Geographically Separating Sectors in Multi-Objective Location-Routing Problems

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Abstract: This paper deals with multi-objective location-routing problems (MO-LRPs) and follows a sectorization approach, which means customers are divided into different sectors, and a distribution centre is opened for each sector. The literature has considered objectives such as minimizing the number of opened distribution centres, the variances of compactness, distances and demands in sectors. However, the achievement of these objectives cannot guarantee the geographical separation of sectors. In this sense, and as the geographical separation of sectors can have significant practical relevance, we propose a new objective function and solve a benchmark of problems with the non-dominated sorting genetic algorithm (NSGA-II), which finds multiple non-dominated solutions. A comparison of the results shows the effectiveness of the introduced objective function, since, in the non-dominated solutions obtained, the sectors are more geographically separated when the values of the objective function improve.

1 Introduction

Location-routing problems (LRPs) have been widely discussed in the literatures. The proposed methods to solve these problems can be used as a principal tool for making decisions in the supply chain management [2, 6-9]. Sectorization is one of the methods that has been used to deal with multi-objective location-routing problems (MO-LRPs) [1, 5]. Based on sectorization, a geographic territory is divided into sectors according to several criteria and after the definition of a certain number of distribution centres (DCs), a route is defined to meet the demands of customers in each sector. The objective functions used in previous studies for sectorization, such as minimizing the number of opened DCs, the variances of compactness, distances and demands in sectors [10-12], are not enough to achieve geographically separated sectors. If the sectors are geographically separated, the total distance on the defined routes is also thought to de decreased. So sectors separation, attending the minimization of the overlap, is relevant for practical purposes. Herewith, different from previous studies, we decided to propose and apply a new objective function, which must be minimized. We solve the five-objective problem using the non-dominated sorting genetic algorithm (NSGA-II) that has been proposed by Deb et al. in 2002 [4, 13]. A new benchmark was generated, for which NSGA-II finds multiple non-dominated solutions. The results show that as the value of the new objective function decreases, the sectors have less overlap. As an example, we present a comparison between two obtained solutions. Although the values of the objective functions of the selected solutions are different, they are between the

acquired non-dominated solutions. Focusing on the values of the new objective function and the separation of sectors for the solutions, we depict the two solutions in a figure and show the efficiency of the new objective function.

In Section 2, we summarize the used objective functions and the solution method. The experimental results and their discussion appear in Section 3. Then, some conclusions and future work are presented in Section 4.

2 Problem Description and Solution Method

In this section, we explain how to handle the problem of geographic separation of sectors when solving MO-LRPs. The solution method starts by creating sectors composed by customers, whose demands are met by a vehicle starting from a DC and returning to the same DC. Since one vehicle is allocated for each sector, the total demand of customers in a sector can not exceed the capacity of one vehicle. When the sectors become formed, the locations of DCs also become settled at the centroid point of each sector. To define a route for each sector a travelling salesman problem (TSP) is solved with the nearest neighbor heuristic, in which route is defined starting at a DC, visiting the nearest customer at each stage, repeating this process until all customers are visited, and returning to the DC.

Some of the terminology and notations used in the paper are summarized in Table 1.

To solve the problem, we adapt the solution method in an NSGA-II described in references [4, 13]. In this algorithm, at first, an initial population is generated. During iterations, the offspring population is created using the crossover and mutation operators, which forms the merged population with the parent population. Using the non-dominated sorting and crowding distance operators, the merged population is sorted and new individuals are selected as many as the population size. These steps continue until the stopping criterion is met.

As defined in Equation 1, f_1 is the number of opened DCs.

$$f_1 = \sum_{k=1}^{K} x_k \qquad (1)$$

where

$$x_k = \begin{cases} 1, & \text{if } DC \ k \ is \ opened \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

Sectors are expected to be balanced in terms of demand and distance. So, the other objective functions

of the problem are the variances of demands, compactness and distances in sectors, defined as in 3, 5 and 6.

$$f_2 = \frac{1}{M-1} \sum_{m=1}^{M} (DE^m - \bar{DE})^2 \qquad (3)$$

where $DE^m = \sum_{i=1}^{I} DE_i \times y_i^m$ and $\bar{DE} = \frac{\sum_{m=1}^{M} DE^m}{M}$ and

$$y_i^m = \begin{cases} 1, & \text{if } customer \ i \ belongs \ to \ sector \ m \\ 0, & \text{otherwise.} \end{cases} \qquad (4)$$

$$f_3 = \frac{1}{M-1} \sum_{m=1}^{M} (CP^m - \bar{CP})^2 \qquad (5)$$

where $CP^m = \frac{CE^m}{CE_{max}^m}$ and $\bar{CP} = \frac{\sum_{m=1}^{M} CP^m}{M}$.

To calculate CE_{max}^m, the coordinates of the centre point of each sector are considered. They are the average of the coordinates of the customers in the sector.

$$f_4 = \frac{1}{M-1} \sum_{m=1}^{M} (DS^m - \bar{DS})^2 \qquad (6)$$

It is assumed that all customers are connected with each other.

$DS^m = \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{j=1}^{I} DS_{ij} \times z_{ij}^m$ and $\bar{DS} = \frac{\sum_{m=1}^{M} DS^m}{M}$.

$$z_{ij}^m = \begin{cases} 1, \text{if } the \ path \ from \ customer \ i \ to \ customer \ j \ is \ on \\ \quad a \ defined \ route \ in \ sector \ m \\ 0, \text{otherwise.} \end{cases} \qquad (7)$$

To reach geographically separate sectors, the fundamental aim of this work, we propose a fifth objective function to minimize, defined as $f_5 = \bar{D}$. It represents the average of the maximum overlap between sectors. This measurement is similar to the Davies-Bouldin index that is used to evaluate clustering algorithms [3]. The overlap between sectors m and n, D^{mn}, is defined as the ratio of the sum of the farthest distances between the centers of the sectors and their customers to the distance between the centers of the sectors. For each sector m, its maximum overlap with other n sectors, D^m, is calculated. The objective function is the average D^m, $\forall \ m \in \bar{M}$. The steps to calculate the proposed objective function are also outlined in Figure 1.

The objective function, f_5, tries to reduce the farthest distance in each sector and to increase the distance between the centre of the sectors, as much as possible.

Table 1: Used notations

f_1	Number of opened DCs
f_2	Variance of demands in sectors
f_3	Variance of compactness of sectors
f_4	Variance of distances in sectors
f_5	Average of the maximum overlap between sectors
$i, j \in \bar{I} = \{1, ..., I\}$	Index of all customers
$k \in \bar{K} = \{1, ..., K\}$	Index of DCs
$m \in \bar{M} = \{1, ..., M\}$	Index of sectors
CE^m	Total distance between the centroid and customers in sector m
CE^m_{max}	Distance between the centroid and farthest customer in sector m
CE^{mn}	Distance between the centres of sectors m and n
CP^m	Compactness of sector m
D^{mn}	Overlap between sectors m and n
DE_i	Demand of customer i
DE^m	Total demand of customers in sector m
DS_{ij}	Distance of path from customer i to customer j
DS^m	Total distance along the defined route in sector m
SE^m	Sector m
VC	Capacity of each vehicle
x_k	Decision variable about if DC k opened or not
y_i^m	Decision variable about if customer i belongs to sector m or not
z_{ij}^m	Decision variable about if there is a path from customer i to customer j on the defined route for sector m

So, to be minimized based on the Pareto optimality concept, the objective function of the problem is as in Equation 8.

$$f = Min(f_1, f_2, f_3, f_4, f_5) \qquad (8)$$

Some constraints are considered. Each customer must be assigned to only one sector, which is imposed by Constraint 9.

$$\sum_{m=1}^{M} y_i^m = 1, \ \forall i \in \bar{I} \qquad (9)$$

It is assumed that the fleets are homogeneous, i.e. the capacity of the vehicles is the same. Therefore, there is no need for a decision variable for assigning the vehicles. However, the total demand of customers in each sector must be less than or equal to the capacity of each vehicle, which is imposed by Constraint 10.

$$DE^m \leq VC, \ \forall m \in \bar{M} \qquad (10)$$

3 Experimental Results and Discussion

Since a DC is opened in the centre of each sector and a vehicle is assigned to it, the number of vehicles shows the maximum number of sectors that can be formed, which is also the maximum number of DCs that can

be opened. We create a benchmark as 15×8, which is indicated as *Number of customers × Number of vehicles*. The customer demands and their coordinates are generated according to distributions $U(10, 100)$ and $U(0, 1000)$, respectively, which are discrete uniform distributions. When the demands of customers are defined, the total demand is calculated and then the capacity of each vehicle is defined as *1.3 × round (total demand / number of vehicles)*. To solve the problem, we implement NSGA-II algorithm in MATLAB R2019b environment on an Intel Core i7 processor, 1.8 GHz with 16 GB of RAM. The parameters of NSGA-II are: *population size=500, number of iterations=100, crossover rate=60* and *mutation rate=0,1*. NSGA-II finds multiple non-dominated solutions for this benchmark. Table 2 includes two solutions, which are randomly selected from the non-dominated ones, to illustrate that the new objective function can improve separation between sectors. To better understand such improvement, we visualize the solutions as in Figure 2, where the blue circles are customers, the yellow squares are opened DCs and the coloured lines represent the routes in each sector. As shown in Table 2, the value of f_5, the new objective function, for solution 2 is less than the value for solution 1 and, as seen in Figure 2, sectors are more separate in solution 2.

Table 2: Two obtained results

	Solution 1					Solution 2				
	f_1	f_2	f_3	f_4	f_5	f_1	f_2	f_3	f_4	f_5
15×8	6	2360.99	0.002	8427.24	13.70	6	621,51	0,005	24592.51	1.32

As seen in Table 2, in terms of f_4, which is the variance of distances in sectors, solution 1 has a better value compared to the other one. But it should be considered that while the total distance on the routes is 8392 for this solution, it is 2554 for solution 2, which indicates that when sectors are geographically separate, better solutions can be acquired in terms of distance. This matter is an advantage of the geographic separation of the sectors.

Let us mention that 375 non-dominated solutions are attained for the benchmark, in which as the value of the new objective function decreases, the sectors are less overlap. All details about the benchmark, the obtained non-dominated solutions and their ranking are accessible via the corresponding author's email address.

4 Conclusions and Future work

In this study, we focused on the geographical separation of sectors in MO-LRP. Common objectives used to solve this problem based on sectorization are minimizing the number of opened DCs, the variances of compactness, distances and demands in sectors. However, by only using these objective functions, separate sectors may not be achieved. Therefore, different from previous studies, we proposed a new objective function to geographically separate sectors. We generated a benchmark and applied NSGA-II to solve it. The algorithm found multiple non-dominated solutions, which had different values in terms of the new objective function. A comparison showed that the sectors were more separated as the value of the new proposed objective function decreased. As an example, we selected and presented two non-dominated solutions, considering the value of the new objective function. We visualized states, in terms of separation, which also showed that the proposed objective function is effective for geographic separation. It was also observed that if the sectors are geographically separated, the total distance on the defined routes is reduced. This matter shows the importance of the geographical separation of the sectors.

In the defined objective function, at first, for each sector, its maximum overlap with other sectors is calculated. Then, the average of this value for all sectors is calculated and minimized. This doesn't guarantee to reduce the farthest distance in each sector and as well as to increase the distance between the centres of the sectors. But since it reduces the average overlap, it can provide good results in terms of separation of the sectors.

In future studies, we plan to investigate how different objective functions affect geographic sector separation.

Furthermore, with the integration of multi-objective evolutionary optimization algorithms and multi-criteria decision-making techniques, we intend to design a decision support system to solve various MO-LRPs.

Acknowledgments
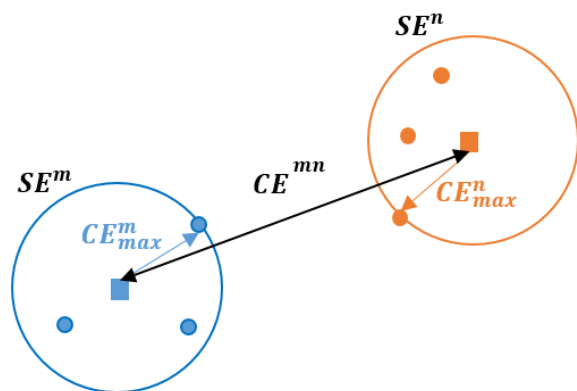
1) For each sector SE^m, for all SE^n that $m \neq n$, calculate

$$1.1) \ D^{mn} = \frac{CE^m_{max} + CE^n_{max}}{CE^{mn}}$$

$$1.2) \ D^m = Max \ D^{mn}$$

2) Calculate $\bar{D} = \frac{1}{M} \sum_{m=1}^{M} D^m$

3) As an objective function **Minimize** $\bar{D}$

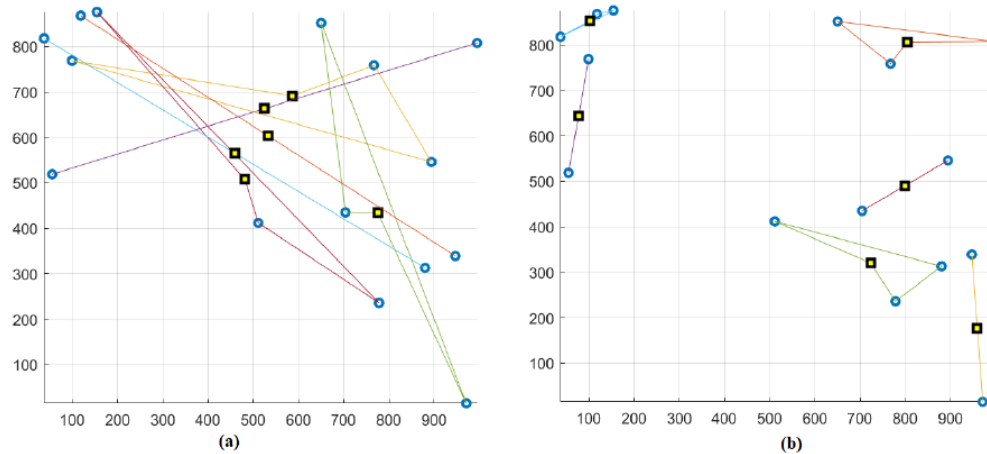Figure 1: The new objective function calculation steps

Figure 2: Results obtained for (a) solution 1 (b) solution 2 of Table 2

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Contribution of Authors

Aydin Teymourifar, Ana Maria Rodrigues, and José Soeiro Ferreira: conceptualization, investigation, implementation of the computer code and supporting algorithms, writing, review and editing