New Modeling Methods of Spiral Bevel and Hypoid Gear Based On the CAD Surface Design Features

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Abstract: Based on the comprehensive analysis of previous research about the spiral bevel and hypoid gear, principle of forming spherical involute tooth surface was proposed with the spherical involute that was new theory applied in the field of their modeling. According to precise shape of tooth surfaces, each part of the quick and accurate derivations of the basic parametric equation of the tooth profile curve was made. Especially, it focused on the generating-line and tooth trace. In addition, on the base of some CAD features on curves surface design namely by rotating to constitute surfaces, by sweeping to found surfaces, and creating a surface from the point cloud, some new approaches were correspondingly proposed ,which were modeling with rotating generating line, sweeping from tooth profile and discrete point cloud on the tooth surface . Respectively, each approach was given certain optimization, in order to get higher accuracy, and faster efficiency, better flexibility, and provide new theoretical foundation and means for fast and accurate modeling of spiral bevel and hypoid gear.

Keywords: the spiral bevel and hypoid gear; the spherical involute; tooth profile curve; CAD features; Modeling.

1 Introduction

Bevel gears are still indispensable parts of drive systems of various machinery and equipment, in the event that there is a need for power transmission between intersecting axes [1]. Accurate tooth surface and good surface quality are critical to achieve the low-noise bevel gear drives [2]. Because of a relatively complex geometry, continuous efforts are made to streamline the design and manufacturing process [3].So accurate models of spiral bevel and hypoid gear provides the basis for digitized manufacturing such as supporting the tooth contact analysis (TCA) technology, the error correction of tooth surface technology and other key technology [4]. Therefore, model design of the spiral bevel and hypoid gear is still hot topics of research. Recently, modeling of spiral bevel and hypoid gear made a lot of achievement, summarized in three aspects: 1) Point-to-surface modeling by fitting discrete points on the tooth surface, during which the vital steps are the solution of the discrete points on the tooth surface and their obtainment based on derivation of the tooth surface equations [5]. 2) line-to-surface modeling by fitting tooth profile curves, whose core areas are the equation derivations of the tooth profile curves and the output in the three-dimensional graphics software [6]. In such cases, geometricalgear models defined in a CAD environment prove to be a good choice [7].3) Simulation process modeling according the Boolean cutting with virtual reality technology, whose foundations are equivalent conversion based on machine cutter cutting and machine tool motion parameters at the actual status [8], and program control between the cutter and gear blank [9]. There are many deficiencies in the above methods, which complete the modeling of the spiral bevel and hypoid gear in the light of traditional processing methods and gear meshing theory and with the help of three-dimensional graphics and data analysis and processing software:

1) Modeling accuracy was lower. Mainly as follows: they existed interpolation accuracy error in the extraction process of discrete point on tooth surfaces; the derivation of the fillet on the tooth profile and its processing were ignored; lacking of the accordingly optimization and reconstruction for tooth

surface after the simulation processing; there were no Measurement and correction of tooth surface deviation.

2) Modeling efficiency was lower. Theoretical knowledge were too numerous and difficult to master; there were large amount of data need to be processed and complex algorithm; processing was cumbersome, corresponding, and operating was repeated. And often result in data and process uniformity, lack of support from relevant parameters or digitizing software for rapid modeling.

3) Modeling flexible was poor. A large number of derivation calculation and drawing operations made the data fixed and can not be reused; Loss of function of the software platform and the incompatibilities between them, it resulted in the lack of systematic and parametric modeling process; there is a gap between what had got and advocated digitized modeling in the view of appropriate data sharing and integration, the NC, and network information technology.

In the actual transmission of spiral bevel and hypoid gear drives, tooth profile existed somewhere on the spherical surfaces where the base cone vertex as the center. So that, taking full account of the accuracy and completeness of the tooth profile curve, this paper avoided the traditional design theory and processing methods [10], and put up with a principle of forming spherical involute tooth surface in the modeling. Additionally, computer-aided design (CAD) surface design features as the criterion, mainly such as rotating, by sweeping, and creating the curves surface from the point cloud, a variety of new modeling and their accordingly optimization methods were explored. And then, it can make up some deficiencies in the previous modeling, and create new conditions for fast and accurate parametric modeling of the spiral bevel and hypoid gear.

2 Principle of Forming Spherical Involute Tooth Surface

2.1 Principle of forming spherical involute tooth surface

As shown in Fig.1, by means of the spherical involute theory, the principle of the forming spherical involute tooth surface can be expressed as: When the circular plane O is tangent to the base cone OK_0N, it makes pure rolling along the base cone surface, so space trajectory of the round curve K_0K_t on the circular plane forms one side of the tooth surface. During that, the curves K_0K_t is named generating line, whose endpoint K_0 and K_t respectively form the spherical involute curves of heel and toe by rotating. Similarly, if size of the radius R from the plane O and the angle of rotation θ are changed, a variety of available modelling schemes can be provided. Furthermore, the equation of spherical involute K0Kt can be expressed by the spherical deflection angle β_k at any point K as follows [11]:

$$\beta_\kappa = \frac{\arccos \dfrac{\cos\delta_k}{\cos\delta_b}}{\sin\delta_b} - \arccos\frac{\tan\delta_b}{\tan\delta_k} \qquad (1)$$

In the spherical coordinate system, it can be expressed as:

$$\vec{K} = \vec{K}(\rho,\theta,\varphi) = \vec{K}(R,\delta_\kappa,\beta_\kappa) \qquad (2)$$

Where, δ_k is the cone angle at a point of the corresponding spherical involute, and δ_b is base cone angle.

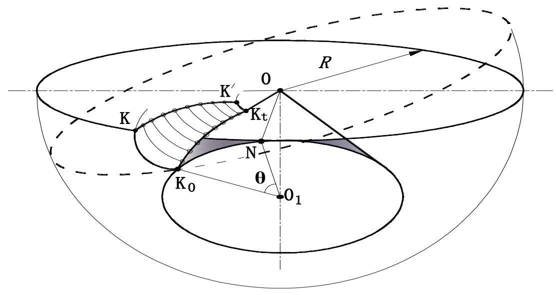

Fig.1 Principle of forming spherical involute tooth Surface

2.2 The geometric parameters between base cone and tangent circle plane

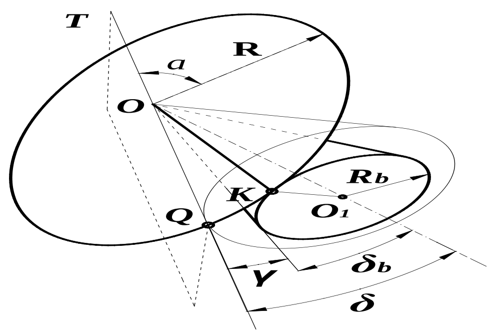

Fig.2 The geometric parameters between base cone and tangent circle plane

As forming principle of the spherical involute tooth surfaces seen, one of the key points of the method is the determination of the position of the tangent circle plane O and base cone OKO_1. What are seen in Fig.2, T represents the pitch plane where exists the pitch line; δ_b indicates the base angle; R means the radius of the tangent circle plane; R_b demotes the radius of the back plane of base cone. The lines OM and OQ show pith cone elements. The pith plane T as shown direction around the pitch cone element OQ is rotated by the angle α, and then there has been a tangent circle plane O. At the same time, the angle α between pitch plane T and tangent circle plane O whose tangent line is the base cone element OK, signifies the back cone pressure angle at the point K. what is more, γ says the angle of the pitch and base cone element. In line with the spatial geometric relations, the geometric parameters between the base cone and the tangent plane can be shown by the following formulas:

$$\delta = \arctan \frac{z_1}{z_2} \tag{3}$$

$$R_b = \frac{mz \cos \alpha}{2 \cos \delta} \tag{4}$$

$$\delta_b = \arctan \sqrt{\frac{\sin^2 \delta}{\tan^2 \alpha + \tan^2 \delta}} \tag{5}$$

$$\gamma = \arccos \frac{\cos \delta}{\cos \delta_b} \tag{6}$$

$$R = \frac{R_b}{\sin \delta_b} \tag{7}$$

Where z_1 and z_2 represents the tooth number of a pair of gear drive; and m is module.

3 Modeling by rotating the generating line to create tooth surface

3.1 The derivation of the generating line

As far as the principle of forming spherical involute tooth surface was concerned, the derivation of the generating line K_0K_t is also the key to generating directly tooth surfaces. Taking the base cone vertex as the center of the sphere to make a series of sphere with different radius which intersects with the base cone spiral curve and then taking these intersection points as starting points of spherical involute, the generating line which constitutes a tooth surface is formed. Therefore, in the process of formation of spherical involute tooth surface, the generating line can be obtained as follows:

As expressed in Fig.3, not only does a moving point K_n make uniform linear motion along the straight element O_1E, but also O_1E constant velocity rotary motion around the axis O_1O, the trajectory of unfixed point K_n known as the spiral curve of the base cone surface. This one is an Archimedean spiral curve with equal pitch of screws. the base cone vertex O as the center of the sphere, do a series of sphere with variable radius R_n intersecting with the spiral curve at a point K_n, a generating line is posed by the space rotation curve of the point K_n, as follows:

$$\begin{cases} x = R_n \sin \delta_b \cos \theta_n \\ y = R_n \sin \delta_b \sin \theta_n \ (n = 0, 1, 2 \cdots t) \\ z = R_n \cos \delta_b \end{cases} \tag{8}$$

The angle of rotation of the spiral curve on base cone, namely the relative spherical declination from the toe to the heel is expressed as:

$$0 \le \theta_n \le \frac{R \cot \delta_b \tan \beta_0}{R_b - B \tan \delta_b} \tag{9}$$

Where R means the radius of circular plane; B represents face width of gear, often select 0.30; β_0 indicates spiral angle of the toe.

Moreover:

$$R_{toe} \le R_n \le R_{heel} \tag{10}$$

Where, the R_{toe} and R_{heel} respectively demote the distance of the toe and the heel from the cone vertex.

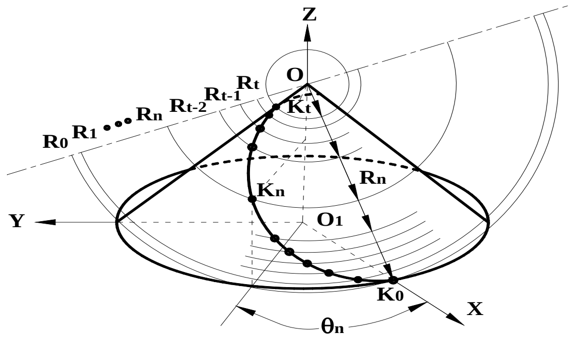

Fig.3 The derivation of the generating line

3.2 Modeling by rotating the generating line

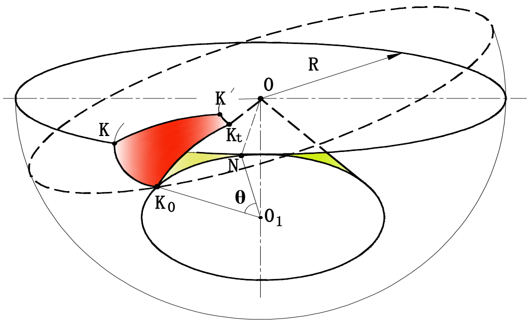

Fig.4 Modeling by rotating the generating line

In the three-dimensional graphics software, the base cone surface and tangent plane are in an assembled state as shown in Fig.4. Taking advantage of the rotation feature, above principle, the solving generating line work as rotation curves, one side of tooth profile can be drawn by rotating accordingly the angle θ. Then, changing the rotation direction, another side will be received. At this point, make Boolean operations between a single tooth surface and the gear blank, then through an annular array function, which can complete the entire gear solid modeling on the gear blank, where angle of rotation θ represents relative moving range from beginning point to the end of the spherical involute, and is obtained by the spherical involute parameter equations:

$$\begin{cases} \theta_a = \dfrac{\arccos\dfrac{\cos\delta_a}{\cos\delta_b}}{\sin\delta_b} - \arccos\dfrac{\tan\delta_b}{\tan\delta_a} \\ \theta_f = \dfrac{\arccos\dfrac{\cos\delta_f}{\cos\delta_b}}{\sin\delta_b} - \arccos\dfrac{\tan\delta_b}{\tan\delta_f} \\ \theta = \theta_f + (\theta_a - \theta_f)t \end{cases} \quad (11)$$

Where δ_a indicates angle of the face cone; δ_f demotes root cone angle; t signifies parameter, $0\le t\le 1$.

4 Modeling by sweeping from the tooth profile curves of transverse plane to create a tooth surface.

4.1 The basic composition of tooth profile curves

As represented in Fig.5, at the heel or the toe, an integral tooth profile is consisted of four segments at least: the tooth top S1; the working tooth profile S2; the tooth root S4; and transition fillet S3 which is the connection segment between S2 and S4. A complete tooth profile is also includes another side which is axially symmetric with what is in Circle as shown. In manufacturing of the gear, the tooth surface covers working tooth profile is generated by the straight edge portion of the tool. And transitional surface posed by the transition fillet is enveloped with tooth top of the cutting tools or the rounded package of tooth top according to different tool shape.

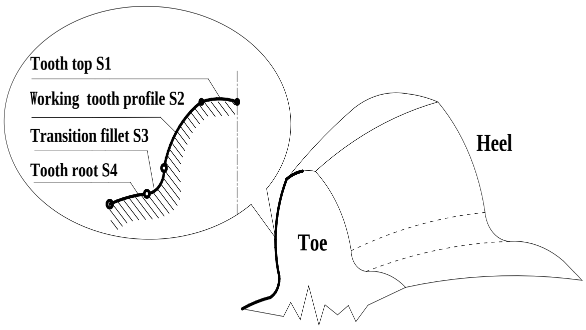

Fig.5 The basic composition of tooth profile curves

4.2 The solution of each part of the tooth profile

1) The solution of the working tooth profile. On the basis of the forming principle spherical involute tooth surface and its derivation, the working tooth profile S2 at the heel is the spherical involute and can be shown as:

$$\begin{cases} \rho = R \\ \theta = \delta_k = \delta_f + (\delta_a - \delta_f)t \\ \varphi = \beta_\kappa \end{cases} \quad (12)$$

Symmetrical side of the tooth profile to the S2 can be expressed as:

$$\varphi = \beta_\kappa - (360/z - \phi_b) \quad (13)$$

Other parameters are consistent with the S2.

2) The solution of the tooth top. As tooth top is just a part of top circle, it is simplified as:

$$\begin{cases} \rho = R \\ \theta = \delta_k = \delta_a \\ \varphi = \beta_\kappa + \delta_{as} t \end{cases} \quad (14)$$

3) The solution of the tooth root. As tooth root S4 is just a part of root circle, it is simplified as:

$$\begin{cases} \rho = R \\ \theta = \delta_k = \delta_f \\ \varphi = \beta_\kappa + \delta_{fs} t \end{cases} \quad (15)$$

Similarly, symmetrical side of the tooth profile to the S4 can be expressed as:

$$\varphi = \beta_\kappa - (360/z - \phi_f) \quad (16)$$

Other parameters are consistent with the S4.

4) The solution of each part of the toe profile curves. Above parametric equations are all compositions of the heel, however, the solution of the toe has not changed except radius vector ρ and the angle φ of rotation, thence its equation is represented as:

$$\begin{cases} \rho = R - B \\ \theta = S_\kappa \\ \varphi' = \varphi \pm (\theta_n)_{\max} \end{cases} \quad (17)$$

Where, B means face width.

4.3 The formation of tooth profile curves

In the light of the basic composition of tooth profile curves, the working tooth profile is spherical involute, so it can be formed in line with the above principle of forming the spherical involute tooth surface. In the three-dimensional graphics software, build models of tangent circle plane and base cone, then make the rotation features, the simulating of space trajectory of endpoint of the heel K_0 and K_t can be received, so as to get tooth profile curves. As for the tooth top, it is an arc on the top that can be drawn directly in accordance with the relevant parametric equations. Tooth root is the same as tooth top. The fillet curve which is a connection part needs some optimized treatments as followings:

1) When radius of the base cone is smaller than the tooth root circle. The fillet curve can be deal with variable radius rounding feature [12]:

$$R_f = 0.3m_{ne} \tag{18}$$

Where m_{ne} represents the normal module, and R_f is radius of the arc.

2) When the base cone radius is more lager than the tooth root circle. Wherefore, it can be reversely extended to create an arc at the starting point, where radius of the arc should be greater than the maximum radius *r '* of tool edge:

$$R_f \geq r' \tag{19}$$

Where, the maximum radius of tool edge can be selected from the relational operational manual in the Gleason machining [10].

4.4 Derivation of the guide line

The intersecting line between the pitch cone and tooth surface is called the pitch trace, which means the longitudinal tooth profile. The pitch line in the certain plane is called the tooth trace which is a true reflection of the arc shape of the gear. It is why that tooth trace can be used as a guide line. As shown in Fig.6, it is the tooth trace of the spiral bevel and hypoid gear, where r_0 is represented the radius of the cutter head, and β is expressed as nominal helix angle, and the distance from point O to O_1 is the cutter location. For the reason that the helix angle β' and cone distance R' at any point of the pitch line are indicated as the equation of tooth trace:

$$\sin\beta' = \frac{1}{2R'}\left[(R')^2 + 2r_0 R\sin\beta - R^2\right] \tag{20}$$

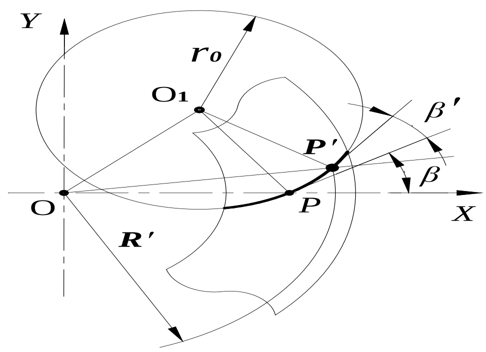

Fig.6 Tooth trace curve

4.5 The general process of accurate modeling

As shown in Fig.7, in the three-dimensional graphics software, according to the parameter equations which had been in the derivation, the general process of accurate modeling can be finished from sweeping the tooth profile curves of the heel to the toe along the guide line. This process has some following steps:

1) to create the basic gear blank;

2) to structure the tooth profile curves of the heel and the toe;

3) to draw the guide line;

4) to make some sweep and array operations, the former can complete single tooth surface, the latter is suited for the model of all gear tooth surfaces.

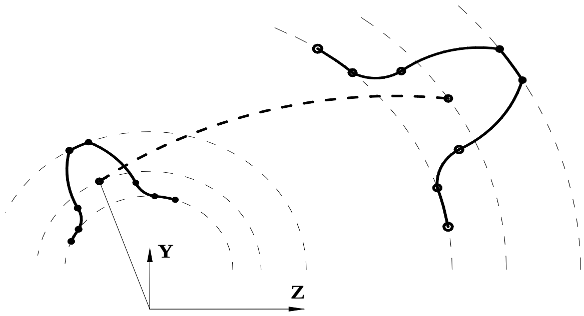

Fig.7 Modelling by sweeping from the tooth profile of the heel and the toe

4.6 Optimization methods

In order to ensure sufficient accuracy of the model design, they can be made the following optimization on the basis of the modeling methods:

a) Interpolation with the cross-section tooth profile curves string. Between tooth profile of the heel and the toe in derivation, taking the tooth trace as the reference standard, changing the amount of radius vector ρ equivalently, there will be a lot of tooth profile curves appear towards the tooth trace direction. And the changed parameter is shown as:

$$\rho = R - \Delta L \cdot n \cdot B\left(n = 1, 2, \cdots, t\right) \tag{20}$$

Where, ΔL is mutative amount for each interpolation distance, and can be chosen value appropriately, *t* is the number of curves to insert them.

2) To add guide curves appropriately. For an instance in Fig.8, the solution of the tooth top curves a and b on the top of the spiral bevel and hypoid gear, is just fast obtained by the rotation from the staring and end point of top arc of tooth profile along the tooth trace. Thence, their parameter equations can be expressed simply as:

$$\begin{cases} \rho = R - (R - B)t \\ \theta = \delta_k = \delta_a \\ \varphi = \beta_\kappa \end{cases} \tag{21}$$

Similarly, in view of the coordinates of endpoint of the fillet and root circle, the corresponding equations of the fillet curve c and d and root curve e are all solved with changing the radius vector ρ.

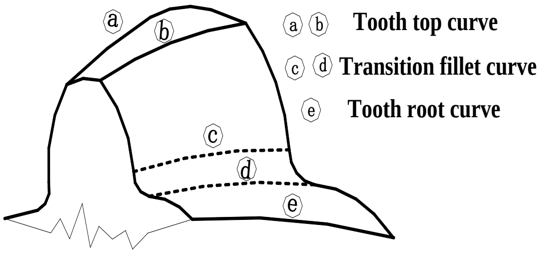

Fig.8 Added guide curves

5 Modeling from discrete point cloud to create a tooth surface

5.1 Obtainment of the discrete points

In the light of related parameter equations, firstly the equations of the U line and V line on tooth surface are solved, then the solutions of the intersecting points can be got through the simultaneous equations of the above two ones, so the discrete points of uniform distribution can be drawn on the tooth surface. As expressed in Fig.9, Select the tooth surface of the 7 × 7 uniformly distributed discrete points where has 7 U and V lines.

a) Parameter equations of the U lines. Along the generating line, N uniform division points can be gained according to the R_n of the variable value. Where it is determined by the formula, as follows:

$$N = \frac{\overline{R_0 R_t}}{R_n R_{n+1}} (n = 0, 1, \cdots, t-1) \quad (22)$$

Then, on the basis of derivative parametric equations, the equation expression of U line can be represented by the space trajectory of the derivate points in the forming process of tooth surfaces.

b) Parameter equations of the V lines. On the basis of derivate parameter equations of face tooth profile curves, similarly, N uniform division points can be acquired. In addition, Then just appropriately change the radius vector ρ, namely:

$$\rho = R - (R - B)t \quad (23)$$

The parameter equation expression of a group of V lines with equal distance at tooth length direction can be quickly done.

c) Solution of the uniformly distributed discrete points. Making the simultaneous parameter equations of the above U line and V line, the equations of the intersecting points will be solved and input the certain data processing software. Now and then, discrete points on tooth surface can be calculated and output, and taking the data of these points to constitute a file with suffix ".dat", the solution is over.

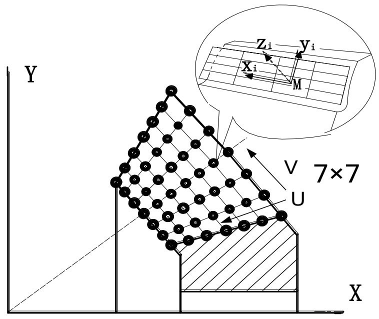

Fig.9 Obtainment of the uniformly distributed discrete points

5.2 Related settings and solution of the boundary of discrete point clouds on tooth surface

In the processes of the creation of the tooth surfaces, the order of the direction U and V can be suitable to select them, generally three or below three order expression. As well as path type, it would better to select the single one, namely to be closed along the U and V. Besides, what is more vital, the boundary of discrete point clouds on tooth surface can be obtained as follows: as seen in Fig.10, in fact, the boundary of tooth surface is surrounded by the four the curves namely the heel, the face cone, the root cone and the toe that are formed by the tooth surface in the intersecting with other four surfaces. Fatherly, four boundary curves can be solved in turn according to parametric equations obtained.

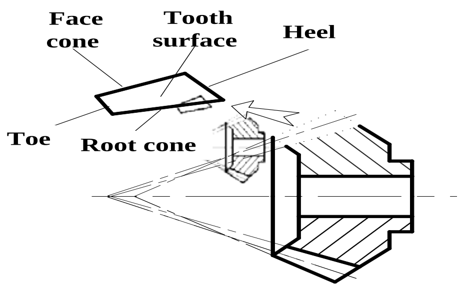

Fig.10 The boundary of tooth surface

5.3 Modeling by creating tooth surface from discrete point clouds and related optimization methods

Generally, the process of modeling by creating tooth surface from discrete point clouds is as follow: at first, they are essential to get information of the discrete points of tooth surface by reading the data files from the above step, and to create a group of its sheet body in turn. Then, it need complete the creation of a single tooth surface by utilizing the suturing techniques, and then obtain gear blanks model by the relevant parameters. At last, applying Boolean operations between a single tooth and gear blanks, a single tooth slot can be completed. And then the modeling of entire gear can be obtained with armillary array function. It is obvious that there is some deviation in above modeling. Thus, it need some following approaches to improve it as following:

1) To complete the integration of acquirement of discrete points of tooth surface and modeling for the entire gear. The discrete point not only include on the tooth concave or convex, but also on the top , the root, the heel, and the toe, which are all as a single tooth surface carrier to get the extract information of uniformly distributed discrete points of the entire tooth, in integration circumstance. Thus, it reduces the repeated extraction process and suturing work of each part of the tooth surface and improves efficiency; and also avoids cumulative error in the suturing process and the repeated processing data and improves the precision.

2）NURBS reconstruction for model of the spiral bevel and hypoid gear is made. After getting the model, it can make use of NURBS skinning technology to get optimized tooth surface and adequate accuracy [13]. There is no doubt that a NURBS surface can be defined according to tooth mesh by the intersection between U line and V line in obtainment process of discrete points. Among that, break rational polynomial can be formed by parameter variables (u, v) at the two directions the U and V line. So the NURBS surface can be defined as:

$$S(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n}p_{ij}N_i,k_u(u)N_j,k_v(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n}w_{ij}N_i,k_u(u)N_j,k_v(v)} \quad (u,v\in[0,1]) \qquad (24)$$

Where p_{ij} is vertex on the controlled mesh namely the intersection point, w_{ij} is weighting factor for the vertex, $N_i,k_u(u)$ is the No. i spline basis functions with k_u order, $N_j,k_v(v)$ is the No. j spline basis functions with k_v order.

6 Examples of Modeling Approach

As shown in Figure 11, according to principle of forming spherical involute tooth surface and CAD feature on creating surface from point cloud, the model of gear and Gaussian cloud drawing of the tooth surface after NURBS fitting which leads to an 12×14 non-uniform grid are acquired. As derived from the Gaussian could drawing with tooth surface analysis, Smoothness of tooth surface is very good, and its precision is very high. And as presented in Figure 12 that is the precision of tooth surface of three modeling approaches using a series of points by drawing compared with the theoretical by solving. What is obtained that precision of three methods are all adequate high, where the largest error of tooth surface is 0.0713mm, and the smallest one is 0.0434mm. Furthermore, the average error of three modeling approaches is respectively 0.0546mm、0.0603mm、0.0529mm, which is quite enough to reach accuracy requirements of the modeling.

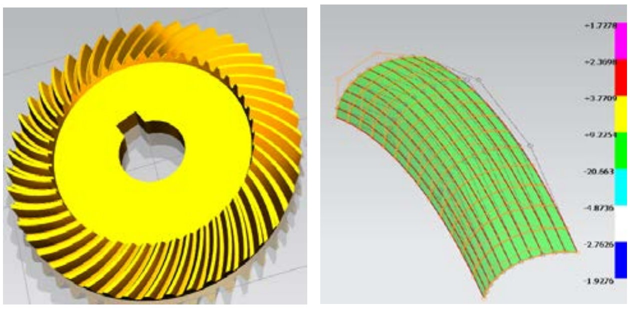

Fig.11 Model of the gear and Gaussian cloud after NURBS fitting

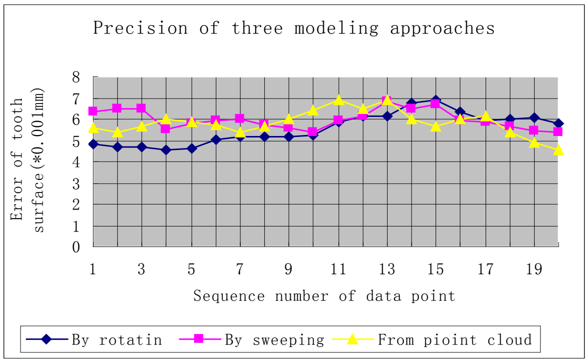

Fig.12 Precision of three modelling approaches

7 Conclusions

1) In this paper, it is able to accurately obtain the complete expression of the equations of tooth surface, based on proposed principle of forming spherical involute tooth surface. Solving overall discrete points and NURBS interpolation and reconstruction for tooth surfaces and the optimizations of the transition fillet can be finished. So they can greatly improve the accuracy of the modeling of tooth surface.

2) Proposed principle of forming spherical involute tooth surface is more simple and practical. Especially, applying CAD features on curves surface design as the main means, it can greatly reduces time of the drawing

operation and repeated calculation. Besides, based on parametric equations can be modified and a variety of optimization methods, it better improve the efficiency of the gear modeling.

3) Modified parametric equations and proposed one-to-one correspondence modeling and optimization programs may provide access to flexible and diversified design. For a example, in the different graphics software platform (such as Pro/E、UG、CATIA), taking NURBS method which is the basic geometric forms of expression and international standards of the data exchange, the parametric and diversified integrated design [14], and integration and sharing of data and information, all can be achieved, What is more, it can offer effective tooth surface information for Tooth Contact Analysis (TCA) and loaded contact analysis (LTCA) [15]. Therefore, it can improve the degree of flexible design during a lot of resources.

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