Outage Performance of Cognitive AF Relaying Networks over generalized $\eta - \mu$ Fading Channels

JING YANG, LEI CHEN, CHUNXIAO LI
Yangzhou University
School of Information Engineering
88 South University Ave., 225009
P. R. CHINA
{jingyang, licx}@yzu.edu.cn
leichen092@163.com

KOSTAS P. PEPPAS
University of Peloponnese
Department of Telecommunication
Science and Technology
Tripoli 22100, GREECE
peppas@uop.gr

P. TAKIS MATHIOPOULOS
National and Kapodistrian
University of Athens
Department of Informatics
and Telecommunications
15784 Athens, GREECE
mathio@di.uoa.gr

Abstract: In this paper, a dual-hop cognitive amplify-and-forward (AF) relay network subject to independent non-identically distributed (i.n.i.d.) $\eta - \mu$ fading channels is investigated. In the considered network, secondary users (SUs) including one secondary user source (SU-S) and one secondary user relay (SU-R) are allowed to share the same spectral resources with the primary user (PU) simultaneously under the premise that the quality of service (QoS) of PU can be guaranteed. In order to guarantee the QoS of PU, the maximum interference power limit is considered to constraint the transmit powers at SU-S and SU-R. For integer-valued fading parameters, a closed-form lower bound for the outage probability (OP) of the considered networks is obtained, whereas the lower bound in integral form for the OP is derived for arbitrary-valued fading parameters. For the special case of the generalized $\eta - \mu$ fading channels, such as Nakagami-m fading channels, the analytical results become the previous published results. In order to obtain further insights on the OP performance, asymptotic expressions for the OP at high SNRs are derived. From the asymptotic results, we also reveal that the diversity gain of the secondary network is only determined by the fading parameters of the secondary network, whereas the primary network only affects the coding gain. Finally, simulation confirms the correctness of our analysis.

Key–Words: Outage probability (OP), amplify-and-forward (AF), cognitive relaying networks (CRN), $\eta - \mu$ fading, spectrum sharing.

1 Introduction

Over the past few years, cognitive radio with spectrum sharing has attracted considerable interest. In underlay cognitive networks, the secondary users (SUs) can simultaneously access the licensed spectrum of the primary user (PU) without causing harmful interference on PU. Thus, in order to ensure PU's quality of service (QoS), the power constraint of the interference on the primary network must be considered. Recently, to further improve the spectrum efficiency, incorporating cooperative relaying into cognitive networks has gained extensive attention owing to its high spectrum utilization [1–7]. With the maximum interference power limits, the exact outage probability (OP) of an underlay cognitive network with amplify-and-forward (AF) relaying has been investigated in [1]. In the presence of the primary users interference, the exact expression for OP of a dual-hop cognitive decode-and-forward (DF) relay network has been obtained in [2]. Bao *et al.* proposed cognitive multihop DF networks and analyzed the system performance with the interference limits in [3]. With maximum transmit power limits, the outage performance of cognitive network using AF relaying in [4] and DF relaying in [5] has been analyzed. Most recently, incorporating multiuser diversity and multiple-input multiple-output (MIMO) technologies into cognitive networks, the outage analysis has been investigated in [7]. While in these works, the channel models are normally assumed as Rayleigh fading distribution [1–3, 7] or Nakagami-*m* fading distribution [4–6].

For small-scale fading, many well-known channels, e.g., Rayleigh, Hoyt, Nakagami-*m* channel, have been widely used to characterize the fading channel. However, in some practical cases, there are no distributions that can match experimental data very well. Due to this, Yacoub [8] proposed the so-called $\eta - \mu$ distribution to better model small-scale fading in non-line-of-sight (NLOS) conditions than those well-known distributions. In addition, the $\eta-\mu$ fading is a general fading including Rayleigh, Hoyt, Nakagami-*m* fading as special cases [9]. In recent years, the $\eta-\mu$ fading model has been paid much attention [11, 12]. In [11], the authors analyzed the error performance by using moment generating function (MGF) over $\eta-\mu$ fading channels, without investigating outage performance. Later, Peppas *et al.* analyzed the conventional dual-hop relaying network over mixed $\eta-\mu$ and $\kappa-\mu$ fading channels in [12].

Despite the wide applicability of the $\eta-\mu$ distribution, to the best knowledge of the authors, the performance of cognitive AF relay networks in $\eta-\mu$ fading environment is still unexplored in the open technical literature. Motivated by this lack, we investigate the outage probability for the dual-hop cognitive AF relay networks over i.n.i.d. $\eta-\mu$ fading channels. The tight lower bound and the asymptotic expressions for outage probability (OP) have been both obtained. From the asymptotic results, the diversity gain and coding gain are achieved indicating that the diversity gain is only determined by the fading parameters of the secondary network, whereas the primary network only affects the coding gain. For the special case of the generalized $\eta-\mu$ fading channels, such as Nakagami-m fading channels, the analytical results become the previous published results in [4]. In order to guarantee the QoS of PU, the maximum interference power limit is considered in this paper. Finally, simulation is presented to verify the correctness of our analysis.

2 Network and Channel Model

Consider a dual-hop cognitive AF relay network including one SU source (SU-S), one AF SU relay (SU-R), one SU destination (SU-D), and one PU destination (PU-D). All nodes are equipped with single antenna and operate in half-duplex mode. The communication from SU-S to SU-D is performed into two times slots. During the first time slot, SU-S transmits signal x to SU-R with transmit power P_S, then the received signal at SU-R can be written as $y_r = g_1\sqrt{P_S}x + n_r$, where g_1 is the channel coefficient of the link SU-S $\rightarrow$ SU-R and n_r is additive white Gaussian noise (AWGN) at SU-R. Whereas during the second time slot, the received signal y_r is amplified with gain factor G and then forwarded to SU-D with transmit power P_R, the received signal at SU-D is $y_d = Gg_1g_2\sqrt{P_SP_R}x + Gg_2\sqrt{P_R}n_r + n_d$, where g_2 is the channel coefficient of the link SU-R $\rightarrow$ SU-D and n_d is AWGN at SU-D. In order to ensure the interference on PU below the maximum tolerable interference power $\mathcal{Q}$, the transmit powers at SU-S and SU-R are governed by $P_S = \mathcal{Q}/|h_1|^2$ and $P_R = \mathcal{Q}/|h_2|^2$, where h_1 and h_2 are the channel coefficients of the interference link SU-S $\rightarrow$ PU-D and SU-R $\rightarrow$ PU-D, respectively. We assume that all AWGN components have zero mean and variance N_0. By setting $1/G^2 = |g_1|^2P_S + N_0$, the end-to-end instantaneous SNR at SU-D can be obtained as [4]

$$\gamma_d = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}, \tag{1}$$

where

$$\gamma_1 = \overline{\gamma}_{\mathcal{Q}}\frac{|g_1|^2}{|h_1|^2}, \gamma_2 = \overline{\gamma}_{\mathcal{Q}}\frac{|g_2|^2}{|h_2|^2}, \tag{2}$$

with $\overline{\gamma}_{\mathcal{Q}} = \mathcal{Q}/N_0$. Throughout this analysis, it is assumed that all links are subject to i.n.i.d. $\eta-\mu$ fading. Thus, $|g_\ell|^2$ and $|h_\ell|^2$ follow the $\eta-\mu$ distribution with parameters μ_{g_ℓ}, η_{g_ℓ} and μ_{h_ℓ}, η_{h_ℓ}, respectively, where $\ell \in \{1,2\}$. Let $E\{|g_1|^2\} = \Omega_1$, $E\{|g_2|^2\} = \Omega_2$, $E\{|h_1|^2\} = \Omega_3$ and $E\{|h_2|^2\} = \Omega_4$.

Therefore, the probability density function (PDF) of X, where $X \in \{|g_1|^2, |g_2|^2, |h_1|^2, |h_2|^2\}$ can be expressed as [8]

$$\begin{aligned} f_X(x) =& \frac{2\sqrt{\pi}\mu^{\mu+0.5}h^\mu x^{\mu-0.5}}{\Gamma(\mu)H^{\mu-0.5}\overline{X}^{\mu+0.5}}\exp\Big(-\frac{2\mu hx}{\overline{X}}\Big) \\ &\times I_{\mu-0.5}\Big(\frac{2\mu Hx}{\overline{X}}\Big), \end{aligned} \tag{3}$$

where $\Gamma(\cdot)$ denotes the Gamma function [15] and $I_\nu(\cdot)$ the ν-th order modified Bessel function [15, eq.(8.431)]. Also, $\overline{X} = E(X), \mu > 0$ is related to the fading severity, $\mu \in \{\mu_{g_\ell}, \mu_{h_\ell}\}$ and $\eta \in \{\eta_{g_\ell}, \eta_{h_\ell}\}$. The parameters h and H are given by [8] $h = (2+\eta^{-1}+\eta)/4, H = (\eta^{-1}-\eta)/4$ with $0<\eta<\infty$, with $h \in \{h_{g_\ell}, h_{h_\ell}\}$ and $H \in \{H_{g_\ell}, H_{h_\ell}\}$.

Assuming integer values of μ, the cumulative distribution function (CDF) of X can be obtained as follows [12],

$$\begin{aligned} F_X(x) = 1 - \frac{1}{\Gamma(\mu)}\Big(\frac{h}{H}\Big)^\mu \Bigg\{ \sum_{k=0}^{\mu-1}\sum_{p=0}^{\mu-k-1}\frac{1}{p!}\big[A^px^pa^{(k)} \\ \times\exp(-Ax) + (-1)^\mu B^px^pb^{(k)}\exp(-Bx)\big]\Bigg\}, \end{aligned} \tag{4}$$

where

$$a^{(k)} = \frac{(-1)^k(\mu+k-1)!H^{-k}}{2^{\mu+k}k!(h-H)^{\mu-k}},$$
$$b^{(k)} = \frac{(\mu+k-1)!H^{-k}}{2^{\mu+k}k!(h+H)^{\mu-k}},$$
$$A = \frac{2\mu(h-H)}{\overline{X}}, \quad B = \frac{2\mu(h+H)}{\overline{X}},$$

and $a^{(k)} \in \left\{a_{g_\ell}^{(k)}, a_{h_\ell}^{(k)}\right\}$, $b^{(k)} \in \left\{b_{g_\ell}^{(k)}, b_{h_\ell}^{(k)}\right\}$, $A \in \{A_{g_\ell}, A_{h_\ell}\}$, $B \in \{B_{g_\ell}, B_{h_\ell}\}$.

For arbitrary values of μ, the CDF of X can be expressed as

$$F_X(x) = 1 - Y_\mu\left(\frac{H}{h}, \sqrt{\frac{2h\mu x}{\overline{X}}}\right) \tag{5}$$

where

$$Y_\mu(x,y) = \frac{\sqrt{\pi}2^{1.5-\mu}(1-x^2)^\mu}{\Gamma(\mu)x^{\mu-0.5}} \times \int_y^\infty e^{-t^2}t^{2\mu}I_{\mu-0.5}(t^2x)\mathrm{d}t \tag{6}$$

denotes the Yacoub integral [8, eq. (20)]. It is noted that $Y_\mu(x,y)$ can be expressed in terms of tabulated functions for integer or half-integer values of μ only. For arbitrary values of μ, an expression of $Y_\mu(x,y)$ in terms of the bivariate confluent hypergeometric functions is available in [10, eq. (2)].

The MGF of X can be deduced in closed form as [14, eq. (3)]

$$\mathcal{M}(s) = [(1+s/A)(1+s/B)]^{-\mu}. \tag{7}$$

Finally, by employing an infinite series representation for the modified Bessel function, [15, eq. (8.447)] as well as the definition of the incomplete gamma function [15, eq. (8.350.2)], the incomplete MGF of X, defined as $\mathcal{M}(x,s) \triangleq \int_t^\infty \exp(-sx)f_X(x)\mathrm{d}x$, can be deduced as

$$\mathcal{M}(x,s) = \frac{2\sqrt{\pi}h^\mu}{\Gamma(\mu)}\sum_{k=0}^{\infty} \times \frac{H^{2k}(\mu/\overline{X})^{2\mu+2k}\Gamma(2\mu+2k, 2\mu\, h\, t\,/\overline{X}) + s\, t}{k!\Gamma(\mu+k+1/2)\left(2\mu\, h\, t\,/\overline{X}+s\right)^{2\mu+2k}}. \tag{8}$$

3 Outage Performance Analysis

In this section, the OP of cognitive AF relaying system over i.n.i.d. $\eta-\mu$ fading will be analyzed. The OP, i.e., $P_{\text{out}}(\gamma_{\text{th}})$, is defined as the probability that the instantaneous SNR at SU-D is below a specified SNR threshold γ_{th}, i.e., $P_{\text{out}}(\gamma_{\text{th}}) = \Pr\{\gamma_d \leqslant \gamma_{\text{th}}\}$.

3.1 Lower Bound Analysis for OP

It can be observed that γ_d in (1) is upper bounded by $\gamma_d \leqslant \min\{\gamma_1, \gamma_2\}$, yielding

$$\begin{aligned} P_{\text{out}}(\gamma_{\text{th}}) &\geqslant \Pr\{\min(\gamma_1,\gamma_2) \leqslant \gamma_{\text{th}}\} \\ &= 1-(1-F_{\gamma_1}(\gamma_{\text{th}}))(1-F_{\gamma_2}(\gamma_{\text{th}})) \\ &= F_{\gamma_1}(\gamma_{\text{th}}) + F_{\gamma_2}(\gamma_{\text{th}}) - F_{\gamma_1}(\gamma_{\text{th}})F_{\gamma_2}(\gamma_{\text{th}}). \end{aligned} \tag{9}$$

In order to obtain the lower bound expression for OP, the CDFs of γ_1 and γ_2, $F_{\gamma_1}(\gamma)$ and $F_{\gamma_2}(\gamma)$ should be firstly studied, respectively. Then, $F_{\gamma_\ell}(\gamma)$ is given by

$$\begin{aligned} F_{\gamma_\ell}(\gamma) &= \Pr\{\gamma_\ell \leqslant \gamma\} = \Pr\left\{|g_\ell|^2 \leqslant \frac{\gamma}{\overline{\gamma}_Q}|h_\ell|^2\right\} \\ &= \int_0^\infty f_{|h_\ell|^2}(x)\int_0^{\frac{\gamma}{\overline{\gamma}_Q}x} f_{|g_\ell|^2}(y)dxdy \\ &= \int_0^\infty f_{|h_\ell|^2}(x)F_{|g_\ell|^2}\left(\frac{\gamma}{\overline{\gamma}_Q}x\right)dx. \end{aligned} \tag{10}$$

Since $|h_\ell|^2$ and $|g_\ell|^2$ follow the $\eta-\mu$ distribution, $\forall\ell = \{1,2\}$, and for integer values of μ_{g_ℓ} and μ_{h_ℓ}, using [15, eq. (8.467)], the modified Bessel function $I_{\mu-0.5}(z)$ in (4), with $\mu > 0$ being an integer, is expressed in closed form as

$$I_{\mu-0.5}(z) = \frac{1}{\sqrt{\pi}}\sum_{k=0}^{\mu-1}\frac{(\mu-1+k)!}{k!(\mu-1-k)!} \times \left[\frac{(-1)^k\exp(z)-(-1)^{\mu-1}\exp(-z)}{(2z)^{k+0.5}}\right]. \tag{11}$$

Then, by utilizing (3) and (4), $f_{|h_\ell|^2}(\cdot)$ and $F_{|g_\ell|^2}(\cdot)$ can be easily obtained. By substituting these results into (10) and with the help of [15, Eq. (3.351.3)], the CDF of γ_1 can be obtained as (12) given on the top of next page, where

$$\widetilde{\sum_{k,p,q}} = \frac{1}{\Gamma(\mu_{g_1})\Gamma(\mu_{h_1})}\left(\frac{h_{g_1}}{H_{g_1}}\right)^{\mu_{g_1}}\left(\frac{h_{h_1}}{H_{h_1}}\right)^{\mu_{h_1}}\sum_{k=0}^{\mu_{g_1}-1}\sum_{p=0}^{\mu_{g_1}-k-1} \times \sum_{q=0}^{\mu_{h_1}-1}\frac{(\mu_{h_1}+q-1)!(\mu_{h_1}+p-q-1)!}{p!q!(\mu_{h_1}-q-1)!(4H_{h_1})^q}\left(\frac{\mu_{h_1}}{\Omega_3}\right)^{\mu_{h_1}-q}.$$

$$F_{\gamma_1}(\gamma) = 1 - \widetilde{\sum_{k,p,q}} \left(\frac{\gamma}{\overline{\gamma}_Q}\right)^p \Bigg\{ a_{g_1}^{(k)} A_{g_1}^p \Bigg[(-1)^q \left(A_{g_1}\frac{\gamma}{\overline{\gamma}_Q} + A_{h_1}\right)^{-(\mu_{h_1}+p-q)} + (-1)^{\mu_{h_1}} \left(A_{g_1}\frac{\gamma}{\overline{\gamma}_Q} + B_{h_1}\right)^{-(\mu_{h_1}+p-q)} \Bigg]$$
$$+ (-1)^{\mu_{g_1}} b_{g_1}^{(k)} B_{g_1}^p \Bigg[(-1)^q \left(B_{g_1}\frac{\gamma}{\overline{\gamma}_Q} + A_{h_1}\right)^{-(\mu_{h_1}+p-q)} + (-1)^{\mu_{h_1}} \left(B_{g_1}\frac{\gamma}{\overline{\gamma}_Q} + B_{h_1}\right)^{-(\mu_{h_1}+p-q)} \Bigg] \Bigg\}. \quad (12)$$

It can be observed that the CDF of γ_2 is similar to the CDF of γ_1, so $F_{\gamma_2}(\gamma)$ can be directly derived from (12) by substituting the respective parameters by their counterparts (i.e., $\mu_{g_1} \to \mu_{g_2}$, $\mu_{h_1} \to \mu_{h_2}$, $h_{g_1} \to h_{g_2}$, $h_{h_1} \to h_{h_2}$, $H_{g_1} \to H_{g_2}$, $H_{h_1} \to H_{h_2}$, $a_{g_1}^{(k_1)} \to a_{g_2}^{(k_1)}$, $a_{h_1}^{(k_2)} \to a_{h_2}^{(k_2)}$, $b_{g_1}^{(k_1)} \to b_{g_2}^{(k_1)}$, $b_{h_1}^{(k_2)} \to b_{h_2}^{(k_2)}$, $a_{g_1}^{(k)} \to a_{g_2}^{(k)}$, $b_{g_1}^{(k)} \to b_{g_2}^{(k)}$, $A_{g_1} \to A_{g_2}$, $A_{h_1} \to A_{h_2}$, $B_{g_1} \to B_{g_2}$, $B_{h_1} \to B_{h_2}$ and $\Omega_3 \to \Omega_4$).

Finally, for integer values of μ_{g_ℓ} and μ_{h_ℓ}, $\forall \ell = \{1, 2\}$, substituting (12) and the CDF of γ_2 into (9), the lower bound expression for OP can be obtained is given as (13) given on the top of next page, where

$$\widehat{\sum_{k,p,q}} = \frac{1}{\Gamma(\mu_{g_2})\Gamma(\mu_{h_2})}\left(\frac{h_{g_2}}{H_{g_2}}\right)^{\mu_{g_2}}\left(\frac{h_{h_2}}{H_{h_2}}\right)^{\mu_{h_2}} \sum_{k=0}^{\mu_{g_2}-1}\sum_{p=0}^{\mu_{g_2}-k-1}$$
$$\times \sum_{q=0}^{\mu_{h_2}-1} \frac{(\mu_{h_2}+q-1)!(\mu_{h_2}+p-q-1)!}{p!q!(\mu_{h_2}-q-1)!(4H_{h_2})^q}\left(\frac{\mu_{h_2}}{\Omega_4}\right)^{\mu_{h_2}-q}.$$

Specially, for Nakagami-m fading channels, i.e., $\mu_{g_\ell} = m_{g_\ell}$, $\mu_{h_\ell} = m_{h_\ell}$ and $\eta_{g_\ell} = \eta_{h_\ell} = \eta \to 0$ [8], where m_{g_l} and m_{h_l} denote Nakagami fading parameters, hence, $h - H \to 1/2$, $h + H \to \infty$ and $h/H \to 1$, $A \to m/\overline{X}$ and $B \to \infty$. The lower bound expression for OP in (13) becomes

$$P_{\text{out}}(\gamma_{\text{th}}) \geq 1 - \left[\sum_{k=0}^{m_{g_1}-1} \frac{\alpha_3^{m_{h_1}}\Gamma(m_{h_1}+k)\left(\frac{\alpha_1\gamma_{\text{th}}}{\overline{\gamma}_Q}\right)^k}{k!\Gamma(m_{h_1})\left(\alpha_3 + \frac{\alpha_1\gamma_{\text{th}}}{\overline{\gamma}_Q}\right)^{m_{h_1}+k}} \right]$$
$$\times \left[\sum_{k=0}^{m_{g_2}-1} \frac{\alpha_4^{m_{h_2}}\Gamma(m_{h_4}+k)\left(\frac{\alpha_2\gamma_{\text{th}}}{\overline{\gamma}_Q}\right)^k}{k!\Gamma(m_{h_2})\left(\alpha_4 + \frac{\alpha_2\gamma_{\text{th}}}{\overline{\gamma}_Q}\right)^{m_{h_2}+k}} \right], \quad (14)$$

where $\alpha_1 = m_{g_1}/\Omega_1$, $\alpha_2 = m_{g_2}/\Omega_2$, $\alpha_3 = m_{h_1}/\Omega_3$ and $\alpha_4 = m_{h_2}/\Omega_4$ which is identical with [4, eq. (23)].

Analyzing the most general case of $\eta - \mu$ fading channels, i.e., the case with not-necessarily-integer values for the μ parameter of the $\eta - \mu$ distribution, for arbitrary values of the μ fading parameters, the computation of (10) is very difficult, mostly because of the fact that the CDF of the η-μ fading channel is available in integral form only. Consequently, the evaluation of (10) requires a two-fold numerical integration. Instead, it is more convenient to express in the Fourier transform domain, by employing the Parseval's theorem.

By employing the Parseval's theorem [16], the product integral in (10) can be written as

$$F_{\gamma_\ell}(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\mathcal{F}\left\{f_{|h_\ell|^2}(x); x; \omega\right\}}$$
$$\times \mathcal{F}\left\{F_{|g_\ell|^2}\left(\frac{\gamma}{\overline{\gamma}_Q}x\right); x; \omega\right\} d\omega. \quad (15)$$

where $\mathcal{F}\{\cdot\}$ denotes Fourier transform and $\overline{(\cdot)}$ denotes complex conjugate. To this end, the Fourier transforms $\mathcal{F}\{f_X(x); x; \omega\}$ and $\mathcal{F}\{F_Y(Tx); x; \omega\}$ should be deduced. The first Fourier transform, can be readily obtained as

$$\mathcal{F}\left\{f_{|h_\ell|^2}(x); x; \omega\right\} = \mathcal{M}_{|h_\ell|^2}(-\imath\omega), \quad (16)$$

where $\mathcal{M}_{|h_\ell|^2}(s)$ is the moment generating function (MGF) of $|h_\ell|^2$ and $\imath = \sqrt{-1}$.

By employing the following Fourier transforms [16]:

$$\mathcal{F}\left\{\int_{-\infty}^{x} g(\tau)\mathrm{d}\tau; x; \omega\right\} = -\frac{\imath}{\omega}\mathcal{F}\{g(x); x; \omega\}$$
$$+ \mathcal{F}\{g(x); x; 0\}\pi\delta(\omega), \quad (17a)$$

$$\mathcal{F}\{g(Tx); x; \omega\} = \frac{1}{|T|}\mathcal{F}\left\{g(x); x; \frac{\omega}{T}\right\}, \quad (17b)$$

where $T = \frac{\gamma}{\overline{\gamma}_Q}$, one obtains:

$$\mathcal{F}\left\{F_{|g_\ell|^2}(Tx); x; \omega\right\} = -\frac{\imath}{\omega}\mathcal{M}_{|g_\ell|^2}\left(-\imath\frac{\omega}{T}\right)$$
$$+ \pi\delta(\omega). \quad (18)$$

$$P_{\text{out}}(\gamma_{\text{th}}) \geq 1 - \Bigg\{ \widetilde{\sum_{k,p,q}} \left(\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}}\right)^p a_{g_1}^{(k)} A_{g_1}^p \Bigg[(-1)^q \left(A_{g_1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + A_{h_1}\right)^{-(\mu_{h_1}+p-q)} + (-1)^{\mu_{h_1}} \left(A_{g_1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + B_{h_1}\right)^{-(\mu_{h_1}+p-q)} \Bigg]$$
$$+ (-1)^{\mu_{g_1}} b_{g_1}^{(k)} B_{g_1}^p \Bigg[(-1)^q \left(B_{g_1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + A_{h_1}\right)^{-(\mu_{h_1}+p-q)} + (-1)^{\mu_{h_1}} \left(B_{g_1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + B_{h_1}\right)^{-(\mu_{h_1}+p-q)} \Bigg] \Bigg\}$$
$$\times \Bigg\{ \widehat{\sum_{k,p,q}} \left(\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}}\right)^p a_{g_2}^{(k)} A_{g_2}^p \Bigg[(-1)^q \left(A_{g_2}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + A_{h_2}\right)^{-(\mu_{h_2}+p-q)} + (-1)^{\mu_{h_2}} \left(A_{g_2}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + B_{h_2}\right)^{-(\mu_{h_2}+p-q)} \Bigg]$$
$$+ (-1)^{\mu_{g_2}} b_{g_2}^{(k)} B_{g_2}^p \Bigg[(-1)^q \left(B_{g_2}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + A_{h_2}\right)^{-(\mu_{h_2}+p-q)} + (-1)^{\mu_{h_2}} \left(B_{g_2}\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}} + B_{h_2}\right)^{-(\mu_{h_2}+m-q)} \Bigg] \Bigg\}. \quad (13)$$

For arbitrary values of μ_{g_ℓ} and μ_{h_ℓ}, $\forall \ell = \{1, 2\}$, using [14, eq.(3)] as well as the identity

$$(1 + \imath a)^{-\mu} = (1 + a^2)^{-\mu/2} \exp[\imath\mu \arctan(a)] \quad (19)$$

with a being real, a lower bound for OP can be deduced as

$$P_{\text{out}}(\gamma_{\text{th}}) \geq \sum_{\ell=1}^{2} \mathcal{I}(\mu_{g_\ell}, \mu_{h_\ell}, \eta_{g_\ell}, \eta_{h_\ell}, \gamma_{\text{th}}, \overline{\gamma}_{\mathcal{Q}}) - \prod_{\ell=1}^{2} \mathcal{I}(\mu_{g_\ell}, \mu_{h_\ell}, \eta_{g_\ell}, \eta_{h_\ell}, \gamma_{\text{th}}, \overline{\gamma}_{\mathcal{Q}}), \quad (20)$$

where $\mathcal{I}(\mu_{g_\ell}, \mu_{h_\ell}, \eta_{g_\ell}, \eta_{h_\ell}, \gamma_{\text{th}}, \overline{\gamma}_{\mathcal{Q}})$ is expressed as in (21) on the top of the next page. Note that the integral in (20) can be easily evaluated numerically by employing Gaussian quadrature techniques or by employing standard built-in functions for numerical integration, available in popular mathematical software packages such as Matlab, Maple or Mathematica.

3.2 Asymptotic Analysis for OP

In order to obtain further insights on the system performance, the asymptotic expression for OP at high SNRs will be derived in this section, wherefrom the diversity and coding gains can be deduced.

Using the lower bound $P_{\text{out}}(\gamma) = \Pr\{\min(\gamma_1, \gamma_2) \leqslant \gamma\}$, $P_{\text{out}}(\gamma)$ can be approximated at high SNRs as $P_{\text{out}}(\gamma) = F_{\gamma_1}(\gamma) + F_{\gamma_2}(\gamma) - F_{\gamma_1}(\gamma)F_{\gamma_2}(\gamma) \simeq F_{\gamma_1}(\gamma) + F_{\gamma_2}(\gamma)$. From [12, eqs. (14),(15)], for $x \to 0^+$, the asymptotic approximation for $f_X(x)$ can be expressed as [12]

$$f_X(x) \simeq \frac{h^\mu}{\Gamma(2\mu)} \left(\frac{2\mu}{\overline{X}}\right)^{2\mu} x^{2\mu-1}, \quad (22)$$

where $\mu \in \{\mu_{g_\ell}, \mu_{h_\ell}\}$ and $h \in \{h_{g_\ell}, h_{h_\ell}\}$. Employing (22), one can finally obtain the asymptotic approximation for $F_X(x)$ as

$$F_X(x) \simeq \frac{h^\mu}{2\mu\Gamma(2\mu)} \left(\frac{2\mu}{\overline{X}}\right)^{2\mu} x^{2\mu}. \quad (23)$$

Utilizing (3) and (23) to obtain $f_{|h_1|^2}(\cdot)$ and $F_{|g_1|^2}(\cdot)$, respectively, then substituting them into (10) and with the help of [17, eq. (2.15.3.2)], the asymptotic approximation for $F_{\gamma_1}(\gamma)$ can be deduced as

$$F_{\gamma_1}(\gamma) \overset{\overline{\gamma}_{\mathcal{Q}} \to \infty}{=} \frac{\sqrt{\pi}\Gamma(2\mu_{g_1} + 2\mu_{h_1})}{\mu_{g_1}\Gamma(2\mu_{g_1})\Gamma(\mu_{h_1})\Gamma(\mu_{h_1} + 0.5)} \times \frac{h_{g_1}^{\mu_{g_1}}}{(4h_{h_1})^{\mu_{h_1}}} \left(\frac{\Omega_3\mu_{g_1}}{\Omega_1\mu_{h_1}h_{h_1}}\right)^{2\mu_{g_1}} \left(\frac{\gamma}{\overline{\gamma}_{\mathcal{Q}}}\right)^{2\mu_{g_1}} \times {}_2F_1\left(\mu_{g_1} + \mu_{h_1}, \mu_{g_1} + \mu_{h_1} + 0.5; \mu_{h_1} + 0.5; \frac{H_{h_1}^2}{h_{h_1}^2}\right). \quad (24)$$

Similarly, the asymptotic expression for $F_{\gamma_2}(\gamma)$ can be directly derived from (24) after replacing the parameters by their counterparts. Finally, for arbitrary values of μ_{g_ℓ} and μ_{h_ℓ}, $\forall \ell = \{1, 2\}$, when $\overline{\gamma}_{\mathcal{Q}} \to \infty$, utilizing these results, the asymptotic approximation for OP can be expressed as

$$P_{\text{out}}(\gamma_{\text{th}}) \overset{\overline{\gamma}_{\mathcal{Q}} \to \infty}{=} F_{\gamma_1}(\gamma_{\text{th}}) + F_{\gamma_2}(\gamma_{\text{th}}) = \Theta \cdot \left(\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathcal{Q}}}\right)^{\min(2\mu_{g_1}, 2\mu_{g_2})}, \quad (25)$$

where

$$\Theta = \begin{cases} \Theta_1, & \text{if } \mu_{g_1} < \mu_{g_2} \\ \Theta_1 + \Theta_2, & \text{if } \mu_{g_1} = \mu_{g_2} \\ \Theta_2, & \text{if } \mu_{g_1} > \mu_{g_2} \end{cases} \quad (26)$$

$$\mathcal{I}(\mu_{g_\ell},\mu_{h_\ell},\eta_{g_\ell},\eta_{h_\ell},\gamma_{\rm th},\overline{\gamma}_{Q}) = \frac{1}{2}+\frac{1}{\pi}\int_0^\infty \frac{\sin\left[\mu_{h_\ell}\arctan\left(\frac{\omega}{A_{h_\ell}}\right)+\mu_{h_\ell}\arctan\left(\frac{\omega}{B_{h_\ell}}\right)-\mu_{g_\ell}\arctan\left(\frac{\omega}{A_{g_\ell}}\frac{\overline{\gamma}_Q}{\gamma_{\rm th}}\right)-\mu_{g_\ell}\arctan\left(\frac{\omega}{B_{g_\ell}}\frac{\overline{\gamma}_Q}{\gamma_{\rm th}}\right)\right]}{\omega\left(1+\frac{\omega^2}{A_{h_\ell}^2}\right)^{\mu_{h_\ell}/2}\left(1+\frac{\omega^2}{B_{h_\ell}^2}\right)^{\mu_{h_\ell}/2}\left(1+\frac{\omega^2}{A_{g_\ell}^2}\frac{\overline{\gamma}_Q^2}{\gamma_{\rm th}^2}\right)^{\mu_{g_\ell}/2}\left(1+\frac{\omega^2}{B_{g_\ell}^2}\frac{\overline{\gamma}_Q^2}{\gamma_{\rm th}^2}\right)^{\mu_{g_\ell}/2}}d\omega. \tag{21}$$

and Θ_1, Θ_2 are given as

$$\Theta_1 = \frac{\sqrt{\pi}\Gamma(2\mu_{g_1}+2\mu_{h_1})}{\mu_{g_1}\Gamma(2\mu_{g_1})\Gamma(\mu_{h_1})\Gamma(\mu_{h_1}+0.5)} \times \frac{h_{g_1}^{\mu_{g_1}}}{(4h_{h_1})^{\mu_{h_1}}}\left(\frac{\Omega_3\mu_{g_1}}{\Omega_1\mu_{h_1}h_{h_1}}\right)^{2\mu_{g_1}} \times {}_2F_1\left(\mu_{g_1}+\mu_{h_1},\mu_{g_1}+\mu_{h_1}+0.5;\mu_{h_1}+0.5;\frac{H_{h_1}^2}{h_{h_1}^2}\right),$$

$$\Theta_2 = \frac{\sqrt{\pi}\Gamma(2\mu_{g_2}+2\mu_{h_2})}{\mu_{g_2}\Gamma(2\mu_{g_2})\Gamma(\mu_{h_2})\Gamma(\mu_{h_2}+0.5)} \times \frac{h_{g_2}^{\mu_{g_2}}}{(4h_{h_2})^{\mu_{h_2}}}\left(\frac{\Omega_4\mu_{g_2}}{\Omega_2\mu_{h_2}h_{h_2}}\right)^{2\mu_{g_2}} \times {}_2F_1\left(\mu_{g_2}+\mu_{h_2},\mu_{g_2}+\mu_{h_2}+0.5;\mu_{h_2}+0.5;\frac{H_{h_2}^2}{h_{h_2}^2}\right),$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [15].

Based on the asymptotic expression for OP in (25), the diversity gain G_d and the coding gain G_c can be expressed as

$$G_d = \min(2\mu_{g_1}, 2\mu_{g_2}),$$
$$G_c = {\gamma_{\rm th}}^{-1}\Theta^{-1/\min(2\mu_{g_1},2\mu_{g_2})},$$

respectively. As it can be observed from the above formulae, the diversity gain only depends on the more severe fading channel between two hops of the secondary network, whereas the primary network only affects its coding gain.

4 Numerical and Computer Simulation Results

In this section, in order to evaluate the outage performance of cognitive AF relay networks over $\eta-\mu$ fading, some representative numerical results are now

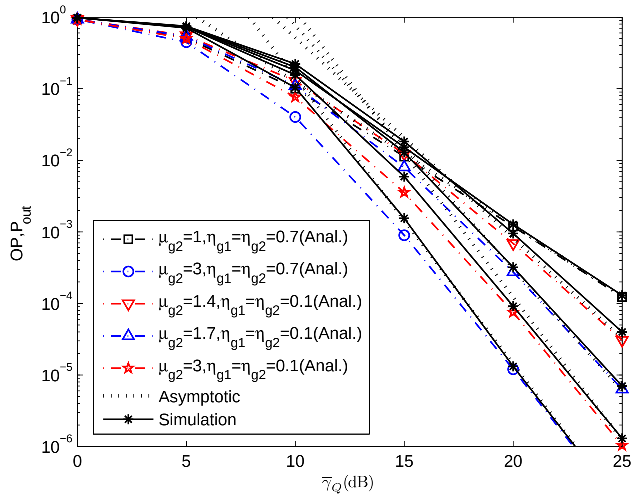

Fig. 1: OP of cognitive AF relay network over $\eta-\mu$ fading channels with parameters $\eta = 0.7, \mu_{g_1} = 2, \mu_{h_1} = \mu_{h_2} = 5$.

presented by using common mathematical softwares such as Matlab or Mathematica. To validate the accuracy of our analytical expressions, we utilize Monte-Carlo computer simulation to demonstrate the aforementioned expressions. For the simulations in Figs. 1 and 2, without loss of generality, we assume that the average channel powers of all links are given by $\Omega_i = \overline{\gamma}_Q, i = \{1,2,3,4\}$. The outage threshold $\gamma_{\rm th}$ is set to 3 dB for all considered analysis. In all figures, the lower bound results are quite tight and the asymptotic results also converge the simulations in high SNR region. This validates the usefulness of our derived analytical and asymptotic expressions.

In Fig. 1, the OP of cognitive AF relay networks over $\eta-\mu$ fading channels is plotted for different μ_{g_2}, η_{g_1} and η_{g_2} with $\eta = 0.7, \mu_{g_1} = 2$ and $\mu_{h_1} = \mu_{h_2} = 5$. Fig. 1 shows the OP for various values of μ_{g_2} between 1 and 3, namely $\mu_{g_2} = \{1, 1.4, 1.7, 3\}$, and $\eta_{g_1} = \eta_{g_2} = \{0.1, 0.7\}$. As observed from Fig. 1, the outage performance improves as μ_{g_2} increases and/or

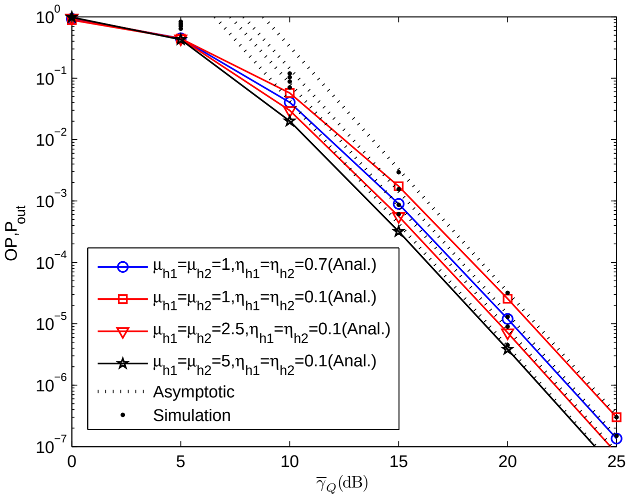

Fig. 2: OP of cognitive AF relay network over $\eta-\mu$ fading channels with parameters $\eta=0.7, \mu_{g_1}=2, \mu_{g_2}=3$.

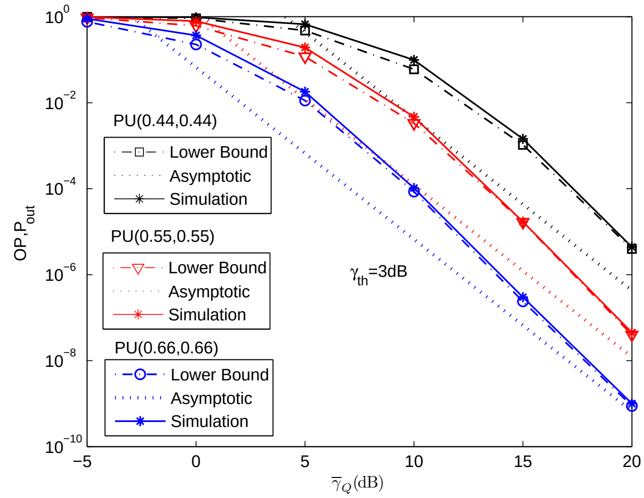

Fig. 3: OP of cognitive AF relay network over $\eta-\mu$ fading channels with parameters $\eta=0.7, \mu_{g_1}=2, \mu_{g_2}=3, \mu_{h_1}=\mu_{h_2}=1$.

η_{g_1}, η_{g_2} increase. It can be also seen that, there is a significant increase in diversity gain when μ_{g_2} increases from 1 to 3, however the same diversity gain can be achieved when $\mu_{g_2}=3$. This is because the diversity gain equals to $\min(2\mu_{g_1}, 2\mu_{g_2})$.

Fig. 2 depicts how the parameters μ_{h_1}, μ_{h_2} and η_{h_1}, η_{h_2} affect the OP performance of the secondary network, respectively. And the OP curves for different μ_{h_1}, μ_{h_2} and η_{h_1}, η_{h_2} with $\eta=0.7$, $\mu_{g_1}=2$ and $\mu_{g_2}=3$ are plotted. It can be observed that the OP performance improves when μ_{h_1} and μ_{h_2} increase from $\mu_{h_1}=\mu_{h_2}=1$ to $\mu_{h_1}=\mu_{h_2}=5$ and/or η_{h_1}, η_{h_2} increase. Moreover, as expected, the fading parameters of interference links only affect the coding gain, without affecting the diversity gain, just as our preceding analysis.

To evaluate the effect of position of PU on SUs' network, Fig. 3 shows the OP of cognitive AF relay network for different PU's position with $\eta=0.7, \mu_{g_1}=2, \mu_{g_2}=3$ and $\mu_{h_1}=\mu_{h_2}=1$. Assume all SUs are located in a straight line, and SU-S, SU-R and SU-D are located at co-ordinates (0,0), (1/2,0) and (1,0), respectively, and PU-D has three co-ordinates (0.44,0.44), (0.55,0.55) and (0.66,0.66). The average channel power Ω_X can be expressed as $\Omega_i=\overline{\gamma}_Q/d_i^4$ [4], $i=\{1,2,3,4\}$, where d_i denotes the distance between the transceivers. From Fig. 3, it can be seen that the position of PU-D significantly affects the OP performance of the secondary network, and when PU-D is located at co-ordinate (0.66,0.66), the best performance can be achieved.

5 Conclusion

In this paper, the tight lower bound as well as the asymptotic expressions of OP for cognitive AF relay network over i.n.i.d. $\eta-\mu$ fading channels have been obtained under the maximum interference power constraint. Based on the newly derived formulae, several important performance metrics can be exhibited to quantify the impact of primary networks on the secondary performance. Our findings reveal that the diversity gain only depends on the more severe fading hop between two hops of the secondary network, whereas the primary network only affects the coding gain. Monte-Carlo computer simulation has confirmed the correctness of our analysis.

Acknowledgements: This work is supported by the National Natural Science Foundation of China (Grant No. 61301111/61401387), China Postdoctoral Science Foundation (Grant No. 2014M56074).

References:

[1] T. Q. Duong, V. N. Q. Bao, and H. -J. Zepernick, Exact outage probability of cognitive AF relaying with underlay spectrum sharing, *Electron. Lett.*, Vol. 47, No. 17, 2011, pp. 1001-1002.

[2] W. Xu, J. Zhang, P. Zhang, and C. Tellambura, Outage probability of decode-and-forward cognitive relay in presence of primary users interference, *IEEE Commun. Lett.*, Vol. 16, No. 8, 2012, pp. 1252-1255.

[3] V. N. Q. Bao, T. Q. Duong, and C. Tellambura, On the performance of cognitive underlay multihop networks with imperfect channel state information, *IEEE Trans. Commun.*, Vol. 61, No. 12, 2013, pp. 4864-4873.

[4] T. Q. Duong, D. B. da Costa, M. Elkashlan, and V. N. Q. Bao, Cognitive amplify-and-forward relay networks over Nakagami-m fading, *IEEE Trans. Veh. Technol.*, Vol. 61, No. 5, 2012, pp. 2368-2374.

[5] M. H. Xia and S. Aïssa, Cooperative AF relaying in spectrum-sharing systems: Performance analysis under average interference power constraints and Nakagami-m fading, *IEEE Trans. Commun.*, Vol. 60, No. 6, 2012, pp. 1523-1533.

[6] J. L. Wang, Z. S. Zhang, Q. H. Wu, and Y. Z. Huang, Outage analysis of cognitive relay networks with interference constraints in Nakagami-m channels, *IEEE Wireless Commun. Lett.*, Vol. 2, No. 4, 2013, pp. 387-390.

[7] F. R. V. Guimarães, D. B. da Costa, T. A. Tsiftsis, C. C. Cavalcante, and G K Karagiannidis, Multiuser and multirelay cognitive radio networks under spectrum-sharing constraints, *IEEE Trans. Veh. Technol.*, Vol. 63, No. 1, 2014, pp. 433-439.

[8] M. D. Yacoub, The $\kappa - \mu$ distribution and the $\eta - \mu$ distribution, *IEEE Ant. and Propag. Mag.*, Vol. 49, No. 1, 2007, pp. 68-81.

[9] M. D. Yacoub, The $\eta - \mu$ distribution: A general fading distribution, in *Proc. IEEE VTC*, Boston, MA, USA, Sep. 2000, pp. 872-877.

[10] D. Morales-Jimenez and J. F. Paris, Outage probability analysis for $\eta - \mu$ fading channels, *IEEE Commun. Lett.*, Vol. 14, No. 6, 2010, pp. 521-523.

[11] H. Yu, G. Wei, F. Ji, and X. Zhang, On the error probability of cross-QAM with MRC reception over generalized $\eta - \mu$ fading channels, *IEEE Trans. Veh. Technol.*, Vol. 60, No. 6, 2011, pp. 2631-2643.

[12] K. P. Peppas, G. C. Alexandropoulos, and P. T. Mathiopoulos, Performance analysis of dual-hop AF relaying systems over mixed $\eta - \mu$ and $\kappa - \mu$ fading channels, *IEEE Trans. Veh. Techno.*, Vol. 62, No. 7, 2013, pp. 3149-3163.

[13] N. Y. Ermolova, Moment generating functions of generalized $\eta - \mu$ and $\kappa - \mu$ distributions and their applications to performance evaluation of communication systems, *IEEE Commun. Lett.*, Vol. 12, No. 7, 2008, pp. 502-504.

[14] K. P. Peppas, F. Lazarakis, T. Zervos, A. Alexandridis, and K. Dangakis, Sum of non-identical independent squared η-μ variates and applications in the performance analysis of DS-CDMA systems, *IEEE Trans. Commun.*, Vol. 9, No. 9, 2010, pp. 2718-2723.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition, Academic Press, 2007.

[16] L. Debnath, *Integral transforms and their applications*, CRC Press, 1995.

[17] A. P. Prudnikov and Y. A. Brychkov and Q. I. Marichev, *Integrals and Series: Special Functions*, 1st ed. Boca Raton, FL, USA: CRC, 1992.