A Markovian Model for Adaptive E-assessment

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Abstract:- The development of Information Communication Technologies (ICT) has increased the popularity of web based learning and E-assessment. The success of any online assessment is largely dependent on the quality of the question bank from which the questions are drawn. Various techniques are available for dynamically generating questions during E-assessment with different difficulty levels. Calibrating the question bank to know the measurement characteristics of the questions is a necessary part of large E-assessment. Classification of a question involves assigning a difficulty level to each question. An adaptive E-assessment strategy has been formulated to test the proficiency in ‘Programming using C’ language. This paper deals with the application of Markov chain to assess the reliability of question classification and to classify the performance of the students based on the attainment of handling difficulty levels over a period of time.

Key-words: Adaptive E-assessment, Question Classification, Multiple Choice Questions (MCQ), Degree of Toughness (DT), Markov Chain, Steady State Probability.

1 Introduction

Student assessment is a vital part in the learning process to categorize them based on the knowledge gained by the students. The advancement of ICT in last few decades has increased use of computers in assessment through online examinations enabling quick and uniform assessment of a very large number of learners. Adaptive assessment is a popular form of computer based assessment. When it comes to assessing the depth of knowledge gained by individual learners, adaptiveness is the key functionality. Adaptive testing has great potential to make learning environment more personalized to the learners. Multiple-choice questions (MCQ) are a widely used method for an adaptive test. Different sets of questions have to be generated for different students, keeping their enthusiasm to face the test steadily [18]. This requires a large set of MCQ stored in a question bank to cater to individual student needs. The bank should be as large as possible and the difficulty level of the questions should be wide enough to cover the entire range of test takers’ ability. A good question bank should have sufficient questions to attain high measurement accuracy throughout the measurement range. Classification of questions is primarily concerned with assigning a difficulty level to each question in the bank. Thus, a high-quality question bank will contain sufficient numbers of useful questions that permit efficient, informative testing. This criterion primarily demands that at all difficulty levels there should be sufficient number of calibrated questions. Hence there is a need to calibrate the questions in the question bank with different difficulty levels. A number of adaptive assessment tools have been extensively used by academic institutions, and well known organizations for specific examinations [9], [15], [21].

An adaptive testing strategy has been formulated to test the proficiency of students in programming using ‘C’ language in an engineering college. This test has been designed to classify the students according to their ability and Intelligence Quotient (IQ). A large number of multiple questions were collected from several course experts and the questions have been classified with different difficulty levels. The purpose of classification is to ensure that students are evaluated consistently. This increases the reliability of the assessment. In most of the literature, classification has been done using Item Response Theory (IRT) models [8], [18].

A Markov chain is a mathematical system that permits transitions from one state to another in a state space. It is a random process usually characterized as memoryless; the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes. Many examples on Markov chain have been discussed in [20]. Markov Chain has been used for predicting the behaviour of customers in terms of their brand loyalty and switching from one telecom service provider to another [4]. An E- Assessment strategy and its implementation have been discussed in [13]. In [14], an overview of Bayesian Network and its application to handle uncertainty in adaptive E-assessment has been studied.

Markov Chain model has been used in different fields including education. Markov chains are especially useful to build prediction models [10], [23], allowing for the establishment of future user behaviour while users are interacting with the web sites. This is done with the analysis of previous users' behaviour with similar interests. In [3], relational Markov model has been applied to model the behaviour of website users to help in personalizing websites. Markov chains have been applied to model the web usage of students in university's website. The results indicate that web usage can be accurately modelled by Markov chain[17]. Student navigation patterns in a web based E-learning system of an educational institution to discover the critical periods of site navigation has been modelled using Markov chain. These usage profiles were used by administrators to personalize the contents of the website and improve the services to satisfy users' expectation[19]. Markov chains are especially useful for predicting models based on continuous sequences of events. Markov analysis has been used to investigate the flow process of students in an university. They have concluded that the probability of dropping out is higher for Science students than for Arts students [2]. The student flow in a higher academic institution has been investigated using Markov analysis. It has been found that the probability of progression to a higher level increases as students move on to a higher level in the system [1]. Hidden Markov models have been used to model the actions of school students for an intelligent tutoring system [7]. Similar work has been carried out to examine the effect of student learning in a computer based learning environment using Hidden Markov models[16]. Markov chain has been applied to study the progress of secondary school students based on their gender [22]. Homogenous Markov chains were used to determine the effect of teaching and learning process in educational institutions [11].

Guessing the answer is one of the important factors associated with adaptive assessment. In an assessment with MCQ, there is a possibility of students making guesses to provide the answer for the questions. Guessing could cover a wider range, from random guessing in which all options are chosen with equal probability to partial uncertainty where the student's probability of choosing some options might be higher or lower than that of choosing other options [5]. If test scores are based simply on the number of questions answered correctly at each DT level, then a random guess increases the chance of a higher score. The system cannot distinguish between the choice of answers based on knowledge and a lucky guess. Hence, students with different levels of knowledge could end up with the same score. Therefore classifying the performance of students based on the DT levels attained over a period of time will be a better measure compared to that of a normal evaluation. The paper focuses on ensuring a reliable classification of questions in the question bank and classifying the students based on their proficiency using Markov chains.

The reminder of the paper is organized as follows: section 2 discusses about Markov model, section 3 explains the adaptive E-assessment procedure, section 4 describes the Markov model for adaptive E-assessment, and section 5 concludes the paper.

2 Markov Model

A Markov chain is a sequence of random variables, which describe the states of a system S denoted by $S = \{s_1, s_2,...,s_n\}$. The process starts in one of these states and moves successively to other states. If the system is currently in state s_i, and moves to state s_j at the next step with a probability denoted by p_{ij}, this probability does not depend upon which state the system was before s_i. The next state depends only on the current state and not on the sequence of events that preceded as represented in equation (1).

$$P(s_n|s_1,s_2\ldots s_{n-1}) = P(s_n|s_{n-1}) \quad (1)$$

This conditional independence property is known as the Markov property.

The changes of states of the system are called transitions. The probabilities associated with various state changes, called transition probabilities, are denoted by P_{ij}. The whole process is characterized by a state space, a transition matrix describing the probabilities of all possible transitions, and an initial state across the state space. The process can also continue to remain in the same state during a specified transition instant and this occurs with probability P_{ii}. An initial probability distribution, defined on S, specifies the starting state. The transition probabilities are represented by a matrix P of nonnegative numbers P_{ij}, where *i* and *j* range over all states in S, which satisfy the conditions denoted in equations (2) and (3).

$$P(s_{n+1} = s_j \mid s_n = s_i) = P_{ij}, i = 1,....n, j = 1,...n \qquad (2)$$

where s_n is the current state and and s_{n+1} is the state after next transition.

$$\sum_{\mathbf{j}} \mathbf{P_{ij}} = \mathbf{1}\text{ , for each state } i \qquad (3)$$

A Markov chain can be constructed with the transition matrix, by using the entries as transition probabilities. The transition matrix shown in equation (4) gives the 1-step probability.

$$P = \begin{bmatrix} P_{11} & P_{12} & .. & .. & P_{1n} \\ P_{21} & P_{22} & .. & .. & P_{2n} \\ P_{31} & .. & .. & ... & P_{3n} \\ ... & .. & .. & .. & .. \\ P_{n1} & .. & .. & .. & P_{nn} \end{bmatrix} \qquad (4)$$

In many applications it is necessary to predict the future states, given the current state [24]. This requires knowledge of the conditional Probability Mass Function (PMF) which is contained in n-step probabilities as given in equation (5).

$$P_{ij}^{(n)} = P(s_{n+1} = j \mid s_n = i) \qquad (5)$$

where s_n is the current state and s_{n+1} is the future state.

The n-step transition probability of a Markov chain is the probability that a process in-state *i* will be in state *j* after ‘n’ additional transitions. The n-step probabilities are calculated using the Chapman-Kolmogorov equation (6).

$$p_{ij}^{(n+m)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)} \qquad (6)$$

This denotes the probability that ‘n’ time units later, the chain will be in state *j* given it is now (at time m) in state *i*. Since the transition probabilities do not depend on the time m ≥ 0, at which the initial condition is chosen, without loss of generality it can be chosen that m=0 and written as in equation (7). Also $P^{(1)} = P$.

$$P_{ij}^{n} = p(s_n = j \mid s_0 = i) \qquad (7)$$

This is denoted in terms of transition matrices as given in equation (8) and in particular as given in equation (9).

$$P^{(m+n)} = P^{(m)} P^{(n)} \qquad (8)$$

$$P^{(n)} = P^{(n-1)} P \qquad (9)$$

where $P^{(n)}$ is given in equation (10).

$$P^{(n)} = P^{n} \quad \text{for n} \geq 1 \qquad (10)$$

The associated transition matrix is depicted in equation (11).

$$P^{(n)} = \{p_{ij}^{(n)}\} \qquad (11)$$

$$P^{(1)} = P \qquad (12)$$

Now P(*i* to *j* in ‘n’ steps) = sum of probability of all paths *i* to *j* in ‘n’ steps.

At an intuitive level, being irreducible means that every point will be visited by our Markov process. Ergodicity is the study of the long term average behavior of systems evolving over time. The ergodic property states that as the number of steps are increased, there exists a positive probability measure at step ‘n’ that is independent of probability distribution at initial step zero [12]. It ensures that all measurable test functions are starting to approach their expectations to average over time. A Markov chain is called an ergodic Markov chain if it is possible to eventually get to every state from every other state with probability greater than zero. If P^n has all positive entries and probability of going from state *i* to state *j* in n steps is positive, then a regular chain is ergodic.

3 Adaptive E-Assessment

This section describes the adaptive E-assessment strategy formulated to test the proficiency of students’ in ‘Programming using C’ language. The application was developed using PHP and MySQL.

3.1 Knowledge Base Creation

A question bank consisting of multiple choice questions (MCQ) were collected from course experts. A conventional test was conducted for a group of students to initially calibrate the questions. Classification was done with the proportion of the examinees who answered each question correctly to the total population. The questions were initially classified into five difficulty levels ranging from DT1 to DT5 and are shown in equation (13).

$$DT_i = \begin{cases} i = 1, VeryEasy \\ i = 2, Easy \\ i = 3, Moderate \\ i = 4, Difficult \\ \mathrm{i = 5,\ Very\ Difficult} \end{cases} \qquad (13)$$

The initial classification of questions based on percentage of students who answered them correctly is shown in Table 1. Each question in the question bank is tagged with a DT. The DT of a question has to be updated periodically, after a broad spectrum of students undergo the tests and the question has been asked sufficiently large number of times. A difficult question is assigned a higher weightage than a less difficult question while assessing the capability of the examinees.

Table 1 - Initial classification of questions

DT	% Answered Correctly	No. of Questions
5	0 – 10	26
4	11 – 29	80
3	30 – 49	97
2	50 – 69	79
1	70 – 100	78

3.2 Procedure

The algorithm for conducting the online test using adaptive strategy is shown in Fig.1. The interesting aspect of this model is that it allows the student to initially opt for the DT of the questions soon after he logs into the system of examination. If he opts for the k^{th} DT (k=1, 2, 3, 4, 5) the system will start displaying the questions randomly from the k^{th} DT for which the candidate answers.

```
Procedure QuestionGenerate (Max_mark,
Coursecode, DT, Time)
//Coursecode - Course of exam (input given)
// maxMark - Max marks of the exam (preset value)
// DT - Degree of Toughness (initially specified by
//the student)
// time - Duration of the exam (preset value)
{for i=1 to 5
  {qCountDT[i] =0;  // qCountDT[i] stores #Que-
                        // generated in  ith DT
   ansCountDT[i]=0;}//ansCountDT[i] stores #Que-
                        //correctly answered in ith DT
   totalScore=0;     // Marks scored by the student
Do
    {count = 0;     // keeps track of #Que. answered
                     //correctly with the given DT
      for i=1 to 3
          Call qGen( );
      If ((count = 3) AND (DT< >5))
 increment DT;
        else if ((count=0) AND (DT<>1))
          decrement DT;
        else{ Call qGen( );
               Call qGen( ) ;
                 If ((count<3) AND (DT< >1))
                   decrement DT;
               else if ((count>=3) AND (DT< >5))
                   increment DT; }
       }while((maxMark>0) AND (timeavailable( )));
}
Procedure qGen( )
 {Do
   {x = RAND(getmaxQ_No(DT))
       //getmaxQ_No(DT)returns max.#Que avai-
       //lable in the db corresponding to the given DT
       //RAND( ) will generate a random Q.No. for-
       // the given DT
    } while(x is already generated for the given DT);
Display the question and Add 'x' to the list;
maxMark = maxMark - mark(DT);
increment qCountDT[DT];
If (answer(x))
          {totalScore=totalScore+mark[DT];
           increment count;
           increment ansCountDT[DT]; }
```

Fig.1. Algorithm for conducting online test

***Case 1**: If the candidate answers the first three questions of the k^{th} DT correctly, the system will automatically shift to $(k+1)^{th}$ DT, provided $k \neq 5$. When $k = 5$, the system continues to ask questions from the same level as long as the expiry of the time* *frame or the examinee has attempted questions for the prescribed maximum marks whichever occurs first*

***Case 2**: In case the candidate answers all the three questions of the k^{th} DT incorrectly, the system will automatically shift to $(k-1)^{th}$ DT, provided $k \neq 1$. It follows from the earlier logic the system continues to display from the 1^{st} DT irrespective of the number of wrong answers provided.*

***Case 3**: This case relates the situation where the examinee answers either one or two questions correctly out of the first three questions from the k^{th} DT. The system exhibits one more question from the same DT. Thus the examinee encounters a total of four questions. A total of three correct answers shifts to $(k+1)^{th}$ DT, provided $k \neq 5$;*

***Case 4**: In case the candidate answers two questions correctly out of the first four questions from the k^{th} DT, one more question from the same DT is given. A total of three correct answers out of five given questions, shifts to $(k+1)^{th}$ DT; otherwise to $(k-1)^{th}$ DT. However shifting to a higher or lower DT does not take place when k=5 or k=1 respectively.*

The score and the number of DT – wise questions asked and answered will get displayed at the end of the test.

Deciding the next question's degree of toughness is based on various factors as shown in equation (14) below:

$$DT_{i+1} = f(Q_i, DT_i, result, nDT) \qquad (14)$$

where,

Q_i is the current question with degree of toughness i,

DT_i is the current degree of toughness,

result is the outcome of the student's answer for the current question,

nDT is the number of questions answered in current session with degree of toughness i.

The system continues with the procedure until the time duration of the assessment elapses or the questions for the prescribed marks have been attempted, whichever occurs first.

3.3 Evaluation Procedure

The marks for a question in each DT are given in Table 2. It is to be noted from the table that marks increase steadily with DT.

Table 2 Marks associated with each context dimension (DT)

DT level	**1**	**2**	**3**	**4**	**5**
Mark	0.2	0.4	0.6	0.8	1.0

The maximum marks and the duration of the assessment can be set according to the needs of the Course. The test will get terminated either on the expiry of the time frame or the examinee has attempted questions for the prescribed maximum marks whichever occurs first. The score and the number of DT – wise questions asked and answered get displayed at the end of the test.

The overall process flow is depicted in Fig.2.

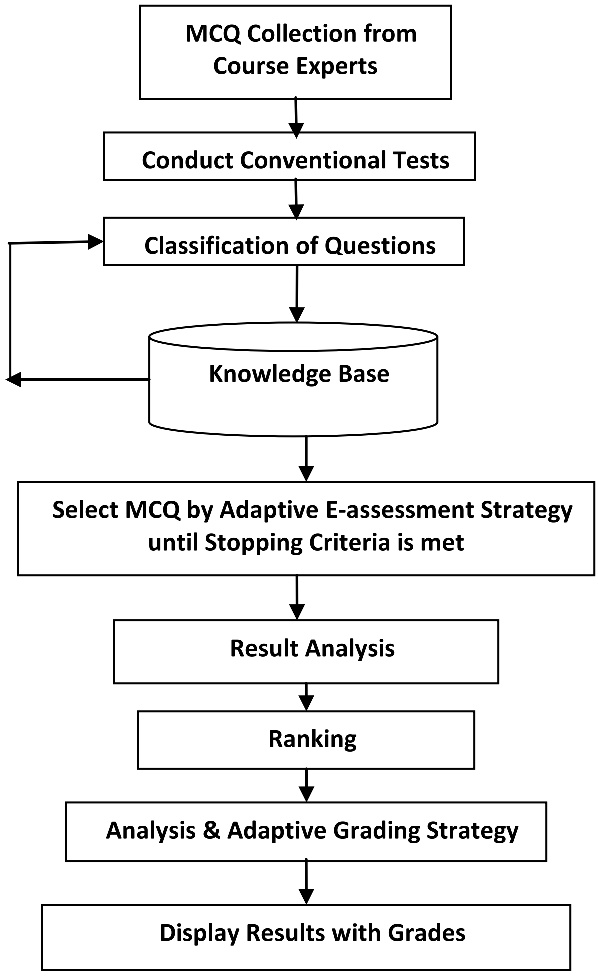

Fig.2 Process Model for Adaptive E-assessment

4 Markov Model for Adaptive E-Assessment

The adaptive E-assessment was conducted for about 200 students of an engineering college and results were collected. The sample data set showing the transition between various difficulty levels are shown in the Table 3.

We employ mathematical modelling to classify the students with appropriate IQ in various DT levels. . In this study, we have classified the students into five groups based on the DT level attained after a period of time as shown in Table 4, with DT5 being the desired level of academic difficulty (the best performers).

Table 3. Transition samples (S-id:Student id, S_1:Start State, S_n:Final State)

S-id	S_1	DT Transitions	S_n
1	1	1-1-1-1	1
2	1	1-2-1-2-1-2-1-2-1-2-1	1
3	1	1-2-1-2-1-2-3-2-1-2	2
4	1	1-2-3-4-3-4-5	5
5	2	2-1-2-1-2-3-2-1	1
6	2	2-3-2-3-2-3-4	4
7	3	3-2-1-2-1-2-1-2	2
8	3	3-4-3-2-3-2-1	1
9	4	4-3-2-3-4-3	3
10	4	4-5-4-5-4-5	5
11	5	5-4-3-2-1-2-1	1
12	5	5-4-3-4-3-4-3	3

Table 4 DT Level Classification

DT level	Student Group
DT5	Very Bright
DT4	Bright
DT3	Mediocre
DT2	Just below Average
DT1	Far below Average

4.1 Transition Probability Matrix of Adaptive E-Assessment

In adaptive E-assessment, a candidate can move across DT levels in a stepwise manner. This can be denoted as a sequence of random variables for each student describing the state of the system DT_1, DT_2,….DT_5. A candidate starting at level '*i*' (*i* = 1, 2, 3, 4, 5) will either move to level *i*+1 or come down to level *i* –1. It is not possible to move to other higher/lower levels. However shifting to a higher or lower DT does not take place when i=5 or i=1 respectively. Hence the probabilities P_{ii}=0 for $2 \le i \le 4$.

The conditional probability of making a random walk from level *i* to level *i+1* is denoted by $P_{i,i+1}$ and that of moving from level *i* to level *i-1* by $P_{i,i-1}$. The transition probability matrix of the finite state Markov chain is given in equation (15).
This follows the discrete Markov model because each state depends only on the previous state. i.e. if a candidate is at DT_i, he can either move to DT_{i-1} or DT_{i+1}. The transition between the states is viewed as a Markov chain.

$$P = \begin{array}{c} \\ DT1 \\ DT2 \\ DT3 \\ DT4 \\ DT5 \end{array} \begin{array}{c} \begin{array}{ccccc} DT1 & DT2 & DT3 & DT4 & DT5 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & 0 & 0 & 0 \\ P_{21} & 0 & P_{23} & 0 & 0 \\ 0 & P_{32} & 0 & P_{34} & 0 \\ 0 & 0 & P_{43} & 0 & P_{45} \\ 0 & 0 & 0 & P_{54} & P_{55} \end{bmatrix} \end{array} \quad (15)$$

A Markov chain is often represented as a graph on S (possibly with self-loops) with an edge going from *i* to *j* whenever transition from *i* to *j* is possible, i.e., $P_{ij} > 0$, and labelled by P_{ij}. The probability of moving from DT_i to DT_j is denoted by P_{ij}. For instance, the probability of moving from DT_1 to DT_2 is denoted by P_{12}, probability of moving from DT_2 to DT_1 is denoted by P_{21} and so on. When the state of the system is either DT_1 or DT_5, the system can remain in the same state and the probabilities are denoted by P_{11} and P_{55} respectively. Markov chain for adaptive E-assessment is shown in Fig.3.
The initial state transition probability matrix is given in equation (16). The probability $P_{i,i+1}$ is calculated as the proportion of the number of students moved from level *i* to level *i+1* to the total number of students who started at level *i.* The probability $P_{i,i-1}$ is calculated as the proportion of the number of students moved from level *i* to level *i-1* to the total number of students who started at level *i*.

$$P^{(1)} = \begin{array}{c} \\ DT1 \\ DT2 \\ DT3 \\ DT4 \\ DT5 \end{array} \begin{array}{c} \begin{array}{ccccc} DT1 & DT2 & DT3 & DT4 & DT5 \end{array} \\ \begin{bmatrix} 0.006 & 0.994 & 0 & 0 & 0 \\ 0.368 & 0 & 0.632 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.56 & 0 & 0.44 \\ 0 & 0 & 0 & 0.875 & 0.125 \end{bmatrix} \end{array} \quad (16)$$

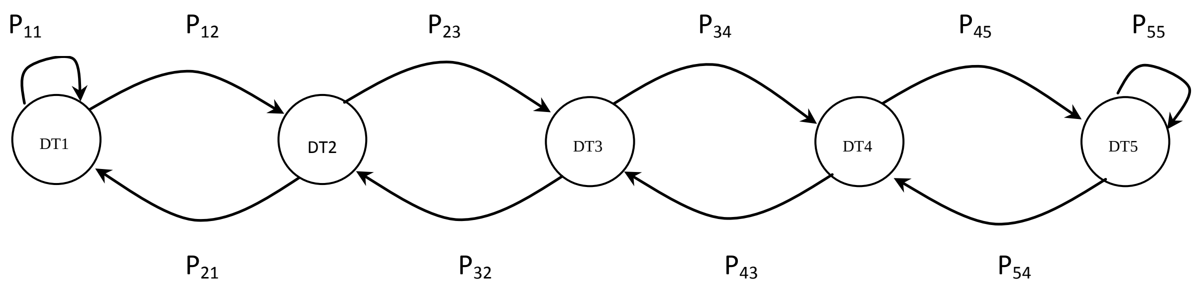

Fig.3 Adaptive E-assessment Markov chain

For levels DT_1 and DT_5 $P_{i,i}$ is the probability that a student continues to remain in the same level. In a batch of 231 students, 181 started at DT_1, out of which 180 moved to DT_2 during the first transition and 1 remained at the same level and hence the probabilities are 0.994 and 0.006 respectively. The other probabilities are calculated in a similar manner.

To predict the future states based on the current state, the n-step probabilities are calculated. High power matrices arrived to observe the candidates DT level after 10 transitions are shown in annexure 1. The conditional probability of the candidates starting at DT_i, to reach the other DT levels is indicated in the n-step probability transitions.

It can be seen that within 5 transitions, the probabilities of all DT levels are greater than zero, which clearly shows that all states are reachable from every other state over a period of time. This is an indication that the Markov chain is ergodic and the questions have been classified correctly.

In the assessment conducted, the number of students who started at each DT level is shown in Table 5.

Table 5. Percentage of students at DT levels initially

DT1	78.35%
DT2	8.23%
DT3	6.06%
DT4	3.9%
DT5	3.46%

After 10 transitions, it can be seen that , out of the students who have started in DT1, 15% could stay in DT1, 51% could move to level DT3, 20% could move to DT5. Of the students started in DT3, 48% stay in DT3 and 14%, 6%, 11%, 21% of the students move to DT1, DT2, DT4 and DT5 respectively. It is observed that most of the students are mediocre.

The comparison between the percentage of students who initially started at each DT level and the percentage of students after 10 transitions at each DT level using Markov chain is shown in Fig. 4.

This classification was arrived by designing questions which provide percentage of students in each category.

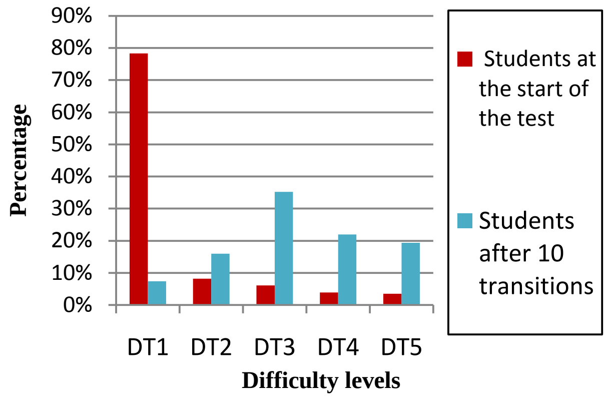

Fig. 4 Percentage of students at each DT level

4.2 Steady state Probabilities

If the state space is finite and the Markov chain is irreducible, then in the long run, regardless of the initial condition, the Markov chain must settle into a steady state. Let 'P' be the transition matrix. Then there exists a vector $\pi = [\pi_1 \ \pi_2 \ldots \ \pi_5]$ such that for any initial state i as shown in equation (17).

$$\lim_{n \to \infty} P_{ij}(n) = \pi_j > 0 \quad (17)$$

where M is the number of states and ,

π_j uniquely satisfy the following steady state equations (18) and (19).

$$\pi_j = \sum_{i=0}^{M} \pi_j \, P_{ij}\,, \quad \text{for } j = 0,1,2,\ldots M \quad (18)$$

$$\sum_{j=0}^{M} \pi_j = 1 \quad (19)$$

The vector $\pi = [\pi_1\ \pi_2\ \pi_3\ \pi_4\ \pi_5]$ is called the steady-state probabilities of the Markov chain. They are independent of the initial probability distribution defined over the states.
The probability of starting at state i, (i = 1,2,3,4,5) in the long run will settle at values that are solutions of equation $\pi P = \pi$ and represented in matrix notation as given in equation (20).

$$[\pi_1\ \pi_{2..}\ \pi_5 \begin{bmatrix} 0.006 & 0.994 & 0 & 0 & 0 \\ 0.368 & 0 & 0.632 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.56 & 0 & 0.44 \\ 0 & 0 & 0 & 0.875 & 0.125 \end{bmatrix} =[\pi_1\ \pi_2..\pi_5] \quad (20)$$

By expanding the above matrix, we have the linear set of equations(21) to (26).

$$0.006\pi_1 + 0.368\pi_2 = \pi_1 \quad (21)$$
$$0.994\pi_1 + 0.5\pi_3 = \pi_2 \quad (22)$$
$$0.632\pi_2 + 0.5\pi_4 = \pi_3 \quad (23)$$
$$0.5\pi_3 + 0.875\pi_5 = \pi_4 \quad (24)$$
$$0.44\ \pi_4 + 0.125\pi_5 = \pi_5 \quad (25)$$

$$\sum_{I=1}^{5} \pi_i = 1 \quad (26)$$

By solving the above equations, we obtain $\pi 1=0.087$, $\pi_2=0.234$, $\pi_3=0.296$, $\pi_4=0.234$ and $\pi_5=0.148$. The probability of students remaining in DT1 is 0.087, DT2 is 0.234, DT3 is 0.296, DT4 is 0.234 and DT5 is 0.148 and this is graphically represented in Fig.5. Steady state probabilities indicate that a large section of students reach DT3 followed by DT2 and DT4.

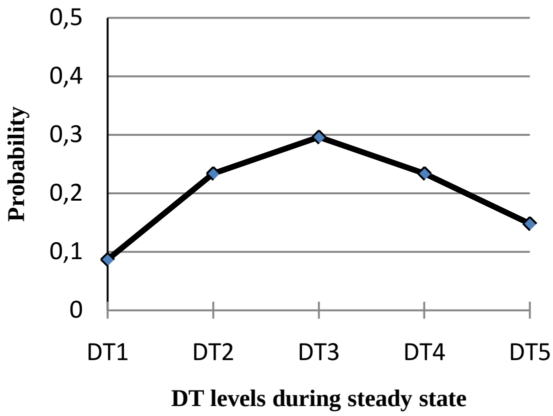

Fig.5. Steady state probabilities for Batch 1

When steady state probabilities were calculated for second batch of students, we obtain $\pi 1=0.057$, $\pi_2=0.188$, $\pi_3=0.352$, $\pi_4=0.274$ and $\pi_5=0.128$ which is graphically depicted in Fig.6. It can be seen that among the five levels, majority of students reach DT3 which is then followed by DT4 and DT2. Thus the figures show the relative performances of the 2 batches of students. In both the batches, a large section of students stay at DT3.

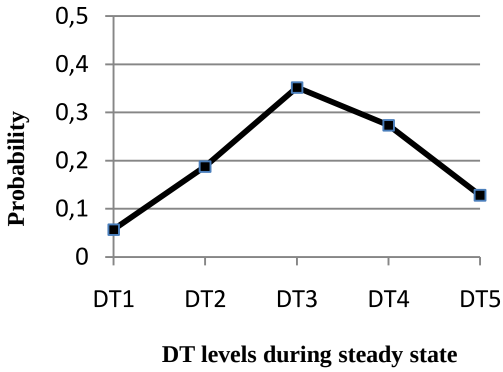

Fig.6. Steady state probability for Batch 2

5 Conclusions

There is a hue and cry about the present evaluation system. At the time when there is a question about the current evaluation system, how to classify the students is a big question. This test procedure has been specially designed to evaluate the students in real terms and to classify them according to their level of attainment. This approach classifies the students based on the DT level attained rather than the score obtained. The probability of students reaching each DT level was also calculated using Markov chains and a comparison was made between two batches of students who took the same assessment. It was found that in both the batches, more number of students could easily reach DT3 which is a mediocre level (29.6%, 35.2% for batches 1 and 2 respectively). However in batch 2 the number of students in DT4 is higher when compared to that of batch 1. The correctness of the classification of questions has also been proved using Markov chains. This classification is a better approach because it uses the transition of DT levels for classification rather than using the score. In this approach the influence of guessing the answers will not have a greater impact on the DT levels attained.

As a part of future work, the questions in the bank can be grouped concept wise and data mining techniques can be used to analyse the students' performance in various concepts. Also a comparison between Markov model and data mining methods can be made.

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Annexure 1
Transition Matrices

Table A1. 1st Level Transition

P_1

	DT1	DT2	DT3	DT4	DT5
DT1	0.006	0.994	0.000	0.000	0.000
DT2	0.368	0.000	0.632	0.000	0.000
DT3	0.000	0.500	0.000	0.500	0.000
DT4	0.000	0.000	0.560	0.000	0.440
DT5	0.000	0.000	0.000	0.875	0.125

Table A2. 2nd Level Transition

P_2

DT1	0.366	0.006	0.628	0.000	0.000
DT2	0.002	0.682	0.000	0.316	0.000
DT3	0.184	0.000	0.596	0.000	0.220
DT4	0.000	0.280	0.000	0.665	0.055
DT5	0.000	0.000	0.490	0.109	0.401

Table A3. 3rd Level Transition

P_3

DT1	0.013	0.661	0.019	0.306	0.000
DT2	0.250	0.002	0.608	0.000	0.140
DT3	0.001	0.510	0.000	0.466	0.024
DT4	0.092	0.000	0.543	0.055	0.310
DT5	0.000	0.112	0.294	0.332	0.262

Table A4. 4th Level Transition

P_4

DT1	0.243	0.023	0.590	0.009	0.136
DT2	0.006	0.565	0.007	0.408	0.015
DT3	0.182	0.001	0.580	0.023	0.213
DT4	0.001	0.311	0.147	0.399	0.143
DT5	0.037	0.067	0.394	0.221	0.282

Table A5. 5th Level Transition

P_5

DT1	0.013	0.548	0.025	0.395	0.019
DT2	0.204	0.009	0.583	0.018	0.185
DT3	0.004	0.456	0.067	0.404	0.070
DT4	0.110	0.034	0.487	0.122	0.247
DT5	0.022	0.165	0.295	0.292	0.226

Table A6. 6th Level Transition

P_6

	DT1	DT2	DT3	DT4	DT5
DT1	0.198	0.026	0.566	0.030	0.180
DT2	0.007	0.489	0.052	0.401	0.051
DT3	0.164	0.020	0.542	0.063	0.212
DT4	0.013	0.310	0.181	0.348	0.148
DT5	0.057	0.112	0.372	0.224	0.235

Table A7. 7th Level Transition

P_7

DT1	0.013	0.475	0.068	0.389	0.055
DT2	0.176	0.022	0.551	0.051	0.200
DT3	0.010	0.413	0.107	0.378	0.093
DT4	0.110	0.066	0.457	0.143	0.223
DT5	0.040	0.192	0.295	0.274	0.200

Table A8. 8th Level Transition

P_8

DT1	0.171	0.036	0.536	0.061	0.196
DT2	0.011	0.435	0.093	0.382	0.079
DT3	0.148	0.041	0.511	0.091	0.210
DT4	0.025	0.302	0.201	0.326	0.146
DT5	0.068	0.141	0.360	0.222	0.209

Table A9. 9th Level Transition

P_9

DT1	0.016	0.423	0.107	0.372	0.083
DT2	0.157	0.039	0.520	0.080	0.205
DT3	0.017	0.378	0.139	0.358	0.108
DT4	0.108	0.091	0.435	0.156	0.210
DT5	0.051	0.206	0.297	0.263	0.184

Table A10. 10th Level Transition

P_{10}

DT1	0.152	0.051	0.507	0.089	0.201
DT2	0.016	0.394	0.127	0.363	0.099
DT3	0.136	0.061	0.484	0.113	0.207
DT4	0.034	0.292	0.218	0.311	0.146
DT5	0.074	0.160	0.352	0.220	0.194