Channel Estimation and Equalization Using Higher Order Cumulant and Constant Modulus Algorithms

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Abstract: This paper present the problem of blind channel estimation of a non-minimum phase system using three algorithms. These algorithms play an important role for blindly parameters channel estimation. Thus studying the problem of blind channel equalization based on the presented algorithm and the filter adaptive equalizer. The simulation results in noisy environment and for different SNR values, demonstrate that the proposed algorithm is more powerful than other algorithms. In addition the proposed algorithm is more powerful in comparison to Constant Modulus Algorithm (CMA) at the blind channel equalization.

Keywords: Adaptive blind equalization, Blind channel Identification, Higher Order Cumulants (HOC), Constant Modulus Algorithm (CMA), Bran A, Proakis (B).

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1. Introduction

Channel system identification from output measurements only is a well-defined problem in several science and engineering areas such as speech signal processing, adaptive filtering, spectral estimation, communication, blind equalization…[1,9,10].

Signal processing techniques using Higher-Order Statistics (HOS) or cumulants have attracted considerable attention in the literature[7]. The information contained in the second-order statistics (autocorrelation functions) would suffice for the complete statistical description of a Gaussian signal. However, there are pratical situation where we have to look beyond the autocorrelation of a signal to extract information regarding deviations from Gaussianity and presence of phase relation. Thus the interest in higher order cumulants (HOC) or higher order statistics (HOS) is permanently growing in the last years[6]. Principally finite impulse response system identification based on HOC of system output has received more attention. Tools that deal with problems related to either nonlinearity, non-Gaussianity, or non-minimum phase (NMP) [4][20] systems are available, because they contain the phase information of the underlying linear system in contrast to second order statistics, and they are of great value in applications, such as radar, sonar, array processing, blind equalization, time delay estimation, data communication, image and speech processing and seismology [16]–[19]. In addition the blind equalization has become an important topic in digital communications. Blind methods use the received signal sequence and some a priori knowledge of the input sequence statistics. Non-minimum phase channel equalization was performed using methods based on high-order statistics or other nonlinearities that are effective only with non-Gaussianly distributed input sequences [15]. In this correspondence, we study the most popular adaptive blind equalization Bussgang algorithm (the Godard algorithm [13]) or constant modulus algorithm (CMA) [14] in the context of non constant modulus data with spatio-temporal diversity. In this paper we describe the three algorithms which are: Safi1, Safi2 and Said1 algorithms, they are used to estimate the parameters of frequency selective channels in the noisy and not noisy case. Moreover we study the blind equalization of channel by the constant modulus algorithm (CMA) and Said1 algorithms.

2. Problem Statement

The output of a FIR channel, excited by an unobservable input sequences, i.i.d. zero-mean symbols with unit energy, across a selective channel with memory p and additive noise $V(k)$ (Fig.1).

$$X(k) = h_p(k)S(k) + V(k) \tag{1}$$

When $h_p = [h(1), h(2), \dots, h(p)]$ represents the channel parameter, $S(k)$ input system and $V(k)$ is the additive colored Gaussian noise.

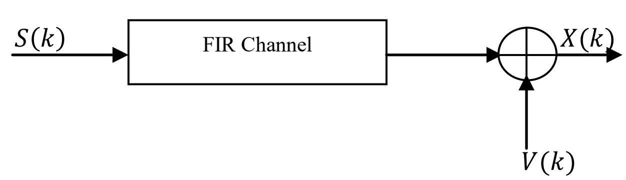

Fig.1 channel model

3. Blind Algorithms

3.1. Safi1 Algorithm[20]

Hypothesis:
Let us suppose that:

- The additive noise $V(k)$ is Gaussian, colored or with symmetric distribution, zero mean, with variance σ^2, i.i.d. with the m[th] order cumulants vanishes for m > 2
- The noise $V(t)$ is independent of $S(k)$ and $X(k)$.
- The channel (FIR system) order p is supposed to be known and $h(0) = 1$.
- The system is causal, i.e., $h(i) = 0$ if $i < 0$.
- The m[th] order cumulant of the output signal is given by the following equation [4], [5], [6]:

$$C_{my}\ (t_1, \dots, t_{m-1}) = \gamma_{mx} \sum_{-\infty}^{+\infty} h(i)h(i+t_1)\cdots h(i+t_{m-1}) \quad (2)$$

Where γ_{mx} represents the m[th] order cumulants of the excitation signal $(S(k))$.
Based on equation (2), Safi1 algorithm is developed by the following relationship:

$$\sum_{i=0}^{p} h(i)C_{3y}(t_1 - i, t_2 - i) = \epsilon \sum_{i=0}^{p} h(i)h(i + t_2 - t_1)C_{2y}(t_1 - i) \quad (3)$$

If we use the property of the ACF of the stationary process, such as $C_{2y}(t) \neq 0$ only for $(-p \leq t \leq p)$ and vanishes else were.

$$\sum_{i=0}^{p} h(i)C_{3y}(-p - i, t_2 - i) = \epsilon h(0)h(t_2 + p)C_{2y}(-p) \quad (4)$$

Else, if we suppose that $t_2 = -p$, Eq. (4) will become

$$C_{3y}(-p, -p) = \epsilon h(0)C_{2y}(-p) \quad (5)$$

Using Eqs. (4) and (5) we obtain the following relation

$$\sum_{i=0}^{p} h(i)C_{3y}(-p - i, t_2 - i) = h(t_2 + p) \quad (6)$$

Else, if we suppose that the system is causal, i.e., $h(i) = 0$ if $i < 0$. So, for $t_2 = -p, \dots, 0$, the system of Eq. (6) can be written in matrix form as:

$$\begin{pmatrix} C_{3y}(-p-1, -p-1) & \cdots & C_{3y}(-2q\ , -2q) \\ C_{3y}(-p-1\ , -p) - \alpha & \cdots & C_{3y}(-2p, -2p+1) \\ \vdots & \ddots & \vdots \\ C_{3y}(-p-1, 1) & \cdots & C_{3y}(-2p, -p) - \alpha \end{pmatrix} \begin{pmatrix} h(1) \\ h(2) \\ \vdots \\ h(p) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -C_{3y}(-p, -p+1) \\ \vdots \\ -C_{3y}(-p, 0) \end{pmatrix} \quad (7)$$

Were $\alpha = C_{3y}(-p, -p)$.
The above Eq. (7) can be written in compact form as

$$Mh_p = d_1 \quad (8)$$

Where M is the matrix of size $(p + 1) \times (p)$ elements, $h_p$is a column vector constituted by the unknown impulse response parameters $h(n)$: $n = 1, \dots, p$ and d_1 is a column vector of size $(p + 1) \times (1)$ as indicated in the Eq. (7).
The least squares solution (LS) of the system of Eq. (8), permits blindly identification of the parameters $h(n)$ and without any "information" of the input selective channel. So, the solution will be written under the following form

$$h_p = (M^T M)^{-1} M^T d_1 \quad (9)$$

3.2. Safi2 Algorithm[20]

From the Eq. (2), the m[th] and n[th] cumulants of the output signal, $\{y(n)\}$, and the coefficients $\{h(i)\}$, where $n > m$, are linked by the following relationship:

$$\sum_{j=0}^{p} h(j)C_{ny(j+t_1,\dots,j+t_{m-1},t_m,\dots,t_{n-1})} = \frac{\gamma_{ne}}{\gamma_{me}} \sum_{i=0}^{p} h(i) \left[\prod_{k=m}^{n-1} h(i + t_k)\right] C_{my}(i + t_1, \dots, i + t_{m-1}) \quad (10)$$

If we take $n = 4$ and $m = 3$ into Eq. (11), we find the basic relationship developed in [7], [8]. If we take $n = 3$ and $m = 2$ into Eq. (9), we find the basic relationship of the algorithms developed in [23]. So, the equation proposed in [4] presents the relationship between different n[th] cumulant slices of the output signal $\{X(n)\}$, as follows

$$\sum_{j=0}^{p} h(j) \left[\prod_{k=1}^{r} h(j + t_k)\right] C_{ny}(\beta_1, \dots, \beta_r, j + \alpha_1, \dots, \alpha_{n-r-1}) = \sum_{i=0}^{p} h(i) \left[\prod_{k=1}^{r} h(i + \beta_k)\right] C_{ny}(t_1, \dots, t_r, i + \alpha_1, \dots, i + \alpha_{n-r-1}) \quad (11)$$

Where $1 \leq r \leq n - 2$.
If we take n=3 we obtain that r=1, so the Eq. (10) will be

$$\sum_{j=0}^{p} h(j)h(j + t_1)C_{3y}\ (\beta_1, j + \alpha_1) = \sum_{i=0}^{p} h(i)h(i + \beta_1)C_{3y}\ (t_1, i + \alpha_1) \quad (12)$$

In the following, we develop an algorithm based only on 4[th] order cumulants. If we take $n = 4$ into Eq. (11) we obtain the following equation:

$$\sum_{i=0}^{p} h(i)h(i + t_1)h(i + t_2)\ C_{4y}(\beta_1, \beta_1, i + \alpha_1) = \sum_{j=0}^{p} h(j)h(j + \beta_1)h(j + \beta_2)C_{4y}\ (t_1, t_2, +\alpha_1) \quad (13)$$

If $t_1 = t_2 = p$ and $\beta_1 = \beta_2 = 0$, Eq. (13) takes the form:

$$h(0)h^2(p)C_{4y}(0, 0, i + \alpha_1) = \sum_{j=0}^{p} h^3\ (j)C_{4y}(p, p, j + \alpha_1) \quad (14)$$

As the system is a FIR, and is supposed causal with an order p, so, the $j+\alpha_1$ will be necessarily into the interval $[0,p]$, this implies that the determination of the range of the parameter α_1 is obtained as follows: $0 \leq j+\alpha_1 \leq p \Rightarrow -j \leq \alpha_1 \leq q-j$, and we have $0 \leq j \leq p$. From these two inequations, we obtain:

$$-p \leq \alpha_1 \leq p. \tag{15}$$

Then, from the Eqs. (13) and (14) we obtain the following system of equations:

$$\begin{pmatrix} C_{4y}(p,p,-p) & \cdots & C_{4y}(p,p,0) \\ \vdots & \ddots & \vdots \\ C_{4y}(p,p,0) & \cdots & C_{4y}(p,p,p) \\ \vdots & \ddots & \vdots \\ C_{4y}(p,p,p) & \cdots & C_{4y}(p,p,2p) \end{pmatrix} \begin{pmatrix} h^3(0) \\ \vdots \\ h^3(i) \\ \vdots \\ h^3(p) \end{pmatrix} = h(0)h^2(p) \begin{pmatrix} C_{4y}(0,0,-p) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,p) \end{pmatrix} \tag{16}$$

and as we have assumed that $h(0)=1$, if, we consider that $h(p) \neq 0$ and the cumulant $C_{my}(t_1,\ldots,t_{m-1})=0$, if one of the variables $t_k > p$, where $k=1,\ldots,m-1$, the system of Eq. (16) will be written as follows:

$$\begin{pmatrix} 0 & & \cdots & 0 & C_{4y}(p,p,0) \\ \vdots & & \ddots & & \vdots \\ 0 & & & & \\ C_{4y}(p,p,0) & \cdots & & & C_{4y}(p,p,p) \\ \vdots & & \ddots & & 0 \\ & & & & \vdots \\ C_{4y}(p,p,p) & 0 & \cdots & & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{h^2(p)} \\ \vdots \\ \frac{h^3(i)}{h^2(p)} \\ \vdots \\ \frac{h^3(p)}{h^2(p)} \end{pmatrix} = \begin{pmatrix} C_{4y}(0,0,-p) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,p) \end{pmatrix} \tag{17}$$

In more compact form, the system of Eq. (17) can be written in the following form:

$$Mb_p = d_2 \tag{18}$$

where M, b_p and d_2 are defined in the system of Eq. (17). The least squares solution of the system of Eq. (18) is given by:

$$\hat{b}_p = (M^T M)^{-1} M^T d_2 \tag{19}$$

This solution give us an estimation of the quotient of the parameters $h^3(i)$ and $h^3(p)$, with $b_p(i) = \left(\frac{\hat{h}^3(i)}{h^2(p)}\right)$, $i=1,\ldots,p$

So, in order to obtain an estimation of the parameters $\hat{h}(i), i=1,\ldots,p$ we proceed as follows:

- The parameters $h(i)$ for $i=1,\ldots,p-1$ are estimated from the estimated values $b_p(i)$ using the following equation:

$$\hat{h}(i) = sign\left[\hat{b}_p(i)\left(\hat{b}_p(p)\right)^2\right]\left\{abs\left(\hat{b}_p(i)\right)(\hat{b}_p(p))^2\right\}^{\frac{1}{3}} \tag{20}$$

where $sign(x) = \begin{cases} 1, & if \quad x>0; \\ 0, & if \quad x=0; \\ -1 & if \quad x<0. \end{cases}$

and $abs(x)=|x|$ indicates the absolute value of x

- The $\hat{h}(p)$ parameters is estimated as follows :

$$\hat{h}(p) = \frac{1}{2} sign\left[\hat{b}_p(p)\right]\left\{abs\left(\hat{b}_p(p)\right) + \left(\frac{1}{\hat{b}_p(1)}\right)^{1/2}\right\} \tag{21}$$

3.3. Proposed algorithm Said1

The general problem of interest here may be characterized using the following model:

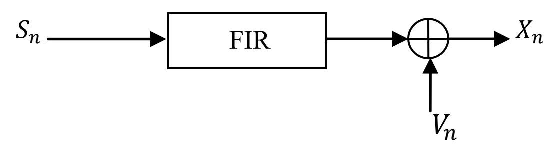

Fig.2 System Model

The convolution model is represented by:

$$X_n = \sum_{l=0}^{L} h_l S_{n-l} + V_n \tag{22}$$

h_l represents the channel parameter, S_{n-l} input system, X_n output system and V_n is the additive colored Gaussian noise. Or model vector of equation (22) is:

$$X_n = [h_0 \ldots h_l] \begin{bmatrix} S_n \\ \vdots \\ S_{n-l} \end{bmatrix} + V_n$$

$$X_n = h^T S_n + V_n \tag{23}$$

The matrix system will be written as follows:

$$\begin{bmatrix} X_n \\ \vdots \\ X_{n-N} \end{bmatrix} = \begin{bmatrix} h_0 & \ldots & h_l & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & h_0 & \ddots & h_l \end{bmatrix} \begin{bmatrix} S_n \\ \vdots \\ S_{n-N-l} \end{bmatrix} \tag{24}$$

Or

$$X(n) = h(n).S(n) + V(n) \tag{25}$$

With $X(n)$ of size $(N+1)\times 1$, $h(n)$ is of size $(N+1)\times(N+L+1)$ and $S(n)$ of size $(N+L+1)\times 1$. The Said1 algorithm is a general method for blind solving estimation problems of channel parameters. The considerations leading to the Said1 algorithm is given below.

- Let $S_0, \ldots, S_M$ be known
  Let N=L,
  Then:

$$E[X(n)S^*_{n-L}] = \begin{bmatrix} h_0 \\ \vdots \\ h_L \end{bmatrix} \tag{26}$$

  Where in practice,

$$E[X(n)S^*_{n-L}] = \frac{1}{M-L+1}\left(\sum_{n=L}^{M} X(n)S^*_{n-L}\right) \tag{27}$$

- Calculate mean square error of channel estimation:
  From equation (26), we have estimation:

$$\hat{h} = \begin{bmatrix} h_0 \\ \vdots \\ h_L \end{bmatrix} \quad (28)$$

Normalization of h and $\hat{h}$ is given by:

$$h = {h}/{\|h\|}, \qquad \hat{h} = {\hat{h}}/{\|\hat{h}\|}$$

So, MSE is calculated by the following equation:

$$MSE = \|h - \hat{h}\| \quad (28)$$

The principle of this algorithm is given by the following diagram:

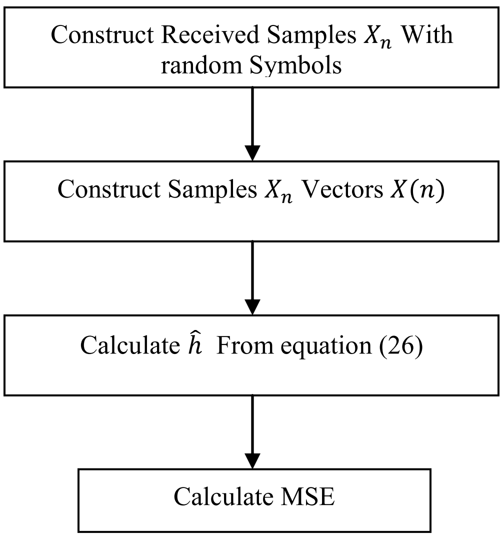

Fig.3 diagram of Said1 algorithm.

4. Said1 Equalization

4.1. Zf Equalization

In order to compensate the effect of power line communication (PLC) channel, we consider the ZF equalizer, wich satisfies the condition shown below.

$$f_{ZF} H = I \quad (29)$$

Where $f_{ZF} = (H^H H)^{-1} H^H$ is the ZF decoding matrix, $(.)^H$ denotes Hermitian transpose, H is the channel matrix and I is the identity matrix. Given the received signal X, the receiver can obtain the estimated signal by using the ZF equalization, which is given by:

$$\hat{S}(n) = f_{ZF}\, X(n) \quad (30)$$
$$\hat{S}(n) = f_{ZF} H\, S(n) + f_{ZF}\, V(n)$$
$$\hat{S}(n) = I\, S(n) + f_{ZF}\, V(n)$$

Where $\hat{S}$ is an estimate matrix of the transmitted signal. If the determinant of H is not zero so that there exists the inverse matrix of H the decoding matrix can be expressed as

$$f_{ZF} = H^{-1} \quad (31)$$

The ZF equalization is ideal when the channel is noiseless. However, when the channel is noisy, the ZF equalization will amplify the noise greatly where the channel has small magnitude in the attempt to invert the channel completely.

4.2. MMSE Equalization

In order to minimize the power of the noise component, we employ the MMSE equalization, which is given by:

$$f_{MMSE} = \underbrace{min}_{f_{MMSE}}\; E\left|\|S_{n-d} - f_{MMSE}^H X(n)\|^2\right| \quad (32)$$

Where f_{MMSE} is the MMSE decoding matrix and $\|.\|$ the norm of $S_{n-d} - f_{MMSE}^H X(n)$.

Or:

$$f_{MMSE} = R_X \hat{h}_d \quad (33)$$

Where $R_X = E[X(n)X^H(n)]$ and $\hat{h}_d = E[X(n)S_{n-d}^*]$. In practice the expression of R_X and $\hat{h}_d$ is:

$$R_X = \frac{1}{k}\sum_{n=1}^{k} X(n)X^H(n) \quad (34)$$

$$\hat{h}_d = \frac{1}{M}\sum_{n=1}^{M} X(n)S_{n-d}^* \quad (35)$$

The MMSE equalization can be used to estimate the channel effect with varying the decoding matrix in accordance with SNR. Besides, it prevents the noise component from being amplified.

So, equalization batch said algorithm is defined by the steps following:

- Transmit k symbols, the first M is assured known,
- Obtain received samples X_n,
- Construct sample vectors $X(n)$,
- Calculate R_X and $\hat{h}_d$,
- Calculate $f_{MMSE} = R_X^{-1}\hat{h}_d$,
- Calculate symbol error rate (SER):
  - ✓ Use $f^H X(n)$ to estimate symbol $\hat{S}_{n-d}$.
  - ✓ Compare $\hat{S}_{n-d}$ with S_{n-d} .

5. Adaptive Equalization

One approach to removing inter-symbol interference in a communications channels is to employ adaptive blind equalization to reduce the Symbol Error Rate (SER). The most popular class of algorithms used for blind equalization are those that minimize the Godard (or constant modulus) criteria [10,11]. In this paper we study the performance of the Constant Modulus Algorithm (CMA).

The Constant Modulus Algorithm (CMA)

The problem with blind adaptation techniques is their poor convergence property compared to traditional techniques using training sequences. Generally a gradient descent based algorithm is used with the blind adaptation schemes. The most commonly used gradient descent based blind adaptation algorithm is the Constant Modulus Algorithm (CMA). The Constant Modulus Algorithm (CMA) [10,12] has gained widespread practical use as a blind adaptive equalization algorithm for digital communications systems operating over inter-symbol interference channels.

The constant modulus (CM) criterion can be expressed by the cost function $J_{CM} = \frac{1}{4}E\{(|X_n|^2 - \gamma)^2\}$, where γ is a positive constant known as the Godard radius [1]. The equalizer update algorithm leading to a stochastic gradient descent of J_{CM} is known as the Constant Modulus Algorithm (CMA) and is specified by [15]:

$$f(n+1) = f(n) + \mu S^*(n) \underbrace{X_n(\gamma - |X_n|^2)}_{\triangleq\ (X(n))} \quad (36)$$

Where μ is a step-size and $S^*(n)$ is the equalizer input vector at time index n. the asterisk denotes conjugation. The function $\psi(.)$ identified in (36) is referred to as the CMA error function.

6. Simulation Results

6.1. Performance of Safi1, Safi2 and Said1 Algorithms in the Without Noise Case.

In this part, we will test the performance of the presented algorithms (Safi1, Safi2 and Said1). In table I we represent the obtained results using different data length for Proakis (B) channel without noise case.

TABLE I
COEFFICIENTS CALCULATED BY SAFI1 , SAFI2 AND SAID1 ALGORITHMS FOR PROAKIS (B) CHANNEL.

N	*Algorithms*				*MSE*
512	*Safi1*	-0.1852	0.3324	0.1456	1.6804
	Safi2	0.669	0.8962	0.7719	1.2283
	Said1	0.3426	0.7944	0.4622	0.1140
1024	*Safi1*	-0.2575	0.5186	0.2792	1.3604
	Safi2	0.6464	0.9017	0.7657	1.1343
	Said1	0.3997	0.8082	0.4286	0.0630
2048	*Safi1*	0.0944	0.1009	0.1135	1.2774
	Safi2	0.5692	0.8614	0.6953	0.7639
	Said1	0.0553	0.8136	0.4065	0.0553
4096	*Safi1*	0.1204	0.4851	0.1675	0.7387
	Safi2	0.5004	0.8067	0.6329	0.5609
	Said1	0.4135	0.8131	0.4096	0.0140
	Real Parameters	0.407	0.815	0.407	

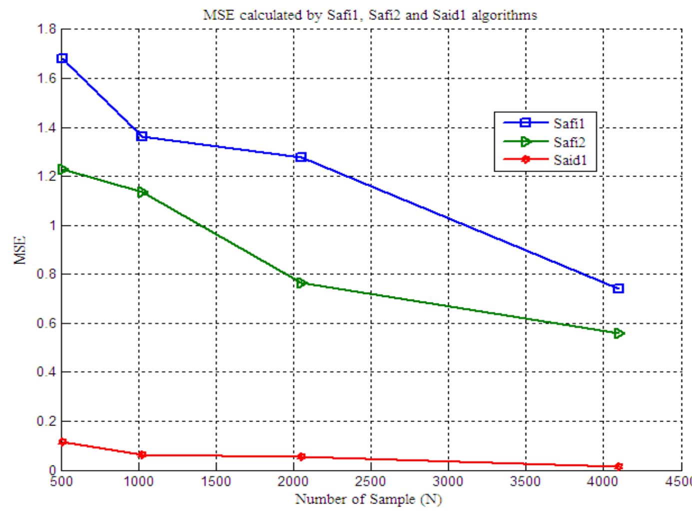

Fig. 4 The variation in the MSE for Safi1, Safi2 and Said1 algorithms with different numbers of samples N.

From table.I we can conclude that for all numbers of samples (N = 512.1024, 2048 and 4096), Said1algorithm gives the results satisfactory with low MSE, that is to say, Said1 gives a good estimate of channel parameters compared to Safi1 and Safi2 algorithms.

6.2. Performance of Safi1, Safi2 and Said1 Algorithms in the Noise Case.

The same as the previous section, we compared both the algorithms, in the case of a noisy additive Gaussian noise (AWGN). We select two values of Signal to Noise Ratio SNR (0 to 16), to test the performance of three algorithms (Safi1, Safi2 and Said1) in a noisy environment.

TABLE II
COEFFICIENTS CALCULATED BY SAFI1, SAFI2 AND SAID1 ALGORITHMS FOR THE PROAKIS (B) CHANNEL IN THE NOISY CASE WITH THE NUMBER OF SAMPLE N = 1024.

SNR	*Algorithms*				*MSE*
0 dB	*Safi1*	-0.1816	0.1849	0.2043	2.9375
	Safi2	0.4383	0.6983	0.6086	0.2717
	Said1	0.4306	0.8051	0.4013	0.0783
16dB	*Safi1*	-0.15650	0.2930	0.2445	2.4863
	Safi2	0.4723	0.6833	0.4295	0.1922
	Said1	0.4040	0.8226	0.4001	0.0142
	Real Parameters	0.407	0.815	0.407	

Similarly we conclude that the third algorithm (Said1) gives a good estimate of channel parameters very close to ones real, if the environment is very noisy (SNR = 0dB), so the Said1 algorithm has a very important advantage compared to the Safi1 and Safi2 algorithms.

6.3. Blind Identification Channel (Bran A)

In this section we consider the amplitude and the phase of the impulse response of the channel BRAN A [3], [4] with the number of samples N = 2048 and the SNR= 16 dB, to verify the performance of the Safi1, Safi2 and Said1 algorithms at the very selective radio mobile channel.

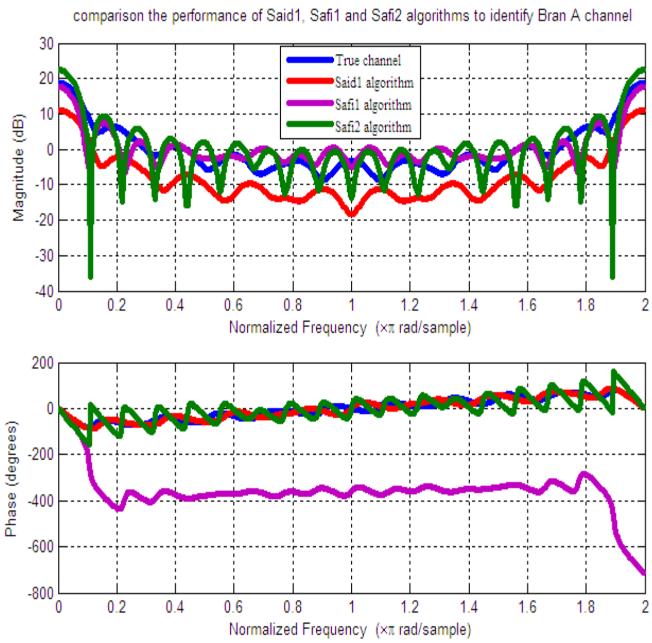

Fig. 5 Estimate the amplitude and the phase of BRAN A channel, with the number of samples N = 2048 and the SNR = 16 dB, using the three algorithms (Safi1, Safi2 and Said1).

The simulation result obtained (Fig. 5) shows that both algorithms have very satisfactory results in terms the amplitude of the impulse response compared to that measured (real). But, the estimation of the phase is different from that measured for Safi1, and to the same phase from the real for Said1 and Safi2 algorithms, so Said1 algorithm gives a good estimate of the phase for the BRAN A channel.

6.4. Equalization of Channel by Said1 and CMMA Algorithms

In this part we test the performance of Said1 and CMA algorithms in order to equalize the output of the channel. They are applied in the following conditions:

The channel complex h= [0.0545+j*0.05 0.2832-.1197*j 0.7676+0.2788*j 0.0641-.0576*j 0 .0566-.2275*j 0 .4063-0.0739*j], with different values of SNR, the total number of data is T=1048 and we use the modulation QPSK or 4 QAM symbol sequence.

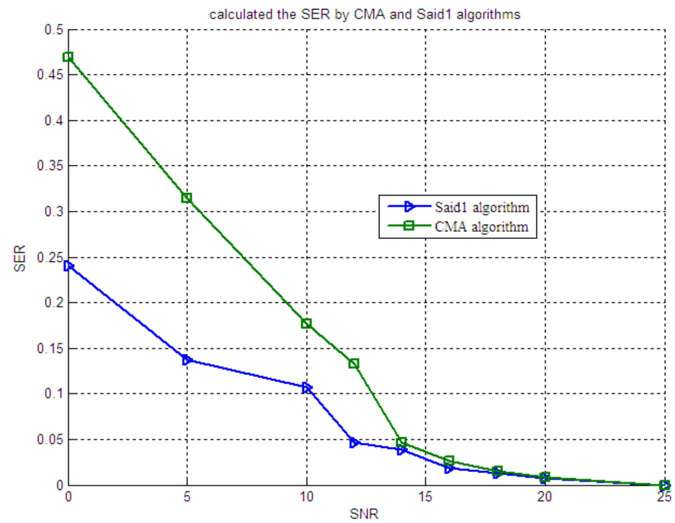

Fig. 8 Calculating the SER in function of SNR by CMA and Said1 algorithms.

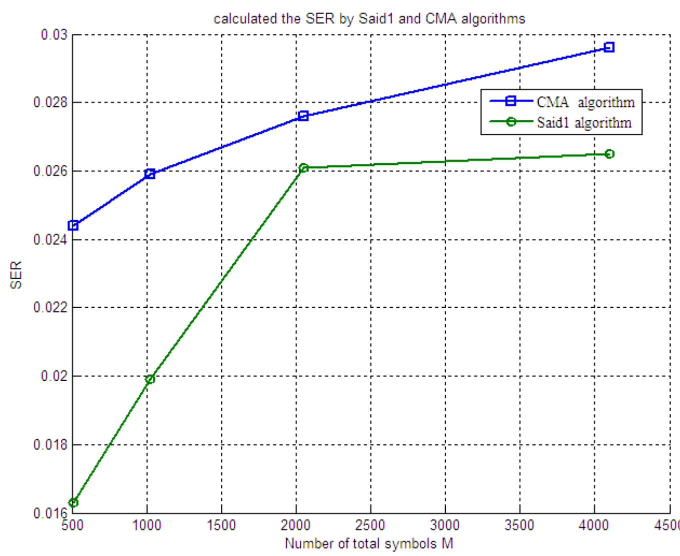

Fig. 9 Calculating the SER in function the number of total symbols M by CMA and Said1 algorithms with SNR=16 dB.

From simulation of results obtained, we notice that the Said1 algorithm gives the right equalization of channel compared to CMA algorithm. In addition to that the SER is decreasing according to SNR (Fig.8) and is increasing according to number of total symbols M (Fig.9) for the two algorithms (CMA and Said1).

7. Conclusion

In this paper we have presented three algorithms which are: Safi1, Safi2 and Said1 algorithms. This algorithms play in important role for blindly estimating the channel's parameters (Prokis B) and identifying the radio mobile channels (Bran A), in addition, we study the problem of blind equalization by the constant modulus algorithm (CMA) and Said1 algorithms. Simulation results show that the Said1 algorithm gives a good estimate of channel parameters very close to ones real, if the environment is very noisy (SNR = 0dB) compared to Safi1 and Safi2 algorithms. Thus Said1 algorithm is more performance compared to CMA algorithm at the blind channel equalization, that to say, gives the right equalization and the good SER according to SNR.

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