Algorithm to Calculate PCB Stepped-Impedance Low-Pass Filters for Microwave Applications

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Abstract: - In this paper, the design, simulation, and implementation of microwave stepped-impedance filters are carried out. This kind of filter can be built using a Printed Circuit Board (PCB) in the radiofrequency (RF) stage of a transmitter or receiver in radiocommunication systems. In the article, a computational tool is described in order to simplify the manufacturing process. The results section of the paper includes the software description in detail, the design of the tridimensional structure of the filter and computation of the patch that is located on the top layer in the PCB, the filter implementation, and the frequency response of the filter. The computational tool employed is a general-purpose software developed in Microsoft Visual Studio.

Key-Words: - Microstrip, PCB, Microwave filter, stepped impedance, low-pass filter, insertion losses, frequency response.

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1 Introduction

Microwave filters are very important in radio-communication systems because they are used to select the frequency interval in the propagation media that is necessary for efficient transmission, [1]. The first stage named Radio Frequency (RF) in the receiver in a radiocommunication system operates at the same frequency as the carrier. In some mobile communication systems, this sort of microstrip filter can be used in combination with the aerial of the receiver, [2], [3].

There are two different approaches to use in the implementation of the RF filters, using lumped elements and distribution elements. Normally, lumped elements can be used in intermediate or low frequencies and distributed elements are used in higher frequencies in many microwave applications. Besides, lumped elements have a lower Q-factor in comparison with distributed elements, but distributed elements take up more space [4], [5]. This technique has been employed in resonators, where the structures are divided into sections with step-transitions of circular or rectangular cavities, [6]. The cavities can be filled using a material with a higher dielectric constant, where this change produces a size reduction of the cavities, [6].

Distributed elements can be built using wave-guide resonators or microstrip lines, but wave-guide resonators need a bigger size. In this paper, a microstrip structure is used to reduce the size of the PCB, [7], [8], [9], [10].

The stepped-impedance resonators are structures that use two different sections with low and high impedance in an alternate sequence (Figure 1). The shape of the patch in the PCB has two widths in the transmission line for two different impedances, where wider lines are used for low-impedances and narrow lines for higher impedances, [9], [10], [11], [12].

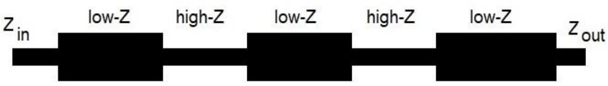

Fig. 1: Stepped-impedance structure

The paper is written in five sections. The first section is an introduction to stepped-impedance resonators, the second section is focused on the problem formulation, and it is dedicated to model description and calculation of the geometry of the filter. In the third section, the algorithm for the filter computation is presented and explained. The fourth section is related to the simulation and experimental results. Finally, the conclusions are presented in section five.

2 Problem Formulation

Distributed elements can be implemented with microstrip lines in a PCB or using stubs, but microstrip lines are smaller than coaxial stubs. The proposed algorithm is focused on microstrip lines, [13], [14].

The first step is to select the order of the filter which determines the selectivity of the circuit. In Table 1 (Appendix), we can see the g_i flat-response coefficients from order 1 to order 10. These coefficients are normalized in input and output impedances, [15], [16].

Using Z-parameters, we can define low-Z (Z_l) and high-Z (Z_h), where the numerical values of these impedances are very different. The typical model of a stepped structure is built with a microstrip line with a low-cost dielectric constant (FR-4 is one of the most common case) 20 Ω is normally equal to low-frequency and 120 Ω is equal to high-frequency.

In this kind of filter, the computation of the length and width numerical values of the microstrip lines is obtained with the approximation in equation (1), where the product obtained with phase constant and line transmission physical length is negligible. In these conditions, the T structure in a filter can be substituted by a single reactance element, [17], [18], [19].

$$\beta l \ll \frac{\pi}{2} \tag{1}$$

The equivalent circuit is simplified as shown in Figure 2. In the ideal model each T-element has been approximated by a single inductor (series element) or capacitor (shunt element). The coefficients related to the order of the filter are the values of the resonant components in the LC electrical network, [11], [12], [20].

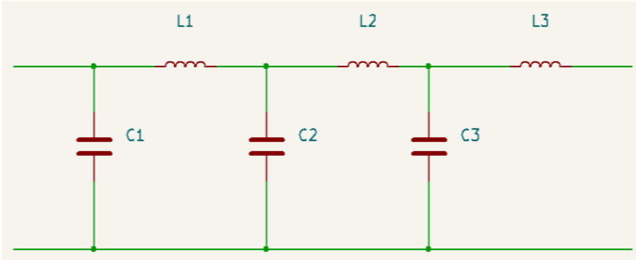

Fig. 2: LC electrical network

The phase constant and line length product are calculated with the modification of the network for specified input and output impedances, see equations (2) and (3) where L is the inductance, H; C is the capacitance, F; β is the phase constant, rad/m; l is the physical length of each microstrip line, m; R_0 is the impedance, Ω; Z_h is the high-impedance, Ω and, Z_l is the low-impedance, Ω.

$$\beta l = \frac{LR_0}{Z_h} \tag{2}$$

$$\beta l = \frac{CZ_l}{R_0} \tag{3}$$

The width of the transmission lines is calculated using very well-known formulas for microstrip lines, [1], [4], [21].

3 Results

The algorithm of the computational tool was implemented in the Visual Basic language using Microsoft Visual Studio. This program calculates the size of each distributed resonant element in the filter and the microstrip lines corresponding to input and output impedance in order to match the measurement instrument.

The phase of the structure in each step is presented in Table 2, for both radians and degrees.

Table 2. Phase of the distributed elements

	Length		Length	
g1	0.206800	Rad	11.85	deg
g2	0.589167	Rad	33.76	deg
g3	0.772800	Rad	44.28	deg
g4	0.805000	Rad	46.12	deg
g5	0.565600	Rad	32.41	deg
g6	0.215417	Rad	12.34	deg
g(in)	0.785398	Rad	45.00	deg
g(out)	0.785398	Rad	45.00	deg

Table 3 shows the results of the filter including the width and length of each resonant element.

Table 3. Electrical length and physical dimensions of the filter

Line	Z Ω	Length deg	W mm	L mm
1	50	45	2.92549	8.19274
2	20	11.85	10.7357	2.01833
3	120	33.76	0.36076	6.61868
4	20	44.23	10.7357	7.54238
5	120	46.12	0.36076	9.04334
6	20	32.41	10.7357	5.52013
7	120	12.34	0.36076	2.41997
8	50	45	2.92549	120

The stepped-impedance filter was simulated using the HFSS software, which is a Finite Element Method (FEM) that is one of the main choices in electromagnetic simulation.

The final structure of the filter is illustrated in Figure 3, where input and output impedance are 50 Ω.

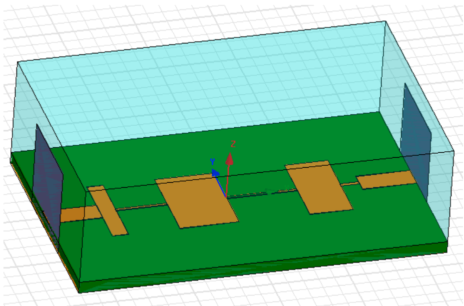

Fig. 3: Stepped-Impedance Filter Model simulated in HFSS

The frequency response of the stepped-impedance filter is shown in Figure 4, where markers 1 and 2 indicate the cut-off frequency and insertion losses, respectively. This plot was obtained in a modal response analysis with the aim of illustrating the selectivity of the low-pass filter.

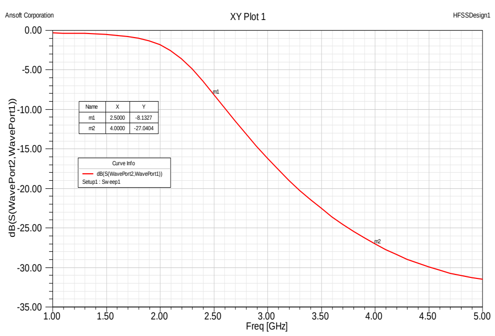

Fig. 4: Transmission parameter (S_{21}) of the simulated filter

The simulation filter was built in PCB with a dielectric constant approximately equal to 4.4, a substrate thickness of 1.544 mm, and a thickness nearly equal to 0.03 mm of copper in the top and bottom layers.

Figure 5 illustrates the appearance of the filter with standardized input and output impedances for RG-59 coaxial cables connected with forward-SMA edge connectors to a Network Analyzer (Field-Fox). This circuit was built by optical and chemical procedures with an accuracy approximately equal to 0.01 mm y the width and length of each stage of the filter.

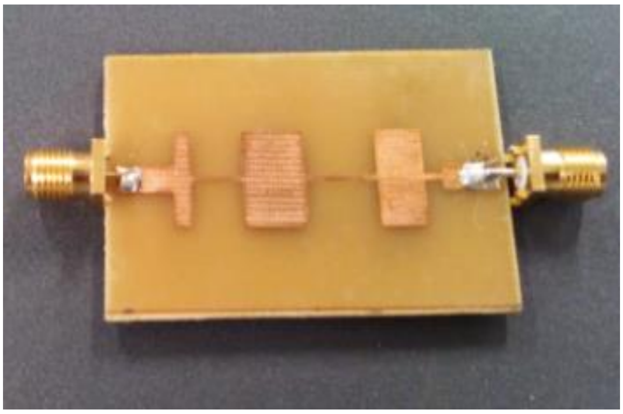

Fig. 5: Photo of the stepped-impedance filter in a low-cost PCB with a FR-4 substrate

The stepped impedance filter was evaluated with the Network Analyzer and compared with the HFSS modal response simulation using S_{21}-Parameter. This result is plotted in Figure 6.

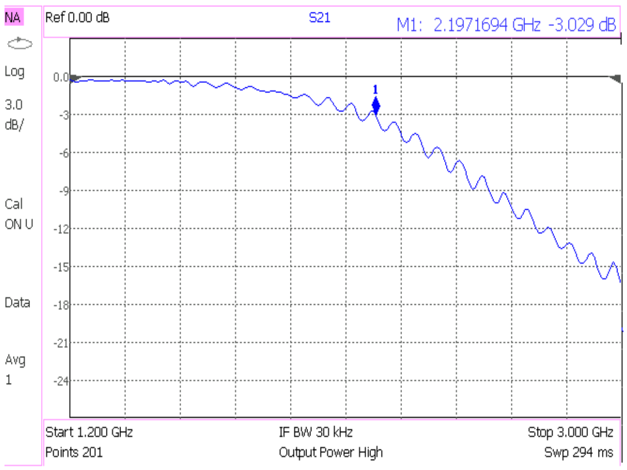

Fig. 6: Frequency response of the transmission parameter (S_{21}) of the filter

The experimental results are quite similar in comparison with the simulation process, where the cut-off frequency is 5% approximately different in comparison with HFSS software.

4 Conclusion

In this work, a microstrip stepped impedance filter was carried out using a computational tool implemented in Microsoft Visual Studio.

The FR-4 substrate was selected to reduce the cost of the PCB and the complexity of the manufacturing process.

The filter obtained in this work has a small size, but it is not compatible with many handset terminals in Personal Communication Systems (PCS). In that case, it is feasible to employ lumped elements, because the frequency is lower in comparison with many applications where distributed elements give a better performance.

There are a lot of applications where the computational tool used in this work can be applied. Right now, the software is updated to give a reduction of the size of the structure, increase the number of stages, considering high-pass and bandpass frequency response and, the addition of some auxiliary structures to modify the response of the stepped-impedance filter, [4], [6], [7], [11], [15], [16].

Another software feature improvement is related to providing the user with the lumped-elements implementation and the stub approach in order to enhance the frequency interval of the tool and, the possibility to adapt better the size of the filter in the terminal equipment.

It is important to emphasize that the main goal of this work is to have an educational tool for undergraduate engineering courses at Metropolitan Autonomous University. There are also many full-wave simulators with some numeric methods, where FEM (Finite Element Method) is one of the most important approaches. HFSS, CST, ADS, etc., [1], [22].

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Mario Castro-Romero: Formal analysis, investigation, methodology, writing – original draft. - Mario Reyes-Ayala: Conceptualization, investigation formal analysis, writing, review and editing.
- Edgar Alejandro Andrade-Gonzalez: Project administration, resources, review, validation.
- Sandra Chavez-Sanchez: Visualization, review validation.
- Rene Rodriguez-Rivera: Visualization, review validation.
- Hilario Terres-Peña: Supervision, validation, review.
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Conflict of Interest
The authors have no conflicts of interest to declare.

APPENDIX

Table 1. Filter coefficients

	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.