Sparse Multipath Channel Estimation for SC-FDE System with Unknown Sparsity

XIAOLING LI, YULANG ZHENG, YONG BAI*
College of Information Science and Technology,
Hainan University,
58 Renmin Ave., Haikou, Hainan 570228,
CHINA
* Corresponding author, email: bai@hainu.edu.cn

Abstract: - The performance of Single Carrier Frequency-Domain Equalization (SC-FDE) system is affected by the precision of channel estimation results. For a pilot-assisted SC-FDE system transmitting over sparse multipath radio channels, we discuss that the sparse multipath channel estimation can be formulated to be an underdetermined compressed sensing (CS) problem or an overdetermined sparse system identification problem under different transmission parameters. For the sparse system identification problem setting, we propose to use Zardoff-Chu sequence as the pilot sequence to form a deterministic circulant Toeplitz observation matrix for signal recovery. To address the unknown sparsity in practical applications, Sparsity Adaptive Matching Pursuit (SAMP) algorithm is investigated to reconstruct the channel impulse response (CIR) instead of other greedy sparse recovery algorithms that need a priori knowledge of channel sparsity. The simulation results demonstrate that using the designed observation matrix and the SAMP algorithm for sparse channel estimation with unknown sparsity achieves better performance than the traditional Least Squares (LS) channel estimation algorithm with reduced length of pilot sequence in the SC-FDE system over 3GPP radio channels.

Key-Words: - SC-FDE, Channel estimation, Compressed sensing, Orthogonal Matching Pursuit, Sparsity Adaptive Matching Pursuit

1 Introduction

In wireless communications, the transmitted signals normally go through multipath channel before reaching the receiver. The time dispersion of different paths often causes serious Inter-Symbol Interference (ISI) to the received signals for broadband wireless transmission. The Single Carrier Frequency-Domain Equalization (SC-FDE) system has been shown to be an attractive transmission technique to combat the ISI [1]. Compared to Orthogonal Frequency Division Multiplexing (OFDM) system, SC-FDE system has similar performance and signal processing complexity. Furthermore, but it has the advantages of lower peak-to-average power ratio (PAPR) and less sensitivity to carrier frequency offsets by employing frequency-domain equalization. The performance of frequency-domain equalization in a SC-FDE system is affected by the precision of channel estimation results [2]. Hence, a precise channel estimation is an important requirement for the SC-FDE system.

There have been some related research works on channel estimation for SC-FDE system [3-5]. In [3], the complementary Golay sequences are used for time-domain channel estimation. In [4], the frequency domain multiplexed pilots are used in the channel estimation. In [5], a lower bound for the MSE of the linear minimum mean-squared error (LMMSE) channel estimator is derived. Most of the works are studied under rich multipath channels. Nevertheless, the physical arguments and growing experimental evidence suggest that many wireless channels encountered in practice tend to exhibit a sparse multipath structure, where only a few channel paths are significant and other channel coefficients are zero or close to zero. When employing traditional channel estimation methods for sparse channel scenarios, it leads to a large number of training sequences which results in loss of energy and bandwidth. Inspired by the success of compressed sensing (CS, also known as compressive sensing) in signal processing [6], where sparse solution of underdetermined linear equations can be accurately and efficiently obtained from relatively fewer number of linear non-adaptive measurements, many sparse channel estimation methods based on CS have been proposed in recent years to exploit the sparse channel structure in SC-

FDE system [7, 8]. In [7], a sparse channel estimation method with sparsity predetermined by wavelet decomposition is proposed. In [8], the performance between channel estimation based on CS and time-domain least squares (*LS*) are compared.

In the pilot-assisted SC-FDE transmission system, the CS problem setting for sparse channel estimation can be formulated when the pilot length is less than the length of the longest channel path. When the pilot length is longer than the channel taps, the sparse channel estimation can be formulated as a sparse system identification problem. The formulated problem becomes an overdetermined (more equations than unknowns) problem instead of an underdetermined (more unknowns than equations) problem based on CS. For the SC-FDE transmission over mobile radio channel specified by 3GPP channels, it is a more normal setting when the transmit data rate is not too high. Under this overdetermined problem setting, the observation matrix can be designed in a deterministic approach and can be solved with simplified algorithms. We propose to use Zardoff-Chu sequence as the pilot sequence to form a deterministic circulant Toeplitz observation matrix for the sparse system identification problem setting.

Next, we evaluate the sparse recovery algorithms for the sparse system identification problem setting. One popular class of sparse recovery algorithms is based on the idea of iterative greedy pursuit, including Matching Pursuit (MP) [9], and Orthogonal Matching Pursuit (OMP) [10], Stagewise OMP (StOMP) [11], Regularized OMP (ROMP) [12], Compressive Sampling Matching Pursuit (CoSaMP) [13]. However, these algorithms assume that the sparsity K is known, whereas K may not be available in many practical applications, such as channel estimation. Do et. al. proposed an iterative greedy reconstruction algorithm, Sparsity Adaptive Matching Pursuit (SAMP) [14]. Compared with other greedy algorithms, the SAMP has the capability of signal reconstruction without prior information of the sparsity. This makes it a promising candidate for many practical applications when the number of significant coefficients of a signal is not available. Extensive experiment results confirm that SAMP is very appropriate for reconstructing compressible sparse signal where its magnitudes are decayed rapidly.

In this paper, we mainly investigate the sparse system identification setting for the sparse channel estimation problem by simulation. A simulation comparison is made for the *LS*, OMP, and SAMP channel estimation algorithms under the sparse system setting with proposed observation matrix. Both NMSE (Normalized Mean Square Error) of channel impulse response estimation and BER (Bit Error Rate) of the SC-FDE transmission are analyzed and compared. The simulation results show that both OMP and SAMP algorithms perform better than the LS algorithm over 3GPP multipath channels even with reduced pilot length. The SAMP algorithm performs slightly lower than OMP with advantage of adaptive spasity.

The rest of this paper is organized as follows. In Section 2, the system model of pilot-assisted SC-FDE transmission system is introduced. Section 3 discusses two different formulations of the sparse channel estimation problem, i.e., the underdetermined CS setting and the overdetermined sparse system identification setting, under different transmission parameters. For the sparse system identification setting, a deterministic circulant Toeplitz matrix is proposed to be the observation matrix by using Zardoff-Chu sequence as the pilot sequence. The traditional LS algorithm, and the sparse recovery algorithms (OMP and SAMP) are presented in Section 4 for multipath channel estimation. The simulation results and discussions are given in Section 5. Section 6 concludes this paper.

2 System Description

The block diagram of SC-FDE transmission system is shown in Fig.1, and the frame structure of the SC-FDE transmission is shown in Fig. 2.

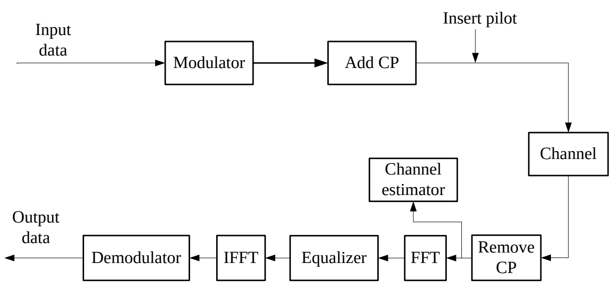

Fig. 1 SC-FDE transmission system

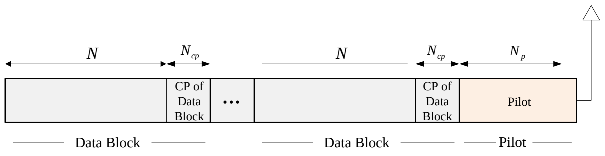

Fig. 2 Frame structure of SC-FDE transmission

At the transmitter, the information bits are mapped into data symbols depending on the modulation type. Then each length-N data symbols appended with a length-N_{cp} CP (Cyclic Prefix) forms a data block. The CP is assumed to be longer than the known channel impulse response (CIR) length L to eliminate inter-block interference (IBI). The pilots are periodically inserted among the transmitted data blocks for synchronization and channel estimation purposes. The pilot is assumed to be inserted in the time domain with length N_p. Then, the SC-FDE blocks is transmitted though the mobile radio channel. A multipath channel with additive white Gaussian noise is considered in this paper. The receiver first removes the CP of the received signals, and utilizes the pilot for channel estimation. Then the equalization in the frequency domain is applied on the received signals. Finally, the equalized signals are demodulated to recover the original data.

At the transmitter, the time-domain samples of one SC-FDE signal can be written as

$$x(n) \qquad 0 \le n \le N-1 .$$

Let the channel impulse response (CIR) be $\boldsymbol{h} = [h(1), h(2)\cdots, h(L)]$, where L is the length of the CIR. When only K ($K<L$) channel paths are significant and other channel coefficients are zero or close to zero, the channel can be called a K -sparse channel.

The baseband CIR can be described as

$$h(t) = \sum_{l=0}^{L-1} \alpha_l(t)\delta(\tau - \tau_l) \qquad (1)$$

where τ_l is the delay of the l th path and $\alpha_l(t)$ is the channel coefficient of the l th path. It can be viewed as a LTI (Linear Time Invariant) or LTV (Linear Time Varying) system impulse response depending on whether the channel coefficients are time invariant or nor. Actually, the baseband CIR specified in (1) is a tapped-delay-line model, where the tap spacing is *1/W*, where W is the signal bandwidth. Each significant path corresponds to a significant tap at some multiples of *1/W*. If the RMS delay spread of the channel is τ_{RMS} , the longest significant tap (corresponding to the Lth path) is close to $\tau_{RMS}W$.

Assume that the CP length N_{cp} is larger than L ($N_{cp}>L$), the baseband model of the SC-FDE system can be written as

$$y(n) = \sum_{l=0}^{L-1} x(n-l)h(l) + z(n) \quad (0 \le n \le N-1) \qquad (2)$$

where $x(n)$ is the input signal, $y(n)$ is the received signal, $z(n)$ is the AWGN samples with zero mean and variance of σ_w^2 .

The resulting input-output relation of data blocks can be expressed as a matrix-vector product

$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{z}$ which represents

$$\begin{pmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N+L-3) \\ y(N+L-2) \end{pmatrix}_{(N+L-1)\times 1} = \begin{pmatrix} x(0) & \cdots & 0 \\ x(1) & \ddots & \vdots \\ \vdots & \ddots & x(0) \\ x(N-1) & \ddots & x(1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x(N-1) \end{pmatrix}_{(N+L-1)\times L} \begin{pmatrix} h(0) \\ h(1) \\ h(2) \\ \vdots \\ h(L-1) \end{pmatrix}_{L\times 1} + \begin{pmatrix} z(0) \\ z(1) \\ z(2) \\ \vdots \\ z(N+L-2) \end{pmatrix}_{N+L-1} \qquad (3)$$

3 Sparse Channel Estimation Problem Formulation

In the pilot-assisted SC-FDE transmission system, the sparse channel estimation can be formulated as a compressed sensing problem (underdetermined setting) or a sparse system identification problem (overdetermined setting) under different transmission parameters.

3.1 Compressed Sensing Problem Formulation

In the pilot-assisted SC-FDE transmission system, when the pilot length N_p is less than the channel taps L (i.e., $N_p < L$), the sparse channel estimation can be formulated as a CS problem. The formulated problem based on CS is an underdetermined problem.

CS is a novel sampling theory that one can recover certain signals from far fewer samples or measurements than traditional Nyquist sampling methods. CS theory is suitable to the situation when the signal is compressible or the signal is sparse in a transform domain.

Suppose that there is a signal $\boldsymbol{g}$ whose length is N, and $\boldsymbol{g}$ is sparse in the space $\Psi = [\psi_1, \psi_2, \cdots \psi_N]$, a $N \times N$ matrix. Namely

$$\boldsymbol{g} = \sum_{i=0}^{N} \psi_i x_i = \Psi^T \boldsymbol{x} \tag{4}$$

where $\boldsymbol{x}$ has only K significant elements and the rest of the elements are zero or close to zero. Thus, $\boldsymbol{x}$ is K -sparse in the space Ψ. Under this assumption, the signal is projected by a $M \times N$ measurement matrix Φ to reduce the dimensionality and obtain an $M \times 1$ observation vector $\boldsymbol{y}$, which is a compressed vector corresponding to $\boldsymbol{x}$. Namely,

$$\boldsymbol{y} = \Phi \boldsymbol{g} = \Phi \Psi^T \boldsymbol{x} = \Theta \boldsymbol{x} \tag{5}$$

CS encodes a signal into a relatively small number of incoherent linear measurements. In theory, the optimal incoherence is achieved by completely random measurement matrices. Hence, to recover $\boldsymbol{x}$ from Θ by applying the CS algorithm, the measurement matrix has to satisfy certain properties, such as Restricted Isometry Property (RIP) [15]. Recently, CS methods have been developed for the estimation of the multipath channels taking into account the sparseness characteristic [16][17].

By examining (3) for the pilot-assisted SC-FDE transmission system, it can be seen that the channel estimation problem corresponds to obtaining an estimate of the CIR from the “full” set of observations described by (3) when the pilot sequence is immediately preceded and succeeded by zeros (i.e., a guard interval of length exists between the data and the pilot sequence). When there is lack of a “guard interval” of length between the data and pilot sequence, it resembles the canonical CS observation model where the number of observations N_p is far fewer than the length of the channel taps L. In this case, the sparse channel estimation can be formulated as a CS problem as follows,

$$\boldsymbol{y}_p = \Phi_p \boldsymbol{h} + \boldsymbol{z}_p \tag{6}$$

where Φ_p is observation matrix.

When designing the observation matrix in the CS setting, it favors random matrices. It is demonstrated that random Toeplitz matrices works well in the CS reconstruction methods. The pilot sequence with the random property can be used to form a Toeplitz matrix. As described in Fig.2, the transmit block consists of unknown data block and pilot block. In such a setting, a pseudo-random sequence can be used to probe a channel, and it has been proved that the Toeplitz matrix generated by the pseudo-random input probe satisfies the RIP [18].

In this case, the resulting input-output relation of data blocks can be expressed as $\boldsymbol{y}_p = \boldsymbol{Ph} + \boldsymbol{z}$ which represents

$$\begin{pmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N_p+L-3) \\ y(N_p+L-2) \end{pmatrix}_{(N_p+L-1)\times 1} = \begin{pmatrix} p(0) & \cdots & 0 \\ p(1) & \ddots & \vdots \\ \vdots & \ddots & p(0) \\ p(N_p-1) & \ddots & p(1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p(N_p-1) \end{pmatrix}_{(N_p+L-1)\times L} \begin{pmatrix} h(0) \\ h(1) \\ h(2) \\ \vdots \\ h(L-1) \end{pmatrix}_{L\times 1} + \begin{pmatrix} z(0) \\ z(1) \\ z(2) \\ \vdots \\ z(N_p+L-2) \end{pmatrix}_{N_p+L-1} \tag{7}$$

Notice that when the pilot sequence is immediately preceded and succeeded by the data sequence, the observation matrix Φ_p is a “partial” Toeplitz matrix in (3). When $N_p < L$, every row of the observation matrix Φ_p in this setting has at least one zero entry. When $N_p \geq L$, the sparse channel estimation can be formulated as a overdetermined sparse system identification problem as described in the next subsection.

3.2 Sparse System Identification Problem Formulation

In the pilot-assisted SC-FDE transmission system, when the pilot length N_p is longer than the channel taps L (i.e., $N_p \geq L$), the sparse channel estimation can be formulated as a sparse system identification problem. The formulated problem becomes an overdetermined (more equations than unknowns) problem instead of an underdetermined (more unknowns than equations) problem based on CS. For the SC-FDE transmission over mobile radio channel specified by 3GPP channels, it is a more normal setting when the data speed is not too high. Under such an overdetermined problem setting, the observation matrix can be designed in a deterministic approach and it can be solved with simplified algorithms.

An overdetermined system is almost always inconsistent (it has no solution) when constructed with random coefficients. The method of ordinary least squares (LS) can be used to find an approximate solution to overdetermined systems. When the system (channel) non-zero coeffcients are fewer than the observation vector, sparse approximate solutions can be found by using greedy algorithms such as OMP. By exploiting the channel sparsity characteristic, the sparse approximate solutions may outperform the traditional LS algorithm.

In the case that the length of pilot sequence is smaller than the CP, the equation (3) of the pilot sequence can be written as

$$\boldsymbol{y}_p = \mathcal{X}_p \boldsymbol{h} + \boldsymbol{z}_p \qquad (8)$$

where $\boldsymbol{y}_p$ is the received pilot sequence, $\boldsymbol{z}_p$ is the noise symbol.

Here we propose to adopt the Zadoff-Chu sequence as the pilot to form the observation matrix. $\mathcal{X}_p$ is a $N_p \times N_L$ Toeplitz matrix formed by the pilots as follows,

$$\mathcal{X}_p = \begin{bmatrix} x_0 & x_{N_p-1} & \cdots & x_{N_p-L+1} \\ x_1 & x_0 & \cdots & x_{N_p-L+2} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ x_{N_p-1} & x_{N_p-2} & \cdots & x_{N_p-L} \end{bmatrix}_{N_p \times L} \qquad (9)$$

where N_p is the length of pilot, L is the length of channel. In $\mathcal{X}_p$, each column is a shifted version of the first column $[x_0\ x_1\ \cdots x_{N_p-1}]^T$, which is the pilot sequence.

The pilot we choose is Zadoff-Chu sequences. Assuming that the length of Zadoff-Chu sequence is N_p. Zadoff–Chu (ZC) sequence can be expressed as

$$ZC_{N_p}(k) = \begin{cases} \exp(-j\dfrac{\pi k^2}{N_p}) & \text{for } N_p \text{ integer even} \\ \exp(-j\dfrac{\pi k(k+1)}{N_p}) & \text{for } N_p \text{ integer odd} \end{cases} \qquad (10)$$

Zadoff–Chu (ZC) sequence is a complex-valued mathematical sequence which, when applied to radio signals, gives rise to an electromagnetic signal of constant amplitude, whereby cyclically shifted versions of the sequence imposed on a signal result in zero correlation with one another at the receiver [19][20]. Zadoff–Chu sequences are CAZAC sequences (constant amplitude zero autocorrelation waveform). Hence, these sequences exhibits the useful property that cyclically shifted versions of itself are orthogonal to one another, provided, that is, that each cyclic shift is greater than the combined propagation delay and multi-path delay-spread of that signal between the transmitter and receiver.

Zadoff–Chu sequences are used in the 3GPP LTE Long Term Evolution air interface in the Primary Synchronization Signal (PSS), random access preamble (PRACH), uplink control channel (PUCCH), uplink traffic channel (PUSCH) and sounding reference signals (SRS) [21].

The ZC sequences have important properties that are appropriate for forming observation matrix in the SC-FDE channel estimation.

A ZC sequence has a constant amplitude. Also its N_p-point DFT has a constant amplitude. This property limits the Peak-to-Average Ratio and generates bounded and time-flat interference to other uses.

The second property shows that the cyclic auto-correlation of each ZC sequence results in a single dirac-impulse at time offset zero.

$$\phi_{kk}(\tau) = \sum_{n=0}^{Z_p-1} x_k(n) x_k^*(n+\tau) = \delta(\tau) \qquad \tau \in [0\ N_p] \qquad (11)$$

In addition, the cross-correlation of each ZC sequence with its cyclic shifted version with l shifts results in a single dirac-impulse at time offset l.

$$\phi_{k(k+l)}(\tau) = \sum_{n=0}^{Z_p-1} x_k(n) x_{k+l}^*(n+\tau) = \delta(\tau+l) \quad \tau \in [0\ N_p] \qquad (12)$$

This property allows the receiver to easily find the timing offset by correlation.

Additionally, each ZC sequences also holds a Fourier dual. This means, that the DFT of a ZC sequence $x_u[k]$ is a weigthed cyclicly-shifted ZC sequence $X_w[k]$ such that $w = -\frac{1}{u} \bmod N_p$. This property is useful in practical systems, as it allows the generation of ZC sequences directly in frequency domain without any DFT operation. Even more important, the correlation may be done in frequency domain and/or in time domain accordingly. In the channel estimation of SC-FDE, the channel estimation can be implemented either in the time domain or in the frequency domain with such a circulant Toplitz matrix formed by shifted ZC sequences.

With such a design of the observation matrix, the pilot vectors form a complete set for orthonormal transform. In the context of sparse component analysis (SCA), $\mathcal{X}_p$ is called sensing dictionary and the column vector in $\mathcal{X}_p$ is called atom. With our design, all the atoms are orthogonal to each other and form a complete or over-complete set for signal recovery. Classical linear algebra show that any unknown signal can be recovered exactly using such a set of test vectors if there is no noise terms in the observation.

4 Sparse Channel Estimation Algorithms

In this paper, we mainly investigate the second setting (overdetermined sparse system identification) of the sparse channel estimation problem in the SC-FDE system. Under this setting, the traditional channel estimation algorithm (*LS*), and two sparse channel estimation algorithms (OMP, and SAMP) for signal recovery are presented as follows. Specifically, the SAMP algorithm can deal with the scenario that the channel sparsity is unknown.

4.1 LS Algorithm

The sparse multipath channel can be estimated by using *LS* algorithm either in the time domain or in the frequency domain. In the time domain, the estimated channel result $\hat{\boldsymbol{h}}$ is the *LS* solution of $\min\|\boldsymbol{y}-\Phi_p\boldsymbol{h}\|_2$, which is a l_2-norm solution. The solution can be expressed as

$$\hat{\boldsymbol{h}} = \Phi_p^{\dagger}\boldsymbol{y} \tag{13}$$

where the pseudoinverse $\Phi_p^{\dagger 1} = (\Phi_p^T\Phi_p)^{-}\ \Phi_p^T$, and Φ^{T} indicates a matrix transpose.

Since the SC-FDE system performs the channel equalization in the frequency domain, by doing FFT of $\hat{\boldsymbol{h}}$, we obtain the estimated channel frequency response $\hat{\boldsymbol{H}}$ of the pilot sequence. That is, $\hat{\boldsymbol{H}} = \mathcal{F}\hat{\boldsymbol{h}}$, where $\mathcal{F}\hat{\boldsymbol{h}}$ denotes the Fourier transform of $\boldsymbol{h}$.

The sparse multipath channel can also be estimated by using *LS* algorithm in the frequency domain. In this case, after FFT transformation, the system equation becomes $\mathcal{F}\boldsymbol{y}_p = \mathcal{F}\Phi_p\mathcal{F}\boldsymbol{h}_p + \mathcal{F}\boldsymbol{z}_p$, where $\mathcal{F}\boldsymbol{y}_p$, $\mathcal{F}\Phi_p$, $\mathcal{F}\boldsymbol{h}_p$, $\mathcal{F}\boldsymbol{z}_p$ denote the Fourier transform of $\boldsymbol{y}$, Φ_p, $\boldsymbol{h}$, and $\boldsymbol{z}_p$, respectively. The channel estimation obtained in the frequency domain is $\hat{\boldsymbol{H}} = \dfrac{\mathcal{F}\boldsymbol{y}_p}{\mathcal{F}\boldsymbol{x}_p}$.

Note that the sparsity characteristic of multipath channel is not exploited in the traditional *LS* algorithm.

4.2 OMP Algorithm

MP is a greedy iterative algorithm for approximately solving the original l_0-norm problem. MP works by finding a basis vector in the dictionary that maximizes the correlation with the residual, and then recomputing the residual and coefficients by projecting the residual on all atoms in the dictionary using existing coefficients. OMP is similar to MP, except that an atom once picked, cannot be picked again. The algorithm maintains an active set of atoms already picked, and adds a new atom at each iteration. The residual is projected on to a linear combination of all atoms in the active set, so that an orthogonal updated residual is obtained. By using a circulant Toplitz matrix as the observation matrix, the atoms in the observation matrix are all orthogonal already. Hence, the OMP is equivalent to MP in this case. The pseudo code of the OMP algorithm is given as follows.

OMP algorithm

INPUT: received pilot vector $\boldsymbol{y}$, sensing matrix Φ, Sparsity *K*;
OUTPUT: A K-sparse approximation $\hat{\boldsymbol{h}}$ of the channel

Initialization:
$\hat{\boldsymbol{h}} = 0$ { Trivial initialization }
$\boldsymbol{r}_0 = \boldsymbol{y}$ { Initial residue }
$F_0 = \emptyset$ {Empty finalist }
for *k=1:K*
$J = Max(|\Phi^*\boldsymbol{r}_{k-1}|)$ { Candidate Test }
$F_k = F_{k-1} \cup J$ { Make Finalist }
$\boldsymbol{r}_k = \mathbf{y} - \Phi_F\Phi_F^{\dagger}\boldsymbol{y}$ { Compute Residue }
if halting condition true (e.g., $\|\mathbf{r}_k\|_2 < \varepsilon$) **then**
quit the iteration;
Output: $\hat{\boldsymbol{h}} = \Phi_F^{\dagger}\boldsymbol{y}$

Here, the pseudoinverse $\Phi_F^{\dagger 1} = (\Phi_F{}^T\Phi_F)^{-}\ \Phi_F{}^T$, which is the *LS* solution of $\min\|\boldsymbol{y}-\Phi_F\boldsymbol{r}\|_2$.

4.3 SAMP Algorithm for Unknown Sparsity

The sparsity adaptive matching pursuit (SAMP) is designed for blind recovery when the sparsity *K* is not available. It follows the “divide and conquer” principle through stage by stage estimation of the sparsity level and the true support set of the target signals. The pseudo code of the SAMP algorithm is given as follows.

SAMP algorithm

INPUT: received pilot vector $\boldsymbol{y}$, sensing matrix Φ, step size s;
OUTPUT: A K-sparse approximation $\hat{\boldsymbol{h}}$ of the channel

Initialization:
$\hat{\boldsymbol{h}} = 0$ { Trivial initialization }
$\boldsymbol{r}_0 = \boldsymbol{y}$ { Initial residue }
$F_0 = \emptyset$ {Empty finalist }
$v = s$ { Size of the finalist in the first stage }
$k = 1$ { Iteration index }
$j = 1$ { Stage index }
repeat
$S_k = Max(|\Phi^* \boldsymbol{r}_{k-1}|, v)$ { Preliminary Test }
$C_k = F_{k-1} \cup S_k$ { Make Candidate List }
$F = Max(|\Phi^\dagger_{C_k} \boldsymbol{y}|, v)$ { Final Test }
$\boldsymbol{r} = \boldsymbol{y} - \Phi_F \Phi^\dagger_F \boldsymbol{y}$ { Compute Residue }
if halting condition true (e.g., $\|\mathbf{r}\|_2 < \varepsilon$) **then**
quit the iteration;
else if $\|\boldsymbol{r}\|_2 > \|\boldsymbol{r}_{k-1}\|_2$ then { stage switching }
$j = j + 1$ { Update the stage index }
$v = j \times s$ { Update the size of finalist }
else
$F_{k-1} = F$ {Update the finalist}
$\boldsymbol{r}_k = \boldsymbol{r}$ { Update the residue }
$k = k + 1$
end if until halting condition true;
Output: $\hat{\boldsymbol{h}} = \Phi^\dagger_F \boldsymbol{y}$

Here, $v = |F_k|$ represents the size of finalist; for a vector $\boldsymbol{a}$, function $Max(\boldsymbol{a}, \text{I})$ returns v indices corresponding to the largest absolute values of $\boldsymbol{a}$. For a set $\Lambda \in \{1,\ldots,N\}$, Φ_Λ is the submatrix of Φ with indices $i \in \Lambda$. At the k-th iteration, S_k, C_k, F_k, $\boldsymbol{r}_k$ represent the short list, the candidate list, the finalist and the observation residual, respectively.

The recovery process in the SAMP algorithm is divided into several stages, each of which contains several iterations. In the kth iteration of the SAMP algorithm, the sizes of candidate set $|C_k|$ and finalist $|F_k|$ are adaptive. This innovation enables the SAMP to conduct blind recovery without priori information of K. $|F_k|$ is kept fixed for iterations in the same stage and increased by a step size $s \leqslant K$ between two consecutive stages. There is a trade-off between s and the recovery speed as smaller s requires more iterations. Empirical results suggest that small s is preferable for signal with (exponentially) decayed magnitude, while large s is advantageous for binary sparse signal.

5 Simulation Results

In this section, we investigate the sparse channel estimation for the pilot-assisted SC-FDE system under the sparse system identification problem setting. We compare the performance of the proposed sparse channel estimation and the traditional *LS* channel estimation in the SC-FDE communication system. The OMP and SAMP algorithms are adopted as the sparse channel estimations.

In the considered SC-FDE system, one transmitted frame is 1024 samples with 896 data symbols and 128 CP samples. The Zadoff-Chu sequences is used as the pilots for the traditional LS channel estimation and the sparse channel estimation methods (OMP and SAMP). The pilot length is 128 for *LS* channel estimation, and the pilot length is 64 for sparse channel estimation methods. For channel equalization at the SC-FDE receiver, the minimum mean square error (MMSE) scheme is used. The signal bandwidth in our simulation is 10MHz, and the symbol sampling rate is also 10M samples per second. The simulation parameters are listed in Table I.

TABLE I. Simulation assumptions and parameters

Sampling rate	10 mega-samples per second
Data modulation format	BPSK
Pulse shaping	None
Pilot length	128 (LS); 64 (OMP,SAMP)
Pilot type	Zadoff-Chu sequence
FFT/IFFT size	1024 samples
Channel estimation	LS; OMP; SAMP
Equalization	MMSE
Channel coding	None

In our simulation, we utilize the 3rd generation partnership project (3GPP) Pedestrian A channel and Vehicular A channel models. The two channel models are shown in Table II and Table III, respectively [22]. There are 4 dominant taps in

Pedestrian A channel and 6 dominant taps in Vehicular A channel. Hence, the spasity of these two channel is 4 and 6 respectively. Under the 10MHz sampling rate, the time resolution of the channels is 100ns. The length of channel we choose is L=31. In the simulation, the channel coefficients are normalized such that ith channel coefficient α_i is normalized to be $\tilde{\alpha}_i = \alpha_i / \sum_{j=1}^{L} \alpha_j^2$.

TABLE II. Channel delay profiles of 3GPP Pedestrian A channel

Tap	Ped. A	
	Relative delay(ns)	Average relative power(dB)
1	0	0
2	110	-9.7
3	190	-19.2
4	410	-22.8

TABLE III. Channel delay profiles of 3GPP Vehicular A channel

Tap	Veh. A	
	Relative delay(ns)	Average relative power(dB)
1	0	0
2	310	-1.0
3	710	-9.0
4	1090	-10.0
5	1730	-15.0
6	2510	-20.0

With the above simulated assumptions and parameters, because $N_p > L$, the channel estimation problem can be formulated as a sparse system identification problem (overdetermined setting) instead of a CS problem.

The normalized mean square error (NMSE) can be used to measure the performance of the estimator as it reflects both the bias and the variance of the estimator. The NMSE [16] of $\hat{\boldsymbol{h}}$ is described as

$$\text{NMSE}\{\hat{\boldsymbol{h}}\} = \frac{E\{\boldsymbol{h} - \hat{\boldsymbol{h}}\}}{E\{\hat{\boldsymbol{h}}\}} \quad (14)$$

Fig. 3 and Fig. 4 show the original and estimated channel delay profiles of 3GPP Pedestrian A channel at SNR= 5dB and 10dB, respectively. It can be seen that four main paths of channel delay profile can be estimated by the channel estimation algorithms. The estimation is more precise at 10 dB than that at 5 dB. Fig. 5 and Fig. 6 show the original and estimated channel delay profiles of 3GPP Vehicular A channel at SNR= 5dB and 10dB, respectively. It can be seen that six main paths of channel delay profile can be estimated by the channel estimation algorithms. The estimation is more precise at 10 dB than that at 5 dB. It can be observed that the estimation errors are higher under 3GPP Vehicular A channel than that under 3GPP Pedestrian A channel.

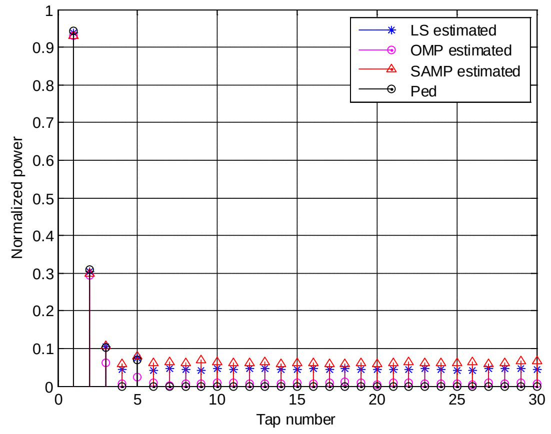

Fig.3. 3GPP Pedestrian A channel delay profile and estimated results (SNR=5dB).

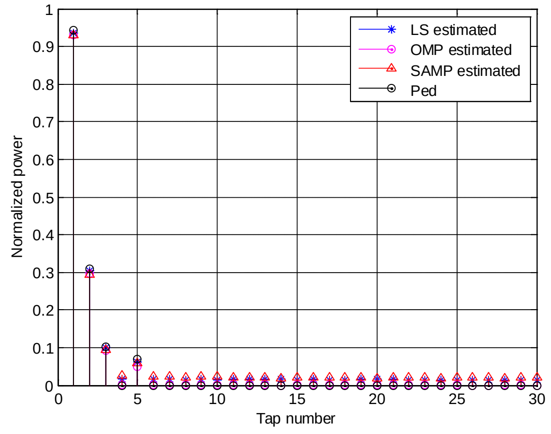

Fig.4. 3GPP Pedestrian A channel delay profile and estimated results (SNR=15dB).

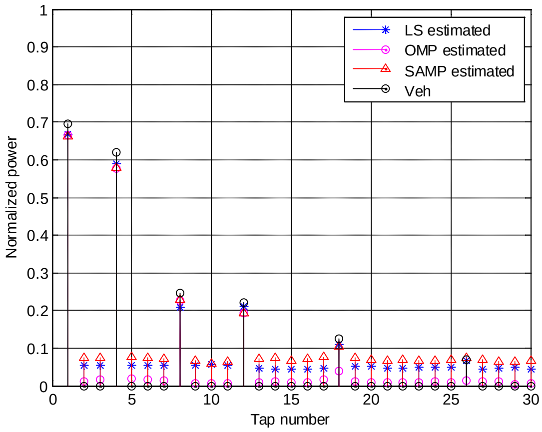

Fig.5. 3GPP Vehicular A channel delay profile and estimated results (SNR=5dB).

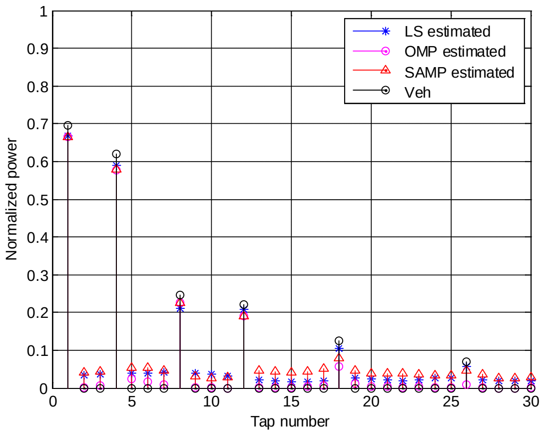

Fig.6. 3GPP Vehicular A channel delay profile and estimated results (SNR=15dB).

Fig. 7 and Fig. 8 show the NMSE versus SNR (Eb/N0) with different channel estimation algorithms under 3GPP Pedestrian A channel and Vehicular A channel, respectively. It can be seen that the NMSE of the sparse channel estimation (OMP & SAMP) methods is smaller than that with the traditional *LS* channel estimation. Furthermore, the NMSE value of the SAMP algorithm is higher than that of the OMP algorithm over 3GPP Pedestrian A channel and Vehicular A channel. Hence, there is drawback of the SAMP algorithm in terms of achieved NMSE though it has advantage of sparse adaptivity comparing with the OMP algorithm. In addition, it can be observed that the NMSE value under 3GPP Vehicular A channel is higher than that under 3GPP Pedestrian A channel. Thus, the NMSE increases with the increase of channel sparsity with the sparse estimation algorithms.

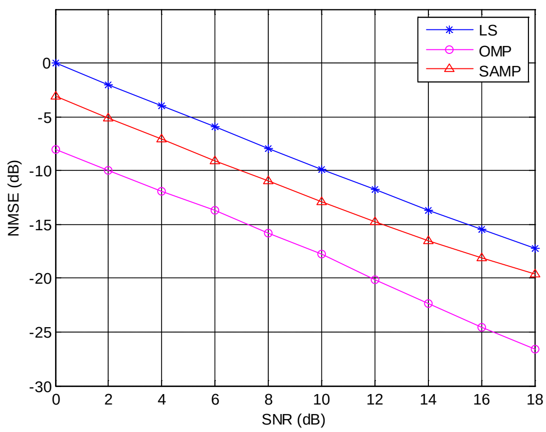

Fig.7. NMSE of channel estimation in Pedestrian A channel.

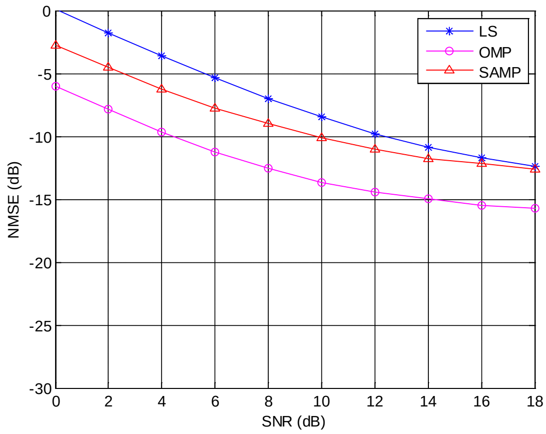

Fig.8. NMSE of channel estimation under Vehicular A channel.

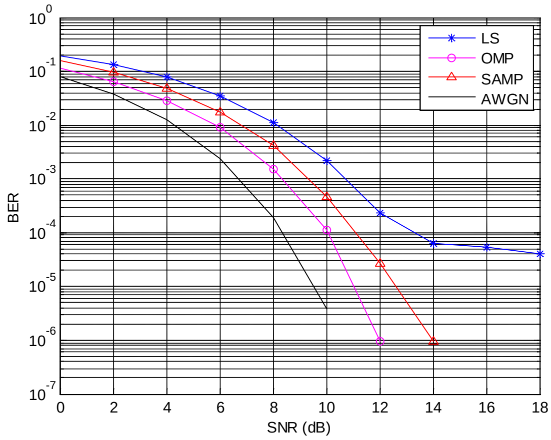

Fig.9. BER performance of SC-FDE system under Pedestrian A channel.

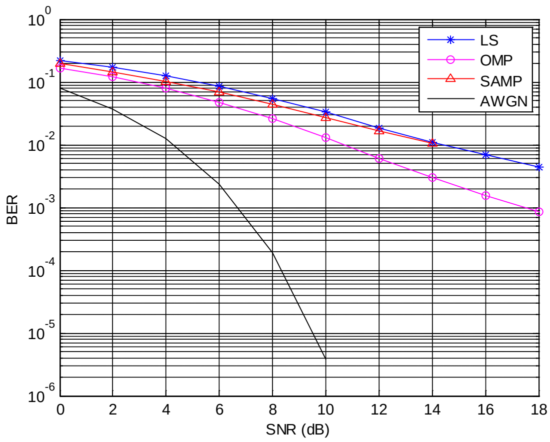

Fig.10. BER performance of SC-FDE system under Vehicular A channel.

Fig. 9 and Fig. 10 shows the BER performance of different channel estimation algorithms in the SC-FDE transmission system under 3GPP Pedestrian A channel and Vehicular A channel, respectively. It can be seen that the sparse channel estimation algorithms perform better than the traditional LS algorithm under 3GPP Pedestrian A channel and Vehicular A channel. Furthermore, all the algorithms performs better under 3GPP Pedestrian A channel than that under the Vehicular A channel. It can be seen that sparse channel estimation algorithms performs better than the traditional *LS* channel estimation even much reduced pilot length.

6 Conclusions

In the pilot-assisted SC-FDE transmission system, the sparse channel estimation can be formulated to be an underdetermined Compressed Sensing (CS) problem or an overdetermined sparse system identification problem under different transmission parameters. Specifically, when the pilot length N_p is longer than the number of channel taps L (i.e., $N_p \geqslant L$), the sparse channel estimation can be formulated as an overdetermined sparse system identification problem. For SC-FDE transmission over mobile radio channel specified by 3GPP, the latter is a more normal setting when the data speed is not too high. Under such an overdetermined problem setting, the observation matrix can be designed in a deterministic approach and can be solved with simplified algorithms. We propose to use Zardoff-Chu sequence as the pilot sequence to form a deterministic circulant Toeplitz observation matrix. With such a design of the observation matrix, the pilot vectors are formed as orthonormal atoms in an over-complete dictionary. In practical system applications, the greedy sparse signal recovery algorithms that need a priori knowledge of channel sparsity can not be employed directly. To address this issue, Sparsity Adaptive Matching Pursuit (SAMP) algorithm is investigated to reconstruct the channel impulse response (CIR) with the proposed observation matrix. The simulation results show that both OMP and SAMP channel estimation algorithms perform better than the traditional LS algorithm for SC-FDE system transmitting over a sparse multipath channel even with reduced pilot length. Comparing with the OMP algorithm, the SAMP algorithm has the advantage of sparse adaptivity, but the drawback of the SAMP algorithm is slightly decreased precision of the estimated CIR.

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