Communication Channel Model between two Neighbors in UAV Networks

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Abstract: - Unmanned Aerial Vehicle "UAV" networking is evolving as a major state of the art research field, thanks to the foreseen revolution in its applications. Enhancing the performance of such networks requires developing a channel model between any couple of neighbors within the UAV network. In this paper, we present a statistical signal reception model for the shadowed communication channel between two neighbors in a UAV network. In this model, we consider both the shadowing effect and the small scale fading effect. Focusing on the applications where a UAV network could be considered as a rural environment, the shadowing is considered as a Line Of Sight (LOS) lognormal one, where the line-of-sight (LOS) component is shadowed by obstacles present within the path between two network nodes. The presented model relates the power spectral density of the received signal to noise ratio per symbol with the physical path parameters, which could be measured in real time.

Key-Words: - UAV network, Rice fading, Shadowing, Loo's model, Path Loss, Nakagami model, Lognormal shadowed distribution.

1 Introduction

Tremendous advantages of UAV networking pave the way towards untethered applications. Major concerned areas include research, commercial, and public domains. Greenhouse monitoring can be accomplished using a set of UAVs as a mobile sensory platform. The design, construction, and validation of such approach are predicted in [1]. Another scenario of applications is about the task of locating a targeted object continuously using one or more UAVs, whereby; the track should be rapidly updated and shouldn't be lost for any reason. Such scenario is sketched in [2]. In [3], detecting springs using a team of UAVs is described.

Many research centers are working on enhancing the operation of the UAV network at every composing sector. Among these sectors, the wireless channel presents a serious challenge for researchers, as it is a fundamental part of the communication system, which –in turn– is the main part of the network. For example, each node within the network needs to find the best next hop in order to forward data, this need to have, as exact as it gets, knowledge about the links between nodes and the qualities of those links, which means determining the effects of the wireless channel between nodes [4]. The precise channel model will give accurate information about the routes, hence, forwarded information will reach its final destination within optimized delay and minimum corruption. Also, It is mandatory to find the type of fading within the channels (slow/fast, flat/frequency selective) which will play a main role in determining the best techniques for the communication system of the nodes. Because of all of those reasons and others (decide about the fading margin in the link budget, calculate the channel capacity which affect the used bit rate, etc.), a lot of work has been conducted in order to find the optimal channel model between network nodes [5-6].

Using the information collected in real time from the propagation environment, fade statistics for communication paths between network nodes could be calculated with the help of the available fundamental theory. Actually, in most cases the knowledge about fade dynamics is so little; so one should use measurements or simulation in order to gather data which is often case specific. On the other hand, in order to simulate the operation of the network and how information will be routed between nodes, it is very important to create a channel simulator. As a main reason for this simulator, a routing metric, which is related to the channel model, should be studied using simulators in order to be evaluated and enhanced before actually used [4-7].

The remaining sections in this paper are organized as follows. In section II we present a study for the physics and statistics included within the communication channel between any two network nodes. In Section III, the complex received signal model is introduced and the Probability Density Function "PDF" of the signal to noise ratio per symbol is calculated. In order to use this PDF in real time, we re-parameterize it into other format; this format uses the parameters that could be estimated in real time. In section IV, BER for the simplest BPSK ideal coherent modulation is calculated. Discussion of the obtained results is shown in section V. Finally, we finish this paper with some concluding remarks and future aspects.

2 Physics and Statistics of UAV Networks Propagation Channel

Because of the dynamic nature of the communication channel between nodes within UAV networks, the transmitted signals within this type of channel are affected by random shadowing effects. The main sources of this shadowing are the nonhomogeneous obstacles that exist within the direct LOS signal path and causes changes in the transmitted electromagnetic waves. Considering no multipath effects, which is generated within the local environment, the received signal will be composed of a number of dominant components resulted from the diffraction and refraction of the waves within the wireless link. But as these components reach the receiving node, they will be affected by the multipath effects that result from local reflections and scattering resulted from the geometry and design of the UAV node [8].

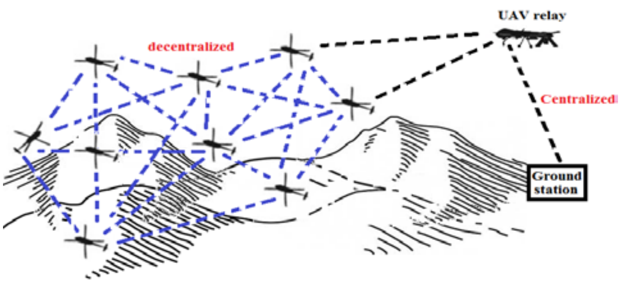

Fig. 1: Decentralized UAV network with centralized UAV relay.

Most applications use the decentralized control scheme in order to deploy multiple UAVs as a network [9-10]. But a lot of applications need to have the UAV network close to the earth surface (low altitudes) in order to have clear and more precise information about the area they cover, besides, the area to be covered will mostly be far away from the control center or base station, thus, hybrid centralized and decentralized solutions are used where a repeater UAV (may be more than one) is used to connect the UAV network to the control center [11-12-13]. The overall topology is illustrated in Fig. 1. As the UAV network will be close to the ground surface, then the wireless link between the UAV nodes could suffer from strong shadowing effects; this will actually affect the LOS or specular component. It is clear that shadowing caused by obstacles within the environment where the network works will have great effects on the network performance. For that, in order to reach the perfect design for the hardware of each node such as:

- The antenna types and receiver technologies [14].
- Optimize routing protocols [15-16].
- Enhance task allocation procedures [4].
- Increase the performance of positioning schemes to be used in UAV network communications [7].

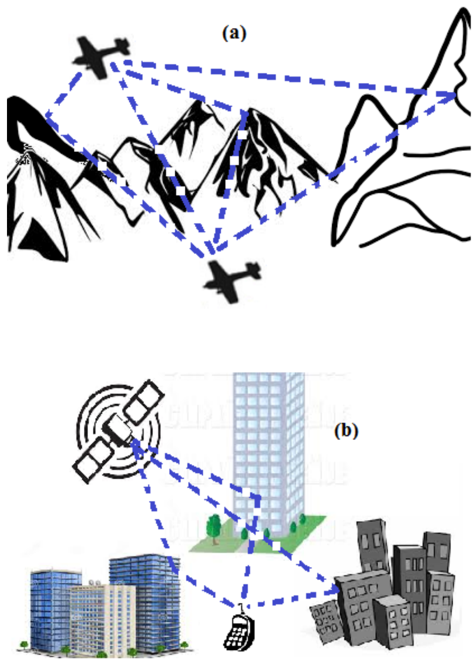

Fig. 2: Proposed approach. (a) The wireless channel within UAV networks in rural environments. (b) The LMS wireless channel.

The shadowing effects should be taken into account when we formulate the statistical model that describes the received signal envelope. Looking at the results which was found in Land Mobile Satellite (LMS) communications, where the channel suffer from both shadowing and small scale fading, and where the shadowing affects only the LOS components [17-18], we can notice that the studied communication channel is the same as it is between UAV nodes, see Fig. 2. Both could be considered as rural environments, both suffer from shadowing on the LOS component, and both suffer from same multipath conditions. Using these results we derive a model for the communication channel between two neighbour nodes within the UAV network and relate it with real time estimated parameters.

In the model proposed in this paper, we assume the presence of scattered waves together with the LOS signal component; this is identical to the phenomenon observed in rician fading [19]. The difference between the proposed model and the rician one is that the LOS component is assumed to be random with lognormal distribution because of the shadowing effect.

This study is aiming at finding a channel model that could be used to calculate the channel quality depending on the channel parameters estimated in real time. The resulting performance metric, which is the BER, will be used by different network nodes in order to build their routing tables efficiently. Using this routing metric will increase the total network efficiency.

3 A Statistical Model for Shadowing in UAV Network Communications Channels

In order to build a communication channel model for the wireless channel between the network nodes, as we have seen in the previous section, we have to consider three different types of fading models; these models shall be used to describe different fading phenomenon within the wireless channel:

- The first model we have to consider is the large scale path loss model, which is used to characterize the average received signal strength at the receiving node.
- The second model we have to include is the shadowing model. This model is used in order to estimate the fluctuation in the received signal power because of the electromagnetic large obstacles within the wireless channel.
- Finally, the small scale fading model should be involved. Actually, it is used to characterize the fluctuation in the received signal envelope which is resulted because of scattering near the receiving node.

3.1 Large-scale fading

According to the free space path loss model, the received signal power formula can be written according to the Friis power transmission formula as follows [20]:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \tag{1}$$

where d is the distance between the two nodes, G_r and G_t are the antenna gains of receiving node and the transmitting node, respectively, P_r, P_t are the transmitted and received power, respectively, and λ is the wavelength of the carrier frequency. This means that the received signal power decays as d^{-2}.

In most cases, the network will work in a non-free space environment, because of that it is better to use the generalized path loss model which could be written in decibel units as:

$$PL(dB) = PL_{ref}(dB) + \alpha 10 \log(\frac{d_{ref}}{d}) \tag{2}$$

where PL is the path loss for a given distance d, α is the path loss exponent which depends on the environment clutter and could be estimated previously using collected information about the area where the UAV will be placed and work and information about their heights, and PL_{ref} refers to the path loss at a reference distance d_{ref} which is also determined according to the used antennas at the two communicating nodes [21]. Usually PL_{ref} is the free space path loss gain at the distance d_{ref} when we use omnidirectional antennas (this is the case of UAV nodes), so we can write the following formula for the average received signal power:

$$\bar{S} = P_t \left(\frac{\lambda}{4\pi d_{ref}}\right)^2 \left(\frac{d_{ref}}{d}\right)^\alpha \tag{3}$$

where G_t=G_r=1 considering omnidirectional antennas to be used on nodes.

The value of α ranges from 2 to 6 for outdoor environments [21], where it equals 2 in free space (assuming there is a line of sight between the communicating nodes). It depends on several factors [21, 22]: the heights of the two communicating nodes, the situation of the ground (flat or other), quantity of particles in the air, atmospheric conditions, the volume of obstacles within the communication channel environment, and others. Usually it is assumed to be equal to 4 if we cannot estimate its value, where it is better to use empirical methods to estimate its value as these methods take into consideration the used frequency and antenna heights [22, 23]. In the case of UAV networks it is usually estimated taking both the surrounded environment and the application into consideration. As there is always a LOS between UAVs, its value will be typically between 2 and 4. As an example, in [24] they set the path loss exponent to 3.5 to consider the effect of obstacles between two UAVs, while in [16-25] they set it to 2 considering good LOS conditions between UAVs. In [26-27] they set it to 3.

3.2 Received signal envelope

According to the discussion above, it is principal to consider that the received signal envelope in UAV network communications channels suffers from the same effects that exist in rician fading [19]. Actually, a lot of researches have used this assumption to model the small scale fading between UAVs [5, 14, 16, and 28]. Also, we are going here to use rician fading to model the multipath fading between network nodes, except that in our situation, the LOS component is a random variable with lognormal distribution, and this is in order to take the shadowing effects into consideration within the channel model. Assuming narrowband stationary model, and according to the definition of the scatter and the LOS components provided earlier, the low pass equivalent complex envelope of the received signal could be written as follows [29]:

$$\mathbf{R}(\mathbf{t}) = W(t)\exp\big(j\emptyset(t)\big) + A(t)\exp(j\emptyset_0) \tag{4}$$

where *W(t)* is the amplitude of the scatter component, and it is a stationary random process that follows a Rayleigh distribution, this will be shown in the hereafter, and *A(t)* is the amplitude of the LOS component and it is assumed to be lognormal distributed. In this model, ϕ_0 is the deterministic phase of the LOS component and $\phi(t)$ is the stationary random phase process with uniform distribution over the range [-π,π). *A(t)* and *W(t)* are independent random processes, and they are also independent of $\phi(t)$.

3.3 Rician effect

If *A(t)* is initially held constant, then the conditional pdf of the received signal envelope $R(t) = |\mathbf{R}(\mathbf{t})|$ is a PDF of Rician distribution [19]:

$$f_{R|A}(r|a) = \frac{r}{b_0}\exp\left(-\frac{r^2+a^2}{2b_0}\right) I_0\left(\frac{ar}{b_0}\right) \tag{5}$$

where $2b_0 = E[W^2]$ represents the average scattered power due to the multipath components, and $I_0(.)$ is the modified Bessel function of the first kind and zeroth order.

In order to find the distribution of W, we can let a tend to 0. Using the approximation of the value of Bessel function when its argument is small [30, eq. (9.6.7)]:

$$I_{\nu-1}(x) \approx \frac{\left(\frac{x}{2}\right)^{\nu-1}}{\Gamma(\nu)} \tag{6}$$

Taking $x = \frac{ar}{b_0}$ and $\nu = 1$, then as a tend to 0; we have:

$$I_0\left(\frac{ar}{b_0}\right) \approx \frac{1}{\Gamma(1)} = 1 \tag{7}$$

Using (7) into (5) and after some mathematical manipulation gives:

$$f_{R|A}(r|a) = \frac{r}{b_0}\exp\left(-\frac{r^2+a^2}{2b_0}\right) \tag{8}$$

When we let a tend to 0, this means that we concentrate the power only in the scatter components, where no power will be exist in the LOS component. This means that the received signal in (4) will be related only to W, and so the conditional probability density function we have in (8) will equal the probability density function of W, and this gives that:

$$f_W(w) = \frac{w}{b_0}\exp\left(-\frac{w^2}{2b_0}\right), \quad w \geq 0 \tag{9}$$

Using (8) and letting a equals zero, we find that the above equation is equivalent to the Rayleigh distribution [19, eq. (2-6)]:

$$f_W(w) = \frac{2w}{\mathrm{p}}\exp\left(-\frac{w^2}{\mathrm{p}}\right) \tag{10}$$

3.4 Shadowing effect

In order to determine the distribution of the received signal envelope at the time where the LOS component is a lognormal distributed random variable, we can use the conditional mathematical expectation "theorem of total probability":

$$f_R(r) = E_A\left[f_{R|A}(r|a)\right]$$

$$f_R(r) = \int_0^\infty f_{R|A}(r|a) f_A(a)da \tag{11}$$

where $E_A[.]$ Is the expectation with respect to *A*. Equation (11) gives:

$$f_R(r) = \frac{r}{b_0}\int_0^\infty \exp\left(-\frac{r^2+a^2}{2b_0}\right) I_0\left(\frac{ar}{b_0}\right) f_A(a)da \tag{12}$$

where

$$f_A(a) = \frac{1}{a\sqrt{2\pi d_0}}\exp\left[-\frac{(\ln a-\mu)^2}{2d_0}\right] \tag{13}$$

Here, μ=E[ln(A)] and $d_0 = Var[\ln(A)]$, $Var[.]$ is the variance, are the mean and the variance of the lognormal distribution, respectively.

Equation (12) is related to $2b_0$, the mean of $\ln(A)$, and the variance of $\ln(A)$. In practice and in real time situations we need to estimate the channel behaviour using measurements that are collected by the network node, this is done in order to calculate the cost of the link between a node and its neighbours, which will be then used to choose the best path to forward data. From these measurements we can estimate the shadowing variance [31-32] and the rician factor [33-34-35-36]. So we have to relate the parameters of this equation with the shadowing variance ($\sigma^2_{X_{dB}}$) and the rician factor (K).

As the LOS component is a random variable, then the LOS power component is also a random variable, and so the rician factor is also a random variable, this is because the rician factor K, which is related to a and b_0 through the relationship $K=a^2/2b_0$, is simply the ratio of the total power of the dominant components (a^2) to the total power of the scattered waves ($2b_0$). Thus, in real time, we actually estimate the average rician factor (K_r), and we have to relate the parameters of (12) with this average value.

As proved in [22], the shadow fading component in the received power is a zero-mean Gaussian random variable added to the path loss when it is expressed in dB. In order to characterize the shadowing effect we have to estimate or measure the variance of the path loss which will also be expressed in dB. Taking the shadowing effect into consideration, without the multipath effect, we can rewrite equation (2) as follows:

$$PL(dB) = PL_0(dB) + \alpha 10\log(\frac{d_0}{d}) + X(dB) \quad (14)$$

where X(dB) is the zero-mean Gaussian distributed random variable that represents the shadowing effect and whose variance will be denoted $\sigma^2_{X_{dB}}$. The shadowing variance ($\sigma^2_{X_{dB}}$) could be estimated by empirical measurements in real time [22]. We have to relate all our parameters with $\sigma^2_{X_{dB}}$ and K_r in order to evaluate any required quantity using the actual measured parameters.

In order to relate the parameters of (12) which are (d_0, μ, b_0) with the parameters which we can estimate in real time (d, $\sigma^2_{X_{dB}}$, K_r), we first need to find the average rician factor in the channel, this could be done by taking the first moment of the rician factor probability density function. For that we will start by finding the PDF of the rician factor in the channel.

Assuming that the variation of the LOS component follows (13), it is possible to perform a transformation of variables to find the distribution of k. Using the relationship $k=a^2/2b_0$, it follows that $a^2=2b_0k$. To obtain the pdf of the transformed variable k, we must evaluate:

$$f_K(k) = f_A\left(\sqrt{2b_0k}\right)\left|\frac{da}{dk}\right| \quad (15)$$

which, in turn, gives

$$f_K(k) = \frac{1}{2k\sqrt{2\pi d_0}} \times \exp\left[-\frac{\left(\ln\sqrt{2b_0k}-\mu\right)^2}{2d_0}\right] \quad (16)$$

or,

$$f_K(k) = \frac{1}{k\sqrt{2\pi(4d_0)}} \times \exp\left[-\frac{\left(\ln k-(2\mu-\ln(2b_0))\right)^2}{2(4d_0)}\right] \quad (17)$$

We can see that this is the PDF of a log-normal variable and we can deduce that:

$$\mu_{\ln k} = \ln K_r - 2d_0 \quad (18)$$

$$Var[\ln(k)] = 4d_0 \quad (19)$$

Using the first moment relation of the log-normal PDF [37], we can find that:

$$K_r = \frac{\mu_{s_a}}{2b_0} \quad (20)$$

where μ_{s_a} is the average power of the LOS component. It is also the second moment of the LOS component.

Recall that the Rayleigh and lognormal random processes are additive, then the mean of the received power is the sum of the mean power of the LOS component and the average power of the multipath component, $\bar{S} = \mu_{s_a} + 2b_0$, and using (20), then:

$$\mu_{s_a} = \frac{K_r}{1+K_r}\bar{S} \quad (21)$$

$$2b_0 = \frac{\bar{S}}{K_r+1} \quad (22)$$

Using equation (3) we find:

$$2b_0 = \frac{P_t\left(\frac{\lambda}{4\pi d_{ref}}\right)^2\left(\frac{d_{ref}}{d}\right)^\alpha}{K_r+1} \quad (23)$$

The second order moment of the LOS received component can be written as [37]:

$$\mu_{s_a} = \exp[2\mu + 2d_0]$$

which gives:

$$\mu = \frac{\left(\ln\mu_{s_a} - 2d_0\right)}{2} \quad (24)$$

and so,

$$\mu = \frac{1}{2}\ln\left(\frac{K_r}{1+K_r}P_t\left(\frac{\lambda}{4\pi d_{ref}}\right)^2\left(\frac{d_{ref}}{d}\right)^\alpha\right) - d_0 \quad (25)$$

Now we have to find the relation between d_0 and the real time estimated parameters. The shadowing variance is the variance of the ratio $\frac{P_t}{s_a}$ in dB [21-22-38-39], where S_a is the received LOS component power, so, in order to relate d_0 with the shadowing variance we can estimate in real time, it is sufficient to find the distribution of the ratio $\Psi_{dB} = 10\log\frac{P_t}{s_a}$.

As it is proved in Appendix (I), the pdf of Ψ_{dB} is:

$$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{1}{\sqrt{2\pi(4\zeta^2 d_0)}} exp\left[-\frac{\left(\Psi_{dB}-(10\,log\,P_t - 2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right] \quad (26)$$

which means, as it is expected, that Ψ_{dB} is normally distributed with variance: $\sigma^2_{X_{dB}}=4\zeta^2 d_0$, where ζ=10/ln(10). This means that we can relate d_0 to $\sigma^2_{X_{dB}}$ using the relation:

$$d_0 = \sigma^2_{X_{dB}}/4\zeta^2 \quad (27)$$

Equations (23), (25), and (27) could now be used to calculate all of the required measurements (PDF, CDF, BER, and others) depending on measurements in real time operation.

Substituting (13) into (12) gives:

$$f_R(r) = \frac{r}{\sqrt{2\pi d_0}b_0}\int_0^\infty \frac{1}{a}\exp\left(-\frac{b_0(\ln a-\mu)^2 + d_0(r^2+a^2)}{2d_0b_0}\right) \times I_0\left(\frac{ar}{b_0}\right)da \quad (28)$$

Equation (28) is the pdf of the received signal envelope within the shadowed rician communication channel observed in UAV networks between any two neighbours. This is the PDF of the Loo's statistical model for land mobile satellite communications channels derived in [17]. But here we could relate the Loo's parameters with those we can estimate in real time working.

Using the change of variable ($S=r^2$) where S is the received signal power, and as $\frac{dr}{ds}=\frac{1}{2r}$ we can find the PDF of the received power as follows:

$$f_S(s) = \frac{1}{2b_0\sqrt{2\pi d_0}}\int_0^\infty \frac{1}{a}\exp\left[-\frac{(\ln a-\mu)^2}{2\times d_0}\right]\exp\left(-\frac{s+a^2}{2b_0}\right)\times I_0\left(\frac{a\sqrt{s}}{b_0}\right)da \quad (29)$$

In order to calculate the BER or the SER within the wireless link, we need to find the PDF of the signal to noise ratio (SNR) or equivalently the PDF of the ratio of symbol energy to the noise power spectral density ($\gamma=\frac{sT_s}{N_0}$). This can be done by using change of variable on (29). The change of variable we could use is:

$$f_\gamma(\gamma)d\gamma = f_S(s)ds \quad (30)$$

and so,

$$f_\gamma(\gamma) = \frac{N_0}{T_s}f_s(s) = \frac{N_0}{T_s}f_s\left(\frac{N_0}{T_s}\gamma\right) \quad (31)$$

Knowing that $\bar{\gamma}=\frac{T_s}{N_0}\bar{S}$ we have $\frac{T_s}{N_0}=\frac{\bar{\gamma}}{\bar{S}}$ and so, $f_\gamma(\gamma)=\frac{\bar{S}}{\bar{\gamma}}f_s\left(\frac{\bar{S}}{\bar{\gamma}}\gamma\right)$ which gives:

$$f_\gamma(\gamma) = \frac{\bar{S}}{2\bar{\gamma}\sqrt{2\pi d_0}b_0}\times \int_0^\infty \frac{1}{a}\exp\left[-\frac{(\ln a-\mu)^2}{2d_0}\right]\exp\left(-\frac{\frac{\bar{S}}{\bar{\gamma}}\gamma+a^2}{2b_0}\right)\times I_0\left(\frac{a}{b_0}\sqrt{\frac{\bar{S}}{\bar{\gamma}}\gamma}\right)da \quad (32)$$

Equation (32) is the probability density function of the SNR per symbol when the average SNR per symbol is $\bar{\gamma}$, the shadowing variance is $\sigma^2_{X_{dB}}$, and the average rician factor is K_r.

4 BER Calculation

The following relation [19] could be used to calculate the BER depending on the PDF of the SNR per bit:

$$P_b(E) = \int_0^\infty P_b(E;\gamma)f_\gamma(\gamma)d\gamma \quad (33)$$

For the case of the simplest PSK modulation with ideal coherent detection, we know that [19]:

$$P_b(E;\gamma) = Q\left(\sqrt{2\gamma}\right) \quad (34)$$

So the BER could be found from (32) as follows:

$$BER = \frac{\bar{S}}{2b_0\bar{\gamma}\sqrt{2\pi d_0}}\int_0^\infty Q\left(\sqrt{2\gamma}\right)\times \int_0^\infty \frac{1}{a}\exp\left[-\frac{(\ln a-\mu)^2}{2d_0}\right]\exp\left(-\frac{\bar{S}\gamma+\bar{\gamma}a^2}{2b_0\bar{\gamma}}\right)\times I_0\left(\frac{a}{b_0}\sqrt{\frac{\bar{S}\gamma}{\bar{\gamma}}}\right)da\,d\gamma \quad (35)$$

It is better to normalize the parameters within (35) in order to get a relation which is independent of the received signal power, and so independent of distance, transmitted power, bit rate and bandwidth, and to be related only with the average SNR per bit. Because of that it is better to use the following normalization:

$$a_n = a/\sqrt{\bar{S}} \quad (36)$$

$$2b_{0n} = \frac{1}{K_r+1} \quad (37)$$

$$\mu_n = \frac{1}{2}ln\left(\frac{K_r}{1+K_r}\right)-d_0 \quad (38)$$

The normalization defined by (36-37-38) keeps the statistics properties unchanged (see Appendix II).

Knowing that $\ln(a_n)-\mu_n=\ln(a)-\mu$, we can write (35) as follows:

$$BER = \frac{1}{2b_{0n}\bar{\gamma}\sqrt{2\pi d_0}}\int_0^\infty Q\left(\sqrt{2\gamma}\right)\times \int_0^\infty \frac{1}{a_n}\exp\left[-\frac{(\ln a_n-\mu_n)^2}{2d_0}\right]\exp\left(-\frac{\gamma+a_n{}^2}{2b_{0n}\bar{\gamma}}\right)\times I_0\left(\frac{a_n}{b_{0n}}\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)da_n\,d\gamma \quad (39)$$

Equation (39) is the probability of errors that could exist in the received signal at the receiving node when the used modulation is BPSK, and when the average received signal to noise ratio per bit is $\bar{\gamma}$, knowing that the shadowing variance in the communication channel between the two nodes is $\sigma^2_{X_{dB}}$ and the average rician factor is K_r. All the parameters in (39) could be calculated using (23), (25), and (27).

Actually, it is very difficult to find a closed form formula for (39), and in order to calculate it we have to use numerical solutions as the "trapz" function available in MATLAB. This is very complicated and time consuming when implemented on the UAV boards. For that, it is better if we could find an acceptable approximation to be used in order to write a closed form formula for the BER. In [18], another shadowed rician model has been proposed where the shadowing is also considered as LOS shadowing, which means that the shadowing affects only the LOS component, but it is characterized as a Nakagami-m distributed random variable. Assuming narrowband stationary model, this model uses the same low pass equivalent complex envelope of the received signal shown in (4), except that *A(t)* here follows a Nakagami-m distribution. The multipath component is characterized by the Rayleigh distribution too. It has been shown that the model in [18] provides a similar fit to the Loo's model, and it is considered as an acceptable approximation for it without losing in the characteristics of the channel model. Thus we can use this approximation in order to find a closed form formula for the BER.

The Nakagami-m model has been proposed in [40]. According to this model the amplitude of the LOS component is distributed according to Nakagami-m distribution:

$$f_A(a) = \frac{2m^m a^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{ma^2}{\Omega}\right), a \geq 0 \tag{40}$$

where $\Gamma(.)$ is the Gamma function, Ω is the average power of the LOS component, and $m = \frac{\Omega^2}{Var[S_a]} \geq 0$ is the Nakagami parameter.

In order to model the different types of LOS conditions in a variety of UAV networks channels, we will let m to change from 0 to infinity. In the traditional Nakagami model for multipath fading [19], m changes over the limited range of m≥0.5. The case of m=0, $f_A(a) = \delta(a)$, corresponds to urban areas where the LOS is totally obstructed, while the case of infinity m, $f_A(a) = \delta\left(a - \sqrt{\Omega}\right)$, corresponds to open areas with no LOS obstructions. These two extreme cases are not exist in real practical situations, thus, moderate values of m which corresponds to rural areas where the LOS component is partially obstructed, case of UAV networks, are expected.

In order to use this approximated model, we have to relate its parameters (Ω, m, b_0) with the parameters we can estimate in real time (d, K_r, $\sigma^2_{X_{dB}}$). The main problem here is the difficulty in finding a closed form formula for the variance of Ψ_{dB} whose PDF is (see appendix III):

$$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{\left(P_t \frac{m}{\Omega}\right)^m}{\zeta \Gamma(m)} \exp\left[-\frac{m}{\zeta}\Psi_{dB} - \frac{mP_t}{\zeta\Omega}\exp\left(-\frac{\Psi_{dB}}{\zeta}\right)\right] \tag{41}$$

Because of that, we will relate this approximated model parameters with those of Loo's model, where we have already relate them to the real time estimated parameters. Using the second-order matching used in [18] we get the following relations:

$$d_0 = \frac{\Psi'(m)}{4} \tag{42}$$

$$\Omega = m \exp[2\mu - \Psi(m)] \tag{43}$$

where $\Psi'(.)$ is the first derivative of the psi function $\Psi(.)$ [41].

So from the real time estimated parameters we can find the Loo's parameters using equations (23, 25, and 27), then find m numerically using (42), then find Ω using (43). Now we have to find a closed form formula for the BER. In order to do that, we have to find the PDF of the received SNR per symbol. Using equations (5) and (40), we can use the conditional mathematical expectation $\int_0^\infty f_{R|A}(r|a) f_A(a) da$, which gives the PDF of the received signal envelope:

$$f_R(r) = \frac{r}{b_0}\exp\left(-\frac{r^2}{2b_0}\right)\int_0^\infty \exp\left(-\frac{a^2}{2b_0}\right) I_0\left(\frac{ar}{b_0}\right) \times \frac{2m^m a^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{ma^2}{\Omega}\right) da \tag{44}$$

and so,

$$f_R(r) = \left(\frac{2b_0 m}{2b_0 m + \Omega}\right)^m \frac{r}{b_0}\exp\left(-\frac{r^2}{2b_0}\right) \times {}_1F_1\left(m; 1; \frac{\Omega r^2}{2b_0(2b_0 m+\Omega)}\right), for\ r \geq 0 \tag{45}$$

where ${}_1F_1(.;.;.)$ is the confluent hypergeometric function [41].

The PDF of the received power could be then determined from (45) as follows:

$$f_S(s) = \left(\frac{2b_0 m}{2b_0 m + \Omega}\right)^m \frac{1}{2b_0}\exp\left(-\frac{s}{2b_0}\right) \times {}_1F_1\left(m; 1; \frac{\Omega s}{2b_0(2b_0 m+\Omega)}\right), for\ s \geq 0 \tag{46}$$

As we have seen before, $f_\gamma(\gamma) = \frac{\bar{S}}{\bar{\gamma}} f_s\left(\frac{\bar{S}}{\bar{\gamma}}\gamma\right)$, then we can find the PDF of γ as follows

$$f_\gamma(\gamma) = \left(\frac{2b_0 m}{2b_0 m + \Omega}\right)^m \frac{\bar{S}}{2b_0\bar{\gamma}}\exp\left(-\frac{\bar{S}\gamma}{2b_0\bar{\gamma}}\right) \times {}_1F_1\left(m, 1, \frac{\Omega\bar{S}\gamma}{2b_0\bar{\gamma}(2b_0 m+\Omega)}\right) \tag{47}$$

The Moment Generating Function (MGF) could be found using the table of integrals in [41] as follows:

$$M_\gamma(s) = \frac{(2b_0 m)^m \left(1+\frac{2b_0}{\bar{S}}\bar{\gamma}s\right)^{m-1}}{\left[(2b_0 m+\Omega)\left(1+\frac{2b_0}{\bar{S}}\bar{\gamma}s\right)-\Omega\right]^m} \tag{48}$$

Based on the MGF, we can find the BER for a number of modulation schemes in uncorrelated fading channels [19]. For the ideal coherent BPSK signals the BER is:

$$BER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left(\frac{1}{\sin^2\theta}\right) d\theta \tag{49}$$

Using the Appell hypergeometric function (see Appendix IV):

$$BER = \frac{(2b_0 m)^m \left(1+\frac{2b_0}{\bar{S}}\bar{\gamma}\right)^{m-1}}{4\left(2b_0 m + \frac{2b_0}{\bar{S}}\bar{\gamma}(2b_0 m+\Omega)\right)^m} F_1\left(\frac{1}{2}, 1-m, m; 2; \frac{1}{\left(1+\frac{2b_0}{\bar{S}}\bar{\gamma}\right)}, \frac{2b_0 m}{\left(2b_0 m+\frac{2b_0}{\bar{S}}\bar{\gamma}(2b_0 m+\Omega)\right)}\right) \tag{50}$$

Using the following normalization:

$$b_{0n} = \frac{b_0}{\bar{S}} \tag{51}$$

$$\Omega_n = \frac{\Omega}{\bar{S}} \tag{52}$$

We can find that the Nakagami parameter m will keep its value, as it is the square of the ratio of two power quantities. Then, equation (50) will be independent of the average received signal power, and it can be written as follows:

$$P(E) = \frac{(2b_{0n}m)^m (1+2b_{0n}\bar{\gamma})^{m-1}}{4\left(2b_{0n}m+2b_{0n}\bar{\gamma}(2b_{0n}m+\Omega_n)\right)^m} \times F_1\left(\frac{1}{2}, 1-m, m; 2; \frac{1}{(1+2b_{0n}\bar{\gamma})}, \frac{2b_{0n}m}{\left(2b_{0n}m+2b_{0n}\bar{\gamma}(2b_{0n}m+\Omega_n)\right)}\right) \quad (53)$$

Using Appendix (V) we find that we can rewrite equation (43) using the normalization of (51) and (52) as follows:

$$\Omega_n = m\exp\left[2\mu_n - \Psi(m)\right] \quad (54)$$

Equation (42) remains unchanged as both d_0 and m are not affected by the normalization.

Equation (53) is the probability of errors that could exist in the received signal at the receiving node when the used modulation is PBSK, and when the average received signal to noise ratio per bit is $\bar{\gamma}$, knowing that the shadowing variance in the communication channel between the two nodes is $\sigma^2_{X_{dB}}$ and the average rician factor is K_r.

5 Numerical Results and Discussions

The models discussed in the previous sections predict fade distribution for the signals within the channel between any two nodes in the UAV network and include the effects of both the shadowing and the multipath rician small scale fading. System engineers need to know the fade distributions in order to make a reliable UAV network system. The dynamics of propagation will affect such system specifications such as packet length, coding, best route and others.

Table 1: Parameters values for a rural environment with average shadowing [42].

μ	-0.115	μ_{s_a}	0.8368	$\sigma_{X_{dB}}$	1.3984
b_0	0.126	m	10.14	K_r[dB]	5.2048
$\sqrt{d_0}$	0.161	Ω	0.8354		

In this section, we study the variation of the BER in the wireless link between two neighbours in UAV networks using ideal coherent BPSK with the variation of three parameters: the average SNR per bit, the shadowing standard deviation, and the average rician factor. Table 1 contains the parameter values that had been measured by Loo [42]. These parameters had been measured in a typical rural environment for average shadowing effect which corresponds to the case of the channel between two UAV network nodes. The corresponding estimated parameters are included. Fig. 3 illustrates the variation of the BER with the average SNR per bit for BPSK modulation using the values of Table 1. Two theoretical graphs are added to the figure, one for the ideal (AWGN) channel, and the other is for the non-shadowed channel (only the rician effect); these two cases are added for comparison. It is clear from this figure that for too small values of the average SNR per bit, the shadowing effect is not very important, but when we have moderate to high SNR per bit values, it begins to be more significant, and it should be taken into account during the process of judging the channel performance.

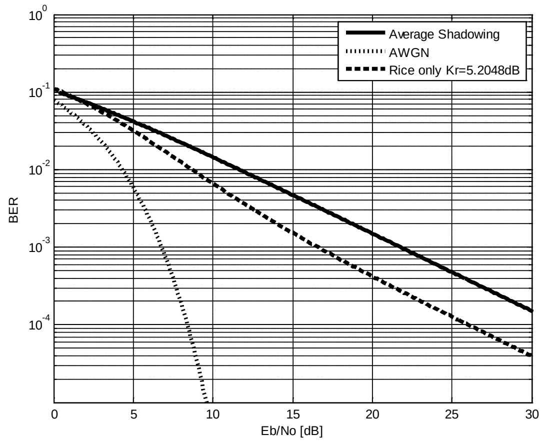

Fig. 3: Variation of the BER with the average SNR per bit.

Actually, the aim of finding the BER is at judging the link performance between a node and its neighbours; this will be used in finding the best link between different redundant links. As in most UAV networks applications the distance between nodes is almost the same, we are generally interested with the effect of shadowing and small scale fading on the BER in order to choose the best link.

Fig. 4 shows the variation of the BER with the shadowing standard deviation for different values of the rician factor and different values of the average SNR per bit. When Eb/No=30dB, which means that the noise power is so small relative to the received signal power, the shadowing has a great effect on the channel performance, where we can see that the BER will change from the order of 10^{-4} for small values of the shadowing variance to the order of 10^{-1} for high values of the shadowing variance. Even for moderate Eb/No values (between 9 and 18 dB) we can see a considerable effect of the shadowing, where the BER changes from the order of 10^{-3} to the order of 10^{-1}. In the case of small Eb/No values, which correspond to high noise powers relative to the received signal power, we can see that the shadowing effect is too small; the BER values stay in order of 10^{-1}. All the previous results are satisfied for different small scale fading effects as the graphs behaves the same when we change the rician factor value. Thus, we can deduce that during the operation of the network and during the estimation of the cost of the communication between two nodes, we can neglect the effect of shadowing if the received SNR is too small; this corresponds to noisy channels or large distances between nodes. But the shadowing effect should be taken into account within the cost estimation for moderate and high Eb/No values.

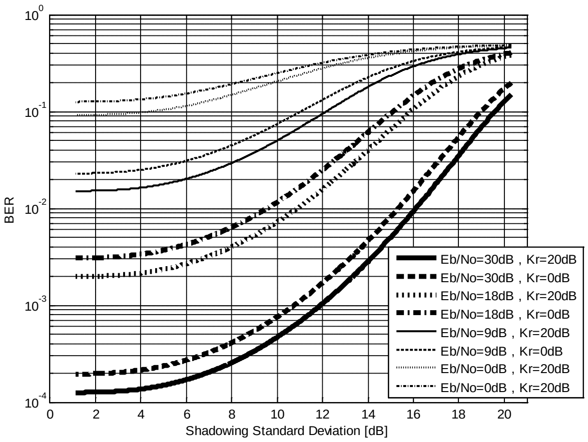

Fig. 4: BER variation with the shadowing standard deviation for different values of the rician factor and different values of the SNR per bit.

For the effect of the small scale fading which is rician in our case, we can see from Fig. 4 that its effect is negligible for high values of the shadowing standard deviation, especially for low SNR per bit values where the graphs become too close.

6 Conclusion

A statistical model for fading in UAV networks communications channels has been presented. In this model, the small scale multipath fading is considered together with the LOS lognormal shadowing. Additionally, the parameterization of this model in terms of the parameters we can estimate in real time operation will provide useful criteria in establishing the routing metric used to judge the link quality between any node and its neighbours in the network. In order to find a closed form formula for the BER, a given acceptable approximation is used. Results show that we can neglect the shadowing effect besides the rician effect when we have small SNR per bit values, and we can neglect the Rician effect in the case of high shadowing standard deviation values.

Further work is underway to enable each node to estimate unknown or changing communication environment parameters such as the path loss exponent, the shadowing standard deviation, the rician factor, and the noise density using online network measurements. These values can be then updated in the BER equation to calculate the communication cost dynamically.

Future work will include finding a general form of the wireless link communication cost function; this function should take all the channel parameters into consideration. This transmission quality function can be used in order to build the routing table at each node within the network. Adaptive transmission quality factor is to be concluded upon the possible approximations of the channel PDF.

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Appendix I: The PDF of Ψ_{dB} in LOO Model.

Using the equation: $\ln(x) = (\ln(10))\log(x)$, we can rewrite equation (28) as follows:

$$f_A(a) = \frac{1}{a\sqrt{2\pi d_0}} \exp\left(-\frac{\left((\ln 10)\log a - \mu\right)^2}{2d_0}\right)$$

$$f_A(a) = \frac{1}{a\sqrt{2\pi d_0}} \exp\left(-\frac{\left(20\log a - \frac{20\mu}{\ln 10}\right)^2}{2\left(\frac{20}{\ln 10}\right)^2 d_0}\right)$$

$$f_A(a) = \frac{2\zeta}{2\zeta a\sqrt{2\pi d_0}} \exp\left(-\frac{(20\log a - 2\zeta\mu)^2}{2\times 4\zeta^2 d_0}\right)$$

where ζ=10/ln(10). Finally:

$$f_A(a) = \frac{2\zeta}{a\sqrt{2\pi(4\zeta^2 d_0)}} \exp\left(-\frac{\left(20\log a - (2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right)$$

Taking $s_a = g(a) = a^2$, we have $a = g^{-1}(s_a) = \sqrt{s_a}$. So $\left|\frac{dg^{-1}(s_a)}{ds_a}\right| = \frac{1}{2\sqrt{s_a}}$

Depending on the relation

$$f_{S_a}(s_a) = f_A\left(g^{-1}(s_a)\right)\left|\frac{dg^{-1}(s_a)}{ds_a}\right|,$$

equation (13) can be rewritten as follows:

$$f_{S_a}(s_a) = \frac{1}{2\sqrt{s_a}} \frac{2\zeta}{\sqrt{s_a}\sqrt{2\pi(4\zeta^2 d_0)}} \exp\left(-\frac{\left(20\log\sqrt{s_a} - (2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right)$$

$$f_{S_a}(s_a) = \frac{\zeta}{s_a\sqrt{2\pi(4\zeta^2 d_0)}} \exp\left(-\frac{\left(10\log s_a - (2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right)$$

As $\Psi_{dB} = 10\log\frac{P_t}{s_a}$, then $\Psi_{dB} = \frac{10}{\ln 10}\ln\frac{P_t}{s_a}$, and so $\left|\frac{ds_a}{d\Psi_{dB}}\right| = \frac{s_a}{\zeta}$.

Knowing that $10\log s_a = 10\log P_t - \Psi_{dB}$, we can find the pdf of Ψ_{dB} from the PDF of s_a as follows:

$$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{s_a}{\zeta} \frac{\zeta}{s_a\sqrt{2\pi(4\zeta^2 d_0)}} \exp\left(-\frac{\left(10\log P_t - \Psi_{dB} - (2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right)$$

or,

$$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{1}{\sqrt{2\pi(4\zeta^2 d_0)}} \exp\left(-\frac{\left(\Psi_{dB} - (10\log P_t - 2\zeta\mu)\right)^2}{2(4\zeta^2 d_0)}\right)$$

Which is a normal distribution PDF with $\mu_{\Psi_{dB}} = 10\log P_t - 2\zeta\mu$ and $\sigma^2_{X_{dB}} = 4\zeta^2 d_0$.

Appendix II: Statistical Properties of a_n.

Applying the change of variable ($a_n = a/\sqrt{\bar{S}}$) to the PDF of (13) we find:

$$f_{A_n}(a_n) = \sqrt{\bar{S}} \frac{1}{\sqrt{\bar{S}} a_n\sqrt{2\pi d_0}} \exp\left[-\frac{\left(\ln(\sqrt{\bar{S}} a_n) - \mu\right)^2}{2d_0}\right]$$

$$f_{A_n}(a_n) = \frac{1}{a_n\sqrt{2\pi d_0}} \exp\left[-\frac{\left(\ln a_n + \ln\sqrt{\bar{S}} - \mu\right)^2}{2d_0}\right]$$

$$\mu - \ln\sqrt{\bar{S}} = \frac{1}{2}\ln\left(\mu_{S_a}\right) - d_0 - \frac{1}{2}\ln(\bar{S})$$

$$\mu - \ln\sqrt{\bar{S}} = \frac{1}{2}\ln\left(\frac{\mu_{S_a}}{\bar{S}}\right) - d_0 = \mu_n \quad \text{(equation (38))}$$

$$f_{A_n}(a_n) = \frac{1}{a_n\sqrt{2\pi d_0}} \exp\left[-\frac{\left(\ln a_n - \mu_n\right)^2}{2d_0}\right]$$

Then A_n is a lognormal distributed random variable, $\mu_n = E[\ln a_n]$, and $d_0 = Var[\ln a_n]$. Thus, we can deduce that:

$\ln\left(E[A_n^k]\right) = \mu_n k + \frac{d_0}{2}k^2$

Appendix III: The PDF of Ψ_{dB} in the Approximated Shadowed Rician Model.

The PDF of the received LOS component envelope is:

$f_A(a) = \frac{2m^m a^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{ma^2}{\Omega}\right)$

Taking Sa=a2, the received LOS component power, we have $\frac{da}{ds_a} = \frac{1}{2\sqrt{s_a}}$, and so:

$f_{S_a}(s_a) = \frac{m^m {s_a}^{m-1}}{\left(\mu_{S_a}\right)^{m}\Gamma(m)} \exp\left(-\frac{m s_a}{\mu_{S_a}}\right), s_a \geq 0$

As $\Psi_{dB} = 10\log\frac{P_t}{s_a}$, then $\Psi_{dB} = \frac{10}{\ln 10}\ln\frac{P_t}{s_a}$, and so $\left|\frac{ds_a}{d\Psi_{dB}}\right| = \frac{s_a}{\zeta}$.

Knowing that $10\log s_a = 10\log P_t - \Psi_{dB}$, we can find the pdf of Ψ_{dB} from the PDF of s_a as follows:

$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{P_t\frac{m}{\Omega}\left(P_t\frac{m}{\Omega}\exp\frac{-\Psi_{dB}}{\zeta}\right)^{m-1}}{\zeta\Gamma(m)} \exp\left[-\frac{mP_t}{\zeta\Omega}\exp\left(-\frac{\Psi_{dB}}{\zeta}\right)\right]\exp\left(-\frac{\Psi_{dB}}{\zeta}\right)$

or,

$f_{\Psi_{dB}}(\Psi_{dB}) = \frac{\left(P_t\frac{m}{\Omega}\right)^m}{\zeta\Gamma(m)}\exp\left[-\frac{m}{\zeta}\Psi_{dB} - \frac{mP_t}{\zeta\Omega}\exp\left(-\frac{\Psi_{dB}}{\zeta}\right)\right]$

Appendix IV: Prove of Equation (50)

$BER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left(\frac{1}{\sin^2\theta}\right)d\theta$

$BER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(2b_0m)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}\sin^2\theta}\right)^{m-1}}{\left[(2b_0m+\Omega)\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}\sin^2\theta}\right)-\Omega\right]^m}d\theta$

Using the change of variable: $t = \cos^2\theta$, and so, $\sin^2\theta = 1-t$.

$dt = -2\cos\theta\sin\theta\, d\theta \Rightarrow d\theta = -\frac{dt}{2\sqrt{t}\sqrt{1-t}}$.

$\theta\in[0,\pi/2] \Rightarrow t\in[0,1]$, but as θ increases t decreases, and so we have to multiply the integral by (-1):

$BER = \frac{1}{\pi}\int_0^1 \frac{(2b_0m)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}(1-t)}\right)^{m-1}}{\left[(2b_0m+\Omega)\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}(1-t)}\right)-\Omega\right]^m}\frac{dt}{2\sqrt{t}\sqrt{1-t}}$

$BER = \frac{(2b_0m)^m}{2\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}\frac{(1-t)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}(1-t)}\right)^{m-1}}{(1-t)^m\left[(2b_0m+\Omega)\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}(1-t)}\right)-\Omega\right]^m}dt$

$BER = \frac{(2b_0m)^m}{2\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}\frac{(1-t)\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{\left[(1-t)(2b_0m+\Omega)\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}(1-t)}\right)-\Omega(1-t)\right]^m}dt$

$BER = \frac{(2b_0m)^m}{2\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{1}{2}}\frac{\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{\left[(2b_0m+\Omega)\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)-\Omega(1-t)\right]^m}dt$

$BER = \frac{(2b_0m)^m}{2\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{1}{2}}\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}\left[(2b_0m+\Omega)\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)-\Omega(1-t)\right]^{-m}dt$

Denoting: $B = 1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}$, we have:

$B = \left(1+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)\left(1-\frac{1}{1+\frac{2b_0\bar{\gamma}}{\bar{S}}}t\right)$

Denoting:

$C = (2b_0m+\Omega)\left(1-t+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)-\Omega(1-t)$,

we have:

$C = 2b_0m+\Omega-(2b_0m+\Omega)t+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)-\Omega+\Omega t$

$C = 2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)-2b_0mt$

$C = \left[2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)\right]\left(1-\frac{2b_0m}{2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)}t\right)$

Then the BER equation could be written as follows:

$BER = \frac{(2b_0m)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{2\pi\left[2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)\right]^m}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{1}{2}}\left(1-\frac{1}{1+\frac{2b_0\bar{\gamma}}{\bar{S}}}t\right)^{m-1}\left(1-\frac{2b_0m}{2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)}t\right)^{-m}dt$

Denoting: $a = \frac{1}{2} > 0$, $b_1 = 1-m$, $b_2 = m$, $c = 2 > a$, $x_1 = \frac{1}{1+\frac{2b_0\bar{\gamma}}{\bar{S}}}$, and $x_2 = \frac{2b_0m}{2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)}$, we have:

$BER = \frac{(2b_0m)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{2\pi\left[2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)\right]^m}\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-x_1t)^{-b_1}(1-x_2t)^{-b_2}dt$

Knowing that:

$F_1(a,b_1,b_2;c;x_1,x_2) = \frac{1}{B(a,c-a)}\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-x_1t)^{-b_1}(1-x_2t)^{-b_2}dt$

where $B(.,.)$ is the Beta function, the BER equation could written as follows:

$BER = \frac{(2b_0m)^m\left(1+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{2\pi\left[2b_0m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0m+\Omega)\right]^m}B(a,c-a)F_1(a,b_1,b_2;c;x_1,x_2)$

Now, by definition: $B(a,c-a) = B\left(\frac{1}{2},\frac{3}{2}\right) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2})}{\Gamma(2)} = \frac{\sqrt{\pi}\frac{\sqrt{\pi}}{2}}{1} = \frac{\pi}{2}$. Thus the BER becomes:

$$BER = \frac{(2b_0 m)^m \left(1+\frac{2b_0\bar{\gamma}}{\bar{S}}\right)^{m-1}}{4\left[2b_0 m+\frac{2b_0\bar{\gamma}}{\bar{S}}(2b_0 m+\Omega)\right]^m} F_1 \left(\frac{1}{2}, 1-m, m; 2; \frac{1}{\left(1+\frac{2b_0}{\bar{S}}\bar{\gamma}\right)}, \frac{2b_0 m}{\left(2b_0 m+\frac{2b_0}{\bar{S}}\bar{\gamma}(2b_0 m+\Omega)\right)}\right)$$

Appendix V: Prove of Equation (54)

Applying the change of variable $a_n = a/\sqrt{\bar{S}}$ to the PDF of (40), using $\frac{da}{da_n} = \sqrt{\bar{S}}$, we find:

$$f_{A_n}(a_n) = \sqrt{\bar{S}} \frac{2m^m \left(a_n\sqrt{\bar{S}}\right)^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\left(a_n\sqrt{\bar{S}}\right)^2}{\Omega}\right)$$

$$f_{A_n}(a_n) = \bar{S}^{2m-\frac{1}{2}+\frac{1}{2}} \frac{2m^m {a_n}^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m {a_n}^2}{\Omega}\bar{S}\right)$$

$$f_{A_n}(a_n) = \frac{2m^m {a_n}^{2m-1}}{\left(\frac{\Omega}{\bar{S}}\right)^m \Gamma(m)} \exp\left(-\frac{m {a_n}^2}{\left(\frac{\Omega}{\bar{S}}\right)}\right)$$

$$f_{A_n}(a_n) = \frac{2m^m {a_n}^{2m-1}}{\left(\frac{\Omega}{\bar{S}}\right)^m \Gamma(m)} \exp\left(-\frac{m {a_n}^2}{\left(\frac{\Omega}{\bar{S}}\right)}\right)$$

Using the normalization of (52) we could write:

$$f_{A_n}(a_n) = \frac{2m^m a_n^{2m-1}}{\Omega_n^m \Gamma(m)} \exp\left(-\frac{m a_n^2}{\Omega_n^m}\right)$$

As it is a Nakagami-m distribution we can deduce that:

$$\ln\left(E[A_n^k]\right) = \frac{1}{2}\left[\ln\left(\frac{\Omega_n}{m}\right) + \Psi(m)\right] k + \frac{\Psi'(m)}{8} k^2 + \frac{\Psi''(m)}{48} k^3 + \cdots \quad \text{(V-1)}$$

Using (Appendix II) we have:

$$\ln\left(E[A_n^k]\right) = \mu_n k + \frac{d_0}{2} k^2 \quad \text{(V-2)}$$

The absolute values of the psi function and its derivatives converge to zero very fast as m increases [41].

Using the second order matching for the equations (V-1) and (V-2), we find:

$$\mu_n = \frac{1}{2}\left[\ln\left(\frac{\Omega_n}{m}\right) + \Psi(m)\right] \quad \text{(V-3)}$$

$$d_0 = \frac{\Psi'(m)}{4}$$

Thus, we can find Ω_n from equation (V-3) as follows:

$$\Omega_n = m \exp\left[2\mu_n - \Psi(m)\right]$$