymmetries as the World harmony law for improving the quality indices of one- and multidimensional cyclic codes with respect to performance reliability, transmission speed, and transmission content, using vector data coding. These design techniques make it possible to configure encoding system with minimized number of digit weights, while maintaining or improving on error protection, security, and function of autocorrelation. Such sets are *t*-dimensional vectors, each of them together with all their modular sums enumerate the set node points grid of the coordinate system with the corresponding sizes and dimensionality. Systemic researches based on remarkable geometric properties of multi-modular mathematical structures such as "Glory to Ukraine Star" (GUS) combinatorial configurations demonstrated.

Keywords: Rotational symmetry, Harmony, Golomb ruler, GUS-combinatorial configuration, Vector data coding, Self-correcting code, Manifold coordinate system.

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1. Introduction

The major goals of the modern information technology are the expansion of big data process engineering design and use concept of optimize teaching approach to the practical tasks and assessment methods. Another goal of the systems is creation of unified information space with intelligent components of upper management levels such as large amount of data, high computing amount, and data flow intensity. In this aspect of very profitable is the development of intelligent components for the practical tasks and lectures studies, using novel interpretations of mathematical principles for transformation content and the other operating characteristics of the system. The torus accepted as the "perfect" shape that useful to describe objects as mathematical model of systems with spatially distributed structural elements of the system. Surface topology is superior to geometry relating such phenomenon because it deals with much more sophisticated and profound spatial and temporal relationships. The toroidal shape used in harmonic resolution analysis is similar to a doughnut but rather than having an empty central "hole", the topology of a torus folds in upon itself and all points along its surface converge together into a zero-dimensional point at the centre called the vertex [2]. A major branch of geometry is the study of geometrical structures on manifolds [3]. A manifold is a curved space of some dimension. Proposed concept involves techniques based on cyclic groups theory [4] and Ideal Ring Bundles algebra [5]. These aspects of the matter the issue are examined about multidimensional Ideal Ring Bundles [5], [6] and properties of "elegant" rotational symmetry and asymmetry relationships [7]-[9]. Next is given research into the mathematical principles relating to optimal placement of structural elements in spatially or temporally distributed systems [5], [6], [10]. The development of new directions in fundamental and applied research in systems engineering [5], [6], [10]-[13], coded design of signals [5], [6], [8]-[10], [14]-[15], advanced information technologies [10], [16]-[19]. The topological model of the coordinate systems regarded as both algebraic constructions, based on cyclic groups in extensions of Galois fields [4], and intelligent non-redundant combinatorial configurations, generated from "elegant" ensembles of rotational symmetry composed from complementary asymmetries [9]. These design techniques make it possible to configure information technology with vector data indexing and processing under basis of two- and multi-dimensional coordinate system, where basis is a sub-set of general number indexed vector data "category-attribute", which belong to mapping nodal coordinate points set of the system. The basis generates indexed vector data "category-attribute" set using modular summing for complete a reference grid of the coordinate system. Moreover, we require each indexed vector data "category-attribute" mutually uniquely corresponds to the point with the eponymous set of the coordinate system. In practice, the set points obtained using optimized basis of the system. This methodology working out harmonious mutual penetration of rotational symmetry and asymmetry as the remarkable property of real space for configure multi-attribute intelligent information management technologies under the coordinate system. Besides, a combination of binary code with vector weight bits of the database allowed, and the set of all values of indexed vector data sets one-to-one correspondents to nodal points set of the t-dimensional coordinate system. The underlying mathematical principle relates to the optimal placement of structural elements in spatially or temporally distributed systems, using novel designs based on t-dimensional combinatorial configurations, including the appropriate algebraic theory of cyclic groups, number theory, modular arithmetic, and geometric transformations. This information technology brought out relationship between the "elegant" ensembles of rotational symmetry and intelligent models of torus coordinate systems.

The role of the models becomes evident if teacher selects methodology obviously to state the physical essence of a studied problem. The aim of the article involves techniques for improving the quality indices of integrating control functions of technological, business processes, creating unified big vector data information space. The main problem of designing big vector data coding systems is development of an approach to configure two- and multidimensional optimum model of the systems. The multidimensional coding systems, is known to be of very important in information technology, for improving the quality indices of the systems with optimum compressed structure (e.g. two-dimensional torus coordinate system). The paper regards innovative techniques for development of vector data coding design based on the idea of "perfect" spatially or temporally distributed systems, using the appropriate combinatorial configurations as a basis of expanded information field for big data coding and processing.

2. Review of Literature

Geometric optimization as known can be performed as follows: optimize the geometry in internal coordinates, in redundant internal coordinates, and in Cartesian coordinates. Each step of the geometry optimization, Gaussian written to the output file the current structure of the system, the energy for this structure, the derivative of the energy with respect to the geometric variables, and a summary of the convergence criteria [20]. In recent times, a great number of new concepts, parallel algorithms, processing tools, platform, and applications are suggested and developed to improve the value of big vector data [21], [24]-[26]. The vector data-sets often involve a number of attributes, such as name, type, length, content, and other indexes, which have led to difficulties in large-scale data processing. In recent times, a great number of new concepts, parallel algorithms, processing tools, platform, and applications are suggested and developed to improve the value of big vector data. The geometric computing algorithms are always very complex and time-consuming [20], [21]. A framework that couples cloud and high-performance computing for the parallel map projection of vector-based big spatial data regarded in [24]. The projection provides large-scale spatial modeling of big vector data under a common coordinate system. High-dimensional datasets can be very difficult to visualize. While data in two or three dimensions can be plotted to show the inherent structure of the data, equivalent high-dimensional plots are much less intuitive. To aid geometric visualization and processing of a dataset, the processing must be optimized in some way with respect to the underlying criteria. In the paper [26] a theoretical foundation of the combinatorial 2D vector field topology set forth. A discrete Morse theory for general vector fields Forman describes in [27]. This theory applied successfully to scalar fields on triangulated manifolds [28]. Classification of digital n-manifolds based on the notion of complexity and homotopic equivalence presents in paper [29]. The another theoretical approach founded on structural perfection of toroidal and multidimensional manifolds, namely the concept of "Glory to Ukraine Star" combinatorial configurations (GUS-configurations) stated in [11], [30].

3. Perfect Golomb Rulers and Ideal Ring Protractors

In mathematics, a Golomb ruler is a set of marks at integer positions along a ruler such that no two pairs of marks are the same distance apart. The number of marks on the ruler is its order, and the largest distance between two of its marks is its length. There is no requirement that a Golomb ruler be able to measure all distances up to its length, but if it does, it called a perfect Golomb ruler. It has been proved that no perfect Golomb ruler exists for five or more marks [31].

A protractor is an instrument used for measuring angles in degrees. An ideal (or perfect) ring protractor is a set of marks at integer positions along a ring such that no two pairs of marks are the same angular distance apart. The number of marks on the ring is its order, and the largest angular distance between two of its marks is equal to $\alpha_{max} = 360^{o} - \alpha_{min}$. In mathematics, and system engineering an ideal ring protractor is known as "Ideal Ring Bundle" (IRB) [5]. IRBs are cyclic sequences of positive integers, which form perfect partitions of a finite interval [1, S] of integers. The sums of connected sub-sequences of an IRB enumerate the set of integers [1, S-1] exactly R- times. For example, the IRB{1,2,6,4} containing four (n=4) elements allows an enumeration of all numbers 1, 2, 3=1+2, 4, 5=4+1, 6, 7=4+1+2,...12=2+6+4 exactly once (R=1). While the IRB {1,1,2,3} of order n = 4, and S=7, enumerates each "circular sum" exactly twice (R=2): 1, 1; 2, 2=1+1; 3, 3=1+2; 4=3+1, 4=1+1+2; 5=2+3, 5=3+1+1; 6=1+2+3, 6=2+3+1. The concept of Ideal Ring Bundles can be used for finding optimal solutions for wide classes of technological problems in applications which need to partition sets with the smallest possible number of intersections, e.g. metering tanks [5]. Unlike perfect Golomb rulers [31] there are exist a priory endless number of IRBs of order n, and the more n the more number P of the n-sequence IRBs invariants. For example, there are only two (P=2) invariants IRBs of order n=4, R=1: {1,2,6,4}, and {1,3,2,7}. While, exists P=159630 invariants of IRBs of order n=984, R=1 [30, p.100].

Let us regard S-fold rotational symmetry as an ability to reproduce the maximum number of combinatorial varieties, using two-part asymmetrical division over a planar space relative to central point of the symmetry.

To extract meaningful information from the underlying data let us apply to rotational S-fold (S=7) symmetry penetrated by two complementary asymmetric sub-systems completions of the planar symmetric system (Fig.1).

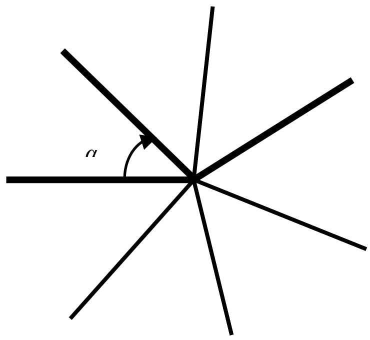

Fig.1 The 7-fold (S=7) rotational symmetry penetrated by two complementary asymmetric sub-systems completions of the planar symmetric system

To see this, we observe that the first sub-system (solid lines) of the complete model forms perfect two-dimensional spatial partitions of a fixed angular interval [α, 6α] by step α= 360°/7 exactly once (R_1=1), while the second (thinner lines) – exactly twice (R_2=2) by the same step.

Our reasoning proceeds from the fact, that the minimal and maximal angular distances relation initiated by S-fold rotational symmetry to be of prime importance for discovery of the S–fold “elegant” symmetry-asymmetry ensemble (Fig.2).

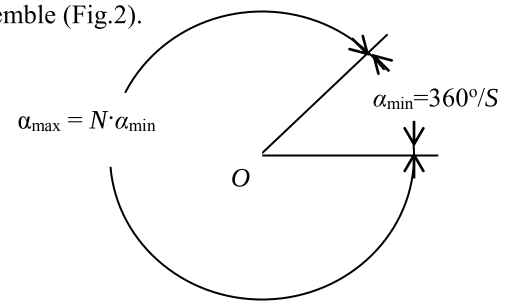

Fig. 2 A chart for discovery of the S –fold “elegant” symmetry- asymmetry ensemble

We require the set of all N angular distances [α_{min} , N - α_{min}] of S-fold harmoniously quantized space divided by a set of n straight lines diverged from a central point O non-uniformly allows an enumeration of all integers [1, S–1] exactly R-times. If these requires request, we call this phenomenon the “perfect” rotational S-fold symmetry. From Fig.2 follows integer relation between of variables S, n, and R [5, p.13]:

$$S = n\,(n\text{-}1)/R + 1 \tag{1}$$

As follows from equation (1), there are exist a priory infinite number of the “perfect” symmetry-asymmetry “elegant” ensembles, and this is a necessary, but not a sufficient condition.

4. Design of Optimal Cyclic Error-Correcting Codes

4.1 Cyclic Codes Design Based on the IRBs

From (1) follows relationships for synthesis of cyclic correcting code. To construct of the code it is necessary to pad “1” into n cells of S-stage array of cells numbered from 1 to S by number x_j in accordance with relation:

$$x_j - 1 \equiv \sum_{i=1}^{j} k_i \,(\mathrm{mod}\, S)\,,\ j=1,2,\ldots,n, \tag{2}$$

where k_i is the i-th element of an IRB with parameters S, n, and R, and the rest (S – n) cells are padded with zeroes. For example, if ring sequence is {1,1,2,3}, n=4, S=7, then k_1=1, x_1= 2; k_2=1, x_2=3; k_3=2, x_3=5; k_4=3, x_4=1, and we obtain the next binary code combination:1110100. This is a basic code word of the cyclic 7-code (S=7), and you can get the rest S-1=6 code words by cyclic shift of the basic one: 1110100, 0111010, …1101001. The code size is doubled if bit positions of code words to appear as opposite bits: 0001011, 1000101, …0010110.

Number of t_d errors to be detected, and corrected t_c are a function of minimal Hamming code distance d_{min} according to the next relationships [5, p.102]:

$$d_{min} = 2(n - R) \tag{3}$$

$$t_d \le d_{min} - 1, \quad t_c \le (t_d - 1)/2 \tag{4}$$

The minimal code distance d_2 for the doubled code size is [5, p.103]:

$$d_2 = S - 2(n\text{-}R) \tag{5}$$

of code words to appear as opposite bits: 0001011, 1000101, …0010110.

One of the indicators of the error-correcting codes is the ratio of Hemming code distance d_{min} to length S of the code combinations, which characterizes the efficiency of the corrective ability of the code: $K_e = d_{min}/S$.

Table 1 provides calculations of quality indices of optimal cyclic IRB error-correcting codes with information parameters n, R, S. Code size P for this coding system is P = 2S.

TABLE I. INDEXES OF THE OPTIMAL IRB CYCLIC ERROR-CORRECTING CODES

n	R	S	d_{min}	K_e	t_d	t_c	P
20	10	39	20	0,513	19	9	78
62	31	123	62	0,504	61	30	246
63	31	127	64	0,504	63	31	247

126	63	251	126	0,502	125	62	502
127	63	255	128	0,502	127	63	510
254	127	507	254	0,501	253	126	1014
255	127	511	256	0,501	255	127	1022
510	255	1019	510	0,5005	509	254	2038
511	255	1023	512	0,5005	511	255	2046
1022	511	2043	1022	0,5002	1021	510	4086
1023	511	2047	1024	0,5002	1023	511	4094

From Table 1, you can see that the optimum cyclic code with has a higher corrective efficiency of the code, which initially falls from 0.513 to 0.504 within the growth of the number of digits from 20 to 62, and approach to the value of 0.5 nonlinearly with any large increase in the number of digits of code sequences. At the same time, the maximum possible number comes of detected errors tends to 50%, and to 25 % corrected errors of the code word length, while code size *P* for this coding system is double the *S*.

4.2 Application of optimal IRB codes in data communications

To improve the reliability of communications and cryptographic protection and data transmission systems using code sequences, it is necessary to use the rule for optimizing IRB codes [30, p.129]:

The highest noise resistance acquired by IRB-code sequences, in which the number of different binary characters differs from each other by no more than one character.

Table 2 shows the characteristic of optimal IRB code sequences with a length of $7 \leq S \leq 39$ for signal recognition capacity in relation to signal/noise less than 1.

TABLE II. CHARACTERISTICS OF OPTIMAL IRB CODE SEQUENCES LENGHTS

Parameters of IRBs				Autocorrelation function			
n	R	S	t_c	+1	-1	Δ	$\lvert S/\Delta \rvert$ 100 %
4	2	7	1	3	4	-1	14,286
5	2	11	2	5	6	-1	9,0909
6	3	11	2	5	6	-1	9,0909
7	3	15	3	7	8	-1	6,6667
8	4	15	3	7	8	-1	6,6667
9	4	19	4	9	10	-1	5,2631
10	5	19	4	9	10	-1	5,2631
11	5	23	5	11	12	-1	4,3478
12	6	23	5	11	12	-1	4,3478
13	6	27	6	13	14	-1	3,7037
14	7	27	6	13	14	-1	3,7037
15	7	31	7	15	16	-1	3,2258
16	8	31	7	15	16	-1	3,2258
17	8	35	8	17	18	-1	2,8571
18	9	35	8	17	18	-1	2,8571
19	9	39	9	19	20	-1	2,5641
20	10	39	9	19	20	-1	2,5641

The autocorrelation function is calculated by a set of step-by-step offsets of this sequence based on the summation of all +1 and -1 elements, after a full cycle of step-by-step offsets. We see that optimized cyclic IRB-codes of length *S* provide correcting to (*S*-3)/4, i.e. (25%) errors, while detecting to 50% ones [30, p.124-125]. The results of calculations do not change from reversing the order or changing the signs of elements to the opposite in any of the variants of IRB sequences.

4.3 Gloria to Ukraine Stars Encoding Systems

Next, we consider a new type of optimized encoding systems based on the remarkable properties of two- or more dimensional vector Ideal Ring Bundles, which properties hold for the same *S*-ordered rotational symmetry in varieties permutations of the vectors in the IRBs, namely the ensembles of "Gloria to Ukraine Stars" (GUS)s combinatorial configurations [11]. For example, 43-fold (*S*=43) rotational symmetry creates a set of doubled IRBs two-dimensional (*t*=2) seven-stages (*n*=7) GUSs [30, p.68-69]. Here is one of them (Fig.3).

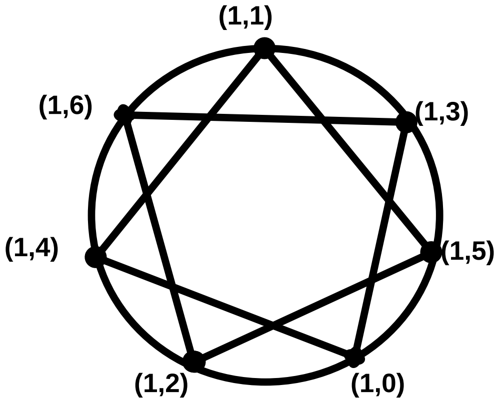

Fig. 3 A chart of two-dimensional optimized encoding system based on the 7–fold doubled IRBs

This we can see a set of two 2D seven-stages (*n*=7) IRBs: {(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)} (ring cycle), and {(1,1), (1,5), (1,2), (1,6), (1,3), (1,0), (1,4)} (star cycle).

In the Table 3 shows forming 7- digit 2D GUS – code created by ring cycle{(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)} under torus coordinate system 6×7 with two (*t*=2) orthogonal ring- axes and common reference point (0,0).

TABLE III. THE 7-DIGIT 2D GUS –CODE CREATED BY RING CYCLEUNDER TORUS COORDINATE SYSTEM 6 × 7

№	Vector	Digit weights of the GUS- code						
		(1,1)	(1,3)	(1,5)	(1,0)	(1,2)	(1,4)	(1,6)
1	(0,0)	1	1	1	0	1	1	1
2	(0,1)	1	1	1	1	1	1	0
3	(0,2)	1	1	0	1	1	1	1
4	(0,3)	1	1	1	1	1	0	1
5	(0,4)	1	0	1	1	1	1	1
…	…	…	…	…	…	…	…	…
41	(5,5)	1	1	1	0	0	1	1
42	(5,6)	1	0	0	1	1	1	1

We can see that 2D vector sequence {(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)} forms complete set of ring code combinations on 2D ignorable array 6×7, and each of them occurs exactly once (R=1).

Table 4 forms 7- digit 2D GUS - code created by star cycle {(1,1), (1,5), (1,2), (1,6), (1,3), (1,0), (1,4)} under torus coordinate system 6×7 with two (t=2) orthogonal ring- axes and common reference point (0,0).

TABLE IV. THE 7-DIGIT 2D GUS –CODE CREATED BY STAR CYCLE UNDER TORUS COORDINATE SYSTEM 6 × 7

№	Vector	Digit weights of the GUS- code						
		(1,1)	(1,5)	(1,2)	(1,6)	(1,3)	(1,0)	(1,4)
1	(0,0)	1	1	1	1	1	0	1
2	(0,1)	1	1	1	0	1	1	1
3	(0,2)	1	0	1	1	1	1	1
4	(0,3)	1	1	1	1	1	1	0
5	(0,4)	1	1	1	1	0	1	1
…	…	…	…	…	…	…	…	…
41	(5,5)	1	1	1	0	0	1	1
42	(5,6)	1	1	0	0	1	1	1

We can see that 2D vector sequence {(1,1), (1,5), (1,2), (1,6), (1,3), (1,0), (1,4)} forms complete set of ring code combinations on 2D ignorable array 6×7, and each of them occurs exactly once (R=1).

Hence, each of the regarded GUS encoding system provides minimizing basis as two-dimension binary vector code of fixed sizes with the same 2D digit weights but differ its cyclic ordering.

We have tabulated optimized monolithic code, which forms massive arranged (solid parts of bits) both symbols “1” and/or “0” for each code combinations as being cyclic. This property makes such vector ring codes useful in applications in high performance vector coding systems and information technology with improving of noise immunity, advanced vector data coded design of signals.

Next, we consider an example of non-redundant two-dimensional binary GUS code under torus coordinate system 3×5 with n=4, and R=1 (Table 5).

TABLE V. THE 4-DIGIT NON-REDUNDANT 2D GUS –CODE UNDER TORUS COORDINATE SYSTEM 3×5

№	The 4- digit 2D GUS- code under torus grid 3 × 5				
	Vector	Digit weights of the binary GUS -code			
		(1,2)	(2,4)	(1,3)	(2,1)
1	(0,0)	1	1	1	1
2	(0,1)	1	1	0	0
3	(0,2)	0	1	1	0
4	(0,3)	1	0	0	1
. . . .					
14	(2,3)	0	1	1	1
15	(2,4)	0	1	0	0

Table 5 illustrates forming 4- digit (n=4) 2D GUS-code {(1,2), (2,4), (1,3), (2,1)} under torus coordinate system 3×5. The code size is $P(n) = 2^n - 1 = 15$ with $n = 4$ and $R = 1$. Note, the basis of the torus coordinate grid created under the non-redundant 2D GUS {(1,2), (2,4), (1,3), (2,1)}.

Clearly, a t- dimensional toroid coordinate system designed for vector data coding t attributes and m_i categories of each of them (i= 1,2,…, t) requires t concurrent disjointed axes m_1, m_2,…,m_i, …,m_t with common reference point for forming t- dimensional coordinate grid of the system with sizes $m_1 \times m_2 \times \ldots \times m_t$. So, the underlying multidimensional information manifold coordinate system can be described by parameters S, n, R, t, m_i (i = 1, 2, …, t). Here t is dimension of vector data array, number of attributes, and number of significant digits of t - dimensional code, m_i is a number of categories of i- th attribute, and number of reference points on i- th ring axis in a toroidal coordinate system. Besides, information about vector data array depends of geometric sizes $m_1 \times m_2 \times \ldots \times m_t$, of the coordinate system.

5. Conclusion and Outlook

The remarkable geometric property of ring symmetry containing two complementary asymmetries is the World Harmony law, which provides improving the quality indices of one- and multidimensional cyclic codes with respect to performance reliability, transmission speed, and transmission content, using optimized vector data encoding design, based on the idea of “perfect” vector combinatorial constructions such as t-dimensional Ideal Ring Bundles (IRB)s and Gloria to Ukraine Stars (GUS)s. These design techniques make it possible to configure encoding system with minimized basis, while maintaining or improving on error protection, security, and function of autocorrelation. Two main classes of optimized binary vector codes regarded, including optimized cyclic error-correcting IRB-sequences, and “Gloria to Ukraine Stars” (GUS)s encoding ensembles. In this turn, GUS-codes involve self-correcting, so-called “monolithic” cyclic codes, and non-redundant t-dimensional vector data GUS-codes for encoding sets of two- or more attribute categories at the same time under *t*-dimensional manifold coordinate systems of appropriate spatial array. Moreover, we require each indexed vector data "category-attribute" mutually uniquely corresponds to the point with the eponymous set of the system. Study the properties allows a better understanding of the role of geometric structure in the behaviour of artificial and natural objects in different dimensionalities. Besides, a combination of binary code with vector weight discharges of the database is allowed, and the set of all values of indexed vector data sets are the same that a set of numerical values. These design techniques make it possible to configure big vector data coding systems with smaller number of code words than at present. The underlying skills are useful at high schools and universities for in-depth training of students, which study computer sciences and information technologies, involving contemporary combinatorial and algebraic theory for increasing interest to scientific researches.

«... harmony that the human mind is ediable to reveal in nature is the only objective reality, the only truth we can achieve; and what I will add is that the universal harmony of the world is the source of all beauty, it will be clear how we should appreciate those slow and difficult steps forward that little by little open it to us..."
Jules Henri Poincaré

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