A Low Complexity Individual Process Fix-LLL Algorithm for MIMO System

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Abstract: - The Lenstra-Lenstra-Lovász (LLL) algorithm is an effective lattice reduction method in multi-input-multi-output (MIMO) systems. Its modified version Fix-LLL (F-LLL)unilaterally aims to largely decrease the computation complexity in size reduction, but size reduction in LLL algorithm is closely associated with column swap procedure. This characteristic is harmful to independently design an optimum proposal for these two algorithmic parts. In this paper, a novel individual process F-LLL algorithm divides the overall process of LLL algorithm into two individual procedures. The column swap procedure of the novel algorithm is modified by searching for the potential column swaps through the whole basis. Until no basis is selected, the procedure terminates and global size reduction is achieved. We mainly design two schemes for selecting potential column swaps: offset coefficient selection criterion (OCSC) and maximum slope selection criterion (MSSC). Simulation results show both OCSC-F-LLL and MSSC-F-LLL can reduce computation complexity and performance loss compared to the LLL algorithm.
Key-Words: - LLL algorithm; lattice reduction; individual process; MIMO system.

1 Introduction

Multi-input multi-output (MIMO) systems have been adopted in several 3G and 4G standards. Owing to its high spectral efficiency and large coverage, MIMO will still be the key technology in future 5G standards [1].In MIMO communication systems lattice reduction (LR) represents a main stream of decoding techniques [2]-[3]. Now LR plays a crucial role in developing different computationally efficient algorithms and achieves reasonably perfect performance [4].In 1982, the Lenstra-Lenstra-Lovász (LLL) algorithm, the most practical and commonly-used LR algorithm, was introduced [5] .The LLL algorithm is featured by polynomial complexity with respect to dimension n and several LLL-aided detectors can collect full diversity like the maximum likelihood (ML) decoding. The performance gap between LLL and ML decoding has been identified by a proximity factor method [6]. A complex LLL (CLLL) algorithm straightforwardly performs the LR with a complex-valued matrix [7]. CLLL expands the definition of basis reduction to a complex field and thus further reduces (nearly a half) the algorithmic complexity. Simulation results show CLLL requires fewer arithmetic operations than LLL and also satisfies the performance of several combinations of linear equalizers [8]-[9].

However, the pursuit of optimum performance results in the extremely high whole complexity of LLL algorithm. Moreover, the LLL algorithm is featured by polynomial complexity with respect to the dimension n, which may not be strong enough for combination of a few equalizers [10]. The LLL algorithm consists of a column swap procedure and a size reduction procedure. The basis can do column swap only after the Lovász condition is satisfied. Plenty research aims at changing the Lovász condition to largely improving the arithmetic operating speed. The effective LLL (E-LLL) algorithm loosely imposes an ascending order on diagonal elements of channel matrix [11]. E-LLL is a weaker version of LLL since it has a provable complexity bound $O\left(n^3 \log n\right)$, which is one order lower than $O\left(n^4 \log n\right)$ of LLL algorithm [12]. Furthermore, an even weaker criterion called diagonal reduction [13] only imposes one single constraint on diagonal elements. The diagonal reduction algorithm, when combined with the successive interference cancelation (SIC) decoding, has identical performance as the LLL algorithm.

Meanwhile, besides modifying conditions of size reduction and column swap procedure, algorithm redesigning is also a research focus. A possible swap LLL algorithm (PSLLL) for lattice reduction is modified by searching for the next column swap through the whole basis instead of the sequential procedure in the original LLL algorithm [14]. This complex-value-based scheme absolutely changes the whole process of LLL. Other schemes combined with LLL algorithm, such as greedy column traverse strategy [15] and fast-Givens rotation scheme [16], still hold the same designation idea. Some other details of the MIMO technology and lattice reduction based on the LLL algorithm can be seen in Refs. [17]-[19].

Based on the statistical law, a novel F-LLL algorithm [20] tries to modify the convergence criterion of μ which is strongly related to size reduction and column swap procedure. F-LLL applies a fix method to substitute the original round method in LLL such that the whole algorithm will have higher probability of skipping two steps of reduction. However, no compromise between complexity and performance is satisfied. Usually there is a big performance loss compared to original LLL, especially when the number of iterations and the size of the MIMO system increase. Also F-LLL is not stable enough.

In this paper, we put forward a series of individual process version of F-LLL algorithms to make a better trade-off between complexity and algorithmic performance. In original LLL, size reduction and column swap procedure are closely related. A proper optimization scheme in size reduction may not work best for the subsequent column swap procedure. Accordingly, the main idea of individual process version of F-LLL algorithms is to divide the overall process of LLL algorithm into two individual procedures. The reduction process of the novel algorithm is modified by searching for the potential column swaps through the whole basis. The basis which satisfies a searching criterion is recorded as the coordinate basis. Then we mainly discuss how to design such a searching criterion. Random select (RS) means to randomly choose a basis from the coordinate basis set. This scheme sometimes may not be guaranteed as the optimal. We design two criteria. First, offset coefficient selection criterion (OCSC) is based on choosing the largest deviation from all the coordinate pairs. The other criterion, maximum slope selection criterion (MSSC), receives the best trade-off. It is similar to get the maximum slope that indicates the fast decline direction. After an iteration of column swap, we continue searching for the potential column swap until there is no proper basis satisfying the searching criterion. When no basis is selected, column swap terminates and global size reduction is applied to basis.

The rest of paper is organized as follows. Section Ⅱ presents the universally-acknowledged MIMO system model and gives a brief introduction to the LLL algorithm. Section Ⅲ provides details regarding F-LLL and novel individual processes versions, and lists ideas of the different designations of searching criterion and theoretical statements .Section Ⅳ demonstrates our simulation results and Section Ⅴ shows the conclusions.

Notation: The real and imaginary parts of x are denoted as $\Re(x)$ and $\Im(x)$ respectively. The inner product in the complex Euclidean space between vectors u and v is defined as $\langle u,v\rangle = u^H v$ and the Euclidean length is $\|u\| = \sqrt{\langle u,u\rangle}$ in $\Re^n$ .The arbitrary integer closest to x is denoted as $\lceil \bullet \rfloor$. The transpose, Hermitian transpose, and inverse of a matrix H are defined as H^T , H^H and H^{-1} respectively. Expectation of H is represented by $E(H)$ with variance σ^2 . A large O notation $f(x) = O(g(x))$ means that for sufficiently large x , $f(x)$ is bounded by a constant time $g(x)$ in absolute value.

2 MIMO System Model and LLL Algorithm

2.1 MIMO System Model

Here we use an $N \times M$ channel matrix to denote a MIMO system with M transmit antennas and N receive antennas. It is assumed that the transmitted signal at the m-th transmit antenna is x_m , and data received at the n-th receive antenna is y_m ($x_m \in \mathbb{Z} + j\mathbb{Z}$, $y_n \in \mathbb{Z} + j\mathbb{Z}$). A common signal alphabet $\mathbb{S}$ is used for all x_m . Over the MIMO channel, the received signal vector is represented as follows:

$$y = Hx \quad (1)$$

Matrix H consists of $M \times N$ independent and identically-distributed ($i.i.d$) complex Gaussian coefficients with zero mean and unit variance. Note that n is assumed to be an $i.i.d$ complex Gaussian vector with unit variance, $E[nn^H] = 2\sigma^2 I$.

2.2 LLL Algorithm

The LLL algorithm is proposed to find a matrix with nearly orthogonal column vectors so as to generate the same lattice. With the LLL algorithm, the lattice reduction can be performed for the M-basis MIMO system with the $N \times M$ channel matrix. We concentrate on real-valued matrix for lattice reduction in MIMO system.

Definition 1 (LLL Reduction [18]): A basis $H \in \mathbb{C}^{m \times n}$ is an LLL reduction process with $\delta(\frac{1}{4} < \delta < 1)$, and the upper triangular factor $R \triangleq [r_{i,j}]$ in its QR decomposition $H_r = Q_r \times R_r$ satisfies the following inequalities:

$$\left|[R_r]_{l,k}\right| \le \frac{1}{2}\left|[R_r]_{l,l}\right| \quad , \quad 1 \le l < k \le 2M \tag{2}$$

$$\delta [R_r]^2_{k-1,k-1} \le [R_r]^2_{k,k} + [R_r]^2_{k-1,k} \ , \ k = 2,...,2M \tag{3}$$

Where $[R_r]_{l,k}$ denotes the $(l,k)th$ entry of R_r. Ineq. (3) is the Lovász condition, and basis will do the column swap procedure only when Ineq. (3) is violated. δ is a real number arbitrarily chosen from $(\frac{1}{4},1)$ while $\delta = \frac{3}{4}$ is widely shared as meeting a good complexity-quality trade-off coefficient. At last, the LLL algorithm generates a LLL-reduced matrix from the real-valued channel matrix H_r.

3 Individual Processes Version of Fix-LLL Algorithm

3.1 Fix-LLL algorithm

Detailed procedure of F-LLL algorithm is showed in Table 1.

The complexity of LLL or F-LLL algorithm is mainly located at size reduction (line 5-11 in Table 1) and column swap procedure (line 12-20). Parameter l ranges from $(1, k-1)$ (line 5) which is called global size reduction.

In F-LLL algorithm, we introduce a novel criterion to substitute the round method. The fix method rounds the elements to the nearest integers towards zero. When the value of $R_r(k-l,k)/R_r(k-l,k-l)$ is within the interval $(-1,+1)$, the value of μ will be forced to zero.

$$\mu = fix(R_r(k-l,k)/R_r(k-l,k-l)) \tag{4}$$

If μ has larger probability of converging to zero, there is higher probability of directly skipping the size reduction. Consequently, a multiple of computation complexity in size reduction will be saved (line 8-9).

Table 1 Process of Novel F-LLL Algorithm based on QR decomposition

Input: Qr，Rr，δ Output: The F-E-LLL-reduced basis	
No.	Algorithm process
1	initialize $T_r \leftarrow I_n$
2	$n \leftarrow size(H_r, 2)$
3	$k \leftarrow 2$
4	while k ≤ n
5	for $l = 1 : k-1$
6	$\mu \leftarrow fix(R_r(k-l,k)/R_r(k-l,k-l))$
7	if $\mu \ne 0$
8	$R_r(1:k-l,k) \leftarrow R_r(1:k-l,k) - \mu \times R_r(1:k-l,k-l)$
9	$T_r(:,k) \leftarrow T_r(:,k) - \mu \times T_r(:,k-l)$
10	end if
11	end for
12	if $\delta R_r^2(k-1,k-1) > R_r^2(k-1,k) + R_r^2(k,k)$
13	swap columns $k-1$ and k in R_r and T_r
14	find a Givens rotation G to restore the upper triangular structure of R
15	$R_r(k-1:k, k-1:n) \leftarrow GR_r(k-1:k, k-1:n)$
16	$Q_r(:,k-1:k) \leftarrow Q_r(:,k-1:k)G^H$
17	$k \leftarrow \max(k-1,2)$
18	else
19	$k \leftarrow k+1$
20	end if
21	end while

We should point out that in the size reduction procedure, μ is used to adjust the elements in matrices R_r and T_r. In the subsequent judge of Lovász condition (line12), only terms $R_r(k-1,k-1)$, $R_r(k,k)$ and $R_r(k-1,k)$ are needed. During size reduction (line 7-10), $R_r(k-1,k-1)$ and $R_r(k,k)$ remain unchanged but $R_r(k-1,k)$ will be modified. As an example when $l = 1$ is showed below:

$$\mu = fix(R_r(k-l,k)/R_r(k-l,k-l)) \tag{5}$$

In a word, size reduction is closely related to the subsequent column swap procedure. Changes of any key parameter will influence the column swap procedure. Accordingly, based on the original LLL algorithm, although F-LLL changes the criterion of size reduction, the whole process of column swap is influenced. In other words, complexity reduction in size reduction does not mean complexity reduction in column swap. The results usually depend on the channel matrix H used. However, the simulation results show that if an algorithm has higher probability of skipping size reduction, it will possess lower probability of doing column swap. But there is no clear tendency of complexity reduction between size reduction and column swap.

3.2 Individual Process Version of Fix-LLL Algorithm

Here we introduce a novel individual process version of F-LLL algorithm which is designed with a real-valued matrix. The main idea of this novel algorithm is to divide the overall process of LLL into two independent procedures. The reduction process is modified by searching for potential column swaps through the whole basis (Line 3-7 in Table 2). Each potential column swaps corresponding to a value of k. The entire potential column swaps are recorded as coordinates (line 8).We use a random selection scheme (RS) to randomly select a value of k in the corresponding column swap (line 12-20 in Table 1). After the swap is accomplished, the basis is updated. We should re-search the entire basis to find if there is any column not satisfying the Lovász condition. We do the procedure: search, select and swap repeatedly until all the values of k brought into Lovász condition (line 4) is not violated, we finish the independent column swap procedure (line 3-7).

Table 2 Individual Processes Version of Fix-LLL Algorithm

Input: H_r ,δ Output: Q_r, R_r, T_r	
No.	Algorithm process
1	$T_r \leftarrow I_n$,$[Q_r, R_r] = QR(H_r)$
2	$n \leftarrow size(H_r, 2), k \leftarrow 2$
3	for $k = 2:1:n$
4	if $\delta R_r^{\,2}(k-1,k-1) > R_r^2(k-1,k) + R_r^{\,2}(k,k)$
5	Record all these basis which isn't satisfied with Lovász condition
6	end if
7	end for
8	Randomly select a value of k and do column swap procedure
9	Repeat line 3 to line 8 until all the values of k are satisfy the Lovász condition
10	Global size reduction procedure based on Fix method
11	Obtain matrix Q_r, R_r, T_r

Line 10 in Table 2 leads in global size reduction which is equivalent to line 5-10 in Table 1.When we apply the fix method into global size reduction, only size reduction is affected. This global size reduction is taken to achieve a fully-reduced basis for linear detection, such as zero forcing (ZF) and minimum mean-square error (MMSE).

We should point out that the random select (RS) scheme is improper for selection of a basis. RS may not obtain a promising trade-off between performance and computation complexity.

3.3 Offset Coefficient Selection Criterion

OCSC is introduced in this section .In Lovász condition $\delta R_r^{\,2}(k-1,k-1) > R_r^2(k-1,k) + R_r^{\,2}(k,k)$, A and B represent the left and right sides respectively:

$$A = \delta R_r^2(k-1,k-1) \tag{6}$$

$$B = R_r^2(k-1,k) + R_r^2(k,k) \tag{7}$$

The absolute tolerance between A and B is defined as ς :

$$\varsigma = A - B \tag{8}$$

By searching all values of k, we select the minimum value of ς, marked as $\varsigma_{\min} = (A-B)_{\min}$. The offset coefficient in the Least Mean Square (LMS) algorithm is defined by measuring the rate of deviation:

$$M = \frac{\varsigma - \varsigma_{\min}}{\varsigma_{\min}} = \frac{\varsigma}{\varsigma_{\min}} - 1 \tag{9}$$

The basis having the largest rate of deviation needs to do column swap first. Finding the largest rate of deviation is equal to finding the largest value of ς. We search all the candidate column swap pairs to find out the largest absolute tolerance: $\max(A-B)$.

We will give the detailed proof below. First we introduce the definition of LLL potential:

Definition 2 (LLL Potential [5]):

$$D = \prod_{i=1}^{N_t-1} D_i = \prod_{i=1}^{N_t-1} \left| R_r(i,i) \right|^{2(N_t - i)}$$

(10)

Where $D_i = \det L_i = \left\| R_r(1,1) \right\|^2 \left\| R_r(2,2) \right\|^2 \cdots \left\| R_r(i,i) \right\|^2$ and L_i is defined as the sub-lattice spanned by $R_r(1,1), R_r(2,2), \cdots R_r(i,i)$.

The size reduction won't change the value of D because size reduction won't change the value of diagonal elements in R_r .The value of D only changes during the column swap procedure. During column swap, LLL potential is strictly decreased. Also after finite iteration LLL algorithm will terminate. Our goal is to design a fast convergence criterion which equals to make the potential decreases fastest. If LLL iteration happens at index k, diagonal elements are all updated that $\tilde{R}_r(k,k)$ and $\tilde{R}_r(k-1,k-1)$ .So the updated potential can describe as:

$$\tilde{D} = \prod_{\substack{i=1 \\ i \neq k-1,k}}^{N_t - 1} \left| \tilde{R}_{i,i} \right|^{2(N_t - 1)} \left| \tilde{R}_{k-1,k-1} \right|^{2(N_t - k + 1)} \left| \tilde{R}_{k,k} \right|^{2(N_t - k)} \quad (11)$$

After computation and simplification, $\tilde{D}$ can be represented by original diagonal elements before LLL iteration and potential D :

$$\tilde{D} = \frac{\left| R_r(k-1,k) \right|^2 + \left| R_r(k,k) \right|^2}{\left| R_r(k-1,k-1) \right|^2} D \quad (12)$$

At each round LLL iteration, we want to select the fastest decline of LLL potential. Fastest decline is defined by difference between D and $\tilde{D}$:

$$decline = D - \tilde{D} \quad (13)$$

$$decline = \left(1 - \frac{\left| R_r(k-1,k) \right|^2 + \left| R_r(k,k) \right|^2}{\left| R_r(k-1,k-1) \right|^2} \right) D \quad (14)$$

$$decline = \left(\frac{\left| R_r(k-1,k-1) \right|^2 - \left(\left| R_r(k-1,k) \right|^2 + \left| R_r(k,k) \right|^2 \right)}{\left| R_r(k-1,k-1) \right|^2} \right) D \quad (15)$$

From the definition in (8), $\varsigma = \max(A - B)$ that:

$$\varsigma = \delta R_r^{\,2}(k-1,k-1) - \left(R_r^{\,2}(k-1,k) + R_r^{\,2}(k,k) \right) \quad (16)$$

In real field, $R_r^{\,2}(k,k) = \left| R_r(k,k) \right|^2$ .When $\delta = 1$, that parameter ς equals to the numerator in equation (15):

$$decline = \left(\frac{\varsigma}{\left| R_r(k-1,k-1) \right|^2} \right) D \quad (17)$$

For a certain value of $\left| R_r(k-1,k-1) \right|^2$, by searching the maximum of ς it will get the largest value of $decline$. So it will cause the fastest decline in LLL potential. But selecting $\max(A-B)$ doesn't mean selecting maximum value of A. Only if the $\left| R_r(k-1,k-1) \right|^2$ remains unchanged, searching $\max(A-B)$ will lead to a fast decline in LLL potential. But sometimes we can't guarantee that in each iteration of LLL algorithm terms $\left| R_r(k-1,k-1) \right|^2$ remains the same. So the effect of OCSC will be limited and may not be the optimal sometimes. The detailed proof has been completed.

However, when the system size is small, such as a 4×4 MIMO system, usually not too many pairs are available for the scheme to select. Consequently, the actual effect of OCSC may be unrealized. Usually in this small MIMO system, the performance of OCSC is very likely to be identical with RS scheme. Increasing the size of the MIMO system will improve the effect of OCSC. Simulation details will be showed in section Ⅴ.

3.4 Maximum Slope Selection Criterion

The other criterion called MSSC also modifies the RS scheme. Still the definitions of A, B and ς in section C are used:

$$A = \delta R_r^2(k-1,k-1)$$

$$B = R_r^2(k-1,k) + R_r^2(k,k)$$

$$\varsigma = A - B \quad (18)$$

We still first calculate ς . This time $S(Slope) = \frac{\varsigma}{B}$ is used to measure the proportion of ς occupying $R_r^2(k-1,k) + R_r^2(k,k)$.

$$S(Slope) = \frac{\varsigma}{B} = \frac{\delta R_r^2(k-1,k-1) - R_r^2(k-1,k) + R_r^2(k,k)}{R_r^2(k-1,k) + R_r^2(k,k)} \quad (19)$$

If we get larger S , there is a bigger gap between A and B. Consequently, this pair needs to be modified first. In general, determining S in MSSC is similar to getting the maximum curve slope, which suggests the largest difference between A and B .In a word, OCSC aims at the largest ς and MSSC aims at the largest $\frac{\varsigma}{B}$. From the angle of measurement error, OCSC is absolute tolerance and MSSC is relative tolerance.

Here we will give the detailed proof of efficiency of maximum slope selection criterion. Still follows the flow of proof above:

$$decline = \left(\frac{\left|R_r(k-1,k-1)\right|^2 - \left(\left|R_r(k-1,k)\right|^2 + \left|R_r(k,k)\right|^2\right)}{\left|R_r(k-1,k-1)\right|^2} \right) D \quad (20)$$

From equation (18):

$$\varsigma = S_{\max} \times \left(R_r^2(k-1,k) + R_r^2(k,k) \right) \quad (21)$$

$$decline = \left(\frac{S_{\max} \times \left(\left|R_r(k-1,k)\right|^2 + \left|R_r(k,k)\right|^2\right)}{\left|R_r(k-1,k-1)\right|^2} \right) D \quad (22)$$

When $\delta = 1$, in equation A:

$$A = R_r^2(k-1,k-1) \quad (23)$$

In real field, $R_r^2(k,k) = \left|R_r(k,k)\right|^2$ .Compared to equation (12), terms equation (21) can be substituted:

$$\frac{\tilde{D}}{D} = \frac{\left|R_r(k-1,k)\right|^2 + \left|R_r(k,k)\right|^2}{\left|R_r(k-1,k-1)\right|^2} \quad (24)$$

$$decline = \left(\frac{S_{\max} \times \tilde{D}}{D} \right) D \quad (25)$$

$$decline = S_{\max} \times \tilde{D} \quad (26)$$

Equation in (23) means that $\tilde{D}$ is the value of potential before LLL iteration. This may be a fixed value. The only various factor that influences the term *decline* is parameter S which is defined in (18). When we search the largest $S_{\max}$, this will cause the fast decline of the potential of LLL. The full proof has been completed.

In a word, OCSC aims at largest ς and MSSC aims at largest ratio $\frac{\varsigma}{B}$. From the angle of measurement error, OCSC is absolute tolerance and MSSC is relative tolerance.

4 Simulation Results

We use computer simulations to verify the theoretical claims on F-LLL and its individual process versions. Channel matrix H and white Gaussian noise n are randomly generated .Constellation mapping is settled with 16QAM.The system consists of 6 transmit antennas and 6 receive antennas (6 × 6).Symbol size is 10000.SNR is defined as symbol energy per transmit antenna versus noise power spectral density. We separately use coding gain to measure the performance of each algorithm at a fixed symbol error rate (SER) and float operations (flops) as an evaluation criterion of computation complexity.

4.1 Average Number of Iterations and Computational Complexity

Average number of iterations is measured by the number of times doing size reduction and column swap. The computation complexity is calculated as the number of flops in real field. The results are listed in Table 3.

ML algorithm and ZF-SIC are also simulated for comparison. Since the RS scheme is based on random selection from potential column swaps, we simulate this scheme twice to see that we have the probability of not achieving a fixed result each time. Another point out is that the search of potential column swaps in individual process version of F-LLL requires additional flops. This extra procedure is demonstrated in line 5 in Table 2.

Table 3 Comparison of different algorithms in both iteration and complexity

Number of iteration	LLL	F-LLL	RS-1-F-LLL	RS-2-F-LLL	OCSC-F-LLL	MSSC-F-LLL
Size reduction	**35**	**6**	**22**	**22**	**23**	**22**
Column swap	**29**	**39**	**29**	**28**	**29**	**24**
Total	**64**	**45**	**51**	**50**	**52**	**46**
Computational complexity	LLL	F-LLL	RS-1-F-LLL	RS-2-F-LLL	OCSC-F-LLL	MSSC-F-LLL
Flops of Size reduction	**1886**	**940**	**844**	**838**	**886**	**842**
Flops of column	**2404**	**1892**	**2392**	**2252**	**2356**	**2040**

swap						
Flops of other generations	**0**	**0**	**330**	**319**	**330**	**275**
Total	**4290**	**2832**	**3566**	**3409**	**3572**	**3157**
Flops save to LLL	**0%**	**33.97%**	**16.88%**	**20.54%**	**16.74%**	**26.41%**

As showed in Table 3, LLL is the optimal algorithm and F-LLL terminates with the fewest flops. LLL owns the largest computation complexity. MSSC-F-LLL terminates with the fewest flops among all the individual process version of F-LLL algorithms. Since the random scheme is randomly choosing a pair to do column swap, each algorithm run has probability of achieving uncertain results. OCSC-F-LLL is inferior to MSSC-F-LLL in complexity.

4.2 Simulation results of performance

We simulate the performance of LLL algorithm and the individual process version of F-LLL algorithms in constellation16QAM. Each algorithm uses linear SIC as an aided algorithm. The ML algorithm and ZF-SIC are also included for comparison.

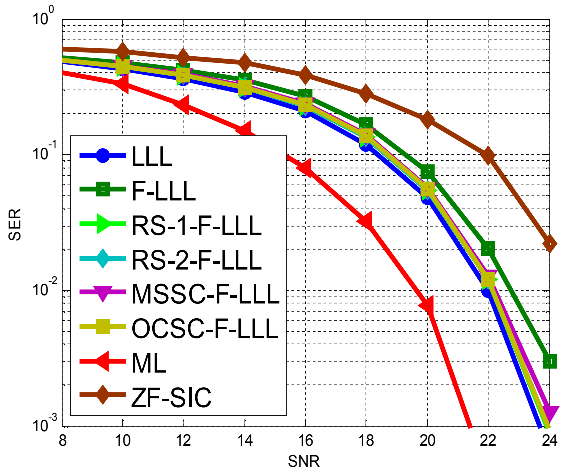

Fig 1 Performance of LLL and family of individual processes version of F-LLL detectors in a 16QAM modulated 6×6 MIMO system

The analysis is combined with iteration and complexity results in *Table 3*, as follows:

Except ML, LLL owns the best performance and highest computation complexity in simulation. Although F-LLL is featured by the lowest flops, it has low stability and performance. F-LLL has a 1.185dB performance gap compared with LLL. The individual process version of F-LLL algorithms try to separate size reduction and column swap apart. On the facet of designing searching scheme, RS-F-LLL algorithm is featured by random selection in coordinate pairs of basis for subsequent column swap procedure. This plan will outperform F-LLL but is limited by uncertain results because the algorithm may not select the same coordinate pair every time.

OCSC-F-LLL and MSSC-F-LLL algorithms reduce 16.74% and 26.41% of flops, respectively, and their performance losses are 0.29dB and 0.51dB, respectively, compared with LLL algorithm. These two searching schemes may enjoy a better trade-off between performance and computational complexity. They can modify the shortcomings of instability and large loss of performance in F-LLL.

Figure 2 shows the complexity comparisons of different algorithms from 2×2 to 8×8 MIMO systems.

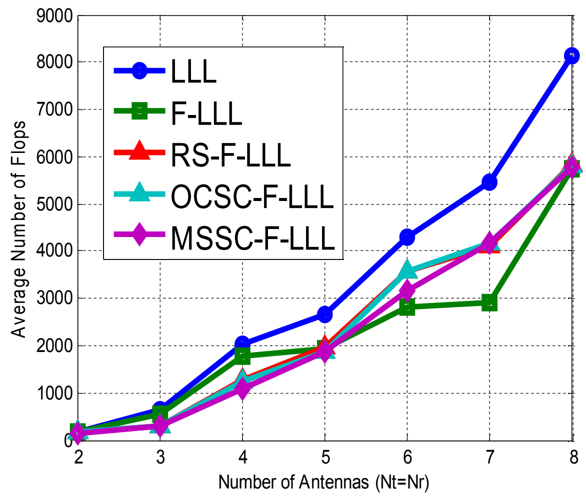

Fig 2 Complexity comparisons of different LLL algorithms from 2×2 to 8×8 MIMO systems

From 2×2 to 8×8 MIMO systems, LLL pursues the best SER performance as well as the highest computational complexity. Although F-LLL often results in larger complexity reduction, its stability is unsatisfactory. In a special case with the 4×4 MIMO system, F-LLL shows the highest computation complexity among all the individual process version of F-LLL algorithms. In general, MSSC-F-LLL owns the lowest complexity and with the increasing size of MIMO systems, OCSC and MSSC gradually exhibit their superiority.

Performance of MSSC-F-LLL from 2×2 to 8×8 MIMO systems is shown in Fig.3. The overall trend of performance variation is that with the system size increasing, that the whole performance of system that combined with MSSC is getting worse. But since the channel matrix in different size of MIMO system is generated randomly, so the detail analysis of the performance variation is needed further research and beyond the scope of this paper.

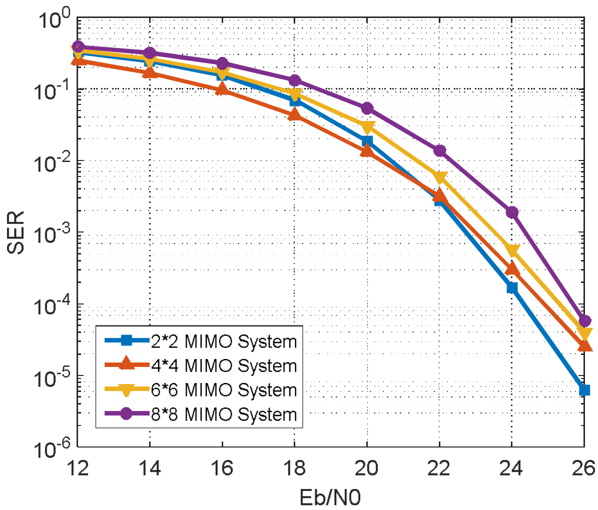

Fig 3 Performance comparisons of MSSC-F-LLL algorithm from 2×2 to 8×8 MIMO systems

5 Conclusions

Families of modified-version F-LLL algorithms are proposed. F-LLL algorithm directly changes the criterion of size reduction and the whole algorithm has higher probability of skipping the size reduction, at the cost of large computation complexity though. F-LLL usually leads to large performance loss especially when the system size or the number of iterations increases. Also F-LLL shows low stability and immeasurable performance.

Consequently, we introduce a new idea of individual process version of F-LLL algorithms. This idea is to treat size reduction and column swap as two individual procedures, so we can optimize them separately. Column swap is first done that all the potential swaps are selected and remain to be chosen for subsequent swap procedure. How to design the selection scheme is mainly discussed. The simplest method is the RS scheme, as it randomly chooses one pair from the coordinate set. However, the algorithm based on the random scheme may have probability of receiving different results every time. OCSC chooses the pair with the largest rate of deviation, which is equal to find the largest difference between $\delta R_r^2(k-1,k-1)$ and $R_r^2(k-1,k)+R_r^2(k,k)$ .MSSC is equivalent to acquiring the largest slope $\frac{\varsigma}{B}$, which means the fastest decline direction and may cause the fast convergence. Simulation results manifest that both OCSC and MSSC can make a better trade-off between performance and algorithm complexity.

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