Reduction of PAPR for SC-FDMA System based on DCT and Hyperbolic Tangent Companding

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Abstract: - SC-FDMA suffers from high Peak-to-Average Power Ratio (PAPR) problem, which restricts its usage in some applications. In this paper, a joint Discrete Cosine Transform (DCT) and hyperbolic tangent companding (HTC) technique has been studied in order to minimize the effects of the peak-to-average power ratio of the SC-FDMA System. The proposed scheme utilizes a DCT rather than the conventional discrete Fourier transform (DFT) as a basis function to implement the single-carrier system. Simulation results show that the new proposed scheme reduces PAPR by about 4.2 dB and 2.6 dB companding respectively with the DCT SC-FDMA without commanding and DFT SC-FDMA with HTC at the same companding parameter. Also, it can be observed that proposed scheme provides a significant BER performance improvement over the DFT SC-FDMA with HTC by about 1.1dB. Simulation results show that, proposed scheme with HTC has the lowest BER degradation, followed by μ-law companding and A law companding by about 1.1dB, 3.1dB and 3.8dB respectively when compared with the original DCT SC-FDMA system without PAPR reduction.

Key-Words: - OFDMA, PAPR, SC-FDMA, DCT, Companding, BER.

1 Introduction

Future communication systems need to provide high data rate services to satisfy the increasing requirements of the next-generation wireless systems. Orthogonal frequency division multiplexing (OFDM) has received a lot of attention in the last few years. Despite the significant advantages offered by OFDM, it has the major inherited drawbacks of fluctuating envelope with high peaks, which lead to high Peak-to-Average Power Ratio (PAPR) and the sensitivity to carrier frequency offsets for the transmitted signal [1-2]. Due to the high PAPR, a linear high-power amplifier is required at the transmitter, however, is not efficiently used. High power consumption, in-band distortion, and spectrum spreading will appeared when an OFDM passes through a nonlinear power amplifier If the linear range of the high-power amplifier is not sufficiently wide [3]. A lot of attention has been directed recently to other broadband wireless communication systems to solve the problems faced in the uplink of OFDM, namely the single carrier with frequency domain equalization (SC-FDE) system [4-5] and the SC-FDMA system [6, 7].

The great advantage of the SC-FDMA technique is its low PAPR, flexible sub-carrier frequency allocation donated by OFDMA and multipath resistance [8]. However, the third generation long-term evaluation standard utilizes OFDMA in the downlink and SC-FDMA in the uplink [9]. There are various methods available to allocate the subcarriers in SC-FDMA systems [10]. The localized FDMA (LFDMA) and the interleaved FDMA (IFDMA) are the two commonly used methods. Users are selected distributed subcarriers over the entire frequency band and exhibits a low PAPR at the expense of a high sensitivity to frequency offsets and phase noise in the IFDMA mode. But in the LFDMA mode, the output data are allocated to consecutive subcarriers and it is more robust to multiple access interference, but it suffers a higher PAPR than the IFDMA mode [11].

There are different PAPR reduction techniques in OFDM systems have been proposed in literature [12–13], such as clipping and filtering [14], companding transforms [15–17], selective mapping [18-19], partial transmit sequence [20], tone injection, tone reservation and linear block coding [20]. In most techniques, PAPR is reduced at the expense of increasing the BER, complexity or data overhead. Companding techniques attenuate the high peaks and amplify the low amplitudes, thus decrease the PAPR of the signal before power amplifier [21-22]. Linear symmetrical transform (LST), linear asymmetrical transform (LAST) and nonlinear companding transform (NLCT) are the three classes of Companding transform [23]. However, it increases the BER due to the distortion increased by the modulating symbols at the transmitter and the expansion of the channel's noise by the decompander at the receiver [23].

In the literature, the PAPR Reduction in SC-FDMA systems with a hyperbolic companding technique is not studied. The objective of this paper is to introduce a new scheme to reduce the PAPR in SC-FDMA systems based upon joint the DCT and a hyperbolic companding technique without degrading the BER system performance. The proposed scheme uses a DCT and IDCT rather than DFT or IDFT, to implement the SC-FDMA system. To achieve this target, we try to optimize the performance of the DCT SC-FDMA with companding system using the minimum BER. The best value of the companding coefficient is chosen and determined through simulations. The performance of the DCT SC-FDMA with companding system is studied and compared with the system without companding.

This paper is organized as follows: In section II, we describe the proposed DCT SC-FDMA system model based on nonlinear companding techniques. In Section III, presents simulation results and show that the proposed scheme outperforms the conventional SC-FDMA schemes. Finally, section IV summarizes and concludes the work presented in this paper. Throughout the paper, Vectors and matrices are symbolized by boldface. Complex conjugate transposition of a matrix, and the inverse of a matrix,are written in $(.)^{H}$ and $(.)^{-1}$, respectively.

2 The proposed DCT SC-FDMA system

In this section, the proposed DCT SC-FDMA with hyperbolic tangent companding system is introduced. The proposed DCT SC-FDMA is similar to DFT SC-FDMA except that it uses DCT and IDCT instead of DFT or IDFT. The transmitter and the receiver parts of the proposed system will be discussed in the following subsections. Frequency selective channels are considered.

2.1 Description of the proposed system

The DCT is a Fourier-related transform that perform the basic criterion of DFT operation but using only real algorithm so, the system can be more stable. A set of trigonometric functions in the form of $\cos(2\pi m F_s t)$ where m = 0, 1,….., M - 1 and 0 < t < T are used. The minimum subcarriers spacing F_s required to attain the orthogonality condition, is 1/2T. This condition can be defined by:

$$\int_0^T \sqrt{\frac{2}{T}}\cos(2\pi b F_s\, t)\sqrt{\frac{2}{T}}\cos(2\pi m F_s\, t)dt = \begin{cases} 1, & b = m \\ 0, & b \neq m \end{cases} \qquad (1)$$

Figure 1. Describe the transceiver block diagram of the proposed DCT SC-FDMA system. At the transmitter, a block of N data symbols is mapped to multilevel symbols in from a certain modulation method, such as the QPSK, then followed by an N points DCT. The subcarrier mapping block assigns the DCT outputs into M≥N subcarrier that can be transmitted, and inserts zeros into any unused subcarriers. The signal after the DCT can be expressed as follows:

$$X_b = \sqrt{\frac{2}{T}}\,\beta_b \sum_{n=0}^{N-1} x_n \cos\left(\frac{\pi b(2n+1)}{2N}\right), \quad b = 0,1,2,\ldots,N-1 \qquad (2)$$

Where x_n is the modulated data symbols, and can be expressed as [24]:

$$\beta_b = \begin{cases} \frac{1}{\sqrt{2}} & b = 0 \\ 1 & b = 1,2,\ldots\ldots,N-1 \end{cases} \qquad (3)$$

After that, the resulting signal is applied to the IDCT, the signal can be expressed as follows [25]:

$$\bar{x}_m = \sqrt{\frac{2}{M}} \sum_{p=0}^{M-1} \bar{X}_p\ \beta_p \cos\left(\frac{\pi p(2m+1)}{2M}\right) \qquad (4)$$

Where $\bar{X}_p$ is the signal after the subcarriers mapping and M is the IDCT length i.e., number of subcarriers (M=B.N). B is the bandwidth expansion factor of the symbol sequence if all the user terminals transmit N symbols per block, the system can handle B simultaneous transmissions without co-channel interference. Finally, companding and a cyclic prefix (CP) appended at the head of IDCT output and the resulting signal is transmitted through the wireless channel via HPA. The high PAPR requires a large back-off which reduces the power efficiency of the amplifier.

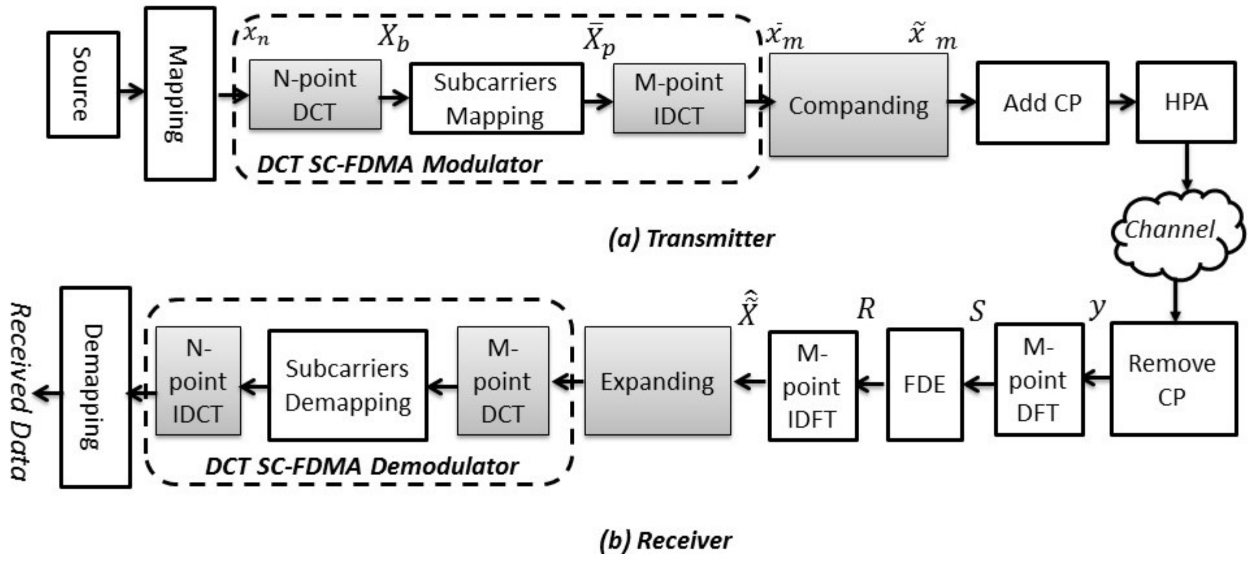

Fig. 1 Structure of the proposed DCT SC-FDMA system with HTC technique over a Vehicular A channels

The PAPR without pulse shaping can be expressed as follows [26]:

$$\text{PAPR (dB)} = 10log_{10}\frac{\max(|\bar{x}_m|^2)}{\frac{1}{M}\sum_{m=0}^{M-1}|\bar{x}_m|^2} \quad (5)$$

we assume fading channel, where the path gains give constant over the duration of each block. Adding a CP of length Ncp is very important in frequency selective channel. The length of the CP selected to be greater than the maximum excess delay of the channel to overcome the inter-block interference (IBI). In this paper, At the receiver, the CP is removed from the received signal, and it can be represented as follows:

$$y = H\bar{x} + n \quad (6)$$

Where $\bar{x}$ is an M×1 vector representing the block of the transmitted samples. y is an M×1 vector of the received samples. n is an M × 1 vector describing the additive noise. H is an M × M circulant matrix describing the multipath channel. The circulant matrix H can be efficiently diagonalized by the DFT and the IDFT. After that, the received signal is transformed into frequency domain as follows:

$$S = \Lambda\bar{X} + N \quad (7)$$

Where Λ is an M × M diagonal matrix containing the DFT of the circulant sequence of H. S, $\bar{X}$ and N are the Fourier transforms of y, $\bar{x}$ and n, respectively. We must perform an FDE in order to detect the modulated symbols, The FDE operator E can be derived according to the minimum mean square (MMSE) criterion as follow:

$$E = (\Lambda^H\Lambda + 1/SNR)^{-1}\Lambda^H \quad (8)$$

Where SNR is the signal to noise ratio. Finally, an M-point IDFT, decopanding and the DCT SC-FDMA demodulation are performed, as shown in Fig.3. The main disadvantage of the DCT SC-FDMA system with respect to the DFT SC-FDMA system is the need for an FDE at the receiver before the demapping process. The fast DCT algorithm proposed in [27] can achieve fewer computational steps than DFT. However, the complexity of the FDE is of O(M) for the DCT SCFDMA system and of O(N) for the DFT SC-FDMA system. This improved that the complexity of the transmitter in the DCT SC-FDMA system is lower than that in the DFT SCFDMA system.

2.2 Hyperbolic tangent (tanh) companding

We propose the use of the hyperbolic tangent companding function with DCT SC-FDMA system to reduce the magnitude of the signal peaks at the transmitter. The hyperbolic tangent (tanh) companding function with two positive parameters k_1 and k_2 is defined by [28]

$$C(\bar{x}_m) = k_1\tanh(k_2\bar{x}_m) \quad (9)$$

Where k_1 and k_2 control the extent of companding applied to the signal $\bar{x}_m$. Figure 2 shows $C(\bar{x}_m) = k_1\tanh(k_2\bar{x}_m)$ versus $\bar{x}_m$ for different values of $\mathrm{k_1}$ and $\mathrm{k_2}$. We will then use the inverse hyperbolic tangent decompanding function at the receiver to recover the original signal.

$$tanh^{-1}\bar{x}_m = a \tanh \bar{x}_m = \frac{1}{2}[\ln(1+\bar{x}_m) - \ln(1-\bar{x}_m)] \quad (10)$$

Note that since the inverse hyperbolic tangent is only real when $\bar{x}_m < 1$, the receiver can be expression as [29]:

$$\tanh^{-1}\bar{x}_m = \frac{1}{2}\ln\frac{1+\bar{x}_m}{1-\bar{x}_m} \quad (11)$$

The best choice of $\mathrm{k_1}$ and $\mathrm{k_2}$ should map the dynamic range of $\bar{x}_m$ into the nonlinear region of the curve. The derivative of C $(\bar{x}_m)$ is given by [30]

$$\mathrm{C}'(\bar{x}_m) = \mathrm{k_1 k_2}\,[1 - \tanh^2\ (\mathrm{k_2}\,\bar{x}_m)] \quad (12)$$

Given that $[1 - \tanh^2\ (\mathrm{k_2}\,\bar{x}_m)] \leq 1$, for all signals $\bar{x}_m$, a sufficient condition to have $\mathrm{C}'(\bar{x}_m) < 1$ is $\mathrm{k_1\ k_2} \leq 1$. In [31], $\mathrm{k_1}$ and $\mathrm{k_2}$ were chosen such that $\mathrm{k_1} = 1/\mathrm{k_2}$, or $\mathrm{k_1\ k_2} = 1$. For values of $\mathrm{k_1}$ and $\mathrm{k_2}$ such that $\mathrm{k_1 k_2} < 1$ the proposed scheme will perform better than that in [31].

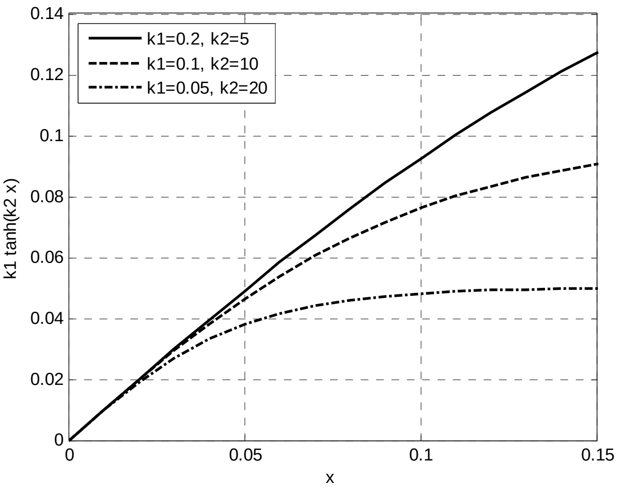

Fig. 2 The hyperbolic tangent companding function $\mathrm{k_1} \tanh (\mathrm{k_2}\,\bar{x}_m)$ versus the signal after the IDFT $\bar{x}_m$ for different values of $\mathrm{k_1}$ and $\mathrm{k_2}$.

3 Results and discussion

In this section, the performance of the proposed scheme is evaluated in terms of the reduction of the PAPR and BER performance through numerical simulations. The simulation parameters are tabulated in Table 1. PAPR statics are shown in terms of CCDF (Complementary Cumulative Distribution Function) which gives the probability that PAPR is greater than a certain PAPR value PAPR0 (Pr{PAPR>PAPR0}).

Table 1 Simulation parameters

Parameter	Description
System bandwidth	5 MHz
Modulation type	QPSK
Cyclic prefix length	20 samples
Transmitter IDCT length (M)	512 symbols
Subcarriers spacing	9.765625 kHz for M=512
SC-FDMA input block length (N)	128 symbols
Subcarriers mapping mode	Localized
SC-FDMA input DCT length (N)	128 symbols
Companding scheme	hyperbolic tangent (tanh) Companding
Channel model	Vehicular A channels
Channel estimation	Perfect
Equalization	MMSE

A solid state power amplifier (SSPA) model is used to model the HPA which produces amplitude conversion without phase distortion and its output can be expressed as follows [15, 23]:

$$x_{out} = \frac{x_{in}}{\left[1 + \left(\frac{x_{in}}{A_{sat}}\right)^{2g}\right]^{1/2g}} \quad (13)$$

Where respectively, the input and the output signals of the amplifier are represented as $\mathrm{x_{in}}$ and $\mathrm{x_{out}}$, g is a positive parameter controlling the nonlinearity level of the power amplifier and A_{sat} is a normalization factor specifying the saturation level of the amplifier. In all simulations, we set $g = 2$ and $A_{sat} = 0.2$.

Figures 3 show the CCDFs of the PAPR for the proposed DCT SC-FDMA with the hyperbolic tangent companding (HTC) system for QPSK modulation formats with M=512, N=128 and localized subcarriers mapping. Clearly, the PAPR performance of the proposed system, with various values of the companding parameters $\mathrm{k_1}$ and $\mathrm{k_2}$, is better than that of the system without companding. For the hyperbolic tangent function, the smaller values of companding parameters $\mathrm{k_1}$ and larger value $\mathrm{k_2}$, the smaller are the values of the PAPR. As a result, the proposed scheme provides a gain of 6.8dB for the companding parameters $\mathrm{k_1} = 0.2$ and $\mathrm{k_2} = 5$, 4.7dB for $\mathrm{k_1} = 0.4$ and $\mathrm{k_2} = 2.5$ and 2.9dB for $\mathrm{k_1} = 0.6$ and $\mathrm{k_2} = 1.6$ at given CCDF=10-4 compared with DCT SC-FDMA without companding .

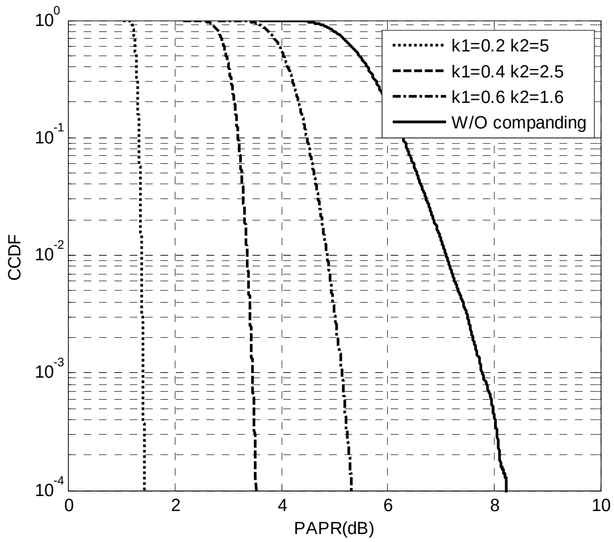

Fig. 3 CCDF of PAPR for the DCT SC-FDMA with HTC for different values of companding parameters k_1 and k_2

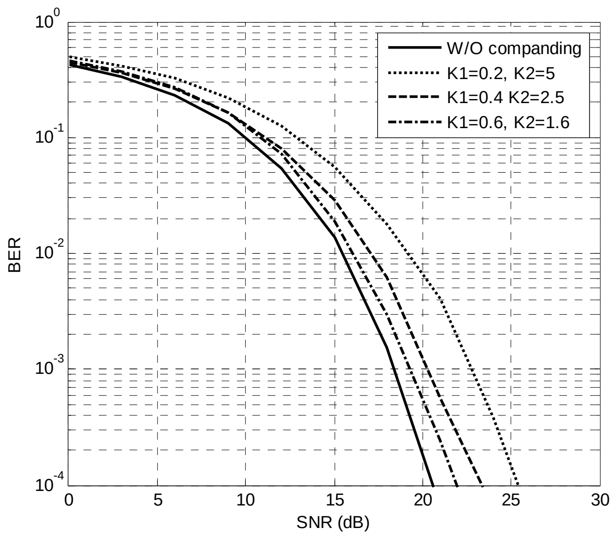

Fig.4 BER for DCT SC-FDMA with HTC for a different values of companding parameters k_1 and k_2 over a Vehicular A channels.

Figures 4 introduces BER performance comparison for the proposed DCT SC-FDMA with HTC for a different values of companding parameters k_1 and k_2 over a frequency selective channel. Simulation results show that, the proposed DCT SC-FDMA with the HTC exhibits degradation in BER performance about 4.8dB for the companding parameters $k_1 = 0.2$ and $k_2 = 5$, 2.7dB for $k_1 = 0.4$ and $k_2 = 1.5$ and 2.9dB for $k_1 = 0.6$ and $k_2 = 1.6$ compared with the original DCT SC-FDMA system without PAPR reduction.

Figures 3 and 4 indicate that, there is a tradeoff between the PAPR performance and BER performance of the proposed scheme with different values of companding parameters k_1 and k_2. The smaller values of companding parameters k_1 and larger value k_2 , the smaller are the values of the PAPR and the largest values of BER. As a result, increasing the amplitude compression will introduce both out-of-band and in-band interference, which leads to degradations in the BER performance. This implies that a companding coefficient should be chosen carefully in order to limit the PAPR without degrading the system performance.

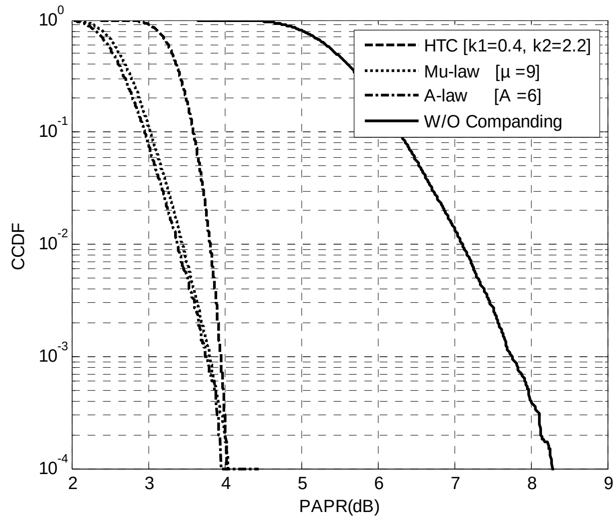

Fig.5 CCDF performance comparison for different companding functions.

Figure 5 shows that the PAPR performance improvement of the DCT SCFDMA with companding system is about 4.4 dB, when compared with that of the system without companding and the complexity is maintained low when μ is set to 9 for the μ -law companding, A is set to 6 for the A -law companding, we use k1 = 0.4 and k2 = 2.2 for the hyperbolic tangent function. It is worth mentioning that the purpose of Fig. 6 is not to compare between different companding functions in terms of PAPR reduction capability, but to show that the proposed schemes achieve a better BER without changing the CCDF performance.

Figure 6 shows the BER performance for the conventional and proposed schemes using the three companding functions in Vehicular A channels. Simulation results show that, hyperbolic tangent companding has the lowest BER degradation, followed by μ-law companding and A-law companding by about 1.1dB, 3.1dB and 3.8dB respectively when compared with the original DCT SC-FDMA system without PAPR reduction. As a result, the proposed scheme with hyperbolic tangent companding provides again of 1.5dB and 2.2dB respectively when compared with μ-law companding and A law companding. We observe that, depending on the companding parameters may have a higher or lower BER compared to conventional scheme. So there is a trade-off between the PAPR performance and BER performance of the proposed DCT SCFDMA with companding system.

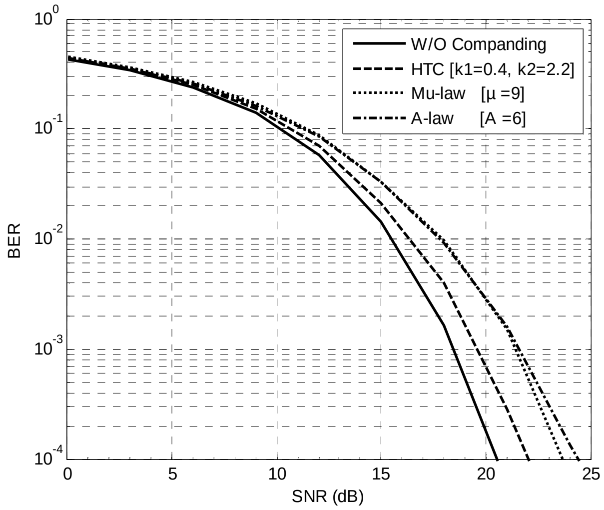

Fig.6 BER performance comparison for different companding functions over Vehicular A channels.

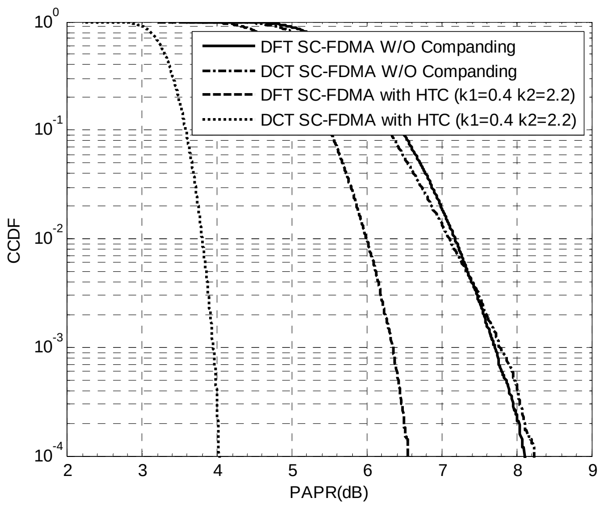

Fig. 7 Comparison of CCDF of PAPR for the proposed scheme of DCT SC-FDMA with HTC, DFT SC-FDMA with HTC, DCT SC-FDMA and DFT SC-FDMA without companding.

In figure 7, plots of CCDF of the PAPR for proposed scheme of DCT SC-FDMA with HTC, DFT SC-FDMA with HTC, DCT SC-FDMA and DFT SC-FDMA without companding are shown with QPSK modulation format. It is clearly seen that, the proposed scheme of DCT SC-FDMA with HTC has lower PAPR than the DCT SC-FDMA without companding by about 4.2 dB and DFT SC-FDMA without companding by about 4.1 dB. It is also noted that, improvements in the PAPR performance can be achieved using the proposed scheme with companding parameter of k1=0.4 and k2=2.2 by about 2.6 dB when compared with DFT SC-FDMA with HTC at the same companding parameter.

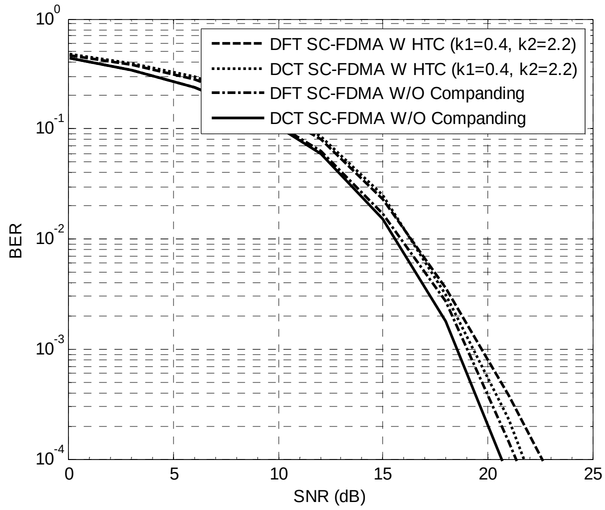

Fig.8 BER for the proposed DCT SC-FDMA with HTC, SC-FDMA with HTC and SC-FDMA without companding.

Finally, figure 8 illustrates the BER performance of the proposed scheme of DCT SC-FDMA with HTC for the companding parameters $k_1 = 0.2$ and $k_2 = 2.2$, DFT SC-FDMA with HTC at the same companding parameters, DCT SC-FDMA and DFT SC-FDMA without companding but over the vehicular A channel. At a BER=10-4, the degradation in BER performance is about 1.1dB, 2.2dB for DCT SC-FDMA with HTC and DFT SC-FDMA with HTC respectively, when compared to that of the DCT SC-FDMA without companding and about 0.3dB, 1.3dB when compared to that of the DFT SC-FDMA without companding. Also, it can be observed that proposed scheme provides a significant BER performance improvement over the DFT SC-FDMA with HTC by about 1.1dB.

4 Conclusion

In this paper, a DCT SC-FDMA system combined with Hyperbolic Tangent Companding is introduced. The effectiveness of the proposed technique is investigated through simulations. Clearly, the PAPR performance of the proposed system, with various values of the companding parameters, is better than that of the system without companding. As a result, a reduction in the PAPR can be achieved by changing the value of the companding parameters k_1 and k_2 for the DCT SC-FDMA with hyperbolic tangent companding. It is found that the proposed scheme must be designed carefully in order to achieve a reduction in the PAPR without degrading the BER performance. On the other hand, a trade-off should be made between the PAPR performance and the BER performance since increasing the value of the companding coefficient leads to a reduction in the PAPR and a degradation in the BER performance. BER performance comparison for the conventional and proposed schemes using the three companding functions in multipath fading show that the proposed scheme with hyperbolic tangent companding has the lowest BER, followed by μ-law companding, A law companding at the same PAPR when μ is set to 9 for the μ -law companding, A is set to 6 for the A -law companding, we use $k_1 = 0.4$ and $k_2 = 2.2$ for the hyperbolic tangent function. It is also noted that, improvements in the PAPR performance can be achieved using the proposed scheme with companding parameter of k_1=0.4 and k_2=2.2 by about 2.6 dB when compared with DFT SC-FDMA with HTC at the same companding parameter and has lower PAPR than the DCT SC-FDMA without companding by about 4.2 dB and DFT SC-FDMA without companding by about 4.1 dB. It can be observed that proposed scheme provides a less degradation compared to DCT SC-FDMA with HTC.

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