An original Continuous Hopfield Network for optimal images restoration

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Abstract: - Image restoration is a very important task in image processing. The Artificial Neural Network (ANN) approach was used to solve this problem, especially the Discrete Hopfield Network (DHN). This approach suffers from the fluctuation problem due to the use of the hard limit function as activation function. To overcome this shortcoming, we use in this work the Continuous Hopfield Network (CHN) that uses a probabilistic density as activation function. Indeed, this kind of function avoids the fluctuation behaviour and permits to extend the research area of the solution. In this regard, we propose our own energy function with appropriate parameters to obtain feasible equilibrium points. The performance of our method is demonstrated by several computational tests.

Key-Words: - Artificial neural network, Image restoration problem, Degraded image, Continuous Hopfield network, Discrete Hopfield network, Linear filtering, Fluctuation problem.

1 Introduction

Image Restoration Problem (IRP) is started since the 50s after many researches carried out to improve the quality of the images received in the space program [1]. These images exposed to many damages due to the sensor and the conditions of the take. Among the defects of the sensors, firstly there is the blur of the optical system and the integrated light on each pixel, secondly the electronic noise which affects the measurement. Image restoration is an useful and a necessary pre-processing step for many applications [2].

Several methods were proposed in the literature to solve this problem, the most important were: firstly, the classical filters used to enhance the image quality based on the convolution operation. Secondly, as image restoration can be seen as a mathematical problem, many authors have proposed analytical methods to solve this problem, among them, the partial differential equation (PDE) [3] [4] [5]. On another side, the (IRP) can be considered as an optimization problem whose finding an approximation of the minimum was the objective of several works [6],[7]. Among these approaches, we mention the use of the Artificial Neural Networks (ANN) to approximate the solution of the IRP [8], [9]. In this context, the Discrete Hopfield Network (DHN) was the first artificial network which has been used for image restoration [10].

This approach suffers from the fluctuation problem due to the use of the hard limit function as activation function. In this work, we propose a new Continuous Hopfield Network (CHN) that uses a probabilistic density as activation function. Indeed, this kind of function avoids the fluctuation behaviour and permits to extend the research area of the solution. To this end, we propose our own energy function with appropriate parameters to obtain feasible equilibrium points using the Hyperplane method [11]. The Continuous Hopfield Network (CHN) was proposed by Hopfield and Tank [12] to solve combinatorial problems; some authors have treated the Quadratic Knapsack Problem (QKP) [13], [14]. Within these papers, the feasibility of the equilibrium points of the *CHN* cannot, for the general case, be assured; Moreover, the solutions obtained are, often, not good enough.

To avoid this problem, a general methodology was proposed to solve the Generalized Quadratic Knapsack Problem (GQKP) [15]. Since the differential equation, which characterizes the dynamics of the CHN, is analytically hard to solve, many researchers used the famous Euler method. Recently, the CHN was used to solve the Travelling Salesmen Problem [16], [17], Constraint Satisfaction Problem [18] and the Placement of the Electronic Circuits Problem [19].
This paper is organized as follow: the second section describes the continuous Hopfield network. In the third section, we model the image restoration problem as a binary quadratic problem. An original continuous Hopfield network is proposed to solve the obtained problem in the fourth section. Finally, some computational experiments are represented.

2 Continuous Hopfield Network

In the beginning of the 1980s, Hopfield published two scientific papers, which attracted much interest. This was the starting point of the new area of neural networks, which continues today. Hopfield showed that models of physical systems could be used to solve computational problems. Moreover, Hopfield and Tank [12] presented the energy function approach in order to solve several optimization problems including the traveling salesman problem (TSP), analog to digital conversion, signal processing problems and linear programming problems. Their results encouraged a number of researchers to apply this network to different problems such as object recognition, graph recognition, graph coloring problems, economic dispatch problems and constraint satisfaction problems[18],[19].

The Continuous Hopfield Networks (CHN) consist of S interconnected neurons with a smooth sigmoid activation function (usually a hyperbolic tangent). The differential equation which governs the dynamics of the CHN is:

$$\frac{du}{dt} = -\frac{u}{\tau} + Tv + i^b \qquad (1)$$

With:

$u = (u_1, \ldots, u_S)^t$,

$v = (v_1, \ldots, v_S)^t$,

$v_i = g(u_i), \quad \forall i, j = 1, \ldots, \mathrm{S}$

$g(u_i) = \frac{1}{2}(1 + \tanh(u_i / u_0)), \quad u_0 > 0$

$T = (T_{ij})_{i,j=1,\ldots,\mathrm{S}}$,

$i^b = (\mathrm{i}_i^b)_{i=1,\ldots,\mathrm{S}}$,

where,

u_i is the current states of the neuron i,

v_i is the output of the neuron i,

T_{ij} is the weight of the synaptic connection from neuron j to neuron i,

i_i^b is the offset bias of the neuron i.

Definition 1

A point u^e is called an equilibrium point of the system (1) if for an input vector u^0, u^e satisfies $u(t) = u^e \quad \forall t \ge t_e$, for some $t_e \ge 0$.

Hopfield has introduced the energy function E on $[0,1]^S$ which is defined by

$$E(v) = -\frac{1}{2}v^tTv - (i^b)^t v + \frac{1}{\tau}\sum_{i=1}^{S}\int_0^{v_i} g^{-1}(x)\,dx \quad . \qquad (2)$$

Hopfield proved that the symmetry of matrix T with zero diagonal are a sufficient conditions for existence of Lyapunov function [20], therefore, the existence of equilibrium point is guaranteed. The Continuous Hopfield Networks (CHN) will solve combinatorial problems that have an energy function taking the following form:

$$E(v) = -\frac{1}{2}v^tTv - (i^b)^t v \qquad (3)$$

Let P be a combinatorial optimization problem with S variables and m linear constraints:

$$
(\mathrm{P}):\begin{cases} Min\,\frac{1}{2}\mathrm{v}^t\mathrm{T}\mathrm{v}+\mathrm{I}^t\mathrm{v} \\ subject\,to \\ Rv=b \\ v_i\in\{0,1\} \quad \forall\,\mathrm{i}=1,...\mathrm{S} \end{cases} \tag{4}
$$

Let define the following sets

- The Hamming Hypercube

$$H=[0,1]^S$$

- The Hamming hypercube corners set

$$H_c=\{0,1\}^S$$

- The feasible solutions set

$$H_F=\{\mathrm{v}\in \mathrm{H}_c /\ \mathrm{Rv}=\mathrm{b}\}$$

The energy function can be assumed as:

$$E(\nu)=\mathrm{E}^c(\nu)+E^p(\nu) \quad \forall\,\nu\in H \tag{5}$$

- $\mathrm{E}^c(\nu)$ is directly proportional to the objective function.

- $\mathrm{E}^p(\nu)$ is a quadratic function that ensure the feasibility of the solution obtained by the CHN.

In the fourth section, we propose an original continuous Hopfield network for image restoration problem. First, we model the said problem as a binary quadratic problem in the next section.

3 Image restoration problem modelling

The main propose of this part is modelling the greyscale image restoration problem in terms of a quadratic optimisation problem with binary variables. First, we define some necessary tools such as convolution operation and linear filtering.

3.1 Convolution operation and linear filtering

As it is well known, the greyscale image can be seen as a two-dimensional function, in the practical situation each image in the grayscale can be represented as a matrix of 2 dimensions where each of its components is a pixel; see the figure 1.

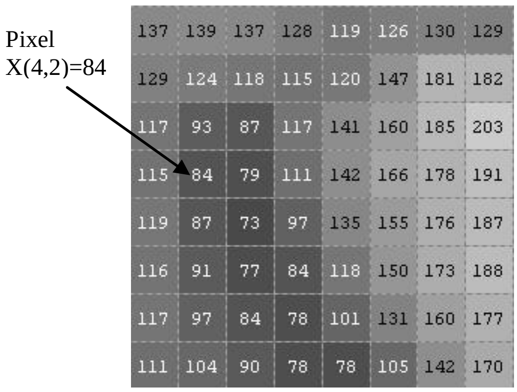

Fig. 1 Matrix representation of the image gray level functions.

The convolution operation

Convolution product is a very common operation in image processing. It presents a useful class of programs simple to use and efficient in its results. This efficacy results from the linearity of the convolution operation. In fact, the convolution is characterized by the linear neighbourhood operation and the invariance translation.

Definition

Let $X(\mathrm{i},\mathrm{j})$ be the gray levels function of image X, and let $H(\mathrm{i},\mathrm{j})$ be an operator. Generally, the convolution product noted by $(*)$ is defined as follow:

$$(H*X)(\mathrm{i},\mathrm{j})=\int_{IR}\int_{IR} H(\mathrm{i}-\mathrm{n},\mathrm{j}-\mathrm{m}).\mathrm{X}(\mathrm{n},\mathrm{m})\,dm\,dn \tag{6}$$

In the continuous case, the integral is substituted by the sum.

The convolution using a kernel

In image processing, a kernel, named convolution matrix or mask too, is a small window used for blurring, sharpening, embossing, edge-detection etc. The said kernel is chosen according to the desired processing, and then a convolution with the image under study is realized. The operation consists of the multiplication of the images pixels and theirs corresponds in the kernel. These products are then summed to find the new value of the block's center pixel; the figure 2 shows how to use the matrix convolution:

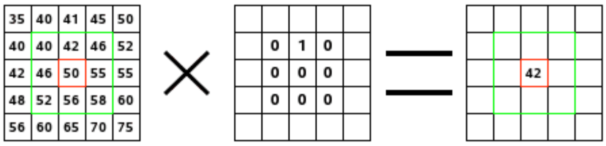

Fig. 2 The convolution using a kernel

Remarks: Two interesting remarks should be pointed out:

- In the practical situations researchers used a kernel of size 3x3 or 5x5.
- In the case of matrix convolution, the set of kernels used for each pixel constitute a block matrix with sub-bocks, therefore, the convolution product ($*$) will be substituted by a simple product.

Linear filtering

Filtering principle is to change the value of the pixels of an image, usually to improve its appearance. In practice, it is about creating a new image by using the pixel values of the original image.

Linear filtering can improve images in many ways: sharpening the edges of objects, reducing random noise, correcting the unequal illumination, correcting the blur and the motion, etc. These procedures are carried out by convolving the original image with a specific kernel.

A linear filter is said a high pass if it permits the passage of high frequencies, else it named pass bas filter.

3.2 Quadratic programming for image restoration

The problem of restoring noisy-blurred images is very important for a large number of applications [2]. In this part, we model a quadratic programming for image restoration. To this end, we define the blur matrix concept.

3.2.1 Image blurring

A blurred image can be seen as a convolution with the blur operator. The next figure shows how blurred image can be constructed

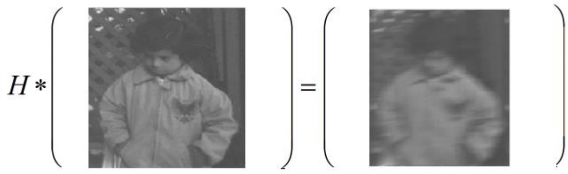

Fig.3 A blurred image using the blur operator H

In the literature, the blur matrix is described using the point spread function (PSF); this later depends on the point source. A blurring is called spatially invariant if the PSF looks the same no matter where the point source is. To take into account the boundary conditions choice (BCs), several structures are proposed [21]. In this context, we cite:

- Zero boundary conditions (ZBC): these conditions put the Dirichlet conditions at the border of the image. His structure is a Block Toeplitz with Toeplitz Block (BTTB).
- Reflexive boundary conditions (RBC): In this category, the continuity of the image is preserved while the continuity of its normal derivative is not. The resulting structure is block Toeplitz + Hankel with Toeplitz + Hankel blocks.
- Periodic boundary conditions (PBC): it's characterizes the discontinuities at the borders of the image, the corresponding matrix is Block Circulant with Circulant Blocks (BCCB) (7).

In our work, we will focus on the periodic conditions by benefiting the computational advantage of the BCCB matrix, and getting rid of the difficulty of adapting the (ZBC's) and (RBC's) matrix's with the IRP problem, for more details see [22], [23].

This is an example of a BCCB matrix:

$$H=\begin{pmatrix} (C) & (C1) & . & . & . & . & . & (C1) \\ (C1) & . & . & & & & & . \\ . & . & . & . & & & & . \\ . & & . & . & . & & & . \\ . & & & . & . & . & & . \\ . & & & & . & . & . & . \\ . & & & & & . & . & (C1) \\ (C1) & . & . & . & . & . & (C1) & (C) \end{pmatrix} \quad (7)$$

C and C1 are two circulant blocks which had their values from the PSF.

3.2.2 Modeling

Practically, the image degradation can be, adequately, modeled by a linear blur (motion, defocusing, atmospheric turbulence...) and an additive white Gaussian process. In this sense, the degradation model is given by

$$Y = H * X + \eta \quad (8)$$

Where

X: The original image.

Y: The degraded image.

H: blur operator.

η : The additive noise.

The purpose of image restoration is to operate on the degraded image Y to obtain a new image that has close to the original image X as possible, subject to suitable optimality criterion. By converting the images to a vectors and using the matrix convolution, the restored image is obtained by the following optimization problem:

$$\min_{x\in\{0,..,255\}^{M\times N}} \frac{1}{2}\|HX - Y\|^2 \quad (9)$$

Where M, N are the image dimensions and $\|.\|$ is the Euclidean norm.

The problem is estimating X from (10). It an ill-posed inverse problem in the sense of Hadamard: the problem is numerically unstable, i.e. if the quantity $\|HX - HX_0\| \to 0$ does not involve $\|X - X_0\| \to 0$ [24].

Several approaches have arisen to regulate the problem [25], [26]. In this work we will focus on the regularization of Tikhonov [27]. It's a technique which is the most used to regularize the ill-posed problems. The method consists of adding a regularization term ($\|DX\|^2$), the idea behind the added term is minimizing the gradient of X which will remove the noise.

The regularized energy function is:

$$\min_{x\in\{0,..,255\}^{M\times N}} \frac{1}{2}\|HX - Y\|^2 + \frac{1}{2}\lambda\|DX\|^2 \quad (10)$$

Where λ is a penalty positive parameter generally associated with the noise [28] , and D is a second-order differential operator; we can choose D as a Laplace operator.

In order to simplify the calculation and to reduce the Complexity, the operator D can be corrected with the same BCs as H .

4 Continuous Hopfield network for image restoration problem

The main purpose of this section is to apply the CHN to solve the IRP. To this end, we define an appropriate energy function which takes into a count the IRP particularities. To be precise, the choice of the parameters of this function must ensure the feasibility of CHN equilibrium points.

4.1 Discrete Hopfield network for image restoration problem

In this subsection, we explain the steps used for solving image restoration using the discrete Hopfield network.

Let X the Gray level function of an image of size $M \times N$, and G its maximal value, by transforming X as a concatenation of the image matrix arrays using the formula:

$$X(m) = X\left(\left(i-1\right).M + j\right) \quad (11)$$

The data is represented via the simple sum scheme which is described in [16] by:

$$X(\mathrm{i}) = \sum_{k=1}^{G} v_{ik} \quad \forall i = 1,........,\mathrm{M}\times\mathrm{N} \quad (12)$$

The network proposed for IRP is a network with $S = M \times N \times G$ neurons mutually interconnected. It is characterized by:

The set of the network state V :

$V = \{v_{ik} \ / \ i = 1,..,M \times N, k = 1,..,\mathrm{G}\}$

The weight matrix:

$T = \{T_{ik,jl} \ / \ i,j = 1,..,M \times N \ \ and \ \ k,l = 1,..,\mathrm{G}\}$

with $T_{ik,jl}$ is the weight between the neurons (i,k) and (j,l),
The activation function Heaviside:

$$v = g(u) = \begin{cases} 1 & if \quad u > 0 \\ 0 & else \end{cases} \tag{13}$$

The first step to solve the IRP using the DHN is calculating for each neuron the current state by the following formula:

$$u_{ik} = \sum_{j=1}^{S}\sum_{l=1}^{G} T_{ik,jl} v_{jl} + I_{ik} \tag{14}$$

By developing (10) and using the expression (11), we obtain the following energy function:

$$\begin{aligned}
E &= \frac{1}{2}\|HX - Y\|^2 + \frac{1}{2}\lambda\|DX\|^2 \\
E &= \frac{1}{2}\sum_{s=}^{S}(y_s - \sum_{i=1}^{S} h_{si}x_i)^2 + \frac{1}{2}\lambda\sum_{s=}^{S}(\sum_{i=1}^{S} d_{si}x_i)^2 \\
E &= \frac{1}{2}\sum_{s=1}^{S}(y_s - \sum_{i=1}^{S} h_{si}x_i)^2 + \frac{1}{2}\lambda\sum_{s=}^{S}(\sum_{i=1}^{S} d_{si}x_i)^2 \qquad (15) \\
E &= \frac{1}{2}(\sum_{s=1}^{S}\sum_{i=1}^{S}\sum_{j=1}^{S}\sum_{k=1}^{G}\sum_{l=1}^{G} h_{si}h_{sj}v_{ik}v_{jl}) + \frac{1}{2}\lambda\sum_{s=1}^{S}\sum_{i=1}^{S}\sum_{j=1}^{S}\sum_{k=1}^{G}\sum_{l=1}^{G} d_{si}d_{sj}v_{ik}v_{jl} \\
&\quad - \sum_{i=1}^{S}\sum_{k=1}^{G}\sum_{s=1}^{S} y_s h_{si} v_{ik} + \frac{1}{2}\sum_{s=1}^{S} y_s^2
\end{aligned}$$

h_{pi} And d_{pi} are, successively, the elements of H and D.
Using the identification between the equation (15) and Lyapunov functions [9], we obtain by neglecting the last term:

$$T_{ik,jl}^{DHN} = -\sum_{s=1}^{S} h_{si}h_{sj} - \lambda\sum_{s=1}^{S} d_{si}d_{sj} \tag{16}$$

And

$$I_{ik}^{DHN} = \sum_{s=1}^{S} y_s h_{si} \tag{17}$$

Two aspects must be mentioned, the first is that the weights are independent of the subscripts k and l which will reduce the complexity by neglecting repeated terms, the second concerns the self-connections $T_{ik,ik}$ which not equal to zero, and then, it contradicts the convergence criterions mentioned before.
To solve this later problem, authors proposed a decision rule [10], it was a stochastic rule which depends on the variation of energy function ΔE and neurons states Δv_{ik}, it is described as follow:

$$v_{ik}^{new} = \begin{cases} v_{ik}^{new} & \text{if } \Delta E \text{ due to state change } \Delta v_{ik} \text{ is less than zero} \\ v_{ik}^{old} & \text{otherwise} \end{cases} \tag{18}$$

Where ΔE and Δv_{ik} are defined by:

$$\Delta E = E_{new} - E_{old}$$
$$\Delta v_{ik} = v_{ik}^{new} - v_{ik}^{old}$$

The next subsection concerns the use of the Continuous Hopfield network for solving the IRP problem.

4.2 Continuous Hopfield network for image restoration problem

In this part we propose the continuous Hopfield network to solve the IRP; firstly, we propose a new energy function that makes compromise between two criterions: the objective function E and the corners constraint characterized by the following quantity:

$$\sum_{i=1}^{S}\sum_{k=1}^{G} v_{ik}(1 - v_{ik}) \tag{19}$$

In this sense, our energy function is

$$E(v) = \frac{1}{2}\alpha\sum_{s=1}^{S}\sum_{i=1}^{S}\sum_{j=1}^{S}\sum_{k=1}^{G}\sum_{l=1}^{G} h_{si}h_{sj}v_{ik}v_{jl} + \frac{1}{2}\lambda\alpha\sum_{s=1}^{S}\sum_{i=1}^{S}\sum_{j=1}^{S}\sum_{k=1}^{G}\sum_{l=1}^{G} d_{si}d_{sj}v_{ik}v_{jl} - \alpha\sum_{i=1}^{S}\sum_{k=1}^{G}\sum_{s=1}^{S} y_s h_{si} v_{ik} + \beta\sum_{i=1}^{S}\sum_{k=1}^{G} v_{ik}(1 - v_{ik}) \tag{20}$$

where α and β are penalty parameters.

It should be noted that the second Term forces the analogue neurons to take finally 0 or 1.

The CHN was proposed for two interesting reasons:

- The use of the CHN permits expanding the feasible solution areas.
- Thanks to the continuous behavior of the CHN, we can improve the image restoration by the resolution of the fluctuation phenomena in which the equilibrium point turn around the solution without stopping.

The network proposed consists of S interconnected neurons with hyperbolic tangent activation function.

A simple comparison between the old DHN (16), (17) and the proposed CHN leads to the following formulas:

$$T_{ik,jl}^{CHN} = \begin{cases} \alpha T_{ik,jl}^{DHN} & if \quad (i,k) \neq (j,l) \\ \alpha T_{ik,jl}^{DHN} + 2\beta & if \quad (i,k) = (j,l) \end{cases} \quad (21)$$

And

$$I_{ik}^{CHN} = \alpha I_{ik}^{DHN} - \beta \quad (22)$$

In the next section, we use the hyperplane method to select the feasible parameters [11].

4.3 Parameter setting

According to the equation (21) and (22), weights and thresholds associated with IRP depend on the parameters α , β , λ. Thus, In order to solve the IRP via the CHN, parameter settings must be performed. In this context we impose the next constraints:

- The constraint below is imposed to minimize the objective function:

$$\alpha \geq 0$$

- The constraint below is necessary to avoid the stability of the interior point $v \in H_c - H_F$:

$$T_{ik,ik}^{CHN} = -\alpha(\sum_{s=1}^{S} h_{si}^2 - \lambda \sum_{s=1}^{S} d_{si}^2) + 2\beta > 0 \quad \forall i = 1,..,S$$

The coming equality is a sufficient condition:

$$\frac{2\beta}{\alpha} > \sum_{s=1}^{S} h_{si}^2 - \lambda \sum_{s=1}^{S} d_{si}^2$$

To liberate this latter, we use consider the constant

$$M = \max_{i=1}^{S}(\sum_{s=1}^{S} h_{si}^2 - \lambda \sum_{s=1}^{S} d_{si}^2)$$

Then, the imposed condition is:

$$\frac{2\beta}{\alpha} > M$$

Finally, the parameters α , β and λ must verify the conditions $\alpha > 0$ and $\frac{2\beta}{\alpha} > M$.

- **Proposed algorithm**

A possible restoration is associated with an equilibrium point. This point is a solution of the equation (1) which is hard to solve analytically. Thus we use the well known Euler Cauchy method. In this regard, and basing on our theoretical studies, we propose the following algorithm for restoring the images under study:

Input: Degraded image Y of size (M,N)
The matrix H and D.
Initialization of *itermax.*
Output: desired clean image.
Begin:
For each pixel use (12), (21) and (22) to Extract T and I;
$Iter \leftarrow 1$;
X=Y;
$E_{old} = E(X)$; $E_1 = E(X)$;
Repeat

For each neuron (i,k) calculate:
Repeat
$u_{ik}^{new} = \sum_{j=1}^{S}\sum_{l=1}^{G} T_{ik,jl} v_{jl}^{old} + I_{ik}$;
$v_{ik}^{new} = \frac{1}{2}(1 + \tanh(u_{ik}^{old} / u_{0,0}))) / u_{0,0} > 0;$
$v_{ik}^{new} = v_{ik}^{old} + \Delta v_{ik}$;
$E_{new} = E(X)$;
$\Delta E = E_{new} - E_{old}$;
$X(i) = X(i) + \Delta v_{ik}$;
Until (Δv_{ik} =0 or $\Delta E > 0$ or $0 \leq X(i) \leq 255$)
End For
$E_2(X) = E(X)$;
if ($E_1 - E_2 = 0$)
Break;
Else
$E_1(X) = E_2(X)$;
$iter \leftarrow iter + 1$;
End if
Until ($iter \leq iter\max$)
Construct the image X using (12).
End

Algorithm1: Image restoration using the continuous Hopfield neural network.

- **Practical algorithm**

Practically the use of the previous algorithm is difficult on machines that have modest configurations and especially a weak processor, for example, in image with $M\times N$ pixels we need $M\times N\times G$ neurons and $\frac{1}{2}(M\times N)^4\times G^2$ interconnections which needs $(M\times N)^4\times G^2$ additions and multiplications at each iteration. For this reason the sequential update was proposed for each neuron [9]. Based on (17) and (18), we can see that weights and bias are independent of subscripts k and l; this permits to reduce the program complexity by neglecting repeated terms. In the following expression, we calculate the state of neuron u_{ik} using the precedent $u_{i,k-1}$:

$$u_{i,k} = u_{i,k-1} + \Delta v_{i,k} T_{i,i} \quad (23)$$

By the same manner, we calculate the variation of energy using the following expression:

$$\Delta E = -u_{ik}\Delta v_{ik} - \frac{1}{2} T_{i.,i.}(\Delta v_{ik})^2 \quad (24)$$

The two latest equations reduce the space and time complexities from $O((M\times N)^4\times G^2)$ and $O((M\times N)^4\times G^2\times K)$ to $O((M\times N)^2)$ and $O((M\times N)^2\times G)$, where K is the number of iterations.

In comparison with the DHN algorithm, the proposed algorithm has a continuous behavior. In fact, to calculate the neurons outputs our algorithm uses the hyperbolic tangent.

5 Computer simulation

In this section, we perform some results by the proposed algorithm. Experimental result will be compared with some filters and algorithms. For this reason, the proposed model has been applied to a reference image of type grayscale. By adopting the periodic boundary conditions, we use the image test "Cameraman", which is blurred by a 1x9 motion and added white Gaussian noise of variance σ . In order to fairly compare the performance with image restoration algorithms, we use the following metrics:

Mean Square Error (MSE):

$$MSE = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}(\mathrm{X}_1(\mathrm{i},\mathrm{j}) - X_2(\mathrm{i},\mathrm{j}))^2}{M\times N}$$

Where X_1 and X_2 denote, respectively, the restored and the original images.

Peak to Signal Noise Ratio (PSNR):

$$PSNR = 10\log_{10}(\frac{255^2}{MSE})$$

For more carrying out of the human system visual, we propose the Structural SImilarity Metric (SSIM) which respects the strong inter-dependencies between pixels.

Structural SImilarity Metric (SSIM):

$$SSIM(\mathrm{X}_1,\mathrm{X}_2) = \frac{(2m_1m_2 + M_1)(2\sigma_{12} + M_2)}{(\mathrm{m}_1^2 + \mathrm{m}_2^2 + M_1)(\sigma_1^2 + \sigma_1^2 + M_2)}$$

Where

m_i : is the averages of X_i.

σ_i : is the variance of X_i.

σ_{12} is the covariance of X_1 and X_2.

M_1 and M_2: are two variables which permits stabilizing the division with weak denominator, they depend on the dynamic range of pixel values.

The performance of the proposed model is compared against some of the most popular image processing filters, such as Median and Winner adaptive filters. In addition, we have compared our algorithm with the DHN [9], [10].

The next figure shows different results of the restored image using some methods including the (DHN) algorithm.

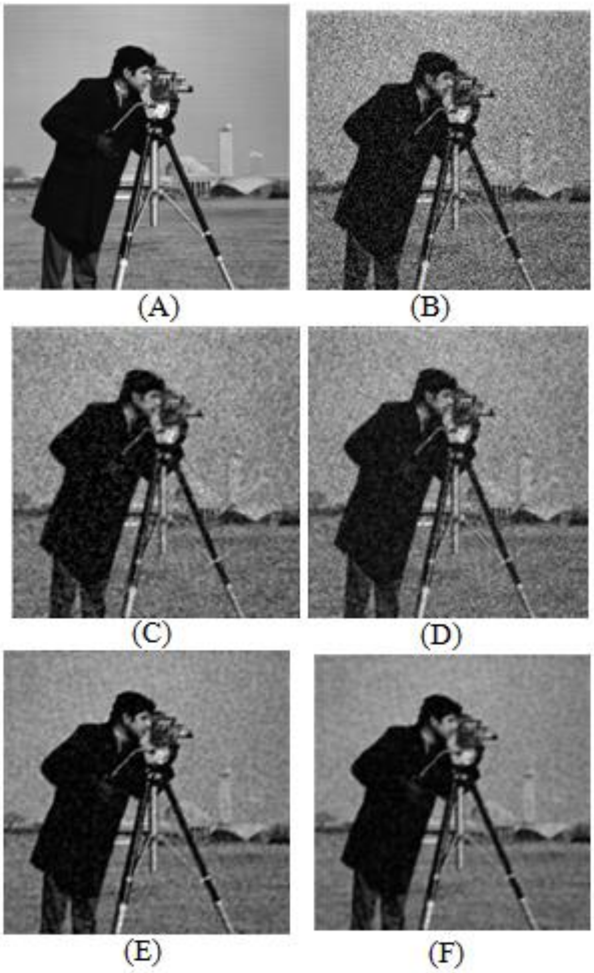

Fig.4: (A) "Cameraman", original image; (B) degraded image by a shift known invariant blur and an additive white Gaussian noise(σ =0.05) ; (C) Restored image with median filter; (D) Restored image with Winner filter; (E) restored image using DHN; (F) Restored image using our method.

In the figure 5, we see that the winner and median filters (C and D) have improved the image quality, but they are much less efficient for images with a high degradation, notably, when the variance of the noise is too wide.

The restored image using the DHN (E) is better than those obtained by the Winner and median filters (C and D); indeed, it can restore the image with a good quality while preserving edge, but it still keeping some noise effects.

Our results show the good performance of our method, especially the preservation of discontinuities. Moreover, the geometric characteristics such as corners and edges and originals contrast are well restored.

To show as close the efficiency of our method, we compare a block of 30x30 pixels from the images A, B, E and F. In comparison with the DHN method, our method makes good noise suppression while keeping as much as possible the information, see the figure 6.

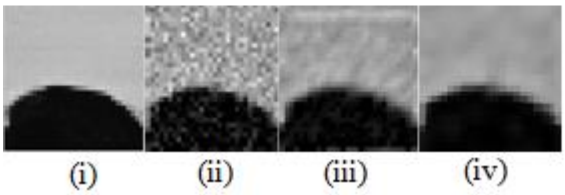

Fig. 5: (i) original block (ii) degraded block (iii) restored block using the DHN (iv) restored block using the proposed method.

For more illustration, the next table proves the performance of our method by some numerical results.

Variance σ	Metrics	Degraded image	Median filter	Winner filter	DHN algorithm	Proposed method
0.05	MSE	$3.06x10^3$	996.2445	10^3	666.70	515.32
	PSNR	13.2673	18.1471	18.128	19.891	21.01
	SSIM	0.1331	0.2717	0.2869	0.4919	0.636
0.03	MSE	$2.04x10^3$	790.6892	767.7110	655.1004	474.5810
	PSNR	15.0186	19.1507	19.2788	19.9677	21.3677
	SSIM	0.1734	0.3349	0.3592	0.5701	0.702
0.02	MSE	$1.48x10^3$	680.8201	646.80	666.7056	482.03
	PSNR	16.4115	19.8005	20.0231	20.21	21.31
	SSIM	0.2178	0.3987	0.4330	0.4919	0.64

Table.1 Metrics values of the restored images using the median, Winner, DHN and CHN method.

In the table 1, we have applied the restoration on "cameraman" degraded with blur motion and variance 0.02, 0.03 and 0.05. The three metrics used for the measurement are the betters in the last column. In one hand, the MSE of our method is the smallest obtained in comparison with the other ones. In the other hand, the proposed method generates a largest PSNR and SSIM than the other methods, which affirms the results in Fig.5.

6 Conclusion & perspectives

In this paper, we have proposed a new model for image restoration based on the continuous Hopfield network. The developed algorithm improves the noise suppression while preserving as much as possible the geometric characteristics of the image. Experimental results show that our restoration algorithm is better than some famous restoration methods. As the CHN converge rapidly to local minima, we can turn our method several times than we preserve the equilibrium points associated with the best restorations. We can use the genetic algorithm to improve the obtained results. In future work, we will investigate different connectionist architectures combined with partial differential equations, and consider different models of degradation.

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