New Multi-Objective Particle Swarm Optimization for Linear Antenna Array Synthesis

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Abstract: Antenna arrays encompass a very vital job in processing and detecting signals received from diverse directions. They are preferred over single element antennas owing to the limitations that exist in the latter in directivity and bandwidth. These limitations are avoided by using the array antennas which associates every element of antenna to different geometrical and electrical configurations to facilitate its beam pattern to be modified with phase and/or amplitude distribution which are called the array weights. The most significant problems to be dealt with in an array antenna design are the control of nulls and the SLL reduction. Lots of researches have used the evolutionary algorithms for obtaining these two objectives. The approaches that were designed for this purpose tackle the objectives simultaneously by creating single objective functions and then taking weighted sum for the objective functions. In this paper, to evade the problems associated with the use of the weighted sum approach, a MO formulation of the problem and a recent approach called Roulette Wheel Multi-objective Particle Swarm Optimization are used. The goal is to obtain the “least standard side lobe level” and a “null reduce” at specific directions. These two goals are contradicting that’s why using the multi-objective optimization is suitable for solving this problem. To test the performance level of the applied method of the multi-objective approach, it is very important to get Pareto optimal which is the way of solving the multi-objective problems. In this paper, the MOPSO is introduced to obtain the Pareto optimal fronts for the two contradicting objectives to show the effectiveness of planned algorithm showing effective results. Improved results for reduced SLLs and null depth are obtained.

Key-words: - PSO, MOPSO, SLL, Null reduction, RP, Roulette Wheel Selection.

1 INTRODUCTION

In an antenna array, outputs of all elements of antenna are processed to give the beam-pattern of the antenna array. The antenna arrays boast many applications for example in radar, sonar, radio and in the third generation wireless communication systems [1]. The main aim in antenna array creation is to find the position of the elements that can construct a radiation pattern as a whole that is close enough to the desired pattern. The blend of linear array elements that are separated in a non-linear fashion has become very popular recently among researchers at electromagnetism field.

To lessen the SLL and to achieve null control are two major important objectives for optimization of antenna. The reason of reducing the SLL is to circumvent the degradation of the total radiation power efficiency. The null formulation is as well essential for the principle of the suppression of the interfering signals at definite directions owing to the increase in the electromagnetic environment pollution. Lots of researches have used the evolutionary techniques to obtain these two objectives [2], [3]. The approaches that were meant for this purpose tackle the objectives simultaneously by creating single objective functions and then taking weighted sum for the objective functions. It is clear that when using a weighted sum method, the obtained solution will depend robustly on values of the specified weights and determining these values isn't a simple task.

To evade the problems associated with the use of the weighted sum approach, a MO formulation of this problem is used and a recent approach done by incorporating Roulette Wheel Selection method with MOPSO is used. These two goals are contradicting that's why using the multi-objective optimization is suitable for solving this problem.

In literature, many works were concerned with the reduction of the SLL in a RP [4, 5]. Other works were concerned with suppressing the regions that are exposed to interfering signals [6]. The above two problems were solved before generally based on controlling complex weights (amplitude and phase), the amplitude only, the phase-only, and the position only of array elements. Interference restraint using complex weights is the most proficient since it has larger degrees of liberty for solution space [7, 8]. However, it is a very pricey method attributable to the expensiveness of both phase shifters besides variable attenuators for all elements. Furthermore, as number of elements in array increases, time of computation of getting the values of the elements amplitude along with phase will increase.
Since several objectives exist which are needed for optimization in the same problem, the problem is treated as a MOO problem.
MOO is an interesting field in diverse applications for researchers. The multi objective problems are a type of problems that are needed to be handled simultaneously. These objectives can either be conflicting or incomparable. A group of solutions for the multi objective problem exist and these solutions may not be put into comparison together. These groups of solutions are called non-dominated solutions or Pareto optimal solutions which are solutions with no more ability of improvement to be achieved in any objective function without causing degradation in any other objective functions [9].

Electromagnetic systems are used widely nowadays. The pollution of the electromagnetic environment was the major problem that provoked the swot of nullifying of the RP methods. They assist to reduce the degradation in the SNR that occurs owing to the interference. The focal concern through the procedure of data transmit is the conventionality of signal received with transmitted signal. This idea is very difficult owing to the fact of revealing transmitted signal to numerous factors which may outcome in either changing the data structure or missing some of that data. Those factors may be planned or because of simple distortion and noise in the environment and may also be caused by device not working properly. Lots of research was made on the modification of definite signal positions into nulls. Some predictable methods were designed for achieving nulls on definite positions. The wide nulls were done by creation of several neighbouring nulls in RP. By means of the EA, they are found to be the most professional in attaining these wide nulls. Wide nulls formulating are attained by the control of array weights called the excitation coefficients. The control of the amplitude only uses a group of changeable attenuators to alter the amplitudes of elements. If the element amplitudes own even symmetry in relation to the centre of array, the number of attenuators and time of computation are halved. The nulls are attained with non uniform excitation coefficients to enforce nulls at definite directions that are encountered by optimization technique used.

The disadvantages of the classical optimization techniques ended up the researches to use the EA that are on the basis of computational intelligence methodology. Lots of EA have been applied before in several optimization problems. Examples for those algorithms are the Genetic Algorithms, the bee colony, the ant colony, the backtracking algorithm and clonal selection.

The linear antenna array is one of the important problems that are faced in electromagnetism. The main aims of a designer when making a design of a linear antenna array, is to get the lowest standard SLLs and null reduction in certain directions. Since linear antenna array RP characteristics depends on the excitation amplitude, inter-element spacing, and phase associated with every element of array [10], by appropriate synthesizing methods needed SLL, beam width which is narrow, main beam navigation with high directivity can be achieved. The goal in antenna array geometry synthesis is to establish the physical design of array which results in producing a RP that is closest to required pattern. The shape of the desired pattern varies widely in relation to the application. Some of the applications are interested in suppressing the SLL whilst keeping the main beam gain as it is [11]. Other methods are interested in the null formulation to lessen the effects of intrusion and jamming. The metaheuristic algorithms are strongly relied on by researchers for those purposes. Those algorithms utilize an objective function for optimization which results in side lobe control and null formulation [12]. Many EA were employed in antenna array design such as GA [13], Simulated Annealing [14], Tabu Search Algorithm (TSA) [15], Memetic Algorithms (MA) [16] and PSO [17].

The used technique for optimization in this presented paper is the Particle Swarm Optimization. It was found to encompass many advantages over other EA for example it doesn't have overlapping and mutation calculation. The result is an optimal Pareto Front showing good results samples.

The article is arranged as follows: At section 2, problem formulation and the fundamental concepts which are used in that paper along with the multi-objective formulation of the problem needed to be solved. In Section 3, The Roulette Wheel Selection method is described, the method's explanation and how to use it for optimization. Also a brief description of the new proposed RWMOPSO is proposed; its steps of operation and a flow chart describing the technique are shown. In section 4, the simulated results of applying the planned algorithm to the antenna RP is proposed. Finally section 5, a conclusion is provided for the proposed work.

2 Problem formulation

2.1 Multi-objective Optimization

MOO is an interesting field in many applications for researchers. The multi objective problems are a type of problems that are needed to be handled simultaneously. These objectives can either be conflicting or incomparable. The multi-objective general design problems are expressed as follows:

$$\min_{x \in \Omega} \{F(x)\} \qquad (1)$$

Where Ω defines search space and F is known as the vector of the objective functions:

$F: \Omega \longrightarrow R^K$,

$F(x) = [(f_1(x), \dots, f_k(x))]^T$, where $f_i: R^n \longrightarrow R$ is an unconstrained function.

In case of the multi-objective optimization, a group of trade-off solutions that signify the best probable compromises among all the objectives is needed to be produced. This means producing solutions such that no objective can be enhanced without deterioration of another.

The traditional techniques that were used were done by converting the MO problem to a single objective problem using a vector of user defined weights. Nowadays, the trend of researchers and practitioners is to optimize multiple objectives problem simultaneously and provide a group of Pareto optimal (compromise, or tradeoff) solutions rather than a single solution.

To describe the idea of optimality the following definitions are introduced [18]:

Definition 1. Let $x, y \in \Omega$; It is said that x dominates y (denoted by $x < y$) if and only if, $f_i(x) \leq f_i(y)$ and $F(x) \neq F(y)$.

Definition 2. Let $x^* \in \Omega$; It is said that x^* is a Pareto optimal solution, if there is no other solution $y \in \Omega$ such that $y < x^*$.

Definition 3. The Pareto Optimal Set PS is specified by:
$PS = \{x \in \Omega / x \text{ is a Pareto optimal solution}\}$.

Definition 4. The Pareto Optimal Front PF is defined by:
$PF = \{F(x) / x \in PS\}$

Figure 1, represents an example of the Pareto optimal curve.

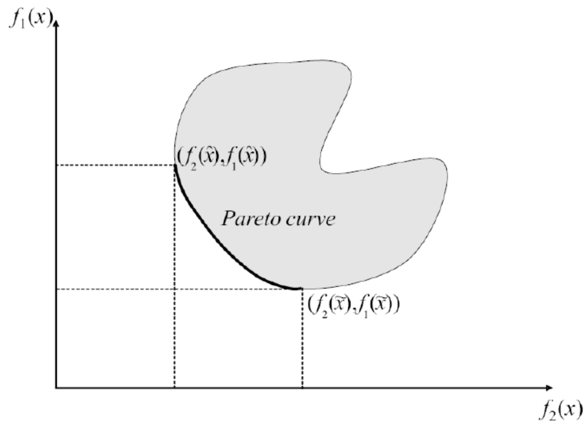

Fig. 1: Pareto optimal curve [18].

3 Multi-objective formulation of the problem

Antenna array is a configuration of single elements of antennas that are placed in certain positions in a space and is used to create a directional RP. In this paper, it is assumed that an antenna of 2N isotropic radiators/elements which are symmetrically placed along the z-axis is used. The array's geometry is shown in Figure (2).

The array factor for this structure is given by:

$$AF(\psi) = 2\sum_{n=1}^{N} I_n^{Re} \cdot \cos(d_n \psi) \qquad (2)$$

Where $\psi = k.\sin\theta$, k is wave number

N represents the number of elements that are placed on each side of the origin.

I_n^{Re} is the real excitation coefficient of each element in the array.

d_n is the inter-element spacing which is the same for all elements ($d = \frac{\lambda}{2}$).

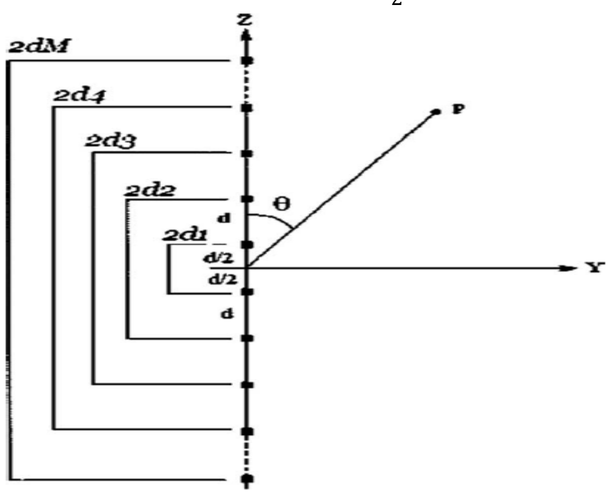

Fig. 2: Design of linear array antenna with 2N elements of equal inter-elements spacing [19].

The aim of using the optimization technique here is to acquire the locations d_n of array's elements that results in achieving a RP with nulls at specific directions and lowest SLL.

Each particle has 2 objective functions, one correlated to the SLL (needed to be lowered so far as possible) and the other correlated to null control. Both objectives to be minimized are given by:

$$f_{SLL} = \sum_i \frac{1}{\Delta\varphi_i}\int_{\varphi l_i}^{\varphi u_i} |AF(\varphi)|^2 d\varphi \qquad (3)$$

Where: φ is the angle measured from the array line

$|AF(\varphi)|^2$ is the difference between the initial radiation pattern and the optimized RP (array factor).

$$f_{NC} = |W_{null}\,(AF - AF_i)|^2 \qquad (4)$$

Where: W_{null} is a function that specifies locations of wide nulls.

Equations (3) and (4) are considered to be two distinct objectives which are conflicting so they are simultaneously optimized in a MO framework. When using the multi-objective optimization, a group of solutions is obtained that signify the best compromises for these objectives. For this reason using the MOEA has a great advantage over single objective optimization which is providing greater flexibility in designing of linear antenna array.

4 Multi-objectiveParticle Swarm Optimizers

4.1 Multi-objective PSO in literature

Using the EA in MOO is one of the methods that are used and that proved being victorious in acquiring good results. This is owing to the capability of the EA to search for several Pareto optimal solutions concurrently besides having better global search in the search space. The PSO proved its ability to achieve successful and good results [20].

In MOPSO, there isn't available a single global best solution that achieves the best solution for all objectives, instead a global best set of particles are represented by a group of non-dominated solutions. It is not granted to get a single local best position for each particle. That's why choosing the global with best local particles that are used as guides for the whole search space isn't a simple task in the case of the multi-objective problems. Two main approaches can be used for designing algorithms of PSO for multi-objectives problems.

Different methods have been presented in literature to choose the best particles to be used as the local and best global particles in MOPSO.

A study on PSO performance in multi-objectives optimization problems was introduced in [21] focused on getting the Pareto front using weighted aggregation method. Using the same concept of the vector evaluated genetic algorithm (VEGA) a vector evaluated PSO (VEPSO) was introduced and examined to be used in MOO problems [22]. However, selecting the particles that improve one objective without considering the other objectives lead to neglecting other particles with relatively good performance that may be useful for compromise solutions [23].

MOPSO technique is presented in [24]; it uses dynamic neighborhood strategy to decide the best local particles for every particle in bi-objectives optimization problems. However, the choice of the best local particles based on one objective from multiple objectives decreases the algorithm's performance because one dimensional optimization is utilized to resolve multiple objectives problems and the selection of a fixed objective requires a previous knowledge about all the objective functions. Also, considering single objective function for optimization with the other one to be fixed, as well as extending the algorithm to objective of higher dimension spaces, are very concerned questions.

Another MOPSO technique is proposed in [25] where search space is separated into hyper cubes before selecting the best local guidance for every particle in the swarm. The suggested method has been examined on four bi-objectives test problems, compared with Pareto Archived Evolutionary Strategy method (PAES) and a competing MOPSO it gives hopeful results at three test problems. This algorithm achieved promising results compared with PAES and non dominated Sorting Genetic Algorithms II (NSGA-II).

A MOPSO technique was proposed [26] where the choosing of the best local guidance for every particle in the swarm is founded on a dominated tree method. However, the performance of the suggested MOPSO techniques in [25] and in [26] was very poor in the multi-frontal fourth test problem with multimodality [27]. In addition, these algorithms have not been verified when applied with objectives of higher dimensional spaces.

Relying on several works in the literature review which were made for the function of the design of antenna arrays, PSO showed its superiority in the ability to obtain best results among other evolutionary algorithms. In [28], a contrast of using genetic algorithm, PSO and DE was proposed. This comparison showed the dominance of the PSO over the others.

The PSO has been applied broadly over the previous decades in so many applications. Those applications were briefly discussed in [29].

The metaheuristic design combine separate objectives which are frequently conflicting to linear weighted sum into one single objective function. However finding the weights required is the most significant thing that affects the optimization results. It is very hard to find a group of weights that achieve all the requirements. Using the multi-objective concept in array antenna design problems, a search was done to find the most recent developed MOEAs (Multi-Objective Evolutionary Algorithms) that can solve linear array blend more proficient than the conventional single-objective approaches. Differential Evolution [30] has been utilized in this optimization problem before. Its main concept is based on decomposing the MO problem to a number of sub-problems to optimize them simultaneously. Each sub-problem is optimized by employing information from its several adjacent sub-problems.

In this paper, the two major problems faced during the array design synthesis are optimized using a new multi-objective optimization method. This is done by applying PSO strategy for solving the multi-objective optimization problem. The two problems are the reduction of the SLL and the null control. These objectives are obviously contradicting. A new MOPSO based Pareto solution is proposed to face the jamming problem to lessen the effect of the jamming in certain directions and to satisfy multiple contradictory objectives. It is very vital to maintain the data sent or received from an antenna secured to prevent the data loss or change. This is the major problem which faces any antenna, however reducing the SLL is also of great importance to prevent unwanted radiations in undesired directions. The SLL levels are of lower power density than that in the main beam.

4.2 Roulette Wheel Selection Method

Using the selection method of Roulette Wheel in the area of function optimization has given an increase in the expansion of resources, dependability and multiplicity of the population and lessens in the doubt of selection process. The Roulette Wheel Selection is originally a genetic operator which is utilized for choosing useful solution for recombination.

If each individual i at population has fitness f_i, its probability of selection is:

$$p_i = \frac{f_i}{\sum_{j=1}^{N} f_j} \qquad (5)$$

Where N is the number of individuals in the population.

The concept of the roulette wheel selection can be understood by imagining a roulette wheel in which each candidate solution is a representation of a pocket on the wheel where the size of the pocket is proportional to solution's probability of selection.

The fundamental part for selection procedure is to randomly choose from one generation to produce the source of the subsequent generation. The requirement is that the fittest individuals have a greater chance of survival than weaker ones. This replicates nature in that fitter individuals will tend to have a better probability of survival and will go forward to form the mating pool for the next generation. Weaker individuals are not without a chance. In nature such individuals may have genetic coding that may prove useful to future generations.

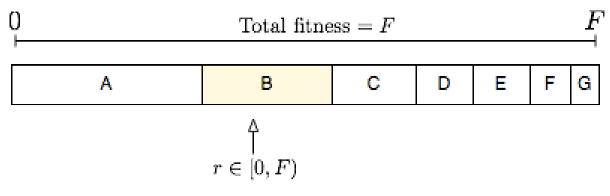

Fig. 3: Example of a single element selection [31].

Parents are selected in proportion to their fitness. When the chromosomes are better, they will have better probability of selection. Imagine a roulette wheel where all chromosomes in the population are placed, all chromosomes have their places according to its fitness function, like in figure (3).

The roulette wheel selection is done using a random cumulative number which is generated to be able to obtain the better fitness.

Given below is the algorithm used for choosing the leader employing Roulette Wheel Selection.

Algorithm: Select Leader based on ROULETTE WHEEL SELECTION

Leader=SelectLeader ()

1. Grid Index of All Repository Members
2. Choose the unique of Occupied Cells
3. Determine Number of Particles in Occupied Cells
4. Selection Probabilities are given by
   (a) P=exp(-beta*N);
   (b) P=P/sum(P);
5. Select Cell Index according to RouletteWheelSelection.
6. Select the Cell.
7. Select the Cell Member.
8. Select the Member Index.
9. The output Leader selected.

Algorithm: ROULETTE WHEEL SELECTION ()

r := random number ,where $0 \le r \le 1$;

C=cumsum (P);

i=find(r<=C); first index of C greater than r

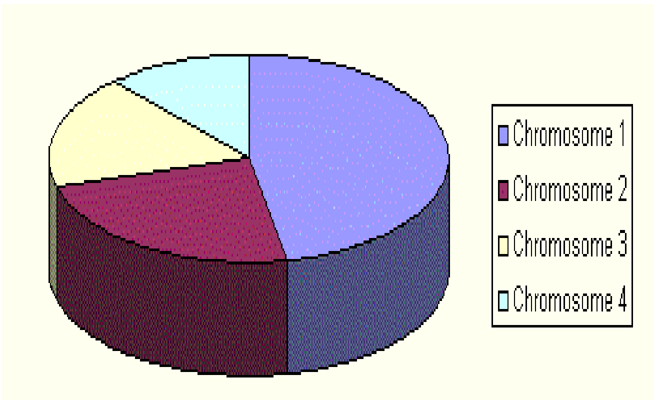

Fig.4: Roulette Wheel illustration of chromosomes placed in relation to their fitness functions.

5 The New Proposed Multi-Objective PSO approach

In this algorithm two external archives are created the first one have best local Pareto set to archive the non-dominant best values for each individual particle from the start of the iterations and the other one contains global best Pareto set to archive the non-dominant best values for all particles from the start of the iterations. In the New proposed MOPSO, a new technique is applied to pick the local and best global particles to be utilized in the update of each particle in the swarm.

The RWMOPSO proposed for the non-dominated Pareto solutions is described as follows:

- Initialize each particle with the initial excitation coefficients which are here supposed for array of 20 elements having 10 excitation coefficients owing to the symmetry of elements from the origin.
- Each particle has 2 objective functions which are one linked to the SLL (needed to for reduction so far as possible) and the other correlated to null control. The two objective functions applied for minimization are (3) and (4).

- If the two fitness functions of a definite particle are fewer than other fitness functions of the other particles then add this particle to a repository.

- Then the repository members are updated founded on region selection (grid index).
- Next, pick a leader particle for the swarm using the Roulette Wheel Selection for the particles at the swarm.

- Apply mutation to get new solution and check if this solution dominates or not.

- Keep repeating the above steps until reaching a group of the non-dominated solutions and select the solutions that formulate the Pareto Front.

- The proposed New MOPSO flow chart is given as follows:

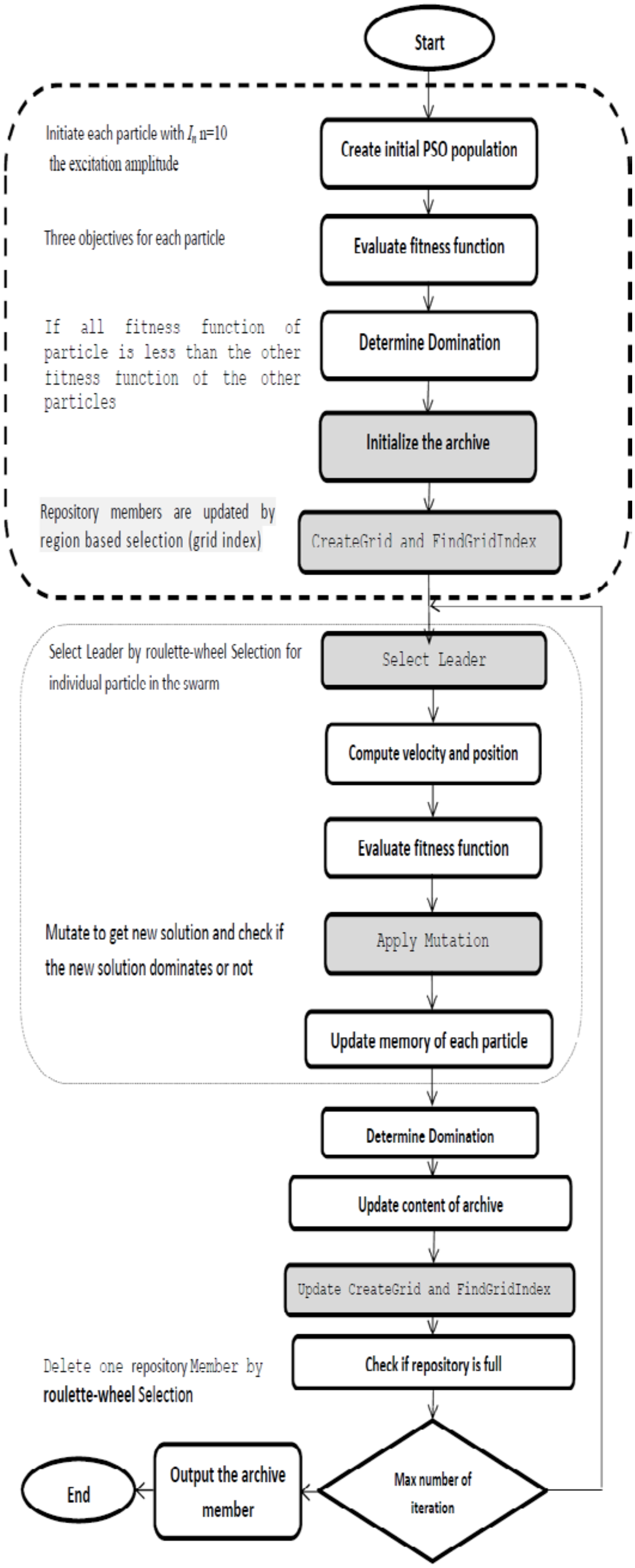

Fig. 5: The proposed New MOPSO algorithm's flow chart.

6 Simulation Results

To prove the efficiency of the used new proposed algorithm MOPSO, it has been employed for solving problem of optimal trade off between the contradictory goals of a linear array antenna design which are control of null and the SLL reduction. The dominance of the proposed algorithm is shown throughout its ability to capture the right Pareto front and the range of the Pareto front optimality using two objectives. The figures (6), (7), (8) and (9) show samples of results for the obtained radiation pattern for two objectives where the first objective is the SLL reduction and second objective is the null control.

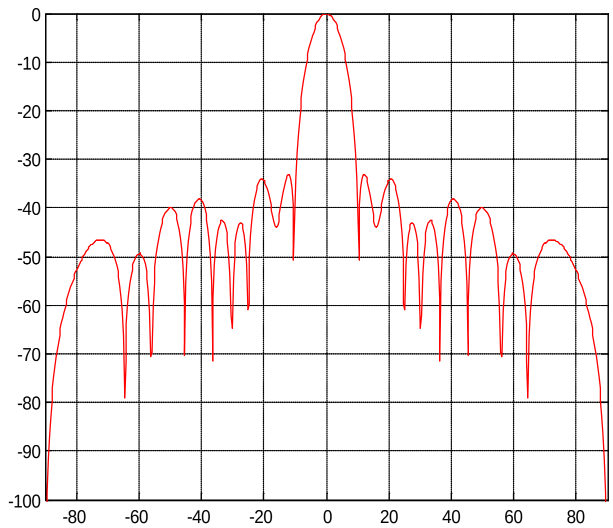

Fig. 6: 1st Sample of results for the obtained radiation pattern after applying RWPSO technique

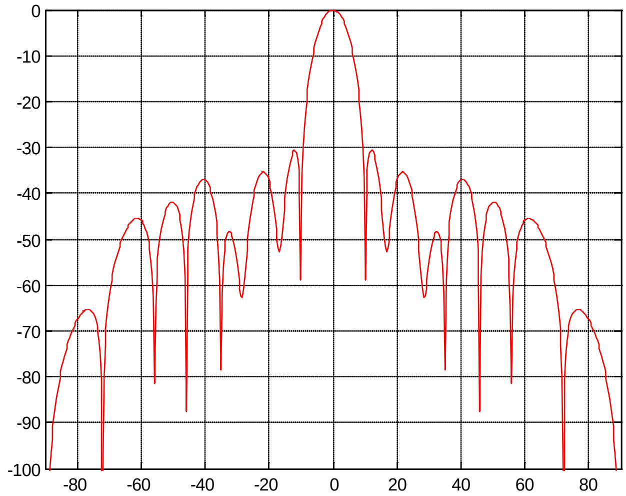

Fig. 7: 2nd Sample of results for the obtained radiation pattern after applying RWPSO

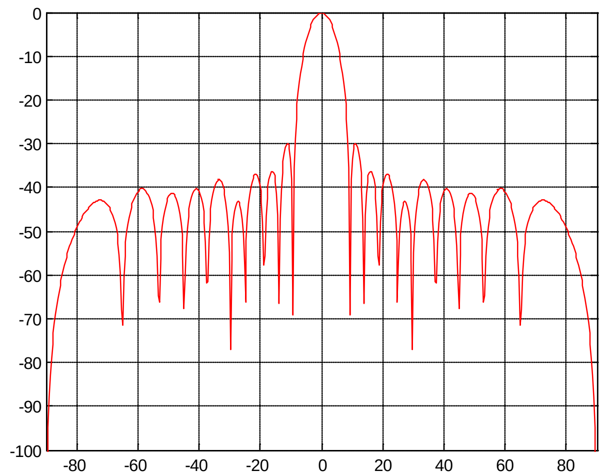

Fig.8: 3rd Sample of results for the obtained radiation pattern after applying RWPSO

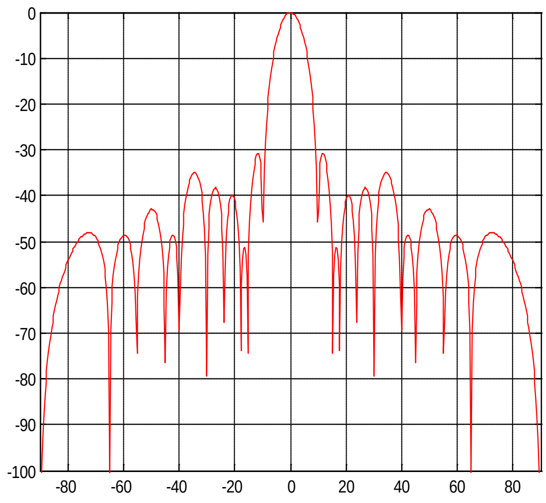

Fig. 9: 4th Sample of results for the obtained radiation pattern after applying RWPSO

As shown in the above figures, these are samples of results that are obtained after the new MOPSO technique is applied. A group of non-dominated solutions are obtained after the optimization process. This group of solutions are all obtained and then the designer/optimizer chooses the solutions that are the best regarding what he/she is searching for. In this multi-objective optimization case, each solution reached has one objective reached to be more improved than the other since both objectives are contradicting. A trade-off of both objectives is obtained. The group of non-dominated solutions formulate the Pareto Front.

The Pareto Front of both objectives the SLL reduction and the null formulation are as follows:

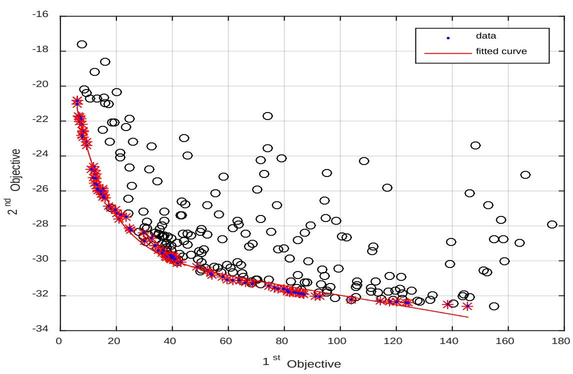

Fig. 10: Fitted Curve of Pareto Front result after applying the RWMOPSO

The first objective is the null control and second objective is the SLL reduction. All the other black dots are the group of non-dominated solutions.

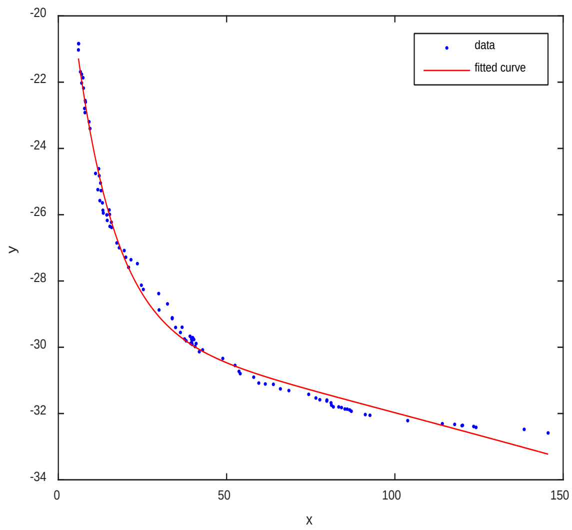

Fig. 11: The Final Fitted curve of Pareto Front of both objectives after the new MOPSO.

X: Represents the null control.

Y: Represents the SLL reduction.

Samples of RWMOPSO Results	Elements Spacing of the array									
Sample 1	±7	±6.992	±6.458	±5.673	±4.930	±4.074	±3.195	±2.417	±1.730	±0.813
Sample 2	±7	±6.992	±6.409	±5.750	±4.909	±4.069	±3.269	±2.399	±1.953	±1.040
Sample 3	±7	±6.990	±6.380	±5.787	±4.914	±4.070	±3.370	±2.279	±1.919	±1.312
Sample 4	±7	±6.989	±6.467	±5.773	±4.908	±4.047	±3.214	±2.416	±1.723	±0.775
Sample 5	±7	±6.972	±6.435	±5.674	±4.907	±4.077	±3.200	±2.426	±1.782	±0.841
Sample 6	±7	±6.981	±6.420	±5.701	±4.923	±4.055	±3.200	±2.430	±1.774	±0.878
Sample 7	±7	±6.977	±6.449	±5.661	±4.906	±4.076	±3.202	±2.427	±1.785	±0.864

Table 1: Samples of optimized results obtained using the new MOPSO technique.

The above simulation results prove that the new MOPSO has a great ability to get accurate Pareto front and possesses superiority to accomplish the optimal trade off of several objectives for changed cases. All simulated results are improved and have a right Pareto front shape. The non dominated optimal Pareto group of solutions contains the values of all best global particles that are accomplished by the proposed new MOPSO technique.

7 Conclusion

Pareto solutions that are founded on the new MOPSO are proposed for optimizing antenna array linearly organized. The optimization is done for two objectives which are: minimizing the SLL so far as possible and null reduction (which is forming nulls at specific directions). The results of the simulation showed the high efficiency of the new MOPSO technique in achieving best results. Proper Pareto front shape was achieved using this technique proving its superiority. The designer can choose the solutions sets that are most suitable for his desire of design.

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