Accelerating column generation approach for the crew pairing problem with time windows

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Abstract: - The airline crew pairing problem with time windows is solved using a mixed linear programming method. The problem is to cover cost of all flights of the company. Given the large size of the problems encountered in industry, these models are solved by an approach based on column generation that can handle implicitly all feasible solutions and a master problem determining the best solution. We present a new approach to improve the acceleration of the method of column generation problem for the construction crew pairing, it is projected in each arc, the resources on a vector of dimension less using a Lagrangean relaxation algorithm to determine the coefficients of the projection arc combined with an algorithm for re-optimization, well generates a subset of complementary solutions to the master problem. The computational experiments were conducted on a broad range of instances, many of them published in the literature. They show a significant reduction of the number of column generation iterations and computing time.

Key-Words: - Column generation, Crew pairing problem, Mathematical programming, Lagrangean relaxation

1 Introduction

In the airline industry, optimization and automation of the crew pairing is a major financial and organizational challenge. The problem is to cover cost of all flights of the company, programmed over a given time, with crews trained staff cockpit (pilot, co-pilots) and flight attendants (stewardesses, stewards) . At intervals of several days (of the order of the week), each crew from the base to which he is assigned, connects a number of flights and returned to base. This sequence of flights back to the base is called rotation. The crew pairing of an airline is extremely restricted by international regulations, national and domestic labor, and the limited availability of resources.

These constraints make the problem particularly difficult to solve. The use of models and optimization software for this problem enables large companies to make substantial financial gains. It is not uncommon that a reduction of one percent on the total cost of rotations translates into tens of millions of dollars of savings for large companies [1], where research, basic and applied abundant on the subject. The general problem of airline crew pairing with time windows (ACPTW) can be formulated as a feasible flow problem minimum cost in a multiple network, with additional variables and constraints of time windows.

Finally, note that time windows make the problem (ACPTW) NP-hard. The terms of network construction rotations eligible for calculating the cost of rotations and the associated mathematical program are presented in Section 2, Section 3 provides an overview of classical techniques of resolution, including the method of column generation, whose associated sub problem is treated in Section 4, in addition to our contribution. Section 5 presents some numerical results. To Section 6 concludes the paper

2 Presentation of the problem

All flights covered by the crews is denoted $V = (1, ..., N)$. The flight schedule and schedules are established partners on an almost-certain about the period of the order of the month or week depending on company size. The term flight Associate each element $i \in V$ in some cases unfair, since i can actually represent a sequence of flights aggregate and indivisible, i.e., can not be covered by a single crew in its entirety. Often too, the task to cover a crew is not the only flight but a flight service may start before and end after the flight itself, in order to include the time of preparing the aircraft and the accompanying time passengers, for example. However, we maintain this terminology theft, for readability. We know for each flight $i \in V$:

(i) the departure time $t^\nearrow(i)$,
(ii) the time of arrival $t^\searrow(i)$,
(iii) the airport of departure $a^\nearrow(i)$,
(iv) the airport of arrival $a^\searrow(i)$.

A rotation should begin and end at one of the bases of the company. The set B of bases is usually composed of large platforms called interconnection hubs. The problem building rotations in the air has often resource constraints on rotations. To address these constraints, valid for each rotation individually, a classical model associates with each crew subnet constructed as follows.

The airline crew pairing with time windows (ACPTW) can be modeled, if the cost function is linear, the Linear Programming in mixed variables. We have a feasible flow problem minimum cost on all subnets with varying binary variables and continuous flow of resources:

$$\min \sum_{k=1}^{K} \sum_{(i,j) \in A^k} c_{ij}^k x_{ij}^k \qquad (1)$$

$$\sum_{k=1}^{K} \sum_{j:(i,j) \in A^k} x_{ij}^k \geq 1 \ for \ i \in V = \{1, \ldots, n\} \qquad (2)$$

$$\sum_{i:(o^k,i) \in A^k} x_{o^k i}^k = 1 \ for \ k \in \mathcal{K} \qquad (3)$$

$$\sum_{i:(i,d^k) \in A^k} x_{id^k}^k = 1 \ for \ k \in \mathcal{K} \qquad (4)$$

$$\sum_{i:(i,j) \in A^k} x_{ij}^k = \sum_{l:(j,l) \in A^k} x_{jl}^k \ for \ j \in V^k \qquad (5)$$

$$T_i^{k,q} + t_i^{k,q} - T_j^{k,q} \leq M(1 - x_{ij}^k) \ for \ (i,j) \in A^k, k \in \mathcal{K} \quad (6)$$

$$a_i^k \leq T_i^k \leq b_i^k \ for \ i \in V^k, k \in \mathcal{K} \qquad (7)$$

$$x_{ij}^k \in \{0,1\}, T_i^k \geq 0 \ for \ (i,j) \in A^k, k \in \mathcal{K} \qquad (8)$$

The objective (1) minimizes the total cost of rotations. Constraints (2) express the coverage of each flight by at least one crew if only one crew is allowed per flight is forced to equal. Constraints (3-5) define a path structure in the subnet G^k: transition from a flow unit (3 or 4) and flux conservation at the vertices (5). Constraints (6-7) are the constraints associated with each rotation. Constraint (1.6), where M > 0 is a parameter very large, can also be found under the following nonlinear form:

$$x_{ij}^k(T_i^k + t_i^k - T_j^k) \leq 0 \ \ (i,j) \in A^k, k \in \mathcal{K} \qquad (9)$$

The inequality in (6) or (9) states that the waiting is allowed for the crew, otherwise the constraint is written to equality. This constraint yields accumulated consumption of resource q at node j, since we have:

$$T_j^k = \max(a_i^k, T_i^k + t_i^k)$$

Constraints (7) are constraint bounds at the nodes of the network (time windows for example). Note that the constraints (3-7) are local constraints valid for one subnet G^k.

Only coverage constraints (2) are global constraints linking the K sub-networks. The relaxation of these binding constraints and the decomposition of the original problem by sub-network will be an interesting option for resolution.

Finally, note that resource constraints (6-7) make the problem (ACPTW) NP-hard. Even the feasibility problem associated is NP-complete.

3 Solving approaches

3.1. Principles of decomposition
There are two types of constraints in the system (2) - (7):
(i) the coverage constraints (2), said bonding or global, binding all crews $k = 1, \ldots, K$,
(ii) the constraints (3) - (7) of each crew $k \in \{1, \ldots, K\}$ *and defining a legal road.*

The matrix associated with constraints (3) - (7) is block diagonal, and the objective (1) is separable (for linear), solving the continuous relaxation of this model may be based on the decomposition of Dantzig-Wolfe . In this type of decomposition, the constraints (3) - (7) define K independent sub-problems and global constraints (2) are stored in the master problem. In a schema type column generation, it is alternately solving the master problem and the K sub-problems. For a complete solution, this scheme can be applied at each node of the search tree. The major difficulty lies in solving sub-problems whose state space can grow exponentially with the number of time windows, making essential use of heuristics. On the other hand, the convergence of the scheme of column generation is sensitive to the quality of solutions provided by the resolution of these sub-problems; the effective resolution of instances from real industry needs to find a good compromise between Quality solutions and time resolution of sub-problems. In what follows, we detail the general principle of column generation for the problem (ACPTW).

3.2 Column Generation, master problem and sub problem

The methods of column generation [3] have been successfully applied to problems of construction of rotations [2], [5]. In this approach, the master problem is reformulated by a coverage problem (PC) (Set Covering and Set Partitioning under the constraint of flight coverage is unequal or equal):

$$(PC) \equiv \begin{cases} \min \sum_{r \in \mathcal{R}} c_r x_r & (10) \\ \sum_{r \in \mathcal{R}} a_{ir} x_r \geq 1 \ \ i \in V = \{1, \ldots, n\} & (11) \\ x_r \in \{0,1\} \qquad r \in \mathcal{R} & (12) \end{cases}$$

Or $\mathcal{R}$ designate all eligible rotations satisfying the resource constraints and sequence between flights, c_r represents the cost of the rotation $r \in \mathcal{R}$, $a_{ir} = 1$ if and only if the rotation r covers flight i, and the variable x_r indicates binary choice whether or not the rotation r in the solution.

3.3 Resolution of sub-problem for column generation

Noting that in the case of several sub-networks $= 1, \ldots, K$, under the resolution of problem (1.21) is decomposed by sub-networks, we omit the index k and the graph of the problem will be denoted as $G = (\{o\} \cup V \cup \{d\}, A)$.

The problem of shortest path with time windows (PSPTW), is Formula as follows:

$$
(PSPTW)\begin{cases}
\min \sum_{(i,j)\in A} c_{ij}x_{ij} & (18) \\[2mm]
\sum_{i:(o,i)\in A} x_{oi} = 1 & (19) \\[2mm]
\sum_{i:(i,d)\in A} x_{id} = 1 & (20) \\[2mm]
\sum_{i:(i,j)\in A} x_{ij} = \sum_{i:(i,j)\in A} x_{ji} \quad j \in V = \{1,\ldots,n\} & (21) \\[2mm]
x_{ij}(T_i + t_i - T_j) \le 0 \quad (i,j) \in A & (22) \\[2mm]
a_i \le T_i \le b_i \quad i \in V & (23) \\[2mm]
x_{ij} \in \{0,1\} , T_i \ge 0 \ (i,j) \in A & (24)
\end{cases}
$$

To resolve this problem, Desrochers and Soumis [8] offer a dynamic programming algorithm type pulling.

Definition 1 A path from each origin o to node j, we associate a label (C_j, T_j) representing the state of its resources and cost.

Definition 2 Let (C_j, T_j) and (C'_j, T'_j) two labels associated with two feasible paths P and P' from o to j. We say that(C_j, T_j) dominates (C'_j, T'_j) (or alternatively that P dominates P' and there $(C_j, T_j) \le (C'_j, T'_j)$ (or as $P \le P'$) if and only if $C_j \le C'_j$ et $T_j \le T'_j$.

Definition 3 A label associated with a feasible path from o to d, is called effective if it is minimal in the sense of the order relation $\le$. A path is said to be efficient if it is associated to a label effective.

The dynamic programming algorithm (DPA) proceeds in three stages. In each node $j \in V$, it does the following:
1. Extension of roads (generation of labels)
2. Filtering (test for feasibility)
3. Dominance (removing labels inefficient).

For a given node j, labels are created by extending those present at the nodes i such that $(i,j) \in A$. Thus, a new label (C_j, T_j) is given by

$$C_j = C_i + c_{ij}$$
$$T_j = \max\{a_j, T_i + t_{ij}\}, q \in \mathcal{Q}$$

Considering that all the predecessors of node $j \in V$ have already milked the dominant node j can be interpreted as determining the Pareto optimal problem of a multicriteria functions:

$$
\begin{cases}
\min_{i;(i,j)\in A} \left(C_i + c_{ij} \ ; \max\{a_j, (T_i + t_{ij})\} \right) \\[2mm]
T_i + t_{ij} \le b_j
\end{cases}
$$

Let v^* its optimal value.

In a recent work Nagih and Soumis [4] propose a method of aggregation of resources for PCC-CR by projection, in each node simultaneously using an algorithm of dynamic programming and Lagrangean relaxation.

3.4 Algorithm

  Step0   initialization
  Step1   Solve PMR^k
  Step2   Solve SP^k
      - calculate feasible solutions $\Phi(u_{ij}^k)$
      -IF $\min \Phi(u_{ij}^k) \ge 0$ then stop, go to Step 5.
        otherwise go to Step 3.
  Step3 generated the best solution $\rightarrow X^k$.
  Step4   put $PMR^k = PMR^k \cup \{X^k\}$.
      Returned to Step 1.
  Step5 $\rightarrow (z_{PM}^k, x^k)$ Optimal solution

4 Numerical results

This section presents the preliminary evaluation of our approach to the problem of construction of vehicle routing with a single resource.

Solomon's 100-customer Euclidean (VRPTW) instances are used to test our algorithm. In these instances, the travel time and the Euclidean distance between two customer locations are the same and this value is truncated to two decimal places. There are six different classes of instances depending on the geographic location of the customers (R : random; C : clustered ;RC : mixed) and width of the scheduling horizon (1 : short horizon ; 2 : long horizon). In this work, instances of type 1 are discarded due to the short horizon that does not allow a significant number of routes to be sequenced to form a workday. Results are thus reported for R2 , C2 and RC2 . Due to the limitations of our exact approach, the computational study focuses on instances obtained by taking only the first 25 customers from each original instance.

Solomon's (VRPTW) test instances are modified to fit our problem. In particular, a value tmax to limit route duration is needed. This value was first set to 100 in the case of R2 and RC2, and 200 in the case of C2. The value is larger for C2 because the service or dwell time at each customer is 90, as opposed to 10 for R2 and RC2. Finally, a gain of 1 is associated with each customer and weighted by an

arbitrarily large constant to maximize first the number of served customers, and then minimize the total distance.

The results for the instances with reduced time windows are shown in Table1. In the table1, a particular instance is identified by its class and its index followed by a dot and the number of customers considered. For example RC202.25 is the second instance of class RC2, where only the first 25 customers are considered. In these table, column Problem is the identifier of the problem instance, ItrGC is the total number of iteration of (PM) solved by Simplex,Col is the total number of columns generated during the branch-and-price algorithm, T(ssp) is the computation time in seconds and Obj. is the total distance.

Problems	ItrGC	Col	T(ssp)	Obj
RC201.25	109	509	0.3	967.9
RC202.25	112	1532	201.0	961.6
RC203.25	712	1589	206.2	751.3
RC205.25	208	744	4.4	974.9
RC206.25	432	1503	3.6	977.1
RC207.25	2999	12009	518.4	819.6
R201.25	218	477	1.0	772.8
R202.25	108	1012	137.0	694.0
R205.25	1326	3211	40.1	761.2
R210.25	71	876	161.4	704.6
R211.25	57	1250	21.7	623.7
C201.25	323	3356	5.2	679.5
C202.25	3023	12430	682.8	677.3

5 Method of separation

The method of column generation is used to solve the relaxed problem at node U. The hybrid of simplex algorithm (method exists in the library ILOG) with a method called Pricing. If the solution is fractional then a separation method is applied to the problem P^u. It is to subdivide the whole set of solutions Su into two disjoint sets, this has the effect of eliminating the achievability of the fractional solution for both new issues that are the son of P^u.

6 Conclusion

In this section devoted to solving the airline crew pairing with time windows (ACPTW), we have mainly developed approaches to column generation and decomposition master problem and sub-problem. The difficulty of solving sub-problem is directly related to the number of resources, we particularly studied the techniques of reduction of space resources, and the concept of reduction is a key element of the effectiveness of the overall resolution of issue. Indeed, if in a strategic planning perspective the computation time may be less critical than the overall cost of rotations, however in an operational setting the gain on the time resolution of sub-problem becomes a major issue. The prospects of research on this problem are numerous.

Among these are the problems of rebuilding a robust solution after disruption by any hazard, planning initially constructed. These re-optimization problem of growing interest among engineers in charge of planning in the large transport companies and open up avenues of research particularly interesting and promising.

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