Wavelength Allotment for All-to-All Broadcast in WDM Optical bidirectional ring with 3-length extension

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Abstract—All-to-all broadcast communication, distributing messages from each node to every other node, is a dense communication pattern and finds numerous applications in advanced computing and communication networks from the control plane to datacenters. A ring network topology is one of the important regular network topologies due to its simple structure, high speed, easy to extend and tolerant to link and node failures. Few researchers recommended connecting the alternate nodes of the ring with additional fibers in the ring network, to support increased call connection probability, higher tolerable to multiple link and node failures, enormous traffic handling capability and improved survivability. To reduce the complicatedness and cost of the network, it is essential to reduce the wavelength-number required to establish all-to-all broadcast in wavelength-division multiplexed ring network. In this paper, a ring network is extended by directly linking all nodes which are separated by two intermediate nodes with additional fibers and this network is referred as ring with 3-length extension. The wavelength allotment methods are proposed for realizing all-to-all broadcast over a WDM optical bi-directional ring with 3-length extension under multiple unicast routing model using a two-stage heuristic algorithm. The heuristic algorithm is developed to identify non-overlapping connections and an explicit wavelength allotment method based on the output of the heuristic technique is given. The result obtained shows that wavelength-number required atmost to establish all-to-all broadcast in a bi-directional ring with 3-length extension is reduced by a minimum of 57% and a maximum of 66% when compared to bi-directional primary ring. Similarly, the wavelength-number required atmost to establish all-to-all broadcast in a bi-directional ring with 3-length extension is reduced by a minimum of 20% and a maximum of 33% when compared to a bi-directional ring with 2-length extension.

Keywords—All-to-All Broadcast, Modified Ring, Wavelength Assignment, WDM Optical Network.

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1. Introduction

OPTICAL Wavelength Division Multiplexing (WDM) technologies have the potential to provide the tremendous bandwidth demand for emerging high performance computing applications. A WDM optical network employs numerous optical nodes and these nodes are interconnected using optical fibers in some fashion. WDM technology permits the passage for multiple wavelength optical signals through the same fiber and thus provides abundant bandwidth. Each optical node employs required optical sources (Ex: laser diodes) at the transmitter section to modulate the input electrical signals with light signal as carrier and required optical detectors (Ex: photo diodes) at the receiver section to demodulate the received signal and extract the input signal that was fed at the transmitter. Though the same fiber can be used for signal transmission in both forward and reverse directions, it is normally assumed that each optical link is a set of two fibers, with one fiber dedicated to forward transmission and another one for reverse transmission. An optical connection (lightpath) (m, n) corresponds to the establishment of an optical path for transfer of a packet from source m to destination n on a unique wavelength. In the absence of wavelength converters at the intermediate optical nodes, each lightpath needs to be on the same wavelength from source to destination.

All-to-all broadcast communication, distributing messages from each node to every other node, finds abundant applications from network control plane to datacenters [1-3]. In general, all-to-all broadcast is employed for numerous applications in advanced distributed computing and communication systems which employ WDM optical networks comprising hundreds of optical nodes at the backbone and involving huge number of operating wavelengths [4-18]. Wavelength need to be assigned for various lightpaths in such a way that no two lightpaths are established using the same wavelength, if they share any common link along entire route. Wavelengths being a scarce and costly resource, its usage need to be restricted to reduce the complexity and cost of the network. Optical WDM all-to-all broadcast communication was extensively analysed by many researchers but still it contains so many research challenges. All-to-all broadcast was studied for numerous topologies like ring, linear array, torus, mesh and tree under all optical routing models. Preceding research works [19-22] proposes interconnecting the alternate nodes of primary ring with additional link, and termed as modified-ring / extended-ring topology to support enormous bandwidth requirement, enlarged call connection probability and improved stability. The link and node failure analysis are studied for the modified/ extended ring networks topology [23-24]. Also, the wide-sense non-blocking multicast communication for modified/ extended ring is studied [25]. In this work, we examine all-to-all broadcast in a ring with 3-length extension network, as it provides lower wavelength usage and eye-catching for optical control plane.

Section 2 gives an overview of the basics required to understand the investigation done in this paper. Wavelength-number required atmost to establish all-to-all broadcast and its associated link load is obtained in section 3 using proposed heuristic algorithm. Finally, section 4 completes the paper highlighting future research avenues.

2. Preliminaries

If Fig.1 illustrate a 12-node (node 0 to node11) ring with 3-length extension. A primary ring network topology is extended by additionally linking two nodes which are separated by two intermediate nodes with additional fibers. This network is referred as ring with 3-length extension. In this network each node x is linked straight to node $(x \oplus 1)$in addition to node $(x \oplus 3)$where $\oplus$stand for addition with modulo$N$. It offers additional paths so as to aid decrease the effectual number of hops and also to decrease the wavelength-number required to establish all-to-all broadcast commendation.

The following definitions are necessary to prove the main results.

Definition 1: A shorter link is one that links the nodes x with $(x \oplus 1)$. A longer link is one that directly links the nodes $x$with $(x \oplus 3)$.

Definition 2: “A connection is the set of all links that joins source node and destination node following a prescribed routing method” [22].

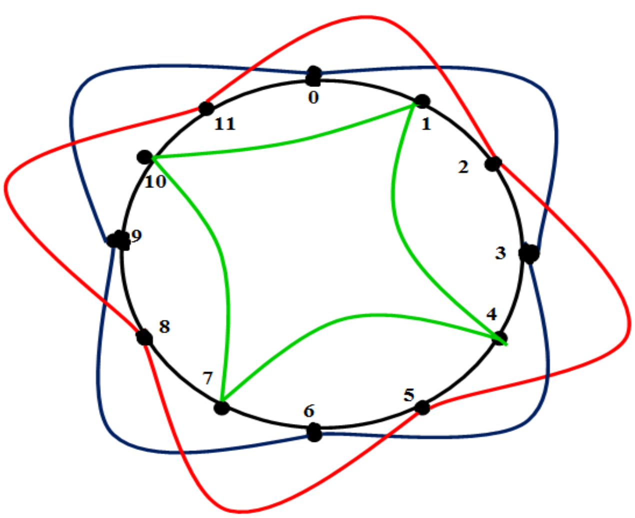

***Fig. 1.** A 12-node ring with 3-length extension*

Definition 3: **“**A connection that selects longest link over a shortest link at the source node and at various intermediate nodes to reach the destination node is said to follow ‘longest link first routing” [22]. For example in Fig. 1, using longest link first routing algorithm, a connection from node 2 to 6 selects first available longest link interconnecting the node 2 with node 5 and then the shortest link interconnecting node 5 with node 6.

Definition 4: **“**If the number of intermediate nodes between the source node and destination node in the primary ring is$l-1$, then the connection is called a length $l$connection. For example, in Fig. 1, if the source node is indexed 2 and the destination node is indexed 5, then the length of the connection is 3”[22].

Definition 5: “Two or more connections are said to be non-overlapping with each other, if they do not share any link along their path”[22].

Lemma 1: In longest link first routing, for $3 \leq l \leq \left\lfloor \frac{N}{2} \right\rfloor$, and $(l \bmod 3 = 0)$, three l length connections starting (source) from any 3 consecutive nodes do not interfere with each other.

Proof: Let a, $a \oplus 1, a \oplus 2$ be the index of the three consecutive nodes where $a \geq 0$. A l length connection starting from node index a, first use the longer links joining the nodes a and $a \oplus 3$, then nodes $a \oplus 3$ with $a \oplus 6$and so on. Similarly, l length connections starting from node index $a \oplus 1$, first use the longer links joining the nodes $a \oplus 1$ and $a \oplus 4$, then nodes $a \oplus 4$ with $a \oplus 7$ and so on. Also, l length connections starting from node index $a \oplus 2$, first use the longer links joining the nodes $a \oplus 2$and $a \oplus 5$, then nodes $a \oplus 5$ with $a \oplus 8$, and so on. Hence, these 3 sets of connections do not share any common link and hence they do not interfere with each other.

Lemma 2: Under longest link first routing, for $4 \leq l \leq \left\lfloor \frac{N}{2} \right\rfloor$, and $(l \bmod 3 = 1)$, three l length connections starting (source) from any 3 consecutive nodes do not interfere with each other.

Proof: Let $a, a \oplus 1, a \oplus 2$ be the index of the three consecutive nodes where $a \geq 0$. A l length connection starting from node index a, first use the longer links joining the nodes a and $a \oplus 3$, then nodes $a \oplus 3$ with $a \oplus 6$, and so on and finally end with one shorter link. Similarly, l length connections starting from node index $a \oplus 1$,first use the longer links joining the nodes $a \oplus 1$ and $a \oplus 4$, then nodes $a \oplus 4$ with $a \oplus 7$, and so on and finally end with one shorter link. Also, l length connections starting from node index $a \oplus 2$, first use the longer links joining the nodes $a \oplus 2$ and $a \oplus 5$, then nodes $a \oplus 5$ with $a \oplus 8$, and so on and finally end with one shorter link. As the longer links involved in the 3 sets of connections are completely different, the shorter link immediately following the last longer link in the 3 sets of connections will also be different (as the source node of shorter links are not same). Hence, these 3 sets of connections do not share any common link and hence they do not interfere with each other.

Lemma 3: Let N be a positive integer. Then, two wavelengths are sufficient to establish all connections of length $l = 2$ in aNnode ring with 3-length extension using longest link first routing.

Proof: Let $a, a \oplus 2$ be the index of the two nodes (where $a \geq 0$) which are separated by exactly one intermediate node indexed $a \oplus 1$. A length 2 connection starting from node indexa, involve two consecutive shorter links, first the link interconnecting the nodes a and $a \oplus 1$, then the link interconnecting the nodes $a \oplus 1$and $a \oplus 2$. Similarly, connections of length 2 originating from node index a $\oplus$ 2, involve two consecutive shorter links, first the link joining the nodes a $\oplus$ 2 and a $\oplus$ 3, then the link joining the nodes a $\oplus$ 3 and a $\oplus$ 4. Hence, these 2 sets of connections do not share any common link and hence they do not interfere with each other. Hence, two wavelengths are sufficient to route all connections of length 2.

Lemma 4: Under longest link first routing, for $5 \leq l \leq \left\lfloor\frac{N}{2}\right\rfloor$, and $(l \bmod 3 = 2)$, connections of same length l, and originating (source) from any 3 nodes which are separated by exactly one intermediate node do not interfere with each other.
Proof: Let $a, a \oplus 2, a \oplus 4$ be the indices of the three nodes (where $a \geq 0$) which are separated by one intermediate nodes. A connection of length l, originating from node index a,first use the longer links joining the nodes a and a $\oplus$ 3, then nodes a $\oplus$ 3 with a $\oplus$ 6, and so on and finally end with two consecutive shorter links. Similarly, connections of length l, originating from node index a $\oplus$ 2, first use the longer links joining the nodes a $\oplus$ 2 and a $\oplus$ 5, then nodes a $\oplus$ 5 with a $\oplus$ 8, and so on and finally end with two consecutive shorter links. Similarly, connections of length l, originating from node index a $\oplus$ 4, first use the longer links joining the nodes a $\oplus$ 4 and a $\oplus$ 7, then nodes a $\oplus$ 7 with a $\oplus$ 10, and so on and finally end with two consecutive shorter links. As the longer links involved in the 3 sets of connections are different, the two consecutive shorter links immediately following the longer links in the 3 sets of connections would also be different (as the indices of the source node of the first shorter link in the 3 set of connections differ exactly by 2). Hence, these 3 sets of connections do not share any common link and hence they do not interfere with each other.

Illustration 1: Wavelength allotment for all-to-all broadcast in a 12-node bi-directional ring with 3-length extension under shortest path/longest link first routing.

Consider the 12-node ring with 3-length extension shown in Fig. 1. The shortest connections in clockwise direction are considered and listed below, the reasons for this are provided in next section.

$(0,1),(0,2),(0,3),(0,4),(0,5),(0,6)$
$(1,2),(1,3),(1,4),(1,5),(1,6),(1,7)$
$(2,3),(2,4),(2,5),(2,6),(2,7),(2,8)$
$(3,4),(3,5),(3,6),(3,7),(3,8),(3,9)$
$(4,5),(4,6),(4,7),(4,8),(4,9),(4,10)$
$(5,6),(5,7),(5,8),(5,9),(5,10),(5,11)$
$(6,7),(6,8),(6,9),(6,10),(6,11),(6,0)$
$(7,8),(7,9),(7,10),(7,11),(7,0),(7,1)$
$(8,9),(8,10),(8,11),(8,0),(8,1),(8,2)$
$(9,10),(9,11),(9,0),(9,1),(9,2),(9,3)$
$(10,11),(10,0),(10,1),(10,2),(10,3),(10,4)$
$(11,0),(11,1),(11,2),(11,3),(11,4),(11,5)$

For $N = 12$, the connection length l for all-to-all broadcast communications are staring from length 1 to length 6. The various connections of all-to-all broadcast communication are grouped as given below:

Group 1: {6,4,1}, Group 2: {3}, Group 3:{2}, Group 4: {5}

The non-overlapping connections in each group can be combined after each group's values are stored in ascending order and a unique wavelength is allocated as shown below:

$$\left\{\begin{matrix}(0,1),(1,5),(5,11),(1,2),(2,6),(6,0),(2,3),(3,7),\\(7,1)\end{matrix}\right\} - \lambda_1$$
$$\left\{\begin{matrix}(3,4),(4,8),(8,2),(4,5),(5,9),(9,3),(5,6),(6,10),\\(10,4)\end{matrix}\right\} - \lambda_2$$
$$\left\{\begin{matrix}(6,7),(7,11),(11,5),(7,8),(8,0),(0,6),(8,9),(9,1),\\(1,7)\end{matrix}\right\} - \lambda_3$$
$$\left\{\begin{matrix}(9,10),(10,2),(2,8),(10,11),(11,3),(3,9),(11,0),\\(0,4),(4,10)\end{matrix}\right\} - \lambda_4$$
$$\left\{\begin{matrix}(0,3),(3,6),(6,9),(9,0),(1,4),(4,7),(7,10),(10,1),\\(2,5),(5,8),(8,11),(11,2)\end{matrix}\right\} - \lambda_5$$
$$\{(0,2),(2,4),(4,6),(6,8),(8,10),(10,0)\} - \lambda_6$$
$$\{(1,3),(3,5),(5,7),(7,9),(9,11),(11,1)\} - \lambda_7$$
$$\{(0,5),(2,7),(4,9),(6,11)\} - \lambda_8$$
$$\{(1,6),(3,8),(5,10),(7,12)\} - \lambda_9$$

Thus, nine wavelengths are required atmost to establish all-to-all broadcast communication of a 12-node ring with 3-length extension

3. Two stage Heuristic algorithm and wavelength allotments

A ring network topology with 3-length extension comprising of N nodes labeled from 0 to N-1 is taken as the source of this investigation. All-to-all broadcast communication is grouped into two clusters of connections with one cluster aggregating all possible connections established in clockwise direction while the other cluster aggregating all possible connections established in anticlockwise direction. All connections are assumed to be routed in the shortest path and by adopting longest link first routing technique. However, clockwise direction is preferred, if the path length of a connection is same in both clockwise and anticlockwise direction. Since each optical link is assumed to be a fiber pair, with each fiber taking care of connections in one particular direction, set of wavelengths employed for connections routed through clockwise direction may also be employed for connections routed through anticlockwise direction. Hence, connections established in clockwise direction are only taken for investigation in this paper.

a. Heuristic algorithm

The identification of non-overlapping connections is the key and is done using a heuristic technique. This technique is executed in two stages namely stage 1 and stage 2. The first stage of the algorithm is presented in Fig. 2. The total number of nodes in the ring N, is inputted to the stage 1 algorithm. Using the value of N, the algorithm generates the lengths of various connections of all-to-all broadcast. Obviously, the length of various connections exists from 1 to till $\left\lfloor\frac{N}{2}\right\rfloor$. Next, these length values are partitioned into two groups. One group comprises of length values, which output a reminder of 2, when it is divided by 3 (List A). Further, the elements namely 2 and 5 are then separated from this group, as wavelength allotment for connections corresponding to it are dealt separately. The other group comprises of length values, which output a reminder not equal to 2, when it is divided by 3 (List B). The stage 1 algorithm then groups the elements of List A and List B separately into multiple disjoint sets (no two sets contain a same element) in such a way that the sum of the elements of each set, excluding the last set, is equal to N. The reason for making the sum of elements of each set equal to N is for an efficient wavelength allotment. This ensures that non-overlapping connections of length equal to each of the value of the elements in the set (one connection for every length equal to the value of the elements in the set), when routed sequentially one after another on a unique wavelength (the destination of one connection being the source for another connection), covers a maximum number of available longest links over a single round along the ring. This strategy ensures that a particular wavelength is used on maximum number of longest links for a single round over a ring and may pertain to optimal wavelength allotment. However, as the sum of the elements in the last set may not be equal to N, adopting the above logic may not pertain to optimal wavelength allotment. If adopted as such, the allotted wavelength may not be present in an entire round over the ring, which paves the way for more wavelength requirement. The pseudocode for stage 1 algorithm is displayed in Fig.3. Further, wavelength allotment for connection lengths equal to the element values of the last set is deliberated in the heuristic algorithm stage 2.

The second stage of the heuristic algorithm is displayed in Fig.4. This algorithm first inputs all the elements of last set obtained through List A of stage 1 algorithm. This algorithm processes the inputs and outputs either the same input set or outputs the input elements in two or more disjoint set. The algorithm first generates a power set with the input elements. Power set is all possible subsets for the given set of input elements. Then the subset-sum, which is the sum of all the element in a subset, is calculated for all the generated subsets but excluding null set which is omitted as it of no relevance for the analysis. For every subset, N is made to divide the subset-sum and the quotient obtained is recorded. The fractional element of the quotient is zero when, N is integer multiples on subset-sum. This ensures that non-overlapping connections of length equal to each of the values of the elements in the subset (one connection for every length equal to the value of the elements in the subset and the destination of one connection being the source for another connection), when routed sequentially one after another and then repeated z times where z is the corresponding quotient value, on a unique wavelength covers a maximum number of available longest links over a single round along the ring. This strategy ensures that a particular wavelength is used on maximum number of longest links for a single round over a ring and may pertain to optimal wavelength allotment.

A value of non-zero in the fractional part of the quotient corresponds to the scenario where N is a not an integer multiple of subset-sum. In this scenario, non-overlapping connections of length equal to each of the values of the elements in the subset (one connection for every length equal to the value of the elements in the subset and the destination of one connection being the source for another connection), when routed sequentially one after another and then repeated z times where z is the corresponding quotient value, on a unique wavelength do not cover a maximum number of available longest links over a single round along the ring. The fractional part of a quotient which has bigger value results in a situation where the number of longest links getting assigned a unique wavelength (so as to result in one full round around the ring) is less. The fractional part of a quotient which has lower value results in a situation where the number of longest links getting assigned a unique wavelength (so as to result in one full round around the ring) is more. However, irrespective of whether the fractional part is lower or bigger, the connections routed on a unique wavelength, as said above, do not make one full round. Hence, among all the subsets generated through power set, the subset which produced lowest fractional part in its quotient value is outputted. In the case of more than one subset producing the same value of fractional part in its quotient, the subset comprising of more element count is outputted. In the scenario of having same values in both fractional part and element count, the subset which was processed first is outputted.

Once a subset is outputted based on the value of the fractional part of the quotient, then a decision has to be made regarding how to employ the elements of the subset towards wavelength allotment. One option is about employing the elements of the subset together for wavelength allotment while the other is passing each and every element of the selected subset separately for wavelength allotment. Remaining options are not considered here to reduce the complexity of this study. This choice to be made is made based on the wavelength-number requirement and the one which require less is selected. The logic for wavelength requirement calculation is, suppose the length of a connection is l_1, and then the maximum number of l_1 length connections that can be established without overlapping with each other on the N node ring with 3-length extension is $3\left\lfloor\frac{N}{l_1}\right\rfloor$. For establishing N connections of length l_1, the total wavelength required is $\left\lceil\frac{N}{3\left\lfloor\frac{N}{l_1}\right\rfloor}\right\rceil$. Let $l_1, l_2, l_3, \dots, l_z$ be the elements of a subset and if they are assumed to be independently employed for wavelength allotment, then the total wavelength-number required is $\left(\left\lceil\frac{N}{3\left\lfloor\frac{N}{l_1}\right\rfloor}\right\rceil + \left\lceil\frac{N}{3\left\lfloor\frac{N}{l_2}\right\rfloor}\right\rceil + \cdots + \left\lceil\frac{N}{3\left\lfloor\frac{N}{l_Z}\right\rfloor}\right\rceil\right)$. On the contrary, if the elements of a subset are assumed to be employed together for wavelength allotment, then the wavelength-number required are $\left\lceil\frac{N}{3\left\lfloor\frac{N}{\sum l_J \, for \; 0\leq J\leq Z}\right\rfloor}\right\rceil$.

Based on the wavelength-number either the elements of the input are outputted as separate sets or together as one output set same as input. This set of elements is removed from the original input list of stage 2 algorithm. Again stage 2 algorithm is executed with the remaining inputs, if the input list A is not empty. Similarly, the same procedure is repeated again for the last group elements obtained from list B at the end of execution of stage 1 algorithm. The pseudocode of stage 2 algorithm is displayed in Fig. 5. The sets outputted from stage 1 and stage 2 of the heuristic algorithm is used to complete wavelength allocation by mapping with appropriate wavelength allotment methods described in the succeeding section. It is required that the elements of each set need to be arranged in non-decreasing order. The following additional points need to be noted here for wavelength allotment: For ($l \bmod 3 \neq 2$) all connections of length l, and originating (source) from any 3 consecutive nodes do not interfere with each other (Lemma 1). Also, for ($l \bmod 3 = 2$), all connections of length l, from nodes $x, x \oplus 2$, and $x \oplus 4$(if such nodes exist) do not interfere with each other (Lemma 4).

b. Wavelength allotment methods

Note: Let $a_1 = l_1, a_2 = l_1 + l_2, a_3 = l_1 + l_2 + l_3, \ldots, a_d = l_1 + l_2 + l_3 + \cdots + l_d$

Category 1: Let $l_1, l_2, l_3, \ldots, l_d$ be the elements of a list such that $a_d = l_1 + l_2 + l_3 + \cdots + l_d = N$

Case i) $N = 3m$

For $x = 0,1,2, \ldots, N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$

Then for $x = 0,1,2, \ldots, \frac{N-3}{3}$

let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left(\frac{N-3}{3} + 1\right) = \frac{N}{3}$

Case ii) $N = 3m + 1$

For $x = 0,1,2, \ldots, N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$

Then for $x = 0,1,2, \ldots, \frac{N-4}{3}$let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$ and $S\left(\frac{N-1}{3}\right) = G(N-1)$.

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left(\frac{N-4}{3} + 1 + 1\right) = \frac{N+2}{3}$

Case iii) $N = 3m + 2$

For $x = 0,1,2, \ldots, N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$

Then for $x = 0,1,2, \ldots, \frac{N-5}{3}$let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$and $S\left(\frac{N-2}{3}\right) = G(N-2) \cup G(N-1)$.

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left(\frac{N-5}{3} + 1 + 1\right) = \frac{N+1}{3}$

Category 2: Let l_1 be the solo element of a list such that $l_1 z = N$ where z is a positive integer.

Case i) $\frac{N}{z} = 3m$

For $x = 0,1,2, \ldots, \frac{N}{z} - 1$, let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus 2a_1), \ldots, (x \oplus (z-1)a_1, x \oplus za_1)\}$

Then for $x = 0,1,2, \ldots, \left\lceil\frac{N}{3z}\right\rceil - 1$, let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left(\left\lceil\frac{N}{3z}\right\rceil - 1 \ + 1\right) = \left\lceil\frac{N}{3z}\right\rceil$

Case ii) $\frac{N}{z} = 3m + 1$

For $x = 0,1,2, \ldots, \frac{N}{z} - 1$ let

$G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus 2a_1), \ldots, (x \oplus (z-1)a_1, x \oplus za_1)\}$

Then for $x = 0,1,2, \ldots, \left\lceil\frac{N}{3z}\right\rceil - 2$,

let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$ also $S\left(\left\lceil\frac{N}{3z}\right\rceil - 1\right) = G\left(\frac{N}{z} - 1\right)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left\lceil\frac{N}{3z}\right\rceil - 2 \ + 1 + 1 = \left\lceil\frac{N}{3z}\right\rceil$

Case iii) $\frac{N}{z} = 3m + 2$

For $x = 0,1,2, \ldots, \frac{N}{z} - 1$ let

$G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus 2a_1), \ldots, (x \oplus (z-1)a_1, x \oplus za_1)\}$

Then for $x = 0,1,2, \ldots, \left\lceil\frac{N}{3z}\right\rceil - 2$ let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$ and $S\left(\left\lceil\frac{N}{3z}\right\rceil - 1\right) = G\left(\frac{N}{z} - 2\right) \cup G\left(\frac{N}{z} - 1\right)$.

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$\left(\left\lceil\frac{N}{3z}\right\rceil - 2 \ + 1 + 1\right) = \left\lceil\frac{N}{3z}\right\rceil$

Category 3: Let $l_1, l_2, l_3, \ldots, l_d$ be the elements of an array such that $(l_1 + l_2 + l_3 + \cdots + l_d)z = N$ where z is a positive integer.

Case i)$\frac{N}{z} = 3m$

For $x = 0,1,2, \ldots, \frac{N}{z} - 1$ let

$$G(x) = \left\{\begin{array}{c} (x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d) \\ (x \oplus a_d, x \oplus a_d \oplus a_1), (x \oplus a_d \oplus a_1, x \oplus a_d \oplus a_2), \ldots, \\ (x \oplus a_d \oplus a_{d-1}, x \oplus 2a_d), \ldots, \\ (x \oplus (z-1)a_d, x \oplus (z-1)a_d \oplus a_1), \\ (x \oplus (z-1)a_d \oplus a_1, x \oplus (z-1)a_d \oplus a_2), \ldots, \\ (x \oplus (z-1)a_d \oplus a_{d-1}, x \oplus za_d) \end{array}\right\}$$

Then for $x = 0,1,2, \ldots, \left\lceil\frac{N}{3z}\right\rceil$ -1

let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $\left\lceil\frac{N}{3z}\right\rceil - 1 + 1 = \left\lceil\frac{N}{3z}\right\rceil$

Case ii) $\frac{N}{z} = 3m + 1$
For $x = 0,1,2,\ldots,\frac{N-1}{z}$ let

$$G(x) = \left\{\begin{array}{c} (x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d) \\ (x \oplus a_d, x \oplus a_d \oplus a_1), (x \oplus a_d \oplus a_1, x \oplus a_d \oplus a_2), \ldots, \\ (x \oplus a_d \oplus a_{d-1}, x \oplus 2a_d), \ldots, \\ (x \oplus (z-1)a_d, x \oplus (z-1)a_d \oplus a_1), \\ (x \oplus (z-1)a_d \oplus a_1, x \oplus (z-1)a_d \oplus a_2), \ldots, \\ (x \oplus (z-1)a_d \oplus a_{d-1}, x \oplus z a_d) \end{array}\right\}$$

Then for $x = 0,1,2,\ldots,\left\lceil\frac{N}{3z}\right\rceil - 2$
let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$ and $S\left(\left\lceil\frac{N}{3z}\right\rceil - 1\right) = G(N-1)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $\left\lceil\frac{N}{3z}\right\rceil - 2 + 1 + 1 = \left\lceil\frac{N}{3z}\right\rceil$

Case iii) $\frac{N}{z} = 3m + 2$
For $x = 0,1,2,\ldots,\frac{N}{z}$, let

$$G(x) = \left\{\begin{array}{c} (x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d) \\ (x \oplus a_d, x \oplus a_d \oplus a_1), (x \oplus a_d \oplus a_1, x \oplus a_d \oplus a_2), \ldots, \\ (x \oplus a_d \oplus a_{d-1}, x \oplus 2a_d), \ldots, \\ (x \oplus (z-1)a_d, x \oplus (z-1)a_d \oplus a_1), \\ (x \oplus (z-1)a_d \oplus a_1, x \oplus (z-1)a_d \oplus a_2), \ldots, \\ (x \oplus (z-1)a_d \oplus a_{d-1}, x \oplus z a_d) \end{array}\right\}$$

Then for $x = 0,1,2,\ldots,\left\lceil\frac{N}{3z}\right\rceil - 2$
let $S(x) = G(3x) \cup G(3x+1) \cup G(3x+2)$ and $S\left(\left\lceil\frac{N}{3z}\right\rceil - 1\right) = G(N-2) \cup G(N-1)$. As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $\left\lceil\frac{N}{3z}\right\rceil - 2 + 1 + 1 = \left\lceil\frac{N}{3z}\right\rceil$

Category 4: Let $l_1, l_2, l_3, \ldots, l_d$ be the elements of a list such that $\frac{N}{2} < (l_1 + l_2 + l_3 + \cdots + l_d) < N$
Wavelength allotment is identical to Category 1.

Category 5: Let l_1 be the solo element of a list such that $l_1 z = N - \epsilon$ where z, ϵ are positive integers and $0 < \epsilon < l_1$
Let $G(0) = \{(0, a_1), (a_1, 2a_1), \ldots, ((\epsilon - 1)a_1, \epsilon a_1)\}$
The remaining connections are grouped as each arrangement no two connections are non-overlapping as per the following procedure. All the connections available in the series $(1, l_1 \oplus 1)$, $(2, l_1 \oplus 2)$ $(3, l_1 \oplus 3), \ldots,$ $(N-1, l_1 \oplus N-1)$ except those incorporated in $G(0)$ are taken one by one as a matrix of l_1 rows first in column wise and then row wise. Here, all connections available in a same row do not overlap with each other. . Hence, an exclusive wavelength required to be assigned for them. So, $l_1 + 1$ wavelength-numbers are required to route all such connections.

Category 6: Let $l_1, l_2, l_3, \ldots, l_d$ be the elements of an array such that
$(l_1 + l_2 + l_3 + \cdots + l_d)z = N - \epsilon$ where z, ϵ are positive integers and
$0 < \epsilon < (l_1 + l_2 + l_3 + \cdots + l_d)$
For $x = 0,1,2,\ldots,N-1$, let $H(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$
Let $S(0) = \{H(0) \cup H(a_d) \cup H(2a_d) \cup \ldots \cup H((\epsilon - 1)a_d)$
The remaining connections are grouped as each arrangement no two connections are noverlapping as per the following procedure. All the groups in the sequence $H(1), H(2), H(3), \ldots, H(N-1)$ except those included in $S(0)$ are written one by one as a matrix of a_d rows first in column wise and then row wise. Here, all the connections present in a same row do not overlap with each other and they can be on the same wavelength. So, $a_d + 1$ wavelengths are required to route all connections of length $l_1, l_2, l_3, \ldots, l_d$.

Category 7: Let $l_1, l_2, l_3, \ldots, l_d$ be the elements of a list such that $a_d = l_1 + l_2 + l_3 + \cdots + l_d = N$
Case i) $N = 6m$
For $x = 0,1,2,\ldots,N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$
Then for $x = 0,1,2,\ldots,\frac{N-6}{6}$
$S_1(x) = G(6x) \cup G(6x+2) \cup G(6x+4)$
$S_2(x) = G(6x+1) \cup G(6x+3) \cup G(6x+5)$
As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-6}{6} + 1\right) = \frac{N}{3}$

Case ii) $N = 6m + 1$
For $x = 0,1,2,\ldots,N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$
Then for $x = 0,1,2,\ldots,\frac{N-7}{6}$
$S_1(x) = G(6x) \cup G(6x+2) \cup G(6x+4)$
$S_2(x) = G(6x+1) \cup G(6x+3) \cup G(6x+5)$
Also, $S_1\left(\frac{N-1}{6}\right) = G(N-1)$
As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-7}{6} + 1\right) + 1 = \frac{N+2}{3}$

Case iii) $N = 6m + 2$
For $x = 0,1,2,\ldots,N-1$ let $G(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \ldots, (x \oplus a_{d-1}, x \oplus a_d)\}$
Then for $x = 0,1,2,\ldots,\frac{N-8}{6}$
$S_1(x) = G(6x) \cup G(6x+2) \cup G(6x+4)$
$S_2(x) = G(6x+1) \cup G(6x+3) \cup G(6x+5)$
Also, $S_1\left(\frac{N-2}{6}\right) = G(N-1)$ and $S_2\left(\frac{N-2}{6}\right) = G(N-2)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-8}{6}+1\right)+2=\frac{N+4}{3}$

Case iv) $N=6m+3$
For $x=0,1,2,\dots,N-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\dots,(x\oplus a_{d-1},x\oplus a_d)\}$
Then for $x=0,1,2,\dots,\frac{N-9}{6}$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
Also,$S_1\left(\frac{N-3}{6}\right)=G(N-3)\cup G(N-1)$and $S_2\left(\frac{N-3}{6}\right)=G(N-2)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-9}{6}+1\right)+2=\frac{N+3}{3}$

Case v) $N=6m+4$
For $x=0,1,2,\dots,N-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\dots,(x\oplus a_{d-1},x\oplus a_d)\}$
Then for $x=0,1,2,\dots,\frac{N-10}{6}$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
Also, $S_1\left(\frac{N-4}{6}\right)=G(N-3)\cup G(N-1)$and $S_2\left(\frac{N-4}{6}\right)=G(N-4)\cup G(N-2)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-10}{6}+1\right)+2=\frac{N+2}{3}$

Case vi) $N=6m+5$
For $x=0,1,2,\dots,N-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\dots,(x\oplus a_{d-1},x\oplus a_d)\}$
Then for $x=0,1,2,\dots,\frac{N-5}{6}$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
Also,$S_1\left(\frac{N-4}{6}\right)=\cup\ G(N-5)G(N-3)\cup G(N-1)$and
$S_2\left(\frac{N-4}{6}\right)=G(N-4)\cup G(N-2)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2\left(\frac{N-5}{6}+1\right)=\frac{N+1}{3}$

Category 8: Let $l_1$be the single element of a list such that $l_1 z=N$ where z is a positive integer.
Case i) $\frac{N}{z}=6m$
For $x=0,1,2,\dots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\dots,(x\oplus (z-1)a_1,x\oplus za_1)\}$
Then for $x=0,1,2,\dots,\left\lceil\frac{N}{6z}\right\rceil-1$ let
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-1\ +1\right)=2\left\lceil\frac{N}{6z}\right\rceil$

Case ii) $\frac{N}{z}=6m+1$
For $x=0,1,2,\dots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\dots,(x\oplus (z-1)a_1,x\oplus za_1)\}$
Then for $x=0,1,2,\dots,\left\lceil\frac{N}{6z}\right\rceil-2$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$Also
$S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2\ \ +1\right)+1=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)-1$

Case iii) $\frac{N}{z}=6m+2$
For $x=0,1,2,\dots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\dots,(x\oplus (z-1)a_1,x\oplus za_1)\}$
Then for $x=0,1,2,\dots,\left\lceil\frac{N}{6z}\right\rceil-2$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
Also, $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)$, $S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2\ \ +1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Case iv) $\frac{N}{z}=6m+3$
For $x=0,1,2,\dots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\dots,(x\oplus (z-1)a_1,x\oplus za_1)\}$
Then for $x=0,1,2,\dots,\left\lceil\frac{N}{6z}\right\rceil-2$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$
Also, $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)\cup G\left(\frac{N}{z}-3\right)$, $S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)$
As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$2*\left(\left\lceil\frac{N}{6z}\right\rceil-2\ \ +1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Case v) $\frac{N}{z}=6m+4$
For $x=0,1,2,\dots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\dots,(x\oplus (z-1)a_1,x\oplus za_1)\}$
Then for $x=0,1,2,\dots,\left\lceil\frac{N}{6z}\right\rceil-2$
$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$
$S_2(x)=G(6x+1)\cup\ G(6x+3)\cup G(6x+5)$

Also, $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)\cup G\left(\frac{N}{z}-3\right)$, $S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)\cup G\left(\frac{N}{z}-4\right)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2+1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Case vi) $\frac{N}{z}=6m+5$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let $G(x)=\{(x,x\oplus a_1),(x\oplus a_1,x\oplus 2a_1),\ldots,(x\oplus(z-1)a_1,x\oplus za_1)\}$

Then for $x=0,1,2,\ldots,\left\lceil\frac{N}{6z}\right\rceil-2$

$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$

$S_2(x)=G(6x+1)\cup G(6x+3)\cup G(6x+5)$

Also $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)\cup G\left(\frac{N}{z}-3\right)\cup G\left(\frac{N}{z}-5\right)$,

$S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)\cup G\left(\frac{N}{z}-4\right)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2+1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Category 9: Let $l_1,l_2,l_3,\ldots,l_d$ be the elements of an array such that $(l_1+l_2+l_3+\cdots+l_d)z=N$ where z is a positive integer.

Case i) $\frac{N}{z}=6m$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let

$$G(x)=\left\{\begin{array}{c}(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\ldots,(x\oplus a_{d-1},x\oplus a_d)\\(x\oplus a_d,x\oplus a_d\oplus a_1),(x\oplus a_d\oplus a_1,x\oplus a_d\oplus a_2),\ldots,\\(x\oplus a_d\oplus a_{d-1},x\oplus 2a_d),\ldots,\\(x\oplus(z-1)a_d,x\oplus(z-1)a_d\oplus a_1),\\(x\oplus(z-1)a_d\oplus a_1,x\oplus(z-1)a_d\oplus a_2),\ldots,\\(x\oplus(z-1)a_d\oplus a_{d-1},x\oplus za_d)\end{array}\right\}$$

Then for $x=0,1,2,\ldots,\left\lceil\frac{N}{6z}\right\rceil-1$ let

$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$

$S_2(x)=G(6x+1)\cup G(6x+3)\cup G(6x+5)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-1+1\right)=2\left\lceil\frac{N}{6z}\right\rceil$

Case ii) $\frac{N}{z}=6m+1$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let

$$G(x)=\left\{\begin{array}{c}(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\ldots,(x\oplus a_{d-1},x\oplus a_d)\\(x\oplus a_d,x\oplus a_d\oplus a_1),(x\oplus a_d\oplus a_1,x\oplus a_d\oplus a_2),\ldots,\\(x\oplus a_d\oplus a_{d-1},x\oplus 2a_d),\ldots,\\(x\oplus(z-1)a_d,x\oplus(z-1)a_d\oplus a_1),\\(x\oplus(z-1)a_d\oplus a_1,x\oplus(z-1)a_d\oplus a_2),\ldots,\\(x\oplus(z-1)a_d\oplus a_{d-1},x\oplus za_d)\end{array}\right\}$$

Then for $x=0,1,2,\ldots,\left\lceil\frac{N}{6z}\right\rceil-2$

$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$

$S_2(x)=G(6x+1)\cup G(6x+3)\cup G(6x+5)$

Also $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2+1\right)+1=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)-1$

Case iii) $\frac{N}{z}=6m+2$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let

$$G(x)=\left\{\begin{array}{c}(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\ldots,(x\oplus a_{d-1},x\oplus a_d)\\(x\oplus a_d,x\oplus a_d\oplus a_1),(x\oplus a_d\oplus a_1,x\oplus a_d\oplus a_2),\ldots,\\(x\oplus a_d\oplus a_{d-1},x\oplus 2a_d),\ldots,\\(x\oplus(z-1)a_d,x\oplus(z-1)a_d\oplus a_1),\\(x\oplus(z-1)a_d\oplus a_1,x\oplus(z-1)a_d\oplus a_2),\ldots,\\(x\oplus(z-1)a_d\oplus a_{d-1},x\oplus za_d)\end{array}\right\}$$

Then for $x=0,1,2,\ldots,\left\lceil\frac{N}{6z}\right\rceil-2$

$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$

$S_2(x)=G(6x+1)\cup G(6x+3)\cup G(6x+5)$

Also $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)$, $S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2+1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Case iv) $\frac{N}{z}=6m+3$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let

$$G(x)=\left\{\begin{array}{c}(x,x\oplus a_1),(x\oplus a_1,x\oplus a_2),\ldots,(x\oplus a_{d-1},x\oplus a_d)\\(x\oplus a_d,x\oplus a_d\oplus a_1),(x\oplus a_d\oplus a_1,x\oplus a_d\oplus a_2),\ldots,\\(x\oplus a_d\oplus a_{d-1},x\oplus 2a_d),\ldots,\\(x\oplus(z-1)a_d,x\oplus(z-1)a_d\oplus a_1),\\(x\oplus(z-1)a_d\oplus a_1,x\oplus(z-1)a_d\oplus a_2),\ldots,\\(x\oplus(z-1)a_d\oplus a_{d-1},x\oplus za_d)\end{array}\right\}$$

Then for $x=0,1,2,\ldots,\left\lceil\frac{N}{6z}\right\rceil-2$

$S_1(x)=G(6x)\cup G(6x+2)\cup G(6x+4)$

$S_2(x)=G(6x+1)\cup G(6x+3)\cup G(6x+5)$

Also $S_1\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-1\right)\cup G\left(\frac{N}{z}-3\right)$, $S_2\left(\left\lceil\frac{N}{6z}\right\rceil-1\right)=G\left(\frac{N}{z}-2\right)$

As the connections available in every $S(x)$ are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2*\left(\left\lceil\frac{N}{6z}\right\rceil-2+1\right)+2=\left(2*\left\lceil\frac{N}{6z}\right\rceil\right)$

Case v) $\frac{N}{z}=6m+4$

For $x=0,1,2,\ldots,\frac{N}{z}-1$ let

$G(x)$

$$= \left\{ \begin{matrix} (x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \dots, (x \oplus a_{d-1}, x \oplus a_d) \\ (x \oplus a_d, x \oplus a_d \oplus a_1), (x \oplus a_d \oplus a_1, x \oplus a_d \oplus a_2), \dots, \\ (x \oplus a_d \oplus a_{d-1}, x \oplus 2a_d), \dots, \\ (x \oplus (z-1)a_d, x \oplus (z-1)a_d \oplus a_1), \\ (x \oplus (z-1)a_d \oplus a_1, x \oplus (z-1)a_d \oplus a_2), \dots, \\ (x \oplus (z-1)a_d \oplus a_{d-1}, x \oplus za_d) \end{matrix} \right\}$$

Then for $x = 0,1,2, \dots, \left\lceil \frac{N}{6z} \right\rceil - 2$

$S_1(x) = G(6x) \cup G(6x+2) \cup G(6x+4)$

$S_2(x) = G(6x+1) \cup G(6x+3) \cup G(6x+5)$

Also, $S_1\left(\left\lceil \frac{N}{6z} \right\rceil - 1\right) = G\left(\frac{N}{z} - 1\right) \cup G\left(\frac{N}{z} - 3\right)$, $S_2\left(\left\lceil \frac{N}{6z} \right\rceil - 1\right) = G\left(\frac{N}{z} - 2\right) \cup G\left(\frac{N}{z} - 4\right)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is $2 * \left(\left\lceil \frac{N}{6z} \right\rceil - 2 + 1\right) + 2 = \left(2 * \left\lceil \frac{N}{6z} \right\rceil\right)$

Case vi) $\frac{N}{z} = 6m + 5$

For $x = 0,1,2, \dots, \frac{N}{z} - 1$ let

$G(x)$

$$= \left\{ \begin{matrix} (x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \dots, (x \oplus a_{d-1}, x \oplus a_d) \\ (x \oplus a_d, x \oplus a_d \oplus a_1), (x \oplus a_d \oplus a_1, x \oplus a_d \oplus a_2), \dots, \\ (x \oplus a_d \oplus a_{d-1}, x \oplus 2a_d), \dots, \\ (x \oplus (z-1)a_d, x \oplus (z-1)a_d \oplus a_1), \\ (x \oplus (z-1)a_d \oplus a_1, x \oplus (z-1)a_d \oplus a_2), \dots, \\ (x \oplus (z-1)a_d \oplus a_{d-1}, x \oplus za_d) \end{matrix} \right\}$$

Then for $x = 0,1,2, \dots, \left\lceil \frac{N}{6z} \right\rceil - 2$

$S_1(x) = G(6x) \cup G(6x+2) \cup G(6x+4)$

$S_2(x) = G(6x+1) \cup G(6x+3) \cup G(6x+5)$

Also,$S_1\left(\left\lceil \frac{N}{6z} \right\rceil - 1\right) = G\left(\frac{N}{z} - 1\right) \cup G\left(\frac{N}{z} - 3\right) \cup G\left(\frac{N}{z} - 5\right)$,

$S_2\left(\left\lceil \frac{N}{6z} \right\rceil - 1\right) = G\left(\frac{N}{z} - 2\right) \cup G\left(\frac{N}{z} - 4\right)$

As the connections available in every $S(x)$are non-overlapping with each other, an exclusive wavelength needs to be assigned for them. As a result, the wavelength-number required is$2 * \left(\left\lceil \frac{N}{6z} \right\rceil - 2 + 1\right) + 2 = \left(2 * \left\lceil \frac{N}{6z} \right\rceil\right)$

Category 10: Let $l_1, l_2, l_3, \dots, l_d$ be the elements of a list such that $\frac{N}{2} < (l_1 + l_2 + l_3 + \dots + l_d) < N$

Wavelength allotment is same as that given for Category 7.

Category 11: Let l_1 be the single element of a list such that $l_1 z = N - \epsilon$ where z, ϵ are positive integers and $0 < \epsilon < l_1$

Let $G(0) = \{(0, a_1), (a_1, 2a_1), \dots, ((\epsilon - 1)a_1, \epsilon a_1)\}$

The remaining connections are grouped as each group no two connections are non-overlapping as per the following procedure. All the connections in the series $(1, l_1 \oplus 1)$, $(2, l_1 \oplus 2)$ $(3, l_1 \oplus 3), \dots$, $(N-1, l_1 \oplus N - 1)$ excluding those available in $G(0)$are taken one by one as a matrix of l_1 rows first in column wise and then row wise. It may be prominent that all connections present in a same row do not overlap with each other. Hence, a unique wavelength required to be allotted for them. So, $l_1 + 1$ wavelengths are required to route all such connections.

Category 12: Let $l_1, l_2, l_3, \dots, l_d$ be the elements of an array such that

$(l_1 + l_2 + l_3 + \dots + l_d)z = N - \epsilon$ where z, ϵ are positive integers and

$0 < \epsilon < (l_1 + l_2 + l_3 + \dots + l_d)$

For $x = 0,1,2, \dots, N-1$, let $H(x) = \{(x, x \oplus a_1), (x \oplus a_1, x \oplus a_2), \dots, (x \oplus a_{d-1}, x \oplus a_d)\}$

Let $S(0) = \{H(0) \cup H(a_d) \cup H(2a_d) \cup \dots \cup H((\epsilon - 1)a_d)$

The remaining groups can be combined as each arrangement no two connections are noverlapping as per the following procedure.

All the groups in the series $H(1), H(2), H(3), \dots, H(N-1)$ except those included in $S(0)$are taken one by one as a matrix of a_d rows first in column wise and then row wise. The connections of all group present in a same row do not overlap with each other and they can be on the same wavelength. So, $a_d + 1$ wavelengths are required to route all connections of length $l_1, l_2, l_3, \dots, l_d$.

Illustration 2: Wavelength allotment for all-to-all broadcast in a 28 node WDM optical ring with 3-length extension:

For, $N = 28$. The output of stage 1 and stage 2 heuristic algorithms are given below:

$G_1 = \{13,12,3\}, G_2 = \{10,9,7,1\}, G_3 = \{4\}, G_4 = \{6\}$

$H_1 = \{14,11\}, H_2 = \{8\}$

The connections of the output groups are sorted in ascending order as shown below,before applying wavelength allotment,

$G_1 = \{1,12,13\}, G_2 = \{1,7,9,10\}, G_3 = \{4\}, G_4 = \{6\}$

$H_1 = \{11,14\}, H_2 = \{8\}$

The following output is obtained when G_1, G_2, G_3, G_4 are passed for wavelength allotment:

For Group G_1: (By Category 1)

$$S(0) = \left\{ \begin{matrix} (0,3), (3,15), (15,0), (1,4), (4,16), (16,1), \\ (2,5), (5,17), (17,2) \end{matrix} \right\} - \lambda 1$$

$$S(1) = \left\{ \begin{matrix} (3,6), (6,18), (18,3), (4,7), (7,19), (19,4), \\ (5,8), (8,20), (20,5) \end{matrix} \right\} - \lambda_2$$

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$$S(8) = \left\{ \begin{matrix} (24,27), (27,11), (11,24), (25,0), (0,12), \\ (12,25), (26,1), (1,13), (13,26) \end{matrix} \right\} - \lambda_9$$

$S(9) = \{(27,2), (2,14), (14,27)\} - \lambda_{10}$

For Group G_2: (By Category 4)

$$S(0) = \left\{ \begin{matrix} (0,1), (1,8), (8,17), (17,27), \\ (1,2), (2,9), (9,18), (18,0), \\ (2,3), (3,10), (10,19), (19,1) \end{matrix} \right\} - \lambda 11$$

$$S(1) = \left\{ \begin{matrix} (3,4), (4,11), (11,20), (20,2), \\ (4,5), (5,12), (12,21), (21,3), \\ (5,6), (6,13), (13,22), (22,4) \end{matrix} \right\} - \lambda 12$$

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$$S(8) = \begin{Bmatrix} (24,25), (25,4), (4,13), (13,23), \\ (25,26), (26,5), (5,14), (14,24), \\ (26,27), (27,6), (6,15), (15,25) \end{Bmatrix} -\lambda 19$$

$S(9) = \{(27,0), (0,7), (7,16), (16,26)\} -\lambda 20$

For Group G_3: (By Category 2)

$$S(0) = \begin{Bmatrix} (0,4), (4,8), (8,12), (12,16), (16,20), \\ (20,24), (24,0)(1,5), (5,9), (9,13), (13,17), \\ (17,21), (21,25), (25,1)(2,6), (6,10), \\ (10,14), (14,18), (18,22), (22,26), (26,2) \end{Bmatrix} - \lambda_{21}$$

$$S(1) = \begin{Bmatrix} (3,7), (7,11), (11,15), (15,19), (19,23), \\ (23,27), (27,3) \end{Bmatrix} - \lambda_{22}$$

For Group G_4: (By Category 5)

$$S(0) = \begin{Bmatrix} \{(0,6), (6,12), (12,18), (18,24), (24,2), \\ (1,7), (7,13), (13,19), (19,25), (25,3), \\ (2,8), (8,14), (14,20), (20,26), (26,4) \end{Bmatrix} - \lambda_{23}$$

$$S(1) = \begin{Bmatrix} (3,9), (9,15), (15,21), (21,27), (27,5), \\ (4,10), (10,16), (16,22), (22,0), \\ (5,11), (11,17)(17,23), (23,1) \end{Bmatrix} - \lambda_{24}$$

For Group H_1: (By Category 11)

$S(0) =$

$\{(0,11), (11,25), (2,13), (13,27), (4,15), (15,1)\}$ $-\lambda 25$

$\{(1,12), (12,26), (3,14), (14,0), (5,16), (16,2)\}$ $-\lambda 26$

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$S(4) = \{(24,7), (7,21), (26,9), (9,23)\}$$-\lambda 33$

$\{(25,8), (8,22), (27,10)(10,24)\}$ $-\lambda 34$

For Group H_2: (By Category 11)

$$S(0) = \begin{Bmatrix} (0,8), (8,16), (16,24), (24,4), (2,10), (10,18), \\ (18,26), (26,6), (4,12), (12,20), (20,0) \end{Bmatrix} -\lambda 35$$

$$\begin{Bmatrix} (1,9), (9,17), (17,25), (25,5), (3,11), (11,19), (19,27), \\ (27,7), (5,13), (13,21), (21,1) \end{Bmatrix} -\lambda 3$$
6

$S(1) = \{(6,14), (14,22), (22,2)\}$$-\lambda 37$

$\{(7,15), (15,23), (23,3)\}$ $-\lambda 38$

For length 2: (Lemma 3)

$$\begin{Bmatrix} (0,2), (2,4), (4,6), (6,8), (8,10), (10,12), (12,14), \\ (14,16), (16,18), (18,20), (20,22), (22,24), \\ (24,26), (26,0) \end{Bmatrix} - \lambda_{39}$$

$$\begin{Bmatrix} (1,3), (3,5), (5,7), (7,9), (9,11), (11,13), (13,15), \\ (15,17), (17,19), (19,21), (21,23), (23,25), (25,27) \end{Bmatrix} - \lambda_{40}$$

For length 5: (Lemma 4)

$$\begin{Bmatrix} (0,5), (2,7), (4,9), (6,11), (8,13), (10,15), (12,17), \\ (14,19), (16,21), (18,23), (20,25), (22,27) \end{Bmatrix} - \lambda_{41}$$

$$\begin{Bmatrix} (1,6), (3,8), (5,10), (7,12), (9,14), (11,16), (13,18), \\ (15,20), (17,22), (19,24), (21,26), (23,0) \end{Bmatrix} -\lambda_{42}$$

C. Link load

Let π indicate the network link load which is the maximum number of paths that share a common link. The link load of a bi-directional ring with 3-length extension is derived as shown below:

Case i) $N = 6m$, where m denotes a positive number.

We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N}{6}$ nodes available at a length $3l$ previous to node $(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\left(\frac{N}{2} - 3l + 1\right)$instant nodes after the node x. Therefore, connections that utilize this longest link = $\sum_{l=1}^{\frac{N}{6}} \left(\frac{N}{2} - 3l + 1\right) \, which \, is \left(\frac{N^2-2N}{24}\right)$

We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N}{6} - 1$, nodes available at a length $3l$ previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$. Similarly nodes available at a length$(3l - 1)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$. Therefore, connections that utilize this shortest link $= 2 * \left(\frac{N}{6} - 1 + 1\right) + \left(\frac{N}{6} - 1 + 1\right) = \frac{N}{2}$

. The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-2N}{24}$.

Case ii) $N = 6m + 1$, where m denotes a positive number.

We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N-1}{6}$ nodes available at a length $3l$ previous to node $(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\left(\frac{N+1}{2} - 3l\right)$instant nodes after the node x. Therefore, connections that utilize this longest link $= \sum_{l=1}^{\frac{N-1}{6}} \left(\frac{N+1}{2} - 3l\right) \, which \, is \left(\frac{N^2-4N+3}{24}\right)$

We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N-1}{6} - 1$, nodes available at a length $3l$ previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$. Similarly nodes available at a length$(3l - 1)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$. Therefore, connections that utilize this shortest link $= 2 * \left(\frac{N-1}{6} - 1 + 1\right) + \left(\frac{N-1}{6} - 1 + 1\right) = \frac{N-1}{2}$. The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-4N+3}{24}$.

Case iii) $N = 6m + 2$, where m denotes a positive number.

We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N-2}{6}$ nodes available at a length $3l$ previous to node $(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\left(\frac{N+2}{2} -\right.$

$3l\Big)$instant nodes after the node x. Therefore, connections that utilize this longest link = $\sum_{l=1}^{\frac{N-2}{6}}\left(\frac{N+2}{2}-3l\right) \, which \, is \left(\frac{N^2-2N}{24}\right)$
We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N-2}{6}$, nodes available at a length $3l$ previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$. Similarly For $0 \le l \le \frac{N-2}{6} - 1$, nodes available at a length$(3l-1)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$.Therefore, connections that utilize this shortest link =$2 * \left(\frac{N-2}{6}+1\right)+\left(\frac{N-2}{6}-1+1\right)=\frac{N+2}{2}$
The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-2N}{24}$.

Case iv) $N = 6m + 3$, where m denotes a positive number.
We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N-3}{6}$ nodes available at a length $3l$ previous to node$(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\Big(\frac{N+1}{2} - 3l\Big)$instant nodes after the node x. Therefore, connections that utilize this longest link = $\sum_{l=1}^{\frac{N-3}{6}}\left(\frac{N+1}{2}-3l\right) \, which \, is \left(\frac{N^2-4N+3}{24}\right)$
We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N-3}{6}$, nodes available at a length $3l$ previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$. Similarly For $0 \le l \le \frac{N-3}{6} - 1$ nodes available at a length$(3l - 1\,)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$. Therefore, connections that utilize this shortest link = $2 * \left(\frac{N-3}{6}+1\right)+\left(\frac{N-3}{6}-1+1\right)=\frac{N+1}{2}$
The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-4N+3}{24}$.

Case v) $N = 6m + 4$, where m denotes a positive number.
We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N-4}{6}$ nodes available at a length $3l$ previous to node $(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\Big(\frac{N+2}{2} - 3l\Big)$instant nodes after the node x. Therefore, connections that utilize this longest link = $\sum_{l=1}^{\frac{N-4}{6}}\left(\frac{N+2}{2}-3l\right) \, which \, is \left(\frac{N^2-2N-8}{24}\right)$

We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N+2}{6} - 1$, nodes available at a length $3l$ previous to node x in the anti clockwise route utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$. Similarly nodes available at a length$(3l - 1\,)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$. Therefore, connections that utilize this shortest link = $2 * \left(\frac{N+2}{6}-1+1\right)+\left(\frac{N+2}{6}-1+1\right)=\frac{N+2}{2}$
The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-2N-8}{24}$.

Case vi) $N = 6m + 5$, where m denotes a positive number.
We can arbitrarily choose one longest link that links the nodes x and $(x \oplus 3)$. For $1 \le l \le \frac{N-5}{6}$ nodes available at a length $3l$ previous to node $(x \oplus 3)$in the anti clockwise route, utilize this longest link to share its information to$\Big(\frac{N+1}{2} - 3l\Big)$instant nodes after the node x. Therefore, connections that utilize this longest link = $\sum_{l=1}^{\frac{N-5}{6}}\left(\frac{N+1}{2}-3l\right) \, which \, is \left(\frac{N^2-4N-5}{24}\right)$
We can arbitrarily select any shortest link that links the nodes x and $(x \oplus 1)$. For $0 \le l \le \frac{N+1}{6} - 1$, nodes available at a length $3l$ previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$ and node $(x \oplus 2)$.

Similarly, nodes available at a length$(3l - 1\,)$previous to node x in the anti clockwise route, utilize this shortest link to share its information to node $(x \oplus 1)$. Therefore, connections that utilize this shortest link = $2 * \left(\frac{N+1}{6}-1+1\right)+\left(\frac{N+1}{6}-1+1\right)=\frac{N+1}{2}$

The link load of longest link is higher than that of shortest link, so the link load of a longest link is the link load of the network. Therefore, the link load of the network $\pi = \frac{N^2-4N-5}{24}$.

It may be noted that for a every value of N, the link load of all longest links and shortest links are same. Hence, the wavelength-number required atmost to establish all-to-all broadcast must be greater than or equal to the link load. The outcome of the Table 1 provides the difference between the wavelength-number required and the link load is less, which proves that the results are either optimal or near optimal. From Table 2, it can be observed that the wavelength-number required atmost to establish all-to-all broadcast in a ring with 3-length extension is reduced by a minimum of 56% and a maximum of 66% when compared to primary ring. Similarly, the wavelength-number required atmost to establish all-to-all broadcast in a bi-directional ring with 3-length extension is reduced by a minimum of 13% and a maximum of 33% when compared to bi-directional ring with 2-length extension.

Table 1 Wavelength-number required atmost to establish all-to-all broadcast along with link load for certain values of node number N in a bi-directional ring with 3-length extension.

Node number N	Wavelength-number required	Link load	Difference between wavelength-number and link load
25	33	22	11
28	42	30	12
30	48	35	13
40	79	63	16
55	138	117	21
60	164	145	19
70	224	198	26
85	317	287	30
90	355	330	25
100	442	408	34
201	1694	1650	44
500	10483	10375	108

Table 2 Comparison of wavelength-number required atmost to establish all-to-all broadcast for certain value of node number Nin a bi-directional ring, bi-directional ring with 2-length extension and bi-directional ring with 3-length extension.

Node number N	Wavelength-number of required atmost to establish all-to-all broadcast		
	Bi-directional ring [4]	Bi-directional ring with 2-length extension [22]	Bi-directional ring with 3-length extension
25	78	41	33
28	98	53	42
30	113	60	48
40	200	105	79
55	378	189	138
60	450	233	164
70	613	315	224
85	903	449	317
90	1013	518	355
100	1250	638	442
201	5050	2500	1694
500	31250	15688	10483

4. Conclusion

A two stage heuristic algorithm is proposed to identify non-overlapping connections among the various connections of all-to-all broadcast in a bi-directional ring with 3-length extension. Explicit wavelength allotment methods are also provided for the same. The result obtained shows that the wavelength-number required is nearly equal to the link load of the network and so the results are either optimal or near optimal. Also, the wavelength-number required atmost to establish all-to-all broadcast in a bi-directional ring with 3-length extension is reduced by a minimum of 57% and a maximum of 66% when compared to bi-directional primary ring. Similarly, the wavelength-number required atmost to establish all-to-all broadcast for a bi-directional ring with 3-length extension is reduced by a minimum of 20% and a maximum of 33% when compared to a bi-directional ring with 2-length extension. This reduction in wavelength-number is at the expense of directly linking two nodes which are separated by two intermediate nodes with additional fibers.

Future works incorporate the examining the similar problem in other network topologies with 2 length and 3 length extension. Wavelength-number requirement needs to be investigated with still higher order extensions, to judge the rate of reduction in wavelength-number requirements. Also, deriving a generalized expression for wavelength-number requirement in a ring network with k-length extension (k is any positive integer and $k < N$ where N is the total number of nodes in the network) is an interesting and challenging future work. The impact of physical layer impairments for these networks needs to be studied. Another issue that requires attention is studying wavelengths requirement under multiple link and node failures in this network.

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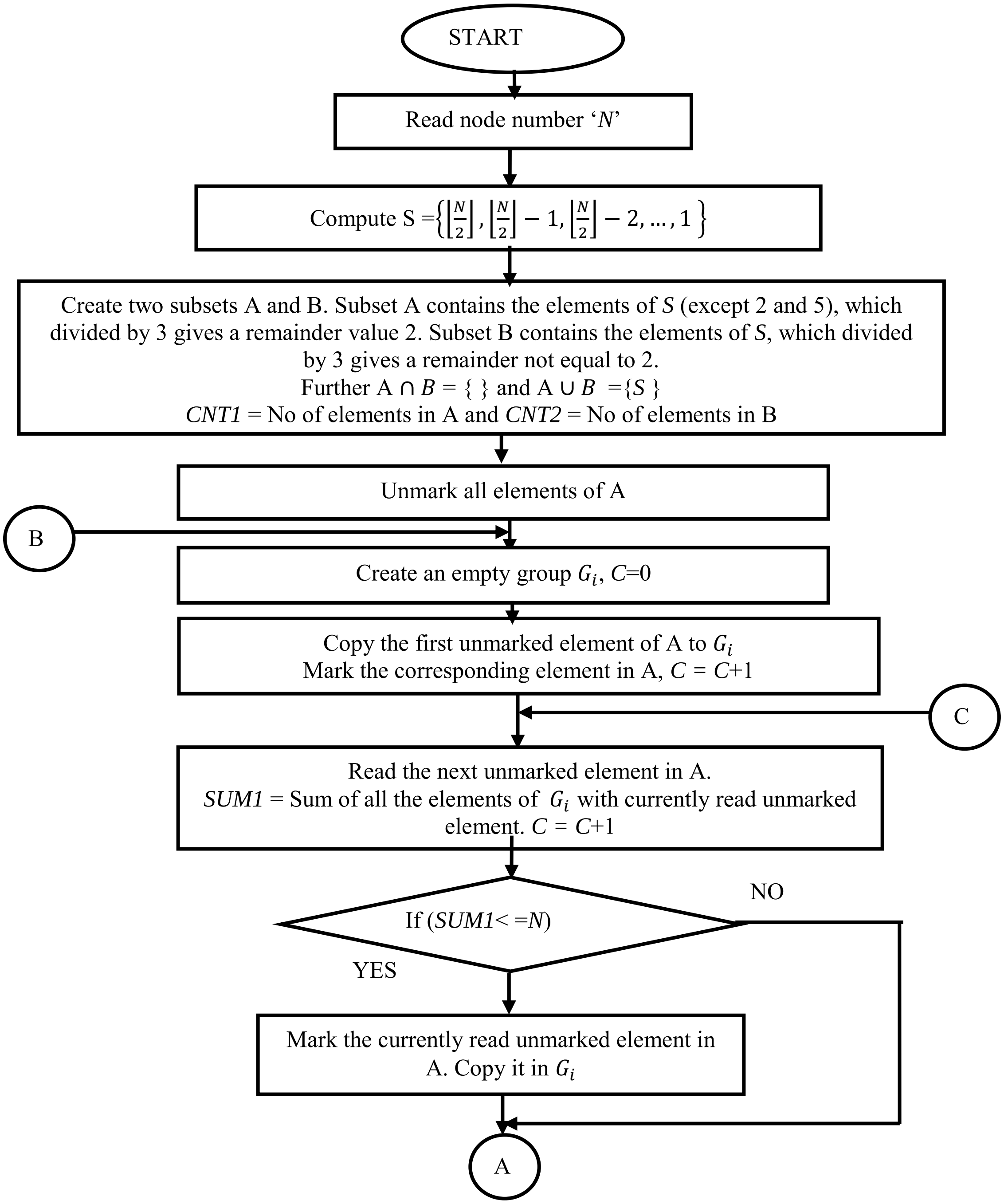

Fig. 2 Flowchart of the stage 1 algorithm for ring with 3-length extension (cont..)

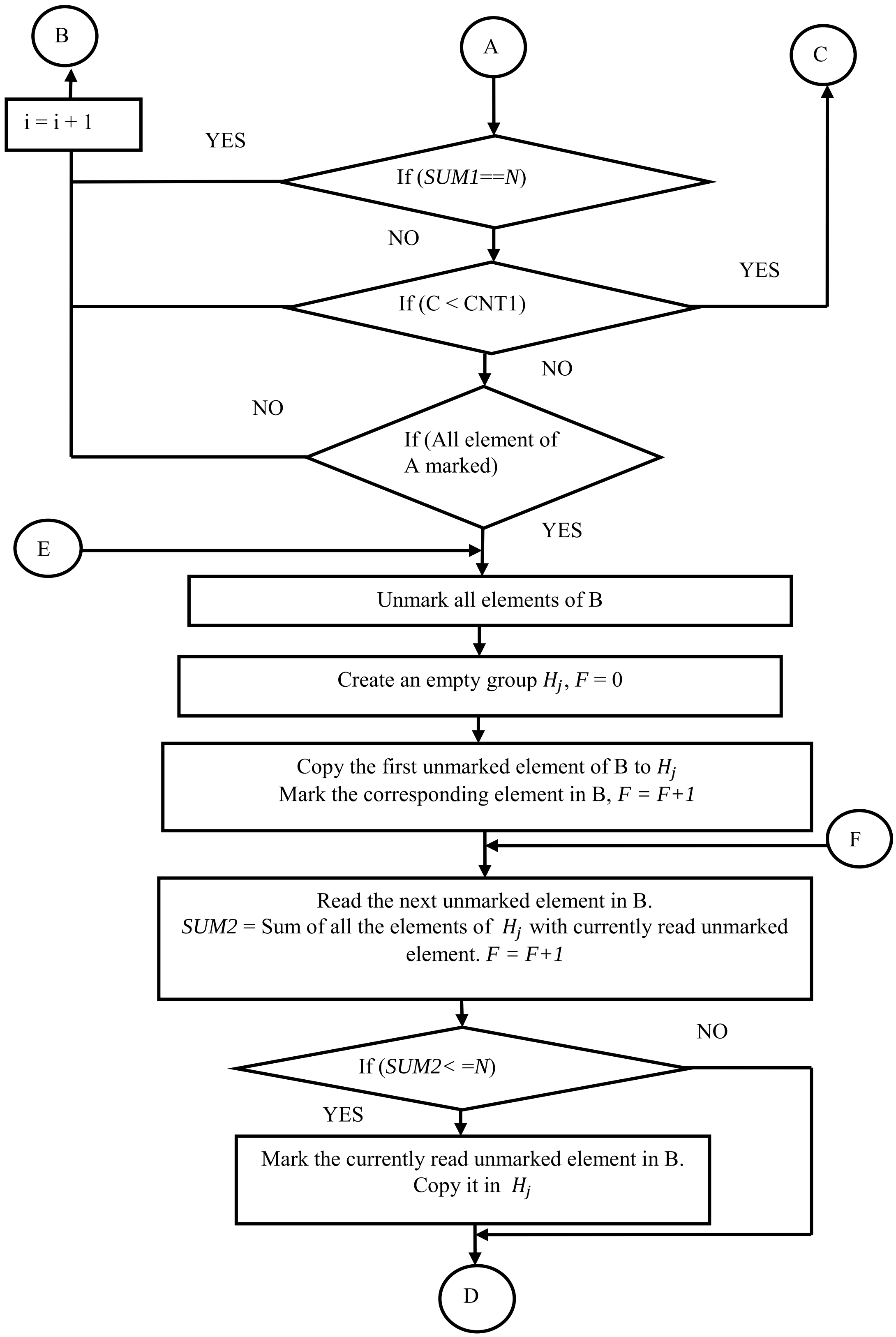

Fig. 2 Flowchart of the stage 1 algorithm for ring with 3-length extension (cont..)

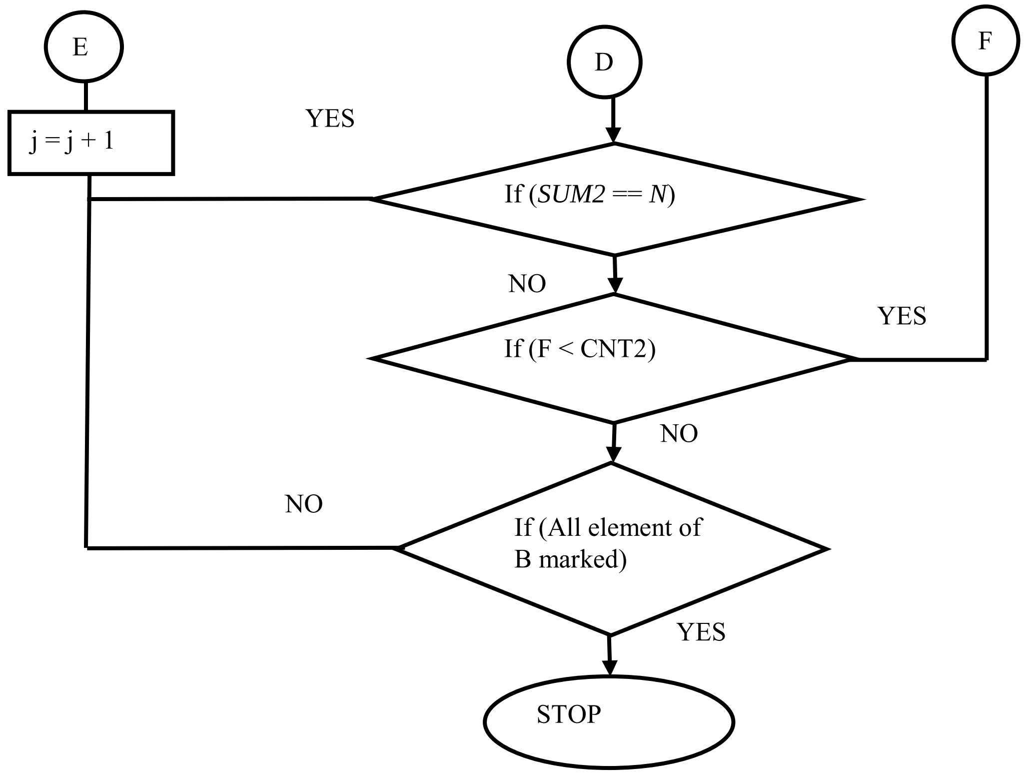

Fig. 2 Flowchart of the stage 1 algorithm for ring with 3-length extension

N- Total No. of nodes available in the network.

$SUM1, SUM2, CNT1 = 0, CNT2 = 0, C, F, I, J, K, L, M = \left\lfloor \frac{N}{2} \right\rfloor, X$are integer variables.

$A[\,]$ →An array A containing length of connections as elements (inputs).

$B[\,]$ →An array B containing length of connections as elements (inputs).

$D[\,]$ → An array D containing elements as flags for elements of array A [ ].

$E[\,]$ → An array E containing elements as flags for elements of array B [ ].

$G[\,][\,]$ → An array G with row as the list number and column as the element index (outputs).

$H[\,][\,]$ → An array H with row as the list number and column as the element index (outputs).

Step 1: While $(M > 0)$

$If\ ((M\%3\ == 2) \&\& (M\ \neq\ 2) \&\&\ (M\ \neq\ 5))$

$A[CNT1] \leftarrow M$

$CNT1 = CNT1 + 1$

Else $If(M\%3\ \neq 2)$

$B[CNT2] \leftarrow M$

$CNT2 = CNT2 + 1$

End if.

$M \leftarrow M - 1$

Step 2: End While.

Step 3: Initialize $I \leftarrow 0, D[\,] \leftarrow 0$

Step 4: Assign $SUM1 \leftarrow 0, J \leftarrow 0,\ C \leftarrow 0$

Step 5: While($C < CNT1$)

$If\ (D\,[C]\ = 0\ \ \&\,((SUM1\ +\ A[C])\ \le\ N)),$

$SUM1\ \leftarrow\ SUM1\ +\ A[C], G[I][J] \leftarrow A[C], J \leftarrow J + 1, D[C] \leftarrow 1.$

$If\ (SUM1 == N),$

$I\ \leftarrow\ I\ + 1;$

Go to Step 4.

End if.

End if.

$C\ =\ C + 1$

Step 6: End while.

Step 7:$If\ (D[X\,] == 1)\ for\ 0 \le X < CNT1$,go to Step 8.

Else $I \leftarrow I + 1$, go to Step 4.

Step 8: Initialize $K \leftarrow 0, E[\,] \leftarrow 0$

Step 9: Assign $SUM2 \leftarrow 0, L \leftarrow 0,\ F \leftarrow 0$

Step 10: While($F < CNT2$)

$If\ (E\,[F]\ = 0\ \ \&\,((SUM2\ +\ B[F])\ \le\ N)),$

$SUM2\ \leftarrow\ SUM2\ +\ B[F], H[K][L] \leftarrow B[F], L \leftarrow L + 1, E[F] \leftarrow 1.$

$If\ (SUM2 == N),$

$K\ \leftarrow K\ + 1;$

Go to Step 4.

End if.

End if.

$F\ =\ F + 1$

Step 11: End while.

Step 12: If $(E[X\,] == 1)\ for\ 0 \le X < CNT2$,the algorithm is terminated.

Else $K \leftarrow K + 1$, go to Step 9.

Fig. 3 Pseudo code of the stage 1 algorithm for ring with 3-length extension

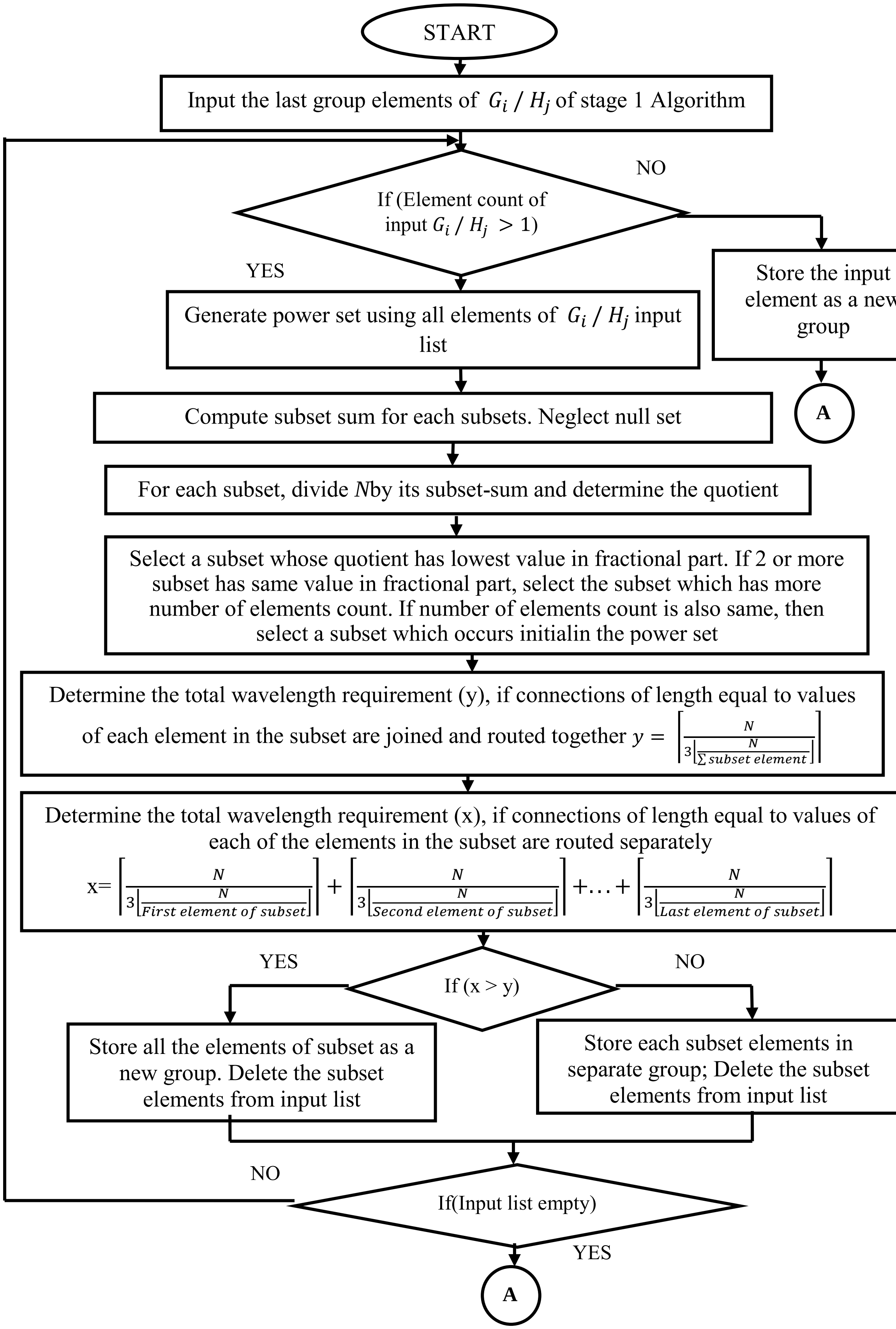

Fig. 4 Flowchart of stage 2 algorithm for ring with 3-length extension (cont..)

N →Total No. of nodes available in the network.

S[ ] → An array containing the last group elements of G[ ][ ]obtained in stage 1 of the algorithm (sorted in descending order)

T[ ] → An array containing the elements of last group of H[ ][ ]obtained in stage 1 of the algorithm (sorted in descending order)

U[ ][ ] →An array U (outputs).

V[ ][ ] →An array V (outputs).

BUF [ ] →An array BUF.

$K \leftarrow 0, L \leftarrow 0, Q \leftarrow 1, Z \leftarrow 0, BUF[\,] \leftarrow 0.$

Step 1: Power set of S[ ] / T[ ]is generated and sorted in descending order based on the number of elements and subset_sum. For each and every subset of S[ ] / T[ ],*N* is divided by its subset sum. Let F be the fractional part of the result. For each subset, If $Q > F, Q \leftarrow F, BUF[J] \leftarrow 0$ for $1 \leq J \leq Z$, $Z \leftarrow$ Number of elements of the corresponding subset, BUF[J] ←Corresponding Subset whose quotient has lowest value in fractional part. (If two or more subset has same value in fractional part, select the subset which has more number of elements count)

Step 2: If

$$\left(\left(\left\lceil\frac{N}{3\left\lfloor\frac{N}{BUF_1}\right\rfloor}\right\rceil+\left\lceil\frac{N}{3\left\lfloor\frac{N}{BUF_2}\right\rfloor}\right\rceil+\cdots+\left\lceil\frac{N}{3\left\lfloor\frac{N}{BUF_Z}\right\rfloor}\right\rceil\right)>\left\lceil\frac{N}{3\left\lfloor\frac{N}{\sum BUF_J \text{ for } 0\leq J\leq Z}\right\rfloor}\right\rceil\right),$$

remove elements of BUF[ ]from S[ ] and store it in U[L][K],

$K \leftarrow K+1$, go to Step 4.

Step 3: $J \leftarrow 0$ Repeat $U[L][K] \leftarrow BUF[\,J\,]$

$J \leftarrow J+1$

$K \leftarrow K+1$

Until $BUF[J\,] = 0.$ Then, the elements of BUF[ ] are deleted from S[ ] / T[ ].

Step 4: If S[ ] / T[ ] is not empty, $L = L+1, Q \leftarrow 1$, go to Step 1. Else, the algorithm is terminated.

Fig. 5 Pseudo code of stage 2 algorithm for ring with 3-length extension