Velocity Estimation of Target in MIMO Radar Environment with Unknown Antenna Locations

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Abstract: - A new algorithm is formulated for MIMO Radar system where positions of target along with that of the transmitter and receiver antennas are unknown. This algorithm considers a widely separated antenna MIMO setup and can be used when transmitters and receivers are either stationary or moving at a very low velocity. Here, the algorithm estimates the location of target with respect to the location of the first transmitter. The TDOA and AOA available for LOS path between the transmitters and receivers along with the reflection path from the target are utilized here. AOA is used only for initialization of antenna positions and target location. Furthermore, accurate estimation using Davidon-Fletcher-Powell (DFP) Algorithm is performed. The paper introduces a new algorithm to compute velocity of a target. Here, first the FDOA is estimated using a novel approach and then velocity is estimated from the FDOA obtained. The velocity estimator for FDOA given tracks CRLB and FDOA estimation tracks its corresponding CRLB upto -12dB. The algorithm uses target and antenna locations to estimate velocity that can be found out from the algorithm introduced in paper.

Key-Words: - Target Localization, Velocity Estimation, MIMO Radars, DFP Algorithm, Antenna Positions Estimation.
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1 Introduction

MIMO radar have attracted significant attention in last few years over conventional radar for the superior performances in higher spatial resolution [1], enhanced parameter identifiability [2], more degrees of freedom (DOFs) [3], detection diversity gain [4], better spatial coverage [5] and possibility of direct application of adaptive array techniques [6]. Two types of MIMO setup have been discussed, MIMO with colocated antennas [7] and with widely separated antennas [8].

MIMO radar with co-located antennas is better for parameter estimation and beam-forming performance as it has more effective spatial degrees of freedom, since its transmitter and receiver antennas are sufficiently close to observe signals reflected from the target. MIMO radar with widely separated antennas, also known as statistical MIMO radar, exploits the diversity of the propagation path, thus can be used where better detection and estimation resolution is needed.

In a Radar system, the detection and estimation of target parameter is the prime application. The parameters of target into consideration are location, velocity, acceleration, Doppler frequency, Radar Cross Section(RCS).etc. Several methodologies have been introduced for target detection [9] and localization. Detection techniques in clutter is also discussed [10],[11]. Several different approaches have been adopted to estimate target location and velocity, based on time of arrival (TOA) [12], time difference of arrival (TDOA) [13], angle of arrival (AOA) [14] or frequency difference of arrival (FDOA) [15].

There is a rapid growth in literature on MIMO radars. In [16], a method for estimating target location when transmitter and receiver location are known was proposed which tracks Cramér-Rao lower bound (CRLB). In [16], the problem of target localization is modeled in MIMO radars using TDOA and AOA measurements. This method solves the maximum likelihood (ML) estimation problem of target position with arena divided into grids and uses steepest descent algorithm (SDA) to further enhance accuracy while maintaing complexity low. In [17] method for estimation of velocity is introduced. It also discusses optimal antenna placements. The paper [18] discuses improvement in performance of estimators when number of antennas are increased. In [19] spare support recovery is used to infer target properties both position and velocity.Optimal Energy allocation is also discussed.

All the available methods stand valid only when the transmitter and receiver position are accurately known beforehand. Thus, the antennas have to be relatively stationary and the positions have to found manually which could consume considerable setup time and if at all by any cause the antenna positions are changed, then the system would have to be setup again. There fore to conceptualize a portable MIMO Radar system, an algorithm is vital which can elude the setup time. We have come up with a new method which estimates the antenna positions and target positions at the same time. Here for example, if we consider a MIMO radar setup where the antennas are kept on a movable mount, then even when the antennas change positions we do not have to pause and obtain the coordinates of antennas, but the algorithm estimates the target position with respect to the first transmitter even when the antennas are in motion.

In the proposed algorithm, Cartesian plane is fixed and the transmitter and receiver positions are initialized used TDOA and AOA measurements. Once transmitter and receiver locations are initialized, the target position is localized to grids by solving ML estimation, inspired from [16], then for precise estimation of all unknown positions Davidon-Fletcher-Powell (DFP) from [20] is used.

This paper also introduces a new algorithm to estimate velocity. Here first FDOA is estimated and then using FDOA as input a new algorithm, inspired from methodology in [16] is used to estimate velocity. The velocity domain is discretized to grids and the grid with nearest velocity values are found using sparsity aware ML estimator.

In this paper, to estimate FDOA, a new approach using iterative use of Non-Uniform DFT is presented. It first finds the nearest frequency with respect to the resolution of the current iteration bandwidth and for next iteration, the bandwidth of interest is reduced and kept around the frequency obtained in the last iteration. This is repeated untill the bandwidth of the iteration matches the required precision.

The paper is arranged as, Section 2 describes system model, describes various parameters of MIMO radar used in estimators. Section 3 elaborates the new method. In Section 3.1 a new algorithm for estimating FDOA from signal is elaborated. Section 3.2 a new approach for approximating the velocity of target from FDOA is presented. Input to velocity estimator is antenna and target locations along with target doppler signature (FDOA). Section 3.3 is new procedure to estimate target position along antenna positions. Section 3.3.1 discuses initialization procedure and Section 3.3.2 elaborates methodologies adopted for more accurate estimation of target as well as antenna positions. Section 4 contains the Numerical simulations results for testing the proposed methods. Section 5 concludes the paper.

2 System Model

Let us consider a MIMO setup with M transmitters and N receivers distributed over a 2-D surface. The surface is divided into K grid points for target localization. The positions of transmitter and receiver are denoted by $\mathbf{x}_m=[x_m,y_m]$, m=1,2,..,M and $x_n=[x_n,y_n]$, n=1,2,..,N respectively. The position of a target is denoted by $\mathbf{x}=[x, y]$ and its velocity $\mathbf{v}=[v_x, v_y]$. This is depicted in Fig.1

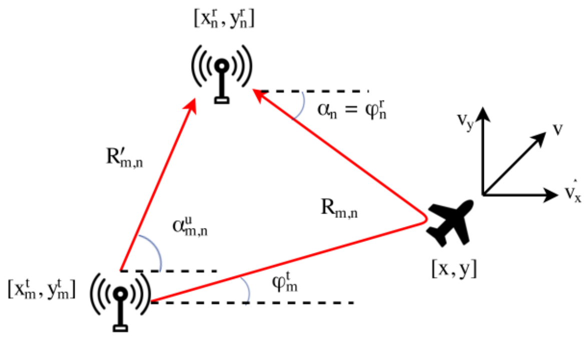

Fig.1 Schematic of MIMO Radar system Arrangement

It needs to be noted that, system considered is 2-D, but can be extended to a 3-D localization. Considering the wave propagation speed (that is, speed of light) by c, the noisy TDOA and AOA measurements due to LOS can be modeled as

$$\tau_{m,n}^L = \frac{1}{c}\|\boldsymbol{x}_{\boldsymbol{m}}^t - \boldsymbol{x}_{\mathrm{n}}^r\| + \epsilon_{m,n}^{\tau} \tag{1}$$

$$\alpha_{m,n}^L = \tan^{-1}\left(\frac{y_m^t - y_n^r}{x_m^t - x_n^r}\right) + \epsilon_{m,n}^{\alpha} \tag{2}$$

where $\tau_{m,n}^L$ and $\alpha_{m,n}^L$ are respectively the measured TDOA abd AOA which contains noise.
The target reflected path TDOA and AOA are

$$\tau_{m,n} = \tau_m^t(\boldsymbol{x}) + \tau_n^r(\boldsymbol{x}) + \omega_{m,n}^{\tau} \tag{3}$$

$$\alpha_n = \tan^{-1}\left(\frac{y - y_n^r}{x - x_n^r}\right) + \omega_n^{\alpha} \tag{4}$$

where $\tau_m^t(\boldsymbol{x}) = \frac{1}{c}\|\boldsymbol{x}_{\boldsymbol{m}}^t - \boldsymbol{x}\|$ for m=1, 2, … ,M and $\tau_n^r(\boldsymbol{x}) = \frac{1}{c}\|\boldsymbol{x}_{\boldsymbol{n}}^r - \boldsymbol{x}\|$ for n=1,2,…, N. TDOA and AOA measurements are disturbed by independent zero mean Gaussian noises of $\epsilon^{\tau}_{m,n}$, $\epsilon^{\alpha}_{m,n}$, $\omega^{\tau}_{m,n}$ and ω^{α}_{n} with the standard deviation of $\sigma^{\epsilon\tau}_{m,n}$, $\sigma^{\epsilon\alpha}_{m,n}$, $\sigma^{\omega\tau}_{m,n}$and $\sigma^{\omega\alpha}_{m,n}$. Here, $\sigma^{\epsilon\tau}_{m,n}$= $\sigma^{\omega\tau}_{m,n}$=$\sigma^{\tau}_{m,n}$ and $\sigma^{\epsilon\alpha}_{m,n}$= $\sigma^{\omega\alpha}_{m,n}$=$\sigma^{\alpha}_{m,n}$. Since all the measurements are obtained in same environment.

The distance between transmitters and receiver antennas can be found from TDOA as $d^{L}_{m,n} = \tau^{L}_{m,n}c$ and the bistatic range with target as $d_{m,m} = \tau_{m,n}c$. The actual distance between transmitters and receiver antennas are $R_{m,n}$=$\|\boldsymbol{x}^{t}_{\boldsymbol{m}} - \boldsymbol{x}\|$+$\|\boldsymbol{x}^{r}_{\boldsymbol{n}} - \boldsymbol{x}\|$ for m=1,2,…,M and n=1,2,…,N. The Frequency difference of arrival (FDOA) can be modelled as

$$\boldsymbol{f}_{mn}(\boldsymbol{v}) = f_{mn}(v_x, v_y) + \epsilon^{f}_{m,n}$$
(5)

where

$$f_{mn}(v_x, v_{y,}) = \frac{v_x}{\lambda}A_{mn} + \frac{v_y}{\lambda}B_{mn}$$
(6)

for m=1,2,…,M and n=1,2,…,N, is the actual frequency difference due to Doppler shift and where,

$$A_{mn} = \cos\phi^{t}_{m} + \cos\phi^{r}_{n}$$

$$B_{mn} = \sin\phi^{t}_{m} + \sin\phi^{r}_{n}$$

$$\phi^{t}_{m} = \tan^{-1}\left(\frac{y - y^{t}_{m}}{x - x^{t}_{m}}\right)$$

$$\phi^{t}_{n} = \tan^{-1}\left(\frac{y - y^{t}_{n}}{x - x^{t}_{n}}\right)$$

and $\epsilon^{f}_{m,n}$independent zero mean Gaussian random variables with standard deviation $\sigma^{f}_{m,n}$.

Let $\hat{s}_m(t) = s_m(t)e^{j2\pi f_c t}$, 0≤t≤T, is signal transmitted from the m[th] transmitter with total energy as E and f_c is the center frequency. Then, the received signal at the n[th] receiver corresponding to the signal transmitted by the m[th] transmitter after reflection from target can be written as

$$r_{mn}(t)=s_r(t)+\Psi_{mn}(t)$$
(7)

for m=,2,…,M and n=1,2,…,N

where

$$s_r(t) = \sqrt{\frac{E}{M}}\zeta_{mn}a_{mn}\hat{s}_m(t - \tau_{mn})e^{j2\pi f_{mn}(\boldsymbol{v})t}$$

and where ζ_{mn} is the unknown complex target reflectivity and τ_{mn} is the time delay of the path R_{mn} and f_{mn} is the Doppler frequency detected by the *n[th]* receiver due to the *m[th]* transmitter. The observation interval T is assumed lengthy enough so that all transmitted signals can be observed, irrespective of their delay. That is, T>>max$\{\tau_{mn}\}$ and the parameter

$$\eta_{mn} = \frac{1}{R^2_{mn}}\rho_{mn}$$
(8)

for m=1,2…,M and n=1,2,…,N. In this, $\rho_{mn} = e^{-j2\pi f_c \tau_{mn}}$. The noise at the n[th] receiver for the signal from m[th] transmitter is denoted as$\Psi_{mn}(t)$ and is a white Gaussian process with mean zero and standard deviation σ_{Ψ}.

3 Proposed Method

3.1 FDOA Estimation

A novel approach to estimate frequency shift is introduced. Let ΔW be the bandwidth of interest. For radar case, ΔW must be chosen such that it can contain the Doppler shift caused by maximum velocity of the target under consideration. Now, the sampling period T_s must be chosen such that T_s≤1/2ΔW so as fulfill Nyquist criteria. Let N_s be the number of samples collected by sampling $r_{mn}(t)$ at sampling period T_s. It is to be noted that $T_sN_s >> \max_{m,n}(\tau_{nm})$. We can write

$$R = [r_{mn}(0), r_{mn}(T_s), \ldots, r_{mn}((Ns - 1)T_s)]$$
(9)

For the iterative algorithm to begin the following parameters are to be considered.

$$f_{min} = f_c - \frac{\Delta W}{2}$$
(10)

$$f_{max} = f_c + \frac{\Delta W}{2}$$
(11)

Now let us define vector w of size $N_s \times 1$ such that

$$w = [w_0, w_1, \ldots\ldots, w_{N_{S-1}}]$$
(12)

where $$w_i = f_{min} + (i - 1)\frac{(f_{max} - f_{min})}{(N_s - 1)}$$
(13)

Let n be another vector

$$n = [0,1,2, \ldots\ldots. N_s - 1]^T$$
(14)

Now to form DFT basis, $E = e^{-j2\pi nwT_s}$, then E can be written as

$$E = \frac{1}{N_s}\begin{bmatrix} 1 & e^{-i2\pi w_0 T_s} & \dots & e^{-i2\pi w_0 (Ns-1)T_s} \\ 1 & e^{-i2\pi w_1 T_s} & \dots & e^{-i2\pi w_1 (Ns-1)T_s} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-i2\pi w_{N_s-1} T_s} & \dots & e^{-i2\pi w_{N_s-1} (Ns-1)T_s} \end{bmatrix} \quad (15)$$

Algorithm 1   **Pseudo-code for FDOA Estimation**

while $\Delta w < \delta$ **do**
 Generate **w** rom (13) and take **n** from (14)
 Generate **E** from (15)
 Y=RE
 $m = \max_i \left(Y\left(w(i)\right)\right)$
 $f_c = w(m) - f_c$
 $f_{min}^{(i+1)} = w(m) - \frac{f_{max}^i - f_{min}^i}{(N_s - 1)}$
 $f_{min}^{(i+1)} = w(m) + \frac{f_{max}^i - f_{min}^i}{(N_s - 1)}$
 $\Delta w = f_{max}^{(i+1)} - f_{min}^{(i+1)}$
 end while

Let Δw be the instantaneous bandwidth for each iteration. Initially $f_{min}^0 = f_C - \left(\frac{\Delta w}{2}\right), f_{max}^0 = f_C - \left(\frac{\Delta w}{2}\right)$ $and\ \Delta w = f_{max}^0 - f_{min}^0$ Let δ be the resolution required for estimation. The iterations are performed till Δw reachesδ. In each iteration the frequency band of interest i.e. $f_{min}^i\ to\ f_{max}^i$ is discretized into N_s frequencies using (12) and peak magnitude in frequency response is found out. The bandwidth around this peak value becomes our new bandwidth of interest. After required number of iterations is performed, the value f_e gives the FDOA estimation with resolution of δ.

3.2 Velocity Estimation

Considering target at position **x** and moving with a velocity **v** as described in Section 2 and FDOA measurements obtained from 3.1. The actual FDOA without the noise is defined by (5).

Let us form a vector **F** of measured doppler shifts (FDOA), i.e. $\boldsymbol{F} = [\boldsymbol{f_{11}}(\boldsymbol{v}), \boldsymbol{f_{12}}(\boldsymbol{v}) \dots \dots \boldsymbol{f_{MN}}(\boldsymbol{v})]^{\boldsymbol{T}}$, F is noised added version of actual Doppler shift, $\boldsymbol{f}(\boldsymbol{v}) = [\boldsymbol{f_{11}^a}(\boldsymbol{v}), \boldsymbol{f_{12}^a}(\boldsymbol{v}), \dots . \boldsymbol{f_{MN}^a}(\boldsymbol{v})]^{\boldsymbol{T}}$. Here the assumption is that we know the target location **x** and antenna locations $x_m^t\ and\ x_n^r$. This can be found using Section 3.3. Using FDOA instead of TDOA in [16], also not using AOA and making necessary changes, a new algorithm for estimating velocity is formulated.

Select **K** number of grid points $\{g^i\}_{(i=1)}^K$in velocity domain to compute the objective function of the ML estimation for all grid points and select the minimum one. Now we can form the matrix **A** by finding **f** (**v**) in different grid points.

$$\mathbf{A}_1=[f(g^1), f(g^2) \dots \dots . f(g^K)\ ] \quad (16)$$

Now, in order to obtain the velocity of a target, the values of **f**(**v**) is compared with the received measurements in all grid points. Thus, the target velocity estimation problem can be written in the sparse representation framework as $\mathbf{F} = \mathbf{A_1 z} + \boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is a MN×1 vector containing the FDOA measurement noise, thus $\boldsymbol{\varepsilon} = [\varepsilon^f]$, where $\varepsilon^f = \left[\varepsilon_{1,1}^f, \varepsilon_{1,2}^f, \dots \varepsilon_{M,N}^f\right]^T$. Vector **z** is a K×1 vector with (K-1) zeros and a one element which is corresponding to the index of the grid point where the target velocity is closest.

Since $\mathbf{F} = \mathbf{A_1 z} + \boldsymbol{\varepsilon}$ has an in-deterministic nature, thus the conventional maximum likelihood (ML) estimation is not viable. Therefore, a simple solution for this problem is to compute the objective function of the ML estimation for all grid points and select the minimum one (brute force). This trivial method is of high complexity and limited positioning accuracy according to grid size in velocity domain. Instead, a compressed sensing technique can be considered taking the sparsity in target's velocity. Thus, the target velocity estimation problem can be expressed using the $l_1$minimization.

$$\hat{\mathbf{z}}=\mathrm{argmin}(\mathbf{A}_1\mathbf{z}-\mathbf{F})\, C_\varepsilon^{-1}(\mathbf{A}_1\mathbf{z}-\mathbf{F})^T + \lambda\|\mathbf{z}\|_1 \quad (17)$$

where λ is a regularization parameter that controls the sparsity of **z** and $\mathbf{C}_{\boldsymbol{\varepsilon}}$ is the covariance matrix of **ε**. Also consider a matrix W such that $\boldsymbol{W_T W} = C_\varepsilon^{-1}\ or\ \boldsymbol{W} = \sqrt{C_\varepsilon^{-1}}$. By applying this, (17) can be expressed as

$$\hat{\mathbf{z}}=\mathrm{argmin}\|\widetilde{A_1}\boldsymbol{z} - \widetilde{\boldsymbol{F}}\|^2+\lambda\|\mathbf{z}\|_1 \quad (18)$$

where $\tilde{A}_1 = \boldsymbol{W A_1}\ and\ \tilde{F} = \boldsymbol{WF}$. We can find a nearest grid index of the target velocity and initialize that as target velocity as v^0.

In grid-based localization, the target velocity which are not located on the grids i.e, off-grid are not accurately localized. In order to resolve this problem, an algorithm based on dictionary learning (DL) techniques can be used which is designed to minimize the following cost function.

$$\Gamma(\mathbf{v})=[\mathbf{F}\text{-}f(\mathbf{v})]^T\boldsymbol{C_\varepsilon^{-1}}[\mathbf{F}\text{-}f(\mathbf{v})]=\|\hat{f}(v) - \tilde{F}\|^2 \quad (19)$$

where $\tilde{\mathbf{F}}$ is the column of $\widetilde{\mathbf{A}}_1$ which corresponds to the estimated target velocity on the grids in velocity domain. It can be seen that he cost function is convex with respect to **v**. Thus, simple steepest decent (SDA) can iteratively estimate the true target's velocity from the matrix $\mathbf{A_1}$. Note that, in the following method we just employ FDOAs in order to satisfy convexity condition. SDA iteration equation is written as

$$\mathbf{v}^{(i+1)}=\mathbf{v}^{(i)}-\mu^{(i)}\,\nabla_v\Gamma(\mathbf{v}^{(i)})$$
(20)

In Appendix 5, the derivation for final recursion equation for updating the estimated velocity vector is explained. Thus, the velocity vector v at the $(i+1)^{th}$ iteration can be written as

$$\boldsymbol{v}^{(\boldsymbol{i}+\mathbf{1})} = \boldsymbol{v}^{(\boldsymbol{i})} - \mu^{(i)} e^T C_\varepsilon^{-1}\psi$$
(21)

where, $e^T=\mathbf{F}\text{-}f(\mathbf{v}^{(i)})$ and $\psi = [\psi_{11}, \psi_{12}, \dots \psi_{MN}]$

$$\psi_{mn} = \left[\frac{\cos\phi_m^t + \cos\phi_n^r}{\lambda_c}, \frac{\sin\alpha_m^t + \sin\alpha_n^r}{\lambda_c}\right]$$
(22)

for m=1,2,..,M and n=1,2,…,N.

The initial value $\mathbf{v}^{(0)}$ of **v** is chosen from the estimate of **v** from previously. The value of $\mu^{(i)}$ is selected according such that $0 < \mu^{(i)} < \frac{2}{\lambda_{max}^{(i)}}$ in which $\lambda_{max}^{(i)}$ is the maximum eigen value of $\hat{f}(v_i)\hat{f}(v_i)^T$. To further clarify the velocity estimation procedure, following pseudo-code describes the estimation procedure in step by step. The Cramer-Rao lower bound (CRLB)) on the estimation error is summarized in Appendix B.1 .

Algorithm 2 **Pseudo-code for Velocity Estimation**
for $n = 1$: Number of blocks **do**
n_{th} block divided into K sub-blocks.
Solve ML estimation for (18) and find the nearest block.
end for
Set $v^{(0)}$from the previous estimated grid point and set $v^{(1)}$ = inf (very large value)
while $\lvert v^{(i+1)} - v^{(i)}\rvert < \delta$ **do**
Compute $v^{(i+1)}$ from (21)
end while

3.2 Target and Antenna Position Estimation

3.3.1 Initialization

Antenna Position

Now, in order to fix a coordinate system, location x_1^t i.e. position of the first transmitter is considered to be origin and the LOS path connecting x_1^t and x_1^r (first receiver) as x-axs. That is, $x_1^t = [0,0]$and $y_1^r = 0$. τ_{mn} and α_n are target TDOA and AOA respectively. If M and N are number of transmitters and receivers considering only single target, then number of unknown quantities for 2-D case will be $N' = 2(M + N) - 1$.

We use AOA information along with TDOA. Let us denote,

$$D' = [d'_{1,1}, d'_{1,2,}, \dots\dots, d'_{M,N}] \tag{23}$$

$$\alpha = [\alpha_{1,1}^u, \alpha_{1,2}^u, \dots\dots, \alpha_{M,N}^u] \tag{24}$$

Let us define M vectors by rearranging D' and α

$$D_m^e = [d'_{m,1}, d'_{m,2,}, \dots\dots, d'_{m,N}] \tag{25}$$

$$\alpha_m^e = [\alpha_{m,1}^u, \alpha_{m,2}^u, \dots\dots, \alpha_{m,N}^u] \tag{26}$$

where m = 1, 2,….,M Now let us initialize the receiver locations as

$$x_N^{r0} = D_1^e[cos\alpha_1^e, sin\alpha_1^e] \tag{27}$$

It should be noted that y_1^r must be forced to zero. Now for transmitters x_1^0 =[0,0].

$$x_m^{to} = \frac{1}{N}(x_N^{ro} - D_m^e[cos\alpha_m^e, sin\alpha_m^e] \tag{28}$$

for m $= 2,3, \dots$ M

Target Position

For initializing target position x, the ML-Estimation concept from [16] is used. The antenna positions considered are x_m^{t0} and x_n^{r0} . K number of grid points $\{g^i\}_{(i=1)}^K$ in spatial domain. Now we can write the Bistatic Range $R_{m,n}(X) = \left\|{X_m^t}^0 - X\right\| + \left\|{X_n^r}^0 - X\right\|$ and by taking TDOA received due to target reflection path d_{mn}, where m $= 2,3, \dots$ M, n $= 2,3, \dots$ N., We can represent $H_R(X) = [R_{1,1}, R_{1,2}, \dots, R_{M,N}]$,$H_A(X) = [\alpha_1, \alpha_2, \dots, \alpha_N]$.
Here, $H(X) = [H_R(X), H_A(x)]^T$ and the measure $B = \left[d_{1,1}, d_{1,2}, \dots\dots d_{M,N}\right]^T$. Matrix A_2 can be computed as,

$$A_2 = [H(g^{(1)}), H(g^{(2)}), \dots\dots H(g^{(K)})] \tag{29}$$

The problem of target localization can be expressed in the sparse representation framework given by $\boldsymbol{B} = \mathbf{A}_2\mathbf{z} + \boldsymbol{\varepsilon}$ where $\boldsymbol{\varepsilon}$ is $(M+1)N\,x1$

vector containing the noises in measurement of TDOA and AOA, i.e., $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_R^T, \boldsymbol{\varepsilon}_\alpha^T]$, where,

$$\boldsymbol{\varepsilon}_{\mathrm{R}} = c.[\omega_{1,1}^{\tau}, \omega_{1,2}^{\tau}, \dots\dots, \omega_{M,N}^{\tau}]^{\mathrm{T}}$$

and $\varepsilon_a = [\omega_1^\alpha, \omega_2^\alpha, \dots \omega_N^\alpha]^{\mathrm{T}}$. Vector $\mathbf{z}$ is a K x 1 vector with (K-1) zeros and a one at to the index which corresponds to the grid point where the target is located.

Since $\boldsymbol{B} = \boldsymbol{A_2 z} + \boldsymbol{\varepsilon}$ has an un-deterministic nature, thus the conventional maximum likelihood (ML) estimation is not viable. Therefore, a simple solution for this problem is to compute the objective function of the ML estimation for all grid points and select the minimum one (brute force). This trivial method is of high complexity and limited positioning accuracy according to grid size in velocity domain. Instead, a compressed sensing technique can be considered taking the sparsity in target's location. Thus, the target localization problem can be expressed using the l1 minimization procedure.

$$\hat{z} = \mathrm{argmin}\,(\mathbf{A_2 z\text{-}B})\, C_\varepsilon^{-1}(\mathbf{A_2 z\text{-}B})^{\mathrm{T}} + \lambda\|z\|_1 \qquad (30)$$

where λ is a regularization parameter that controls the sparsity of z and C_ε is the covariance matrix of ε. A matrix W is so introduced such that $\mathbf{W_T W} = \boldsymbol{C_\varepsilon^{-1}}$ *or* $\boldsymbol{W} = \sqrt{\boldsymbol{C_\varepsilon^{-1}}}$. Now, (30) can be rewritten as

$$\hat{z} = \mathrm{argmin}\,\|\tilde{A}_2 z - B\|_2^2 + \lambda\|z\|_1$$
(31)

where $\tilde{A}_2 = \mathbf{WA_2}$ and $\widetilde{\boldsymbol{B}} = \mathbf{WB}$. We canfind a nearest grid location of the target and initialize that as target location as x^0.

3.3.2 Precise Estimation

For precise estimation of the required parameters we are using DFP algorithm. Let N' be the number of parameters to be estimated. Let F(X) be the cost function and $\nabla F(\mathrm{X})$ the gradient of the cost function. Here $\boldsymbol{C}_\gamma$ is the covariance matrix of γ, *where* $\gamma = [\omega^\tau, \varepsilon^\tau]$, $\omega^\tau = [\omega_{1,1}^\tau, \omega_{1,2}^\tau, \dots\dots, \omega_{M,N}^\tau]$ *and* $\varepsilon^\tau = [\varepsilon_{1,1}^\tau, \varepsilon_{1,2}^\tau, \dots\dots, \varepsilon_{M,N}^\tau]$.

$$F(x) = \left(\boldsymbol{b} - h(\boldsymbol{X})\right)^T C_\gamma^{-1}(\boldsymbol{b} - h(\boldsymbol{X})) \qquad (32)$$

where,

$\mathbf{X}=[x,y,x_2^t,y_2^t,x_3^t,y_3^t,\dots x_M^t,y_M^t,x_1^r,y_1^r,x_2^r,y_2^r,\dots x_N^r,y_N^r]$

$\boldsymbol{b} = [d_{1,1}, d_{1,2}, .. d_{M,N}, \dots d'_{1,1}, d'_{1,2}, \dots d'_{M,N}]$

$h(\boldsymbol{X}) = [R_{1,1}, R_{1,2}, .. R_{M,N}, \dots R'_{1,1}, R'_{1,2}, \dots R'_{M,N}]$

$\nabla F(\mathrm{X})$ *is* computed in Appendix 5

Let us set a small value δ for limiting the convergence. Let $\varphi(\lambda) = \mathrm{F}(\mathbf{x} - \lambda\mathbf{D}\nabla F(\mathrm{X}))$. The iterations are carried out till $\|\nabla F(\mathrm{X})\| < \delta$. Initially $\mathbf{D}_j$ is set as an Identity matrix of order N'. Now, i is varied from 1 to N' computing equation (33), (34), (35), (36) and (37). Thus N' iteration is performed to converge $\mathbf{X}$.

$$d_j = -\mathbf{D_j}(\nabla F(\mathbf{X_j})) \qquad (33)$$

$$\boldsymbol{P}_j = \lambda_j \boldsymbol{d}_j \qquad (34)$$

$$\boldsymbol{X}_{j+1} = \boldsymbol{P}_j + \boldsymbol{X}_j \qquad (35)$$

$$\boldsymbol{q}_j = \nabla F(\mathbf{X}_{j+1}) - \nabla F(\mathbf{X}_j) \qquad (36)$$

$$\boldsymbol{D}_{j+1} = D_j + \frac{\boldsymbol{P}_j P_j^T}{P_j^T q_j} - \frac{\boldsymbol{D}_j q_j q_j^T D_j}{q_j^T D_j \boldsymbol{q}_j} \qquad (37)$$

Once the limiting condition is satisfied the parameters constituting vector $\mathbf{X}$ is precisely estimated.

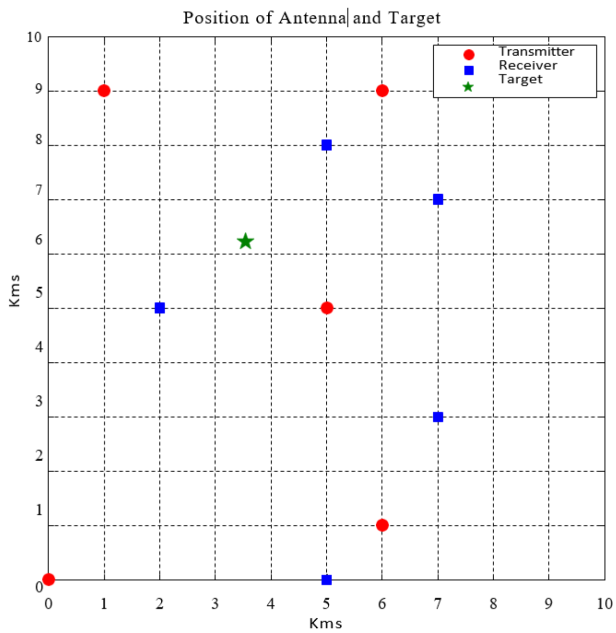

Fig. 2: Transmitters, receivers and target positions with respect to the in spatial domain

Algorithm 3 **DFP Algorithm for Precise Estimation**

while $\|\nabla F(\mathrm{X})\| > \delta$ **do**
Set $\mathbf{D}_0$ = **Identity**(N')
for j<N' **do**
Solve λ_j for $\varphi(\lambda_j) = 0$
$\boldsymbol{d}_j = -D_j(\nabla F(\mathbf{X}_j))$
$\boldsymbol{P}_j = \lambda_j \boldsymbol{d}_j$
$X_{j+1} = P_j + X_j$
$q_j = \nabla F(\mathbf{X}_{j+1}) - \nabla F(\mathbf{X}_j)$
$D_{j+1} = \boldsymbol{D}_j + \frac{P_j P_j^T}{P_j^T \boldsymbol{q}_j} - \frac{\boldsymbol{D}_j \boldsymbol{q}_j \boldsymbol{q}_j^T D_j}{\boldsymbol{q}_j^T D_j \boldsymbol{q}_j}$
end for
end while

4 Simulation and Results

In this section, the proposed algorithms are tested using MATLAB. Fig.9 shows the performance of algorithm presented in Section 3.2. Here first FDOA is estimated as per Section 3.1 and then velocity is estimated. Fig.5 is a plot between MSE of estimated FDOA with respect to different SNR. Fig.6 shows MSE of estimated velocity with respect to noise variance in FDOA (σ_{FDOA}). Here a 2×2 MIMO is tested. The transmitters are at [(2.5712, 3.0642), (1.2968, 8.1879)] and receivers at [(3.1231, 6.6976), (0.8682, 4.9240)] in Kms. The signal energy is taken as 400 and M and N are both 2. Target position is considered [0, 0]. The velocity of target is considered [0.568, 0.081]km/s. Thus, it is tested for higher velocity value.

Fig.7 and Fig.8 shows estimated velocity for 1000 Monte-Carlo trials for SNR=0dB and SNR= -12dB respectively. The carrier signal is considered to be of frequency $f_c = 1GHz$. Bandwidth BW = 50KHz and sampling period $5\mu s$. Number of samples collected is $N_s = 400$ and reflection coefficient $\zeta_{mn} = 1$ is considered.

Testing is done with two MIMO setups, 5×5 and 3×3. In MIMO 5×5, transmitters are located at (0,0), (1,9), (5,5), (6,1) and (6,9), and receivers are
placed at (5,0) , (2,5), (5,8), (7,7) and (7,3). For MIMO 3×3, transmitters are at (0, 0), (5, 8) and (9, 9) and receivers at (4, 0), (1, 9) and (5, 5). Letus consider a target positioned at (3.54,6.23) in both cases. All the distances are to be considered in Kms.

Fig.2 shows antenna and target arrangement in spatial domain. Theestimate of target's position is shown in Figure 3 and Figure 4 for MIMO 5×5 and MIMO 3×3respectively at σ^{α}= 5 deg.

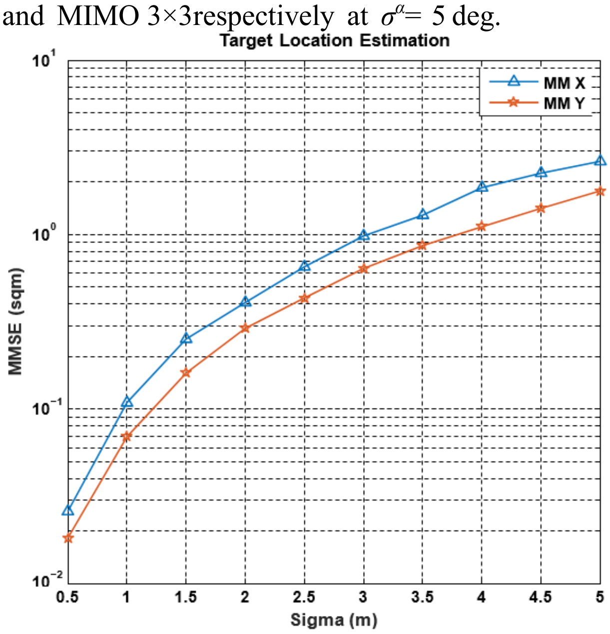

Fig.3: Plot between MSE of target position vs TDOA noise standard deviation $\sigma^{T}_{m,n}$ with MIMO 5×5

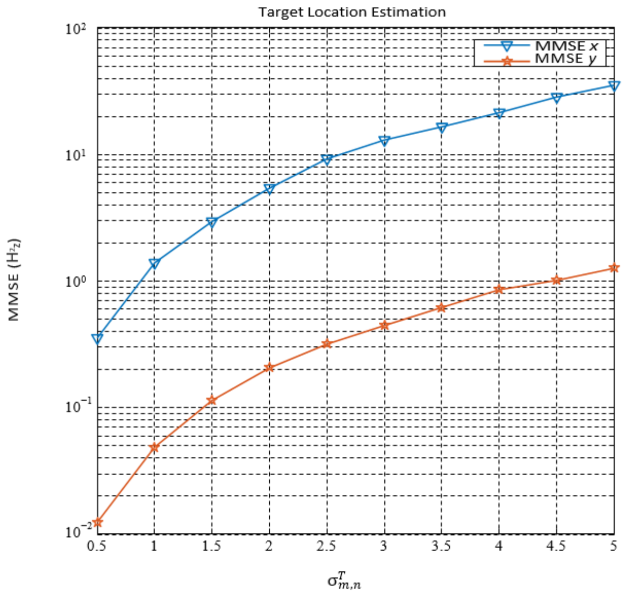

Fig.4: Plot between MSE of target position vs TDOA noise standard deviation $\sigma^{T}_{m,n}$ with MIMO 3×3

Performance of the proposed method is evaluated in the presence of TDOA and AOA noises. The MSE of the target's position versus the standard deviation of TDOA noises was calculated using 1000 - trial Monte-Carlo runs. It can be observed that the MMSE of y is less than that of x in Figure 3. This is because the spatial distribution of antenna are better for y-coordinate than x-coordinate (Figure2).

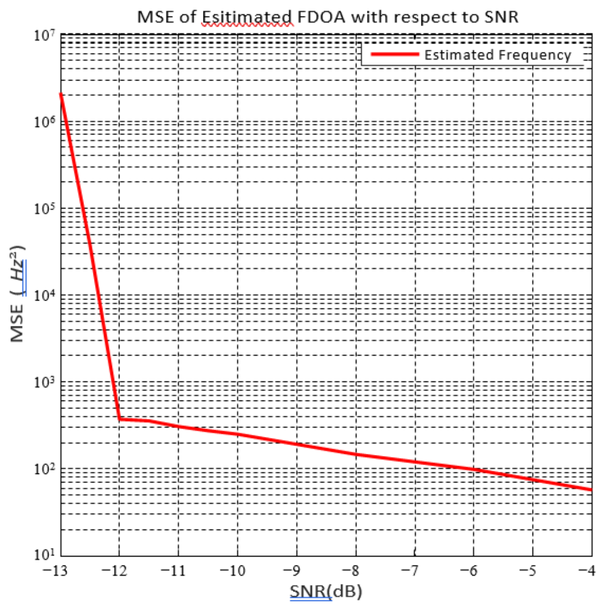

Fig.5: MSE Vs SNR for estimated FDOA

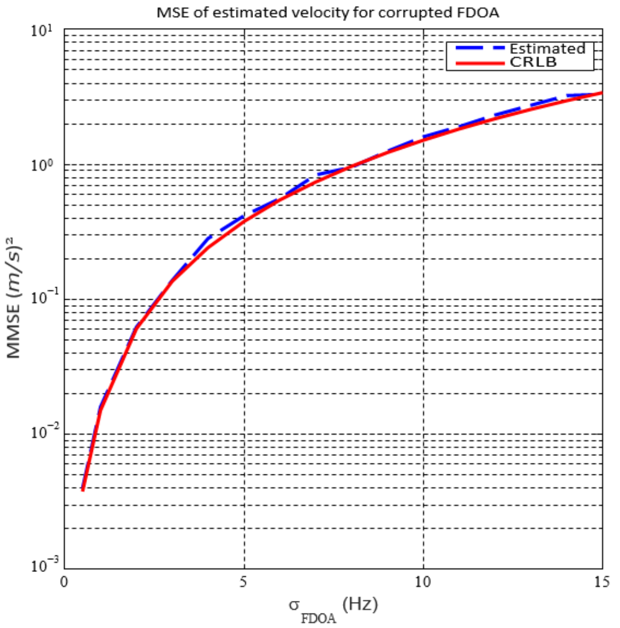

Fig.6: MSE of velocity estimation Vs noise σ_{FDOA} with CRLB

5 Conclusion

In this paper, we formulated the problem of target localization in MIMO radars in widely separated framework with unknown antenna locations. The target localization is done considering that the transmitters and receivers are stationary or moving with a very low velocity such that its positions do not change much with in the estimation interval. Further a new approach for estimating velocity is introduced. Here FDOA is estimated first and then the estimated FDOA with noisy is used to estimate velocity.

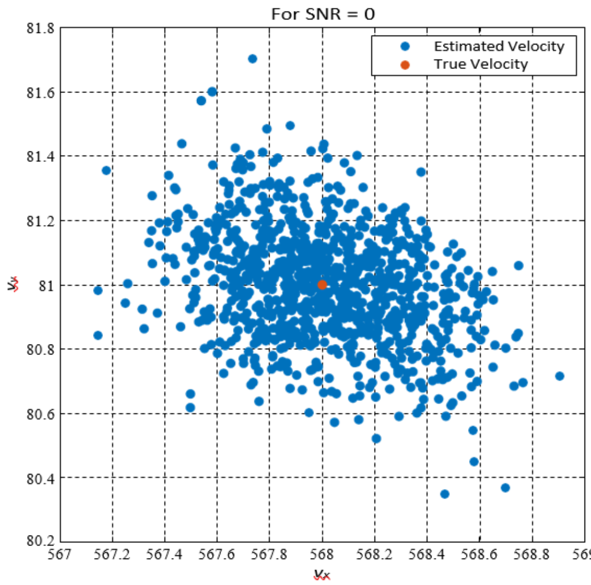

Fig.7: Estimated velocity for 1000 iterations at $SNR = 0$.

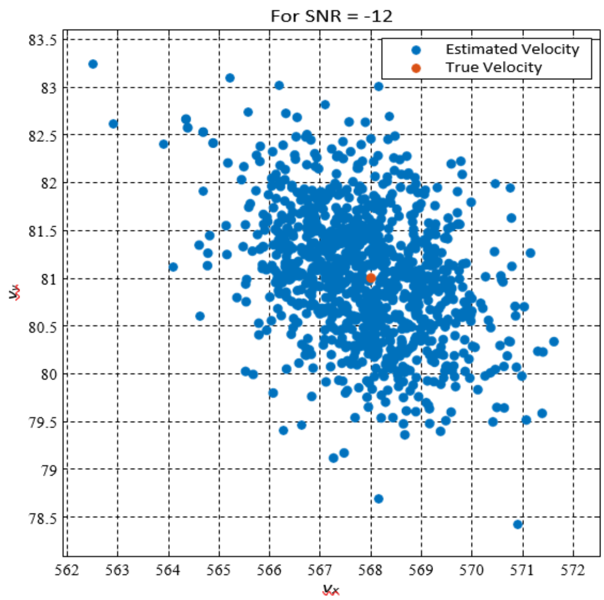

Figure 8: Estimated velocity for 1000 iterations at $SNR = -12$

Appendix A.

Appendix A.1. Cost Function Simplification of 3.2

By some mathematical manipulations, the cost function $\Gamma(\mathbf{v})$ can beformed as:

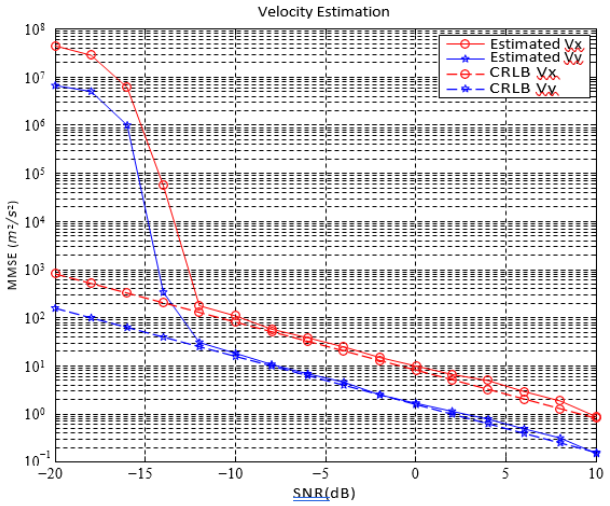

Figure 9: MSE (m^2/s^2) Vs SNR (dB) for a MIMO 2×2 Setup

$$\begin{aligned}\Gamma(\mathbf{v}) &= (\mathbf{F} - f(\mathbf{v}))^T C_\epsilon^{-1}(\mathbf{F} - f(\mathbf{v})) \\ &= (\tilde{f}(\mathbf{v}) - \widetilde{\mathbf{F}})^T(\tilde{f}(\mathbf{v}) - \widetilde{\mathbf{F}}) \\ &= \widetilde{\mathbf{F}}^T\widetilde{\mathbf{F}} - 2\widetilde{\mathbf{F}}^T\tilde{f}(\mathbf{v}) + \tilde{f}(\mathbf{v})^T\tilde{f}(\mathbf{v})\end{aligned}$$

The derivation of the cost function with respect to v_k , $k = 1, 2$ is as follows:

$$\begin{aligned}\frac{\partial\Gamma(\mathbf{v})}{\partial v_k} &= -2\frac{\partial}{\partial v_k}\widetilde{\mathbf{F}}^T\tilde{f}(\mathbf{v}) + \frac{\partial}{\partial v_k}\tilde{f}(\mathbf{v})^T\tilde{f}(\mathbf{v}) \\ &= -2\widetilde{\mathbf{F}}^T\frac{\partial\tilde{f}(\mathbf{v})}{\partial v_k} + \tilde{f}(\mathbf{v})^T\frac{\partial\tilde{f}(\mathbf{v})}{\partial v_k} + \tilde{f}(\mathbf{v})\frac{\partial\tilde{f}(\mathbf{v})^T}{\partial v_k} \\ &= -2\widetilde{\mathbf{F}}^T\frac{\partial\tilde{f}(\mathbf{v})}{\partial v_k} + 2\tilde{f}(\mathbf{v})^T\frac{\partial\tilde{f}(\mathbf{v})}{\partial v_k} \\ &= 2\left(\tilde{f}(\mathbf{v}) - \widetilde{\mathbf{F}}\right)^T\frac{\partial\tilde{f}(\mathbf{v})}{\partial v_k} \\ &= \left(\underbrace{f(\mathbf{v}) - \mathbf{F}}_{e^t}\right)^T C_\epsilon^{-1}\frac{\partial f(\mathbf{v})}{\partial v_k}\end{aligned}$$

For $k = 1, 2$ it can be written as

$$\begin{aligned}\psi_{mn} &= \left[\frac{\partial f(\mathbf{v})}{\partial v_1}, \frac{\partial f(\mathbf{v})}{\partial v_1}\right]^T \\ &= \left[\frac{\cos\phi_m^t + \cos\phi_n^r}{\lambda_c}, \frac{\sin\alpha_m^t + \sin\alpha_n^r}{\lambda_c}\right]^T\end{aligned}$$

for m=1,2,…,M and n=1,2,…,N.

Appendix A.2. Simplification of Gradient of F (X)

The cost function $F(X) = \left(\boldsymbol{b} - h(X)\right)^T C_\gamma^{-1}\ (\boldsymbol{b} - h(X))$. If X is a vector of N number of variables then $X = [x_{-1}, x_{-2}, \ldots, x_{N'}]$. Then Gradient of $F(X)$ can be written as

$$\nabla \mathrm{F}(\mathrm{X}) = \left[\frac{\partial \mathrm{F}(\mathrm{X})}{\partial \mathrm{x}_1}, \frac{\partial \mathrm{F}(\mathrm{X})}{\partial \mathrm{x}_2}, \ldots, \frac{\partial \mathrm{F}(\mathrm{X})}{\partial \mathrm{x}_{N'}}\right]$$

Now,

$$\frac{\partial F(X)}{\partial x_k} = 2\left(\underbrace{((h(X) - b)}_{e}\right)^T C_\gamma^{-1}\left(\frac{\partial h(X)}{\partial x_k}\right)$$

Here,

$$X = [x, y, x_2^t, y_2^t, \ldots, x_M^t, y_M^t, x_1^r, x_2^r, y_2^r, \ldots, x_N^r, y_N^r]$$

$$h(X) = [R_{1,1}, R_{1,2}, \ldots, R_{M,N}, R'_{1,1}, R'_{1,2}, \ldots, R'_{M,N}]$$

Then,

$$\frac{\partial h(X)}{\partial x_k} = \left[\frac{\partial R_{1,1}}{\partial x_k}, \ldots, \frac{\partial R_{M,N}}{\partial x_k}, \frac{\partial R'_{1,1}}{\partial x_k}, \ldots, \frac{\partial R'_{M,N}}{\partial x_k}\right]$$

Computing each partial derivative

$$\frac{\partial R_{m,n}}{\partial x} = \frac{x - x_m^t}{||x - x_m^t||} + \frac{x - x_n^r}{||x - x_n^r||},$$

$$\frac{\partial R_{m,n}}{\partial y} = \frac{y - y_m^t}{||y - y_m^t||} + \frac{y - y_n^r}{||y - y_n^r||},$$

$$\frac{\partial R_{m,n}}{\partial x_m^t} = -\frac{x - x_m^t}{||x - x_m^t||}, \quad \frac{\partial R_{m,n}}{\partial y_m^t} = -\frac{y - y_m^t}{||y - y_m^t||},$$

$$\frac{\partial R_{m,n}}{\partial x_n^r} = -\frac{x - x_n^r}{||x - x_n^r||}, \quad \frac{\partial R_{m,n}}{\partial y_n^r} = -\frac{y - y_n^r}{||y - y_n^r||},$$

$$\frac{\partial R'_{m,n}}{\partial x} = \frac{\partial R'_{m,n}}{\partial y} = 0,$$

$$\frac{\partial R_{m,n}}{\partial x_m^t} = -\frac{\partial R_{m,n}}{\partial x_n^r} = -\frac{x_n^r - x_m^t}{||x_n^r - x_m^t||},$$

$$\frac{\partial R_{m,n}}{\partial y_m^t} = -\frac{\partial R_{m,n}}{\partial y_n^r} = -\frac{y_n^r - y_m^t}{||y_n^r - y_m^t||}$$

Appendix B. CRLB

Appendix B.1. CRLB Derivation For Velocity Estimation

We aim to derive the CRLB for velocity estimation. The velocity v is estimated from the FDOA observations. The CRLB can be calculated usingthe trace of the inverse of Fisher information matrix, denoted by I. For a Gaussian observations, with mean vector μ and covariance matrix C_ϵ, then$I$ can be written as,

$$[I(\mathbf{v})]_{(i,j)} = \left[\frac{\partial \mu}{\partial v_i}\right] C_\epsilon^{-1} \left[\frac{\partial \mu}{\partial v_j}\right] + \frac{1}{2} tr \left[C_\epsilon^{-1} \left[\frac{\partial C_\epsilon}{\partial v_i}\right] C_\epsilon^{-1} \left[\frac{\partial C_\epsilon}{\partial v_j}\right]\right]$$

In the present study, $\mu = f(\mathbf{v})$ and C_ϵ is independent of $\mathbf{v}$. Thus the secondterm in above equation is equal to zero and the first term yields:

$$cov(\hat{\mathbf{v}}) \geq [I(\mathbf{v})]^{-1}$$

$$[I(\mathbf{v})] = \left[\left[\frac{\partial f}{\partial v}\right]^T C_\epsilon^{-1} \left[\frac{\partial f}{\partial v}\right]\right]$$

$$\left[\frac{\partial f}{\partial v}\right] = \psi$$

$$\psi = [\psi_{11}, \psi_{12}, \ldots, \psi_{MN}]$$

CRLB can be plotted using $tr([I(v)]^{-1})$.

Appendix B.2. CRLB Derivation for Combined Velocity Estimation from Signal Using Algorithm

For retrieving noise variance from SNR

$$\sigma_N^2 = 10^{-\frac{SNR}{10}} \left| \frac{\int_{t=0}^{N_s T_s} r_{mn}(t) dt}{N_s} \right|$$

Now the CRLB can be obtained as

$$h_{m,n} = 2\pi \int_{t=0}^{N_s T_s} ((t - \tau_{m,n}) r_{m,n}(t)) dt$$

$$H = [h_{1,1}\psi_{1,1}, h_{1,2}\psi_{1,2}, \ldots, h_{M,N}\psi_{M,N}]$$

Let C_ϵ be the covariance matrix of noise in $r_{m,n}(t)$. Then,

$$cov(\hat{\mathbf{v}}) \geq tr\{\sigma_N^2 inv(H^T C_\epsilon^{-1} H)\}$$

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