

# Approximation of Families of Characteristics of Energy Objects in the Class of Linear Differential Equations

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*Abstract:* - A method is proposed for constructing a fast-computable mathematical model of a family of energy characteristics based on its comparison with a family of integral curves in the class of linear differential equations. The method uses an affine transformation and takes into account the two-level structure of the mathematical model of a family of characteristics (differential operator and boundary conditions). It allows reducing the number of approximating summands and increasing the adequacy by including additional information of a systemic nature about the object under study.

*Key-Words:* - Mathematical Model, Ordinary Linear Differential Equation, Lowering the Order of the Equation, Exponential the Polynomial, Interpolation.

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## 1. Introduction

The use of effective methods for studying of energy systems is associated with a complete analytical description of the family of characteristics (this is especially true for optimizing energy facilities).

Due to the structural and functional complexity and as a result of many connections, energy systems are considered to be comprehensive systems. External operational characteristics of such systems are determined by complex physical models using a numerical method based on computer programs in the form of data array.

The task is to replace a computationally complex physical model with a quickly calculated mathematical model, which is called a surrogate model [1]. This problem is associated with multidimensional approximation technology.

Currently, there are many practical methods for multidimensional approximation, which are conceptually based on polynomial expressions, for example, in the form of a multidimensional power polynomial [2], or in a superposition form of simple functions of one variable and their linear combination [3]. In this case, many common approaches to implementation of multidimensional surrogate models are being utilized, which may include the following common methods: (linear regression, dictionary expansion of nonlinear

parametric functions, splines) [4]; Gaussian processes [1].

But the aforementioned methods are not always convenient for describing the characteristics due to the following good reasons.

First of all, power multidimensional polynomials cannot satisfy the saturation condition for large values of the independent variable (with  $x \rightarrow \infty$ ,  $\lim y(x, \eta)$  there is a finite number for any parameter  $\eta$  in the range of its definition).

Secondly, the methods for implementing such approximations lack the ability to attach additional information to approximation expressions, for example, of a heuristic nature.

Thirdly, the existing approximation methods define a rigid structure of an approximation expression in the form of regular basic functions (polynomials, splines, linear and nonlinear regressions), which have low adaptability when transitioning onto new modes and operational states of the simulated technical system, which narrows the application scope of such approximation models.

The noted drawbacks can be eliminated on the basis of the method described in [5], according to which many hysteresis cycles of ferromagnetic elements are modeled by integral curves of some homogeneous ordinary linear differential equation with constant coefficients (OLDE). This approach allowed to present branches of hysteresis loops in

the form of a finite sum of exponential functions with unregulated poles, which increase adequacy of the model and reduce its dimension. This method made it possible to describe any background of magnetization of a simulated magnetic substance. Moreover, the structure and parameters of the differential operator were determined directly from corresponding conditions of the problem being solved.

But the aforementioned method, based on homogeneous OLDEs, has serious drawbacks, viz.: speculative approaches were used; there was no general methodology for determining integration constants; there was a contradiction between the degrees of freedom of the model and the simulated family of characteristics.

The aim of this work is to develop a methodology for creating surrogate mathematical models of families of characteristics related to quality of magnetic treatment of circulating water based on *heterogeneous* linear differential equations.

The aim of this work is to develop a methodology for creating surrogate mathematical models of families of characteristics on based ordinary differential equations.

## 2. The Theoretical Basis of the Methodology

Ordinary differential equations (ODEs) represent on the  $x, y$  plane a family of curves  $y = f(x, C_1, \dots, C_n)$ . In [6], a classical strict derivation of the  $n$ -th order ODEs directly from a recorded expression of a family of curves is being presented. Let us present this derivation here. To do this, it is required to define a family of curves in implicit form  $\Phi(x, y, C_1, \dots, C_n) = 0$  and to carry out a sequential differentiation  $n$ -number of times, resulting in  $n-1$  equations:

$$\begin{aligned} \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial y} y' &= 0; \\ \frac{\partial^2 \Phi}{\partial x^2} + 2 \frac{\partial^2 \Phi}{\partial x \partial y} y' + \frac{\partial^2 \Phi}{\partial y^2} y'^2 + \frac{\partial \Phi}{\partial y} y'' &= 0; \\ \frac{\partial^n \Phi}{\partial x^n} + \dots + \frac{\partial \Phi}{\partial y} y^{(n)} &= 0. \end{aligned}$$

The exclusion of  $C_k$  parameters from  $n$  equations leads to the following ODE:

$$F(x, y, y', \dots, y^{(n)}) = 0.$$

Conclusion. On the plane, the family of integral curves correspond to both homogeneous and heterogeneous ODEs.

To formalize the simulation method based on DEs, let's select a class of linear ordinary DEs with

constant coefficients, that are completely studied, determined, with their general theory being completed, and where a general solution consists of the sum of exponential functions with complex exponents [7].

*Assertion 1.* To construct a homogeneous DE with constant coefficients, it suffices to approximate the characteristic by the sum of exponential functions.

*Proof.* Let  $\forall t \in [0, t_m]$  by defined by the following exponential expression:

$$S(t) = \sum_{k=1}^n C_k e^{-p_k t}, \quad (1)$$

where the numbers  $C_k$  and  $p_k$  are known.

By virtue of the existence and uniqueness theorem,  $S(t)$  corresponds to a definite homogeneous OLDE with constant coefficients

$$L_n S(t) = 0 \quad (2)$$

and a specific numerical set of initial conditions  $s(0), s'(0), \dots, s^{(n-1)}(0)$  for the Cauchy problem. Since the parameters of the differential operator are

$$L_n = \left( \frac{d}{dt} - p_1 \right) \left( \frac{d}{dt} - p_2 \right) \dots \left( \frac{d}{dt} - p_n \right),$$

then the desired OLDE are expressed in terms of the known exponential indexes, which proves the statement.

Statement 1 implies two important corollaries.

*Corollary 1.1.* The differential operator  $L_n$  is a differential model of the original exponential expression  $S(t)$ .

*Corollary 1.2.* To model a continuous family of characteristics, it is necessary to determine mathematical models simulating variation of a certain set of initial conditions  $s(0), s'(0), \dots, s^{(n-1)}(0)$ , the substitution of which in the exponential expression  $S(t)$ , results in dependences that coincide with each characteristic of the simulated family.

To formalize the process of constructing differential mathematical models, it is proposed to use the method of interpolating the characteristics with the sum of exponential functions and unregulated poles described in [8], which provides smaller dimensionality with better adequacy and determines the structure of the mathematical model based on the conditions of the problem being solved.

The interpolation technique from [8] used in the developed technique ensures high reproduction accuracy of individual characteristics. However, increasing the accuracy of interpolation of individual characteristics increases the number of exponential functions and, accordingly, the order of the OLDE, which leads to the excess

dimensionality of the general solution of the differential mathematical model.

The concept of dimensionality is closely related to the concept of degree of freedom. The generalized number of degrees of freedom is the minimum number of independent variables (generalized coordinates) required for a complete description of a dynamical system. From this definition it follows that the number of degrees of freedom in the differential mathematical model of a technical system corresponds to the number of integration constants of its general solution.

It is obvious that one of the conditions for the adequacy of the differential model and the family of characteristics of the simulated object (process) is the equality of their number of degrees of freedom.

*Assertion 2.* To ensure the adequacy of the differential mathematical model with the family of characteristics of the modeled object (process), it is necessary to reduce the order of the differential model (the number of degrees of freedom of the model) to the number of degrees of freedom of the modeled object (process).

However, the primary mathematical models of the processes of complex technical systems, as a rule, are described by systems of nonlinear ordinary DE or DE in partial derivatives. When constructing a surrogate mathematical model to ensure accuracy of a simulated process in the class of elementary functions (in particular, of exponential ones), it is necessary to increase the number of members of such functions, and hence the order of the differential model - OLDE, which can lead to an excess of its order relative to the nominal degree of freedom of the simulated process. Therefore, increasing accuracy of reproducing individual characteristics leads to an overestimation of the order of the model, which leads to a contradiction between the excessive number of model integration constants and the limited degree of freedom of the simulated family of characteristics. Therefore, there is a need to lower the order of the OLDE.

When lowering the order of differential models, the requirements for preserving the class of differential operator (OLDE with constant coefficients) and the form of the dependent variable  $y(t)$  must be complied with. The known method of direct integration of an incomplete OLDE -  $y^{(n)} = x(t)$  meets these requirements. However, in the general case, the methods for reducing the order of the OLDE described in the classical theory [9–11] do not satisfy the stated requirements. Known methods for lowering the order of DEs, based on replacing the dependent variable, lead to a change

both of the DEs class, and the type of function, as well as the sequence of calculation of the integration constants in the intermediate integrals.

In [12], a technique was developed for converting the initial OLDE into an equivalent inhomogeneous lower-order LDE, which is absent in the classical theory of differential equations. The equivalence condition for the DE transformation implies the invariance of integral curves. This technique of reducing the order of LDEs compared with those presented in the classical theory does not lead to a change in the class of DEs, the form of the  $y(t)$  function, and the sequence of calculation of the integration constants in the intermediate integrals.

Let us present this method of reducing the order of OLDE in a slightly different form, convenient for use in problems of mathematical modeling of a family of characteristics.

*Assertion 3.* Equivalence of general solutions of the original  $L_n y(t) = x(t)$  and reduced by unit  $L_{n-1} y(t) = f_1(t)$  OLDE, where  $L_n, L_{n-1}$  are linear differential operators with constant coefficients of  $n$  and  $n-1$  orders, respectively, is determined by the differential transformation of the right side of  $x(t)$  in  $f_1(t)$ :  $f_1'(t) + p_n f_1(t) = x(t)$  and with the initial condition of  $f_1(0) = y^{(n-1)}(0) + a_{n-2,n-1} y^{(n-2)}(0) + \dots + a_{0,n-1} y(0)$ , where  $p_n$  is a root of the characteristic equation of the original OLDE.

In this case, the original OLDE  $L_n y(t) = x(t)$  decomposes into the following LDE system:

$$\begin{cases} L_{n-1} y(t) = f_1(t); & (3) \end{cases}$$

$$\begin{cases} \left( \frac{d}{dt} + p_n \right) f_1(t) = x(t); & (4) \end{cases}$$

$$y(0), y'(0), \dots, y^{(n-2)}(0), f_1(0).$$

Initially, on the basis of the constructed exponential expressions (1), it is possible to determine a homogeneous OLDE only (2), therefore, as the initial OLDE we use the following DE:  $L_n y(t) = 0$ , and by integrating (4) with  $x(t) = 0$ , we obtain the following expression for the Cauchy problem:

$$f_1(t) = C_{10} e^{-p_n t}.$$

By substituting (5) into the DE (3), we obtain the equivalent OLE with a reduced by one unit order, which is the first integral of the original OLDE.

The second act of reducing the order leads to the following  $\mathcal{D}Y$  system:

$$\begin{cases} L_{n-2} y(t) = f_2(t); & (6) \end{cases}$$

$$\begin{cases} \left( \frac{d}{dt} + p_{n-1} \right) f_2(t) = f_1(t); & (7) \end{cases}$$

$$\begin{cases} \left( \frac{d}{dt} + p_n \right) f_1(t) = 0; & (8) \end{cases}$$

$$y(0), y'(0), \dots, y^{(n-3)}(0), f_1(0), f_2(0),$$

where from (6) it follows that  $f_2(0) = y^{(n-2)}(0) + a_{n-3, n-2} y^{(n-3)}(0) + \dots + a_{0, n-2} y(0)$ ,  $p_{n-1}$  is the root of the characteristic equation of the initial OLDE. By integrating (7) for the Cauchy problem, we obtain:

$$f_2(t) = e^{-p_{n-1}t} \left[ C_{20} + \int e^{p_{n-1}t} f_1(t) dt \right]. \quad (9)$$

By substituting (9) into the differential equation (6), we obtain the equivalent OLDE reduced by two orders, which is the second integral of the initial OLDE.

The initial conditions  $y(0), y'(0), \dots, y^{(n-2)}(0), y^{(n-1)}(0)$  are expressed in terms of the solution parameters (1) of the OLDE for the case of negative roots of its characteristic equation:

$$\begin{aligned} y(0) &= \sum_{k=1}^n C_k, \quad y'(0) = -\sum_{k=1}^n p_k C_k, \dots, \\ y^{(n-1)}(0) &= \sum_{k=1}^n (-p_k)^{n-1} C_k. \end{aligned} \quad (10)$$

By substituting expressions (10) into  $f_1(0)$ , we get:

$$\begin{aligned} f_1(0) &= \sum_{k=1}^n C_k [(-p_k)^{n-1} + a_{n-2, n-1} (-p_k)^{n-2}] + \\ &+ \dots + a_{0, n-1} = C_n H_{n-1}(p = -p_n), \end{aligned} \quad (11)$$

where  $H_{n-1}(p)$  is the characteristic equation of the OLDE (3), which is related to the characteristic equation of the original OLDE (2) in a known manner:

$$\begin{aligned} H_n(p) &= (p + p_n) H_{n-1}(p) = \\ &= (p + p_n)(p + p_{n-1}) H_{n-2}(p). \end{aligned} \quad (12)$$

The integration constant  $C_{10}$  in (5) will take the following form:

$$C_{10} = f_1(0) = C_n H_{n-1}(p = -p_n). \quad (13)$$

Similarly, for  $f_2(0)$  we get:

$$\begin{aligned} f_2(0) &= \sum_{k=1}^n C_k [(-p_k)^{n-2} + a_{n-3, n-2} (-p_k)^{n-3} + \\ &\dots + a_{0, n-2}] = \\ &= C_{n-1} H_{n-2}(-p_{n-1}) + C_n H_{n-2}(-p_n), \end{aligned} \quad (14)$$

where  $H_{n-2}(p)$  is the characteristic equation of the OLDE (5), which is related to the characteristic equation of the original OLDE of  $H_{n-1}(p) = (p + p_{n-1}) H_{n-2}(p)$ .

The integration constant  $C_{20}$  in (9) is determined on the basis of (12) - (14):

$$\begin{aligned} C_{20} &= f_2(0) - \frac{C_{10}}{(p_{n-1} - p_n)} = \\ &= C_{n-1} H_{n-2}(-p_{n-1}). \end{aligned} \quad (15)$$

Therefore, expression (9) takes the following form:

$$f_2(t) = e^{-p_{n-1}t} [C_{n-1} H_{n-2}(-p_{n-1}) +$$

$$\begin{aligned} &+ \int e^{p_{n-1}t} f_1(t) dt] = \\ &= C_{n-1} H_{n-2}(-p_{n-1}) e^{-p_{n-1}t} + \\ &+ C_n H_{n-2}(-p_n) e^{-p_n t}. \end{aligned} \quad (16)$$

The reduction procedure can be applied k-th number of times ( $k \leq n$ ):

$$f_k(t) = \sum_{j=1}^k C_{n+1-j} H_{n-k}(-p_{n+1-j}) e^{-p_{n-j}t}. \quad (17)$$

$$L_{n-k} y(t) = f_k(t); \quad (18)$$

Based on the foregoing, we present the foundations of the methodology for constructing a surrogate model of a family of aperiodic characteristics.

With an excessive number of exponential functions in approximating exponential expression (1), which are equal for all the curves of the family, when generating a differential mathematical model in the class of OLEDs with constant coefficients (CC), the order of the DE in this model can be reduced several times in accordance with (17)-(18). Such a model will describe each characteristic of a given family when specifying a certain set of initial conditions  $y(0), y'(0), \dots, y^{(n-k)}(0)$ . And in order to describe a continuous family of characteristics, it is required to define mathematical models that describe the variation of this set of initial conditions  $y(0), y'(0), \dots, y^{(n-k)}(0)$ , the substitution of which into the exponential expression  $y(t)$ , results in dependencies that coincide with each characteristic of the simulated family. The LDE method allows to guarantee adequacy of the model not only at the points of approximation of the family, but also at all points located in the field of existence of the family (according to continuity theorems with respect to parameters and initial conditions of differential equations [7]).

Accuracy of the approximations depends on the chosen method of interpolation of one-dimensional and two-dimensional dependences, and, in particular, interpolation by exponential polynomials is comprehensively described in corresponding literature, therefore this issue is not being presented here.

### 3. Methodology Implementation Example

Let the characteristics of energy objects be represented as dependencies  $y(x, \eta)$ , where  $x$  is an independent variable,  $\eta$  is some dimensionless parameter. Let the array of source data presented in Fig. 1 by discrete values depicted by various

symbols corresponding to the values of the  $\eta$  parameter (0, 1, 2, 3.5).

It is required to determine a surrogate mathematical model of this family in the variation range of independent variables:  $0 \leq x < \infty$ ;  $0 \leq \eta \leq 3.5$ .

First, by transforming the scales of the independent variable  $x$  using  $\lambda_v^{-1}$  scale factor, we compress the initial characteristics No. 2, No. 3, No. 4 to the basic characteristic No. 1 (affine transformation). The curve with the parameter value  $\eta = 0$  was chosen as the basis characteristic. Each compressed characteristic of the initial family shall be compared with the basic characteristic, which are presented in Fig. 2.

Denote the parameter change number in Fig. 1 with the symbol  $\nu = 1, 2, 3, 4$ .

Using the modified interpolation method (and the least squares method), the basis characteristic  $y_1(x)$  and the deviations of each curve  $y_\nu(\lambda_v^{-1}x)$  from the base curve are approximated by exponential polynomials [8]. As a result, a given family of characteristics will be described by the following exponential polynomial with irregular exponents:

$$y_\nu(x) = \sum_{k=1}^7 C_k e^{-p_k \lambda_\nu x}, \quad (19)$$

the coefficients values of which  $C_k$ ,  $p_k$  are presented in Table 1, and in Fig. 1, the approximated dependences of the initial characteristics, where  $\nu = 1, 2, 3, 4$ , are presented by continuous curves.

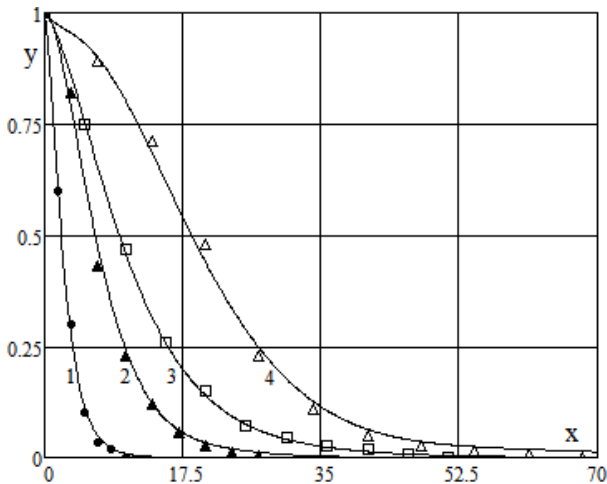


Fig. 1: The family of characteristics  $y(x)$ . Discrete dependences correspond to the source data; continuous dependences are calculated characteristics according to a surrogate mathematical model - with the values of the  $\eta$  parameter: No.1-  $\eta = 0$ ; No.2-  $\eta = 1$ ; No.3-  $\eta = 2$ ; No.4-  $\eta = 3.5$

All the characteristics described by the exponential polynomial (19) can be presented as solutions of a homogeneous OLDE with constant coefficients in the following form:

$$L_7 y = 0.$$

Its characteristic equation has the following form:

$$H_7(p) = (p^2 + \lambda_\nu p + 0.75\lambda_\nu^2)(p + 1.6\lambda_\nu) \times \\ \times (p + 2\lambda_\nu)(p + 0.95\lambda_\nu)(p + 0.82\lambda_\nu) \times \\ \times (p + 0.47\lambda_\nu).$$

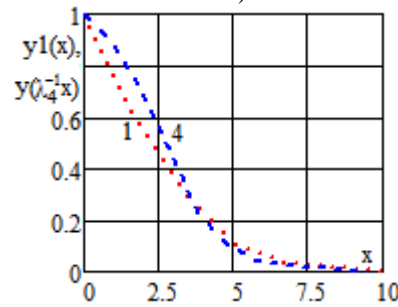
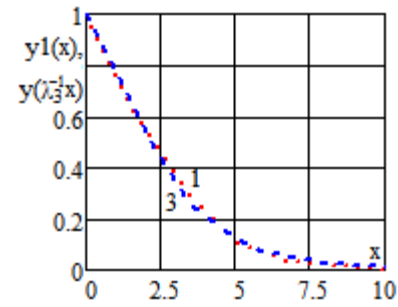
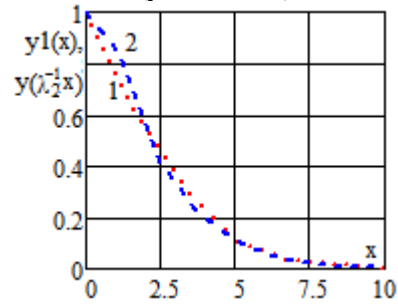


Fig. 2: Functional dependences of each reduced  $y_\nu(\lambda_\nu^{-1}x)$ - characteristics of the family to the basis characteristic  $y_1(x)$ : a -  $\nu = 2$ ; b -  $\nu = 3$ ; c -  $\nu = 4$

Since the scale coefficients  $\lambda_\nu$  are different for different values of  $\eta$ , the coefficients of the OLDE will depend on a corresponding parameter.

The polynomial (19) contains three identical terms for all characteristics (with  $k = 5, 6$ , and  $7$  in Table 1); therefore, let's transfer these summands by the method of reducing the order of the OLDE to the right side of the differential equation. An OLDE of reduced order will take the following form:

$$y^{(4)} + a_{34}y''' + a_{24}y'' + a_{14}y' + a_{04}y = -2.9H_4(-0.82\lambda_\nu)e^{-0.82\lambda_\nu x} + C_5H_4(-0.95\lambda_\nu)e^{-0.95\lambda_\nu x} + f(x); \quad (20)$$

$$f(x) = 2.9H_4(-0.47\lambda_\nu)e^{-0.47\lambda_\nu x} -$$

Table 1: The values of the coefficients of the polynomial (19)

$\nu$	$\lambda_\nu$	$k$	1	2	3	4	5	6	7
1	1	$p_k$	0	0	0	0	0.95	0.82	0.47
		$C_k$	0	0	0	0	-2.9	2.9	1
2	0.36	$p_k$	$0.5 - j0.71$	$0.5 + j0.71$	1.6	2	0.95	0.82	0.47
		$C_k$	$0.072 - j0.072$	$0.072 + j0.072$	0.05	0.05	-2.9	2.9	1
3	0.23	$p_k$	$0.5 - j0.71$	$0.5 + j0.71$	1.6	2	0.95	0.82	0.47
		$C_k$	$0.025 - j0.025$	$0.025 + j0.025$	-0.1	0.1	-2.9	2.9	1
4	0.14	$p_k$	$0.5 - j0.71$	$0.5 + j0.71$	1.6	2	0.95	0.82	0.47
		$C_k$	$0.225 - j0.225$	$0.225 + j0.225$	-0.99	0.99	-2.9	2.9	1

where the fourth-order characteristic equation of the OLDE has the following form:

$$H_4(p) = p^4 + 4.6\lambda_\nu p^3 + 7.55\lambda_\nu^2 p^2 + 5.9\lambda_\nu^3 p + 2.4\lambda_\nu^4.$$

For all OLDEs (20), the value is  $y(0) = 1$ , and the initial conditions are  $y'(0, \eta)$ ;  $y''(0, \eta)$ ;  $y'''(0, \eta)$ , continuously depend on the  $\eta$  parameter; therefore, the power polynomials are first approximated by the varying integration coefficients of expression (19) and the scale factor  $\lambda_\nu$  as a function from  $\eta$ , since, according to the condition of the problem, the working range of variation of the  $\eta$  parameter is limited:

$$B(\eta) = 2C_1(\eta) = 0.444\eta - 0.061\eta^2 + 0.0066\eta^3; \quad (21)$$

$$C_3(\eta) = 0.13\eta - 0.093\eta^2 + 0.013\eta^4 \quad (22)$$

$$\lambda(\eta) = 1 - 1.01\eta^{0.5} + 0.424\eta - 0.061\eta^2 + 0.0066\eta^3. \quad (23)$$

The correspondence results of approximation dependences (21) - (23) with the data in Table 1 are presented in Fig. 3.

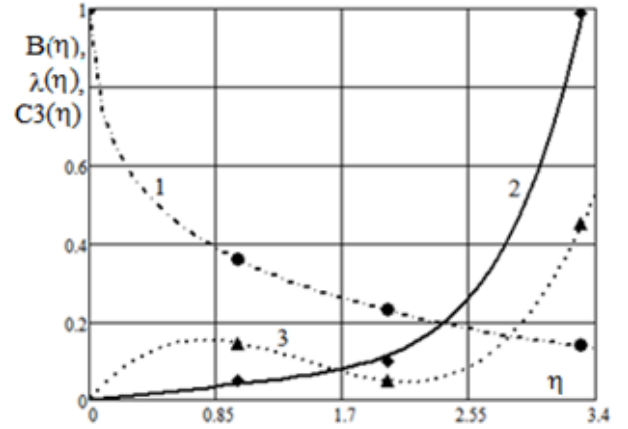


Fig. 3: Approximation dependencies: 1-  $B(\eta)$ ; 2-  $\lambda(\eta)$ ; 3-  $C_3(\eta)$

For completeness of the differential mathematical model based on expressions (20), the mathematical model of variation is determined with the following initial conditions:

$$y(0) = 1; \quad (24)$$

$$y'(0, \eta) = \lambda(\eta)[-0.078 + 0.262\eta - 0.006\eta^2 + 0.0047\eta^3 - 0.0052\eta^4];$$

$$y''(0, \eta) = [\lambda(\eta)]^2[0.47 - 0.127\eta - 0.019\eta^2 - 0.0066\eta^3 + 0.0187\eta^4];$$

$$y'''(0, \eta) = [\lambda(\eta)]^3[0.819 - 0.428\eta + 0.352\eta^2 + 0.0017\eta^3 - 0.0508\eta^4].$$

Since there are theorems on the continuity of solution of ordinary linear DEs from the initial values and parameters [7], with the continuity of the functions (23) and (24) in the range of existence of these approximations, the differential mathematical model (20) and (24) guarantees

continuity of the mathematical model (19) from a change in the  $\eta$  parameter.

Thus, these theorems guarantee, through the differential mathematical model (20) and (24), constructed on the basis of its particular solutions  $y_v(x)$  (expression (19) with  $v = 1,2,3,4$ ), the adequacy of a more general model:

$$y(x, \eta) = \sum_{k=1}^7 C_k(\eta) e^{-p_k(\eta)\lambda^{(\eta)}x}, \quad (25)$$

Model (25) describes a continuous family of characteristics on the initial segment of the change of parameter  $0 \leq \eta \leq 3.5$  and on the positive semi-axis  $0 \leq x < \infty$  of the change of the independent variable. Fig. 4 shows an example of a description by a mathematical model (25) of inverse characteristics  $\varepsilon(x, \eta) = 1 - y(x, \eta)$  of such that correspond to arbitrary parameter values (with values of  $\eta = 0.4; 1.5; 2.7$ ).

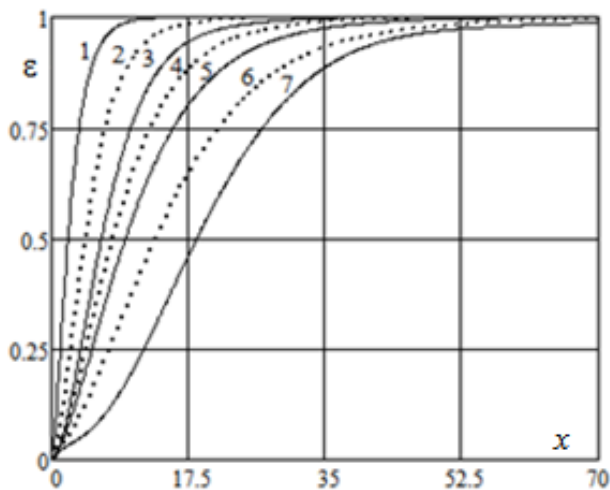


Fig. 4: The calculated characteristics ( $\eta$ ) of the degree of purification for the following  $\eta$ : No.1- 0; No.2- 0,4; No.3- 1; No.4- 1.5; No.5- 2; No.6- 2.7; No.7- 3.5

## 4. Conclusion

A method for constructing a surrogate mathematical model of a family of aperiodic characteristics by integral curves of an inhomogeneous ordinary linear differential equation with constant coefficients has been developed. The methodology involves construction of a two-level mathematical model of a family of characteristics, which consists of a differential operator with constant coefficients and a mathematical model of variation with initial conditions.

The advantage of this approach is that in the class of exponential polynomials it is possible to

achieve adequacy without any strict regulation of the structure of these polynomials by fewer approximating summands and a possibility of including into the model through initial or boundary conditions of additional systemic information about the technical object (technological process) being studied.

The method includes stages of standard interpolation (or approximation based on the least square method) and matching the degrees of freedom of the model and the simulated object based on the equivalent method of reducing the order of a differential equation.

The developed methodology for constructing the aforementioned model and its implementation

using a specific example makes it possible to directly use it in scientific research and engineering developments.

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