An Iterative Re-Weighted Least-Squares Tone Reservation Method for PAPR Reduction in OFDM Systems

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Abstract: - In OFDM systems, the problems associated with a high ratio of peak-to-average power still exist. A search for a simple and practical method to reduce the ratio continues. In this paper, a robust sub-optimal tone reservation method based on iterative re-weighted least-squares minimization of infinity norm is proposed. The method is simple and has a fast quadratic convergence and per iteration complexity $O(LP)$ lower than that of the FFT, where L and P are, respectively, the number of reserved subcarriers, and nonzero elements in the desired peak-reducing signal. In addition, the method does not experience peak re-growth problems and achieves high PAPR reductions of 3.9 dB and 5.6 dB for 1.6% and 5% reserved subcarriers respectively. For 20% reserved subcarriers, the method reaches 7.4 dB PAPR reductions. These reductions are at a small cost of 0.6 dB increase in the average transmitted power. The PAPR reductions from the proposed method compare well with the highly slow and complex optimal tone reservation methods but are far much higher than from sub-optimal methods reported in literature. Simulation results also show that the method has PAPR reductions that are linear with the binary logarithm of the number of subcarriers, and this can help to predict PAPR reductions for different OFDM systems with different number of reserved subcarriers.

Keywords: - Orthogonal Frequency Division Multiplexing (OFDM); High Power Amplifier (HPA); Peak-to-Average Power Ratio (PAPR); Iterative Re-weighted Least-Squares (IRLS); Tone Reservation (TR)

1 Introduction

Multicarrier transmission techniques employ parallel low data rate streams to achieve high data rate on aggregation, and to avoid multipath interference. OFDM is one such technique, which in addition, has mutually orthogonal subcarriers. Orthogonality of subcarriers makes OFDM spectrally efficient and if preserved over the radio channel, only a simple single-tap equalizer is required at the receiver to restore each of the subcarrier signals. In order to eliminate both inter-subcarrier and inter-symbol interferences, the inter-subcarrier spacing and symbol duration values are normally set above the maximum Doppler spread and multipath delay. These noble properties of OFDM have made it the preferred multiplexing technique for high data rate transmissions in many radio systems including Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), IEEE 802.11 Wireless Local Area Networks (WLAN), IEEE 802.16a Wireless Metropolitan Area Networks (WMAN), 4G, and 5G mobile communication networks. Despite the numerous competitive advantages, OFDM signals tend to exhibit high peak-to-average power ratio (PAPR) [1] . Distortionless processing of high PAPR signals through the transmitter section requires the nonlinear transmitter devices mainly the digital-to-analogue converter and high power amplifier (HPA) to have a costly wide dynamic range in order to accommodate all the signal amplitudes.

In addition, the high PAPR signal affects the point of operation of the HPA. Ideally, the HPA should be operated near the saturation region in order to have high power efficiency. However, this will cause nonlinear amplification of the high signal amplitudes and in turn results to in-band and out-of-band radiations, and consequently the degradation of the bit-error rate (BER) and frequency interference in the adjacent channels. To avoid the nonlinear amplification, the HPA can be forced to operate deep in the linear region by providing it with an input power back-off as determined by the PAPR of the input signal. However, this will lower the power efficiency and in turn raise the power consumption and hence the cost of the transmitter in addition to reducing the lifetime of the battery power at the user terminals [2] . For these above reasons, it is desirable to reduce the PAPR to suitable levels, more so for OFDM systems with large number of subcarriers as they are more susceptible to unacceptably high PAPR.

Recently, different methods for PAPR reduction in

OFDM systems have been proposed in literature. They include signal coding and companding [3] , [4] , selective mapping (SLM) [4] , signal scaling [5] , partial transmit sequence (PTS) [7] and tone reservation [8] . The focus of the research is now more on the development of simple practical techniques that have low computational complexity, fast convergence rate, and high PAPR reduction.

The tone reservation (TR) approach is quite promising as it involves reserving a few subcarriers, referred to as peak reduction tones, for use to generate and carry a peak-reducing signal that reduces PAPR. Since the user data and the peak-reducing signal are on separate subcarriers, there is no distortion on the data and hence no degradation of BER. In addition, the technique does not require transmission of any side information because for demodulation, the receiver needs only the locations of the data-bearing subcarriers.

Depending on the derivation of the peak-reducing signal, tone reservation methods are either optimal or sub-optimal. Optimal methods such as the linear programmed TR (LP-TR) [8] and second order cone programmed TR (SOCP-TR) [9] have high computational complexity and slow convergence rate but can achieve high PAPR reduction that can help to benchmark the performance of the sub-optimal methods. The sub-optimal curve fitting TR (CF-TR) [11] iteratively solves a least-squares approximation (LSA) problem to generate the peak-reducing signal. However, in the CF-TR, the PAPR reduction depends on the clipping threshold and degrades if the number of reserved subcarriers is set lower than the number of zeros in the clipping noise. The scaling signal-to-clipping noise ratio (S-SCR and MS-SCR) TR [12] methods utilizes a time-domain kernel together with LSA optimized scaling factor and peak regeneration constraints. However, the two S-SCR and MS-SCR methods are still prone to peak re-growth and their PAPR reduction performance depends on the clipping ratio in use. The sub-optimal weighted TR (WTR) method [13] solves a weighted least squares optimization to generate the peak-reducing signal. However, the WTR method has difficulties finding the optimal weights, experiences peak re-growth and has poor PAPR reduction performance.

In this paper, a fast iterative re-weighted least-squares based tone reservation (IRLS-TR) method that offers high PAPR reductions in OFDM systems is proposed. The method generates the required peak-reducing signal by utilizing a robust iterative re-weighted least-squares algorithm for minimization of the infinity norm to approximate the desired peak-reducing signal. This IRLS-TR technique has low computational complexity and fast quadratic convergence in addition to offering better PAPR reductions than the CF-TR, MS-SCR and WTR methods. The technique has great potential for practical implementations in the current and future multicarrier transmission techniques.

The organization of the rest of the paper is as follows. Section II describes the OFDM signal and statistical distribution of PAPR. Section III outlines the general concept of tone reservation techniques. Section IV describes the proposed technique while simulation results, analysis, and comparison with other techniques are in section V. Section VI concludes the paper and gives suggestion for future work.

2 OFDM Signal and PAPR

The OFDM signal is a superimposition of N mutually orthogonal subcarrier signals. At the baseband level, and during the symbol duration T, the signal has the following analytical expression:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k\,(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi f_k t}. \qquad (1)$$

Here, $x_k(t)$ is the kth modulated subcarrier signal with frequency $f_k = k\Delta f$, and $X(k)$ is the subcarrier-modulating symbol. The modulating symbols are either binary phase-shift keying or M-ary quadrature amplitude modulation symbols [14] . To achieve mutual orthogonality between subcarriers, the subcarrier spacing Δf is set to $1/T$. The relationship given in (1) conveniently ensures the same signal power in the time and frequency domains. Since (1) is similar to the standard inverse discrete Fourier transform (IDFT) equation, signal processing of OFDM signal is via the well-known fast Fourier transform algorithms. Due to the addition of N subcarrier signals in (1), the OFDM signal experiences envelope fluctuations that may have profound effect on the linear processing by the nonlinear devices in the transmitter. The nonlinear device of concern in this work is the high power amplifier (HPA).

Our main interest then, is to find the probability of the maximum instantaneous power of $x(t)$ being out of the linear range of the HPA. A good measure of temporal power fluctuations of a signal is the PAPR, and is defined as the ratio of the maximum instantaneous power to the average power i.e.

$$\text{PAPR}\,\{x(t)\} = \frac{\max\{|x(t)|^2\}}{\mathrm{E}\{|x(t)|^2\}} \qquad (2)$$

where E {·} denotes the expectation operation.

Considering that the subcarrier signals $x_k(t)$ are statistically independent and assuming that N is large, then from the central limit theorem, the real and imaginary parts of $x(t)$ are Gaussian distributed and accordingly, the amplitudes of $x(t)$ are Rayleigh distributed [15] . This in turn implies that $x(t)$ could have high PAPR or in other words, has some high amplitude values that are well above the average value of the signal amplitudes.

In practice, the signal $x(t)$ is processed digitally. Therefore, there is need to approximate the continuous-time PAPR in (2) with its discrete-time counterpart given by

$$\mathrm{PAPR}\{\boldsymbol{x}\} = \frac{\max_{0\le n<N}\{|x(n)|^2\}}{E\{|x(n)|^2\}} \tag{3}$$

where $\boldsymbol{x} = [x(0), x(1), \ldots, x(N-1)]^T$, is the discrete-time signal obtained after sampling of signal $x(t)$.

The signal samples $x(n)$ have magnitudes that are still Rayleigh distributed. Equation (3) is valid if signal $x(t)$ is sampled at a rate sufficiently greater than the Nyquist rate by a factor of at least four in order to avoid missing the peak value [16] . Another important measure of PAPR is the complementary cumulative distribution function (CCDF), which is the probability that PAPR exceeds a certain threshold. The derivation of CCDF has been well treated in [9] and is given by

$$\Pr(\mathrm{PAPR}\{\boldsymbol{x}\} > \gamma) = 1 - (1 - e^{-\gamma})^N \tag{4}$$

where γ is the threshold PAPR, N is the total number of subcarriers, and $\Pr(.)$ denotes the probability operator. The CCDF metric is a performance tool widely used to measure how well a proposed method reduces the PAPR.

3 Tone Reservation Concept

All tone reservation methods follow the same concept of adding one signal, referred here to as peak-reducing signal, to another signal having high PAPR in order to reduce the PAPR. Figure 1 is an illustration of the concept and has $X(k)$ and $C(k)$ as the frequency-domain subcarrier modulating data symbols and peak-reducing coefficients respectively. The modulating symbols form the data vector $\boldsymbol{X} = [X(0), X(1), \ldots, X(N-1)]^T$ with all nonzero values except in L positions reserved for peak-reducing signal. Similarly, the peak-reducing coefficients form the vector $\boldsymbol{C} = [C(0), C(1), \ldots, C(N-1)]^T$ with all zero values except in L positions reserved for peak power reduction. After the IDFT operation on $\boldsymbol{X}$ and $\boldsymbol{C}$, the resulting time signals $\boldsymbol{x} = [x(0), x(1), \ldots, x(N-1)]^T$ and $\boldsymbol{c} = [c(0), c(1), \ldots, c(N-1)]^T$, are combined to generate a low-PAPR signal i.e. $\mathrm{PAPR}\{\boldsymbol{x} - \boldsymbol{c}\} < \mathrm{PAPR}\{\boldsymbol{x}\}$.

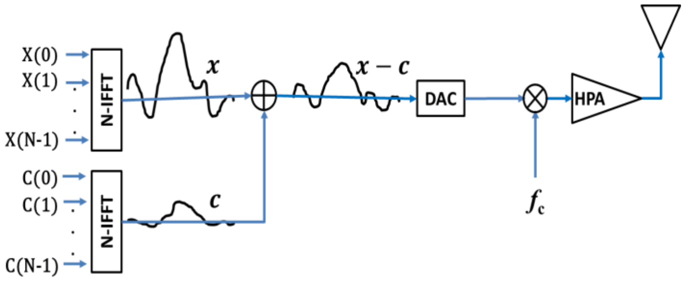

Figure 1: Tone reservation concept

From the foregoing discussion, the signal processing operations can be described by the equation

$$x(n) - c(n) = x(n) - \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} C(k)e^{j2\pi kn/N} \tag{5}$$

or in matrix notation by

$$\boldsymbol{x} - \boldsymbol{c} = \boldsymbol{x} - \boldsymbol{QC} \tag{6}$$

where $\boldsymbol{Q} \in \mathbb{C}^{N\times N}$ is the IDFT matrix with elements $\frac{1}{\sqrt{N}}e^{j2\pi kn/N}$ and all $\boldsymbol{x}, \boldsymbol{c}, \boldsymbol{C} \in \mathbb{C}^N$. Since $\boldsymbol{C}$ has $N-L$ zeros, the computational complexity of (6) can be reduced further by expressing the peak-reducing signal in the form

$$\boldsymbol{c} = \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}. \tag{7}$$

Here $\widehat{\boldsymbol{C}} \in \mathbb{C}^L$ contains only the nonzero values of $\boldsymbol{C}$ and $\widehat{\boldsymbol{Q}} \in \mathbb{C}^{N\mathrm{x}L}$ is a submatrix of $\boldsymbol{Q}$ formed by choosing the L columns corresponding to the reserved subcarriers.

The task now is to find the peak-reducing coefficients or vector $\widehat{\boldsymbol{C}}$ that minimizes the PAPR of $\boldsymbol{x}$. This problem can be formulated as a minimax [17] optimization problem of the form

$$\min_{\widehat{\boldsymbol{C}}} \max|\boldsymbol{x} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}|. \tag{8}$$

Here, $\widehat{\boldsymbol{C}} \in \mathbb{C}^L$ is the optimization variable while $\boldsymbol{x} \in \mathbb{C}^N$ and $\widehat{\boldsymbol{Q}} \in \mathbb{C}^{N\mathrm{x}L}$ are the problem parameters. To date, sub-optimal algorithms to solve the problem in (8) at a reduced computational complexity and short convergence time and at the same time achieve high PAPR reductions continue originating.

Although tone reservation methods offer quite an attractive approach for reducing PAPR, they are not without drawbacks. First, the average power of the PAPR-reduced signal increases. This calls for one to limit the increase to a minimum as the algorithm executes to ensure compatibility with HPA specifications. Second, the number of reserved subcarriers, L, reduces the data rate because the reserved subcarriers do not carry any user data. Therefore, to avoid a high data rate loss, L should be small as can be practically possible.

4 Proposed Method

This paper proposes a novel sub-optimal tone reservation method, hereafter referred to as the iterative re-weighted least-squares tone reservation (IRLS-TR) method, which performs fast approximation of peak-power reducing signals. The method approximates a desired peak-reducing signal with a signal designed in accordance with the frequency allocation constraints as imposed by the tone reservation concept discussed in section III.

4.1 Algorithm

Any PAPR reducing method attempts to reduce the peak power to a value close to the average power. With this consideration, the desired peak-reducing signal that is required to cancel the high peaks of the OFDM signal can be posed as the OFDM signal amplitudes above the average value. Subsequently, the desired peak-reducing signal has the analytical equation

$$d(n) = \begin{cases} \frac{x(n)}{|x(n)|}(|x(n)| - \bar{x}), & |x(n)| > \bar{x} \\ 0 \quad , & |x(n)| \le \bar{x} \end{cases} \qquad (9)$$

$$n = 0,1, \dots, N-1.$$

Here, $\bar{x}$ is the average value of the discrete-time OFDM signal. The elements of $\widehat{\boldsymbol{C}}$ are then determined to have the time-domain peak-reducing signal in (7) equal to the signal in (9) by solving the matrix equation

$$\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} = \boldsymbol{d}. \qquad (10)$$

However, since $\widehat{\boldsymbol{Q}} \in \mathbb{C}^{N\mathrm{x}\mathrm{L}}$, $\widehat{\boldsymbol{C}} \in \mathbb{C}^{L}$ and $\boldsymbol{d} \in \mathbb{C}^{N}$, the system of linear equations in (10) is rectangular and hence overdetermined. For such a system, there is in general no exact solution to $\widehat{\boldsymbol{C}}$ and therefore, the time-domain peak-reducing signal $\boldsymbol{c} = \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}$ can only be determined to approximate the desired signal vector $\boldsymbol{d}$ i.e.

$$\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} \approx \boldsymbol{d}, \qquad (11)$$

by minimizing the residual error

$$\boldsymbol{\epsilon} = \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} - \boldsymbol{d} \qquad (12)$$

using some norm as a measure of the error size.

In addition to the minimization of the error in (12), for the problem at hand, the high peaks of the designed peak-reducing signal, $\boldsymbol{c} = \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}$, should approximate those of the signal $\boldsymbol{d}$ in (9) as practically as is possible in order to cancel all the high peaks in the original OFDM signal and thereby reduce the PAPR. The design of the peak-reducing signal will therefore involve finding the elements of $\widehat{\boldsymbol{C}}$ that minimizes the L_∞ norm of the error. This is equivalent to the minimization of the L_p norm of the error for a large value of p [18] as given by the equation

$$\min_{\widehat{\boldsymbol{C}}} ||\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} - \boldsymbol{d}\,||_p, \quad p \to \infty. \qquad (13)$$

In practice, the solution to the L_p problem approximates that of L_∞ if (13) is solved for $p \ge 10$ [19] . However, there is no analytical method for finding the optimal approximation solution for any norm other than the L_2 norm. For this reason, it is necessary to transform the L_p problem into an equivalent simple weighted least squares (WLS) problem that can be solved analytically. This WLS problem has the form

$$\min_{\widehat{\boldsymbol{C}}} ||\boldsymbol{W}(\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} - \boldsymbol{d})\,||_2 \qquad (14)$$

where $\boldsymbol{W}$ is a real $N \times N$ diagonal weighting matrix that applies large weights on the high signal peaks to emphasize their minimization. If the diagonal weights in $\boldsymbol{W}$ are known, then equation (14) can be solved by a simple method that has the closed form solution [20]

$$\widehat{\boldsymbol{C}} = \left[\widehat{\boldsymbol{Q}}^{*}\boldsymbol{W}^{\boldsymbol{T}}\boldsymbol{W}\widehat{\boldsymbol{Q}}\right]^{-1}\widehat{\boldsymbol{Q}}^{*}\boldsymbol{W}^{\boldsymbol{T}}\boldsymbol{W}\boldsymbol{d}. \qquad (15)$$

Here, the superscripts T and * denote matrix transpose and conjugate transpose respectively.

In order to make (14) to be equivalent to (13), there is need for careful selection of the diagonal weights $w(n)$ of the weighting matrix $\boldsymbol{W}$. Rewriting the L_2 norm of the weighted error, $\boldsymbol{\varepsilon} = \boldsymbol{W}\big(\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} - \boldsymbol{d}\big)$, in (14) as

$$||\boldsymbol{\varepsilon}||_2 = (\textstyle\sum_{n=0}^{N-1} w^2(n)|\epsilon(n)|^2)^{1/2}, \qquad (16)$$

and assigning the error to the weights according to

$$w(n) = |\epsilon(n)|^{(p-2)/2}, \qquad (17)$$

then (16) becomes the L_p norm of the error as in (13), i.e.

$$||\varepsilon||_p = \left(\sum_{n=0}^{N-1} |\epsilon(n)|^{(p-2)}|\epsilon(n)|^2\right)^{1/p}. \qquad (18)$$

Therefore, solving the WLS problem in (14) is identical to solving the L_p problem in (13).

However, solving problem (14) cannot be done in one-step because one needs to find the weights that give the optimal approximation. To this end, this work employs a robust iterative re-weighted least squares (IRLS) algorithm to find the solution to the WLS problem. The algorithm builds from the analytical solution in (15) but with per iteration re-weighting until convergence to the large L_p norm of (13). In its most basic form, the IRLS algorithm starts by solving for $\widehat{\boldsymbol{C}} \in \mathbb{C}^L$ from (15) with all initial weights set to one i.e. $w(n) = 1$. Then, it computes the error vector from (12), followed by new weights from (17) that are for use in the next iteration. Using the new weights, the algorithm finds a new solution $\widehat{\boldsymbol{C}}$ and this process repeats until convergence when the L_p norm of the error is quite small e.g. less than 10^{-4} or until the number of iterations reaches a predetermined maximum iteration number.

The basic IRLS algorithm presented above has two concerns that need to be addressed. Firstly, the algorithm may not converge and/or is numerically unstable for some L_p norms. Secondly, it has linear convergence and therefore if it converges and is numerically stable, it does so very slowly as to be of any practical use. To overcome these two shortcomings, the basic algorithm is transformed into a form of the Newton's method [21] in which the solution is only partially updated at each iteration. With this update, the solution at ith iteration becomes

$$\widehat{\boldsymbol{C}}_i = \mu_i \widehat{\boldsymbol{C}}_i^c + (1 - \mu_i)\widehat{\boldsymbol{C}}_{i-1} \tag{19}$$

where $\mu_i = 1/(p_i - 1)$ is the update parameter [22] [23] and $\widehat{\boldsymbol{C}}_i^c \in \mathbb{C}^L$ is the current WLS solution. However, as is common with most Newtonian methods, this modification makes the algorithm sensitive to initial approximations and this may affect the initial convergence rate.

In order to improve on the initial convergence rate, the value of p is increased gradually from its initial value of two to the final value of the L_p norm that is being used to approximate the L_∞ norm. This modification is quite similar to the homotopy [24] [25] and is done iteratively by multiplying p with a convergence parameter, α, of between one and two depending on the required rate of convergence. The value of p at the i-th iteration is then determined by

$$p_i = \min(p, \alpha p_{i-1}). \tag{20}$$

In addition, the convergence parameter is determine by the value of p in the L_p norm and the maximum iteration number according to

$$\alpha = 10^{\log(p)/M} \tag{21}$$

where p and M are, respectively, the L_∞ approximation norm and maximum iteration number. The settings in (20) and (21) guarantee reliable convergence of the algorithm to the optimal approximation solution. This is because as p progressively increases from the initial value, the difference from one L_p solution to the next is small, and when the algorithm approaches the neighbourhood of the desired L_∞ approximation norm it iterates several times at the same p.

The IRLS-TR algorithm can be summarized as follows.

Algorithm: IRLS-TR

1. Set the L_∞ approximation norm p, initial weights to $w = 1$ and the L_p norm error threshold ε_{th}.
2. Set the maximum iteration number M and convergence parameter $\alpha = 10^{\log(p)/M}$.
3. Choose reserved subcarrier locations and compute the IFFT submatrix $\widehat{\boldsymbol{Q}}$.
4. Generate the original OFDM time-domain signal $\boldsymbol{x}$.
5. Calculate the desired peak-reducing signal $\boldsymbol{d}$ from the signal $\boldsymbol{x}$.
6. Initialize the iteration counter $i = 0$ set the initial p-norm value $p_0 = 2$.
7. Calculate the initial LS solution $\widehat{\boldsymbol{C}}_0 = \widehat{\boldsymbol{Q}}^{-\mathbf{1}}\boldsymbol{d}$.
8. Calculate the error vector $\boldsymbol{\epsilon_i} = \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}_{\boldsymbol{i}} - \boldsymbol{d}$.
9. Calculate the new weighting matrix elements using $w_i(n) = |\epsilon_i(n)|^{(p_i-2)/2}$.
10. Calculate the LS solution $\widehat{\boldsymbol{C}}_i^c = \left[\widehat{\boldsymbol{Q}}^* \boldsymbol{W}_i^T \boldsymbol{W}_i \widehat{\boldsymbol{Q}}\right]^{-1} \widehat{\boldsymbol{Q}}^* \boldsymbol{W}_i^T \boldsymbol{W}_i \boldsymbol{d}$.
11. Calculate the new update parameter $\mu_i = 1/(p_i - 1)$.
12. Update the LS solution to $\widehat{\boldsymbol{C}}_i = \mu_i \widehat{\boldsymbol{C}}_i^c + (1 - \mu_i)\widehat{\boldsymbol{C}}_{i-1}$.
13. Calculate the L_p norm error $||\boldsymbol{\varepsilon_i}||_p = (\sum_{n=0}^{N-1} |\epsilon_i(n)|^p)^{1/p}$.
14. Set $i = i + 1$. If $i < M$ or $||\boldsymbol{\varepsilon_i}||_p > \varepsilon_{th}$, update the p-norm value to $p_i = \min(p, \alpha p_{i-1})$ and go to step 8. Otherwise, transmit $\boldsymbol{x} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}}_{\boldsymbol{i}}$ and terminate algorithm.
15. End

4.2 Convergence

With the modifications in (19), (20) and (21), the algorithm quadratically converges in just a few iterations as illustrated in Fig. 2. The figure depicts a typical plot of the convergence curve of the error in (18) at each iteration as the algorithm was executed. As shown in the figure, the algorithm converged after about 10 iterations.

4.3 Computational Complexity

At each iteration, the algorithm spends most of the computational time to find the solution in (15) for the WLS problem

$$\boldsymbol{W}\widehat{\boldsymbol{Q}}\widehat{\boldsymbol{C}} = \boldsymbol{W}\boldsymbol{d}. \tag{22}$$

Since the matrices $\boldsymbol{W} \in \mathbb{R}^{N\times N}$, $\widehat{\boldsymbol{Q}} \in \mathbb{C}^{N\times L}$, and $\widehat{\boldsymbol{C}} \in \mathbb{C}^{L}$, the computational complexity of the algorithm per iteration is then given by $O(NL)$. This complexity can be reduced further if the algorithm computes the solution from only the non-zero elements in vector $\boldsymbol{d}$. Denoting the number of non-zeros elements in $\boldsymbol{d}$ by P, the computational complexity of the algorithm will then become $O(LP)$. Now, since P is always less than $N/2$, with a few reserved subcarriers for PAPR reduction, the algorithm's complexity of $O(LP)$ is less than $O(N\log_2 N)$ of the fast Fourier transform algorithms.

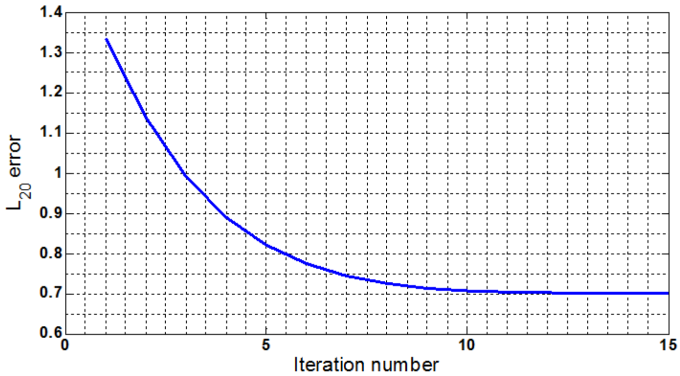

Figure 2: Convergence curve

5 Results and Discussion

The IRLS-TR method proposed in section IV yields the frequency-domain vector $\widehat{\boldsymbol{C}} \in \mathbb{C}^{L}$. The product of this vector and the IDFT submatrix $\widehat{\boldsymbol{Q}}$ is the time-domain signal used to reduce the PAPR of the OFDM signal. Matlab simulations using the OFDM parameters in Table 1 assessed the performance of the method when applied to reduce the PAPR in OFDM systems comprising different subcarrier modulations and different numbers of reserved subcarriers. The locations of the reserved subcarriers were randomly selected in all OFDM symbols.

Table 1: OFDM Parameters

FFT size, N	16, 32, 64, 128, 256, 512, 1024, 2048
Subcarrier modulation	QPSK, 16-QAM, 64-QAM, 256-QAM, 1024-QAM
Number of OFDM symbols	10,000
Number of reserved subcarriers, L	5%, 10%, 15%, and 20% of N
Reserved subcarrier locations	random

To achieve high PAPR reductions, the method generates a peak-reducing discrete-time signal with the highest peaks approximating those of the desired peak-reducing signal as illustrated in Fig. 3. For example, at $n = 137$ the magnitude of the highest peak of the generated peak-reducing signal is 1.214 and this approximates 1.806 of the desired signal. Thus, subtracting the generated signal from the original high PAPR OFDM signal reduces the PAPR. In addition, the peaks of the designed signal are below all the major spikes of the desired signal. This then avoids the peak re-growth problem where the cancellation of one peak regenerates a new peak at a different position as reported in methods that utilize peak-cancelling kernels [7] [26] [27] to reduce PAPR.

The proposed IRLS-TR method achieves high PAPR reduction by utilizing only a small number of reserved subcarriers to generate and carry the PEAK-REDUCING SIGNAL. The CCDF curves in Fig. 4 illustrates this point for the cases where out of 256 total number of subcarriers, 2, 4, 8, 13, 20, and 52 subcarriers were reserved. For example, with 4 and 13 reserved subcarriers, corresponding to 1.6% and 5% of the total subcarriers, the method achieved 3.9 dB and 5.6 dB PAPR reductions at CCDF = 10^{-3}. Compared to the SOCP-TR by Kiambi et al and LP-TR by Tellado, with 5% reserved subcarriers, the reduction of 5.6 dB from the proposed IRLS-TR method is better than the 4.6 dB and 5.3 dB from the two methods respectively.

For the case with 20% reserved subcarriers, the IRLS-TR method proposed here achieved a PAPR reduction of 7.4 dB, and this again is close to the 8.0 dB from the optimal methods.

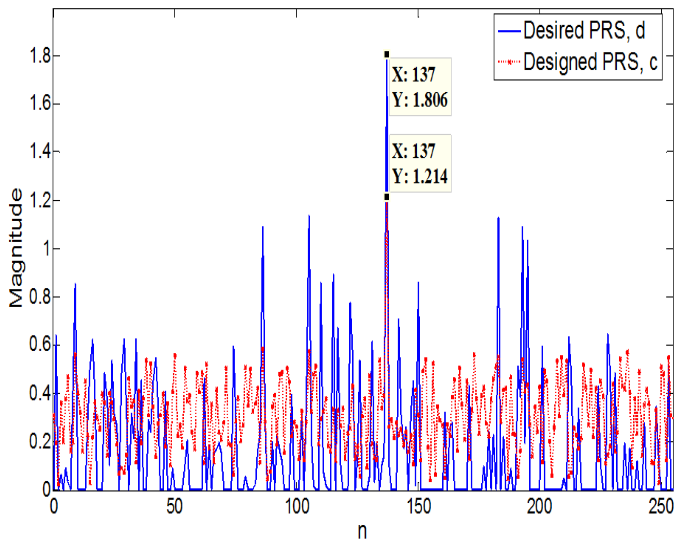

Figure 3: Desired and designed peak-reducing discrete-time signals

To compare the performance of the proposed IRLS-TR method to the sub-optimal methods, the actual results reported in CF-TR by Jiang et al, MS-SCR by Wang et al, and WTR by Xin and Yi were considered. From the results for the PAPR reduction reported by Jiang et al for the CF-TR, it is clear that the proposed IRLS-TR has a better performance. For example, at CCDF=10^{-2} and 6.25% reserved subcarriers in a 16-QAM system, the CF-TR method achieved 4.4 dB that is lower than the 5.6 dB from the proposed IRLS-TR method. In addition, in the proposed IRLS-TR, the number of reserved subcarriers can be freely set unlike in the CF-TR in which PAPR reduction degrades if the number is set to a value lower than the number of zeros, which varies per iteration, in the clipping noise.

The MS-SCR by Wang et al generates a kernel signal to cancel multiple peaks and has fast convergence but still can exhibit peak re-growth and has lower PAPR reductions than the proposed IRLS-TR. For example, for a 16-QAM system with 12.5% reserved subcarriers at CCDF=10^{-3}, the MS-SCR could only manage 4.7 and 5.2 dB PAPR reductions when using high clipping ratios of 2 and 3 respectively, which reductions are quite low compared to the 6.6 dB from the proposed IRLS-TR method for the same system.

For the new WTR method by Xin and Yi, which also tries to mitigate the peak re-growth problem in the traditional WTR method [28] , the PAPR reduction performance is quite poor. The WTR method reported a PAPR reduction of 2.7 dB at CCDF of 10^{-3} for 5% reserved subcarriers in QPSK-modulated OFDM system. This reduction is less than half of the 5.6 dB given by the proposed IRLS-TR method for the same OFDM system.

In summary therefore, based on the results reported for the CF-TR, MS-SCR and new WTR methods, the proposed IRLS-TR method has better and improved PAPR reduction performance than the three methods. In addition, the proposed IRLS-TR method does not experience peak re-growth problem and the required number of reserved subcarriers can be freely set.

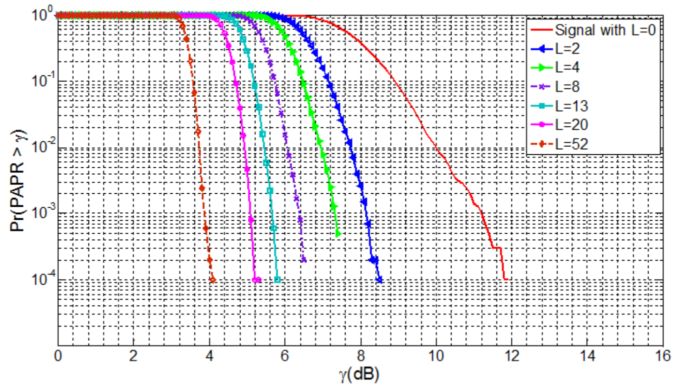

Figure 4: CCDF curves for QPSK-modulated OFDM with 256 subcarriers

On the issue of the increase in the average transmitted power that is expected with all tone reservation methods, the proposed IRLS-TR method exhibits only a small increase of about 0.6 dB. The sub-optimal CF-TR by Jiang et al method reported the same value while the optimal LP-TR method by Tellado had an increase of 1 dB. Due to the high PAPR reductions achieved by the proposed IRLS-TR method, the small increase of 0.6 dB in the transmitted power is tolerable.

Lastly, the simulation results for the same subcarrier modulation but with different number of OFDM subcarriers revealed that the PAPR reduction from the proposed IRLS-TR method increases almost linearly with $\log_2 N$ as illustrated in Fig. 5. The figure shows PAPR reductions for 5%, 10%, 15% and 20% reserved subcarriers against the binary logarithm of the number of OFDM subcarriers. Since all the plots resemble straight-line graphs, each of them can be estimated analytically by a linear function. This makes it easier to estimate the PAPR reductions for different total number of OFDM subcarriers for a fixed number of reserved subcarriers.

Similarly, since the lines have almost the same slope, one can estimate the increase in the PAPR reduction accruing from an increase in the number of reserved subcarriers. For example, for the 5% and 15% reserved subcarriers, the PAPR reductions are respectively given by the linear equations $0.4\log_2(N) + 0.6$, and $0.4\log_2(N) + 1.7$. The difference of 1.1 dB between the two equations is the PAPR reduction achieved due to the increase of the number of reserved subcarriers from 5% to 15%.

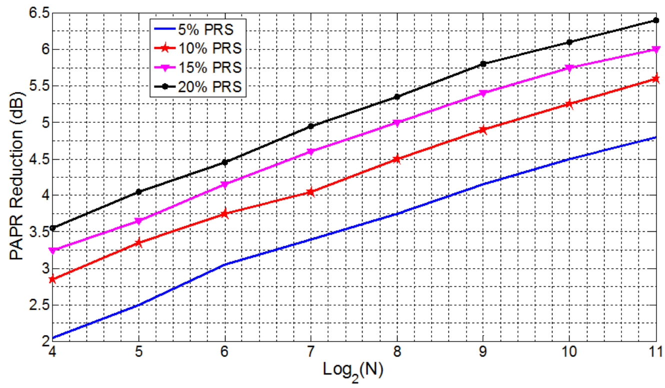

Figure 5: PAPR reduction variation with number of subcarriers

6 Conclusion

In this paper, a new sub-optimal tone reservation technique for PAPR reduction in OFDM systems, referred to as IRLS-TR, is proposed. Investigation into its PAPR reduction capability has found that with only 1.6% and 5% of total subcarriers reserved for PAPR reduction, it respectively achieves 3.9 dB and 5.6 dB of PAPR reduction at a small cost of 0.6 dB increase in the average transmitted power. The method has a fast quadratic convergence and a low computational complexity per iteration of $O(LP)$ which is less than $O(N\log_2 N)$ of FFT. Here, L is the number of reserved subcarriers, N is the total number of subcarriers, and P is the number of nonzero elements in the desired peak-reducing signal.

In addition, the proposed method does not experience the peak re-growth problem associated with TR methods that cancel signal peaks using a time-domain kernel signal and the traditional weighted tone reservation methods. Further investigations revealed that the method has PAPR reductions that are linear with $\log_2 N$ and therefore one can predict PAPR reductions for different OFDM systems with different number of reserved subcarriers.

The effect of the choice of the desired peak-reducing signal on the PAPR reduction capabilities of the proposed method shall be investigated in future work.

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