Cooperative Amplify-and-Forward Relaying Systems with Quadrature Spatial Modulation

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Abstract: Quadrature spatial modulation (QSM) is a recent digital multiple-input multiple-output (MIMO) transmission technique. Combined with cooperative relaying, QSM improves the overall spectral efficiency and enhances the communication reliability. In this paper, we study the performance of QSM amplify-and forward cooperative relaying systems. In particular, a closed-form expression for the average pair-wise error probability (PEP) of the cooperative system is derived, which is used to calculate a tight upper bound of the average bit error probability (ABEP) over Rayleigh fading. In addition, a simple approximate, yet accurate expression is derived and analyzed asymptotically. Simulation results, which corroborate the numerical ones, show the effectiveness of combined QSM and cooperative relaying in improving the overall system performance.

Key–Words: Amplify-and-forward, Quadrature spatial modulation, Spectral efficiency, MIMO.

1 Introduction

The unprecedented demands for huge data rates and high speed wireless communication applications motivated both researchers and industry to investigate several recent technologies, which led to many promising innovations. Such innovations are expected to support many critical services that affect all aspects of human lives.

Quadrature Spatial modulation (QSM) is proposed as a promising technology for the future multiple-input multiple-output (MIMO) wireless networks [1]. It has been shown that QSM increases the spectral efficiency of conventional spatial modulation (SM) system while retaining all SM inherent advantages. The techniques of QSM, SM and space-shift-keying (SSK) are introduced as low-complexity and spectral-efficient implementations of MIMO systems [2], [3], [4] . In addition, they avoid conventional MIMO drawbacks including inter-channel interference (ICI) and complex receivers [1]- [5].

In QSM, the spatial constellation symbols are extended to orthogonal in-phase and quadrature components. One component transmits the real part of a signal constellation symbol and the other transmits the imaginary part. It is worth pointing that, in conventional SM, these two parts are transmitted from a single transmit antenna to avoid ICI at the receiver input. However, in a QSM system, ICI is also avoided entirely since the two transmitted data are orthogonal and modulated on the real part and the imaginary part of the carrier signal.

Very recently, few works studied QSM in conventional MIMO system assuming perfect and imperfect channel state information (CSI) [1], [5], [6]. Authors in [7] proposed Bi-SSK system, which is considered as a special case of QSM without using any modulation schemes. Although QSM has been studied in point-to-point MIMO systems, it is not yet investigated in the context of cooperative systems. As such, this paper studies a QSM single amplify-and-forward (AF) relaying system and analyzes the system error performance.

In this work, the average bit error probability (ABEP) for QSM-AF cooperative systems is analyzed. In particular, a closed-form expression for the average pairwise error probability (PEP) is derived employing the optimal maximum likelihood (ML) detector at the receiver. Based on the derived PEP expression, a tight upper bound expression is obtained using the union bound formula. Moreover, an asymptotic analysis is performed to get insights on key parameters. The results show the effectiveness of cooperative QSM in improving the error system performance.

The remainder of this paper is organized as follow: Section 2 describes system and channel models. Performance analysis is presented in Section 3.

Numerical results are presented in Section 4. Finally, Section 5 concludes the paper.

2 System Model

2.1 Channel Model

We consider a system model comprising a source S with N_t transmit antennas, a single antenna AF relay R, and a single antenna destination D as depicted in Fig. 1 [1]. The cooperative system is operating over Rayleigh fading channels. We assume that $b =$

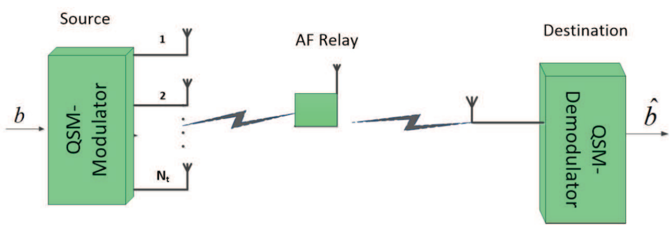

Figure 1: System model of a single RF chain QSM-AF system.

$\log_2(MN_t^2)$ incoming bit stream enters the source at each transmission instant. The incoming data bits are processed and partitioned into three groups. S determines the index of the active transmit antennas by using two groups of $\log_2(N_t)$ bits of b, then maps the remaining $\log_2(M)$ bits onto the corresponding M-ary quadrature amplitude modulation (M-QAM)/ phase shift keying (M-PSK) or other complex signal constellation diagrams. The signal constellation symbol, x, is further decomposed to its real, x_β, and imaginary, x_b, parts. The real part is transmitted from one transmit antenna among the existing N_t transmit antennas, where the active antenna index is determined by the first $\log_2(N_t)$ bits. Similarly, the imaginary part is transmitted by another or the same transmit antenna depending on the other $\log_2(N_t)$ bits. However, the transmitted real and imaginary parts are orthogonal representing the in-phase and the quadrature components of the carrier signal.

An example for QSM bits mapping and transmission is given in what follows assuming $N_r \times N_t, (4 \times 4)$-MIMO system and 4-QAM modulation. The number of data bits that can be transmitted at one particular time instant is $b = \log_2(N_t^2 M) = 6$ bits. Assume that the following incoming data bits, $b = \underbrace{\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}}_{\log_2(M)+\log_2(N_t^2)}$ are to be transmitted. The first $\log_2(M)$ bits $\begin{bmatrix} 0 & 1 \end{bmatrix}$, modulate a 4-QAM symbol, $x = -1 + j$. This symbol is divided further into real and imaginary parts, $x_\beta = -1$ and $x_b = 1$. The second $\log_2(N_t)$ bits $\begin{bmatrix} 1 & 0 \end{bmatrix}$, modulate the active antenna index, $\ell_\beta = 3$ to transmit $x_\beta = -1$ resulting in the transmitted vector $\mathbf{s}_\beta = \begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}^T$. The last $\log_2(N_t)$ bits, $\begin{bmatrix} 1 & 1 \end{bmatrix}$, modulate the active antenna index, $\ell_b = 4$, used to transmit $x_b = 1$, resulting in the vector $\mathbf{s}_b = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$. The transmitted vector is then obtained by adding the real and imaginary vectors, $\mathbf{s} = \mathbf{s}_\beta + j\mathbf{s}_b = \begin{bmatrix} 0 & 0 & -1 & +j \end{bmatrix}^T$.

The transmission protocol consists of two time slots. In the first time slot, the vector, $\mathbf{s}$, is transmitted to the relay , over an $N_t \times 1$ Rayleigh fading wireless channel. In the second time slot, the AF relay forward the received signal to the destination. In the first phase, the received signal at R can be written as

$$y_{s,r} = \sqrt{\frac{P_S}{2}} h_\alpha x_\beta + j\sqrt{\frac{P_S}{2}} h_a x_b + \eta_r, \qquad (1)$$
$$\alpha, a = 1, 2, \cdots, N_t; \beta, b = 1, .., M;$$

where x_β and x_b are symbols in the PAM signal-constellation diagram and h_α and h_a are the activated antenna fading gains, are assumed to be complex Gaussian random variables with zero mean and variances σ_h^2, and $\eta_r \sim CN(0, N_0)$ is the complex Gaussian noise with zero mean and variance N_0 .

In the second phase, in AF relaying, the relayed signal (x_R) is an amplified version of the input signal at the relay node, i.e., $x_R = A \times y_{s,r}$, where A is the amplification factor. Using this technique the amplification process is performed in the analogue domain and consists of a simple normalization of the total received power without further processing. Hence, the received signal at D from R in the second phase can be written as

$$\begin{aligned} y_{r,d} &= Agy_{s,r} + n_{r,d} \\ &= Ag\left(\sqrt{\frac{P_S}{2}} h_\alpha x_\beta + j\sqrt{\frac{P_S}{2}} h_a x_b + \eta_r\right) + \eta_{r,d} \\ &= \underbrace{Ag\sqrt{\frac{P_S}{2}} h_\alpha x_\beta + jAg\sqrt{\frac{P_S}{2}} h_a x_b}_{\text{Signal Part}} + \underbrace{Ag\eta_r + n_{r,d}}_{\text{Noise Part}} \end{aligned} \qquad (2)$$

Then,

$$y_{r,d} = \frac{Ag}{\sqrt{A^2|g|^2+1}}\sqrt{\frac{P_S}{2}} h_\alpha x_\beta + j\frac{Ag}{\sqrt{A^2|g|^2+1}} \times \sqrt{\frac{P_S}{2}} h_a x_b + \hat{n}, \qquad (3)$$

where g is the fading gain between R and D, $A = \sqrt{\frac{1}{P_S\sigma_h^2/2+N_0}}$, and $\hat{n}$ is Gaussian noise with variance N_0.

Since the channel inputs are assumed equally likely, the optimal detector, based on the ML principle, is given as

$$\left[\hat{h}_\alpha, \hat{h}_a, \hat{x}_\beta, \hat{x}_b\right] =$$
$$\arg\min_{\alpha,a,\beta,b} \left|\left| y_{r,d} - \frac{Ag}{\sqrt{A^2|g|^2+1}}\sqrt{\frac{P_S}{2}}\left[h_\alpha x_\beta + jh_a x_b\right]\right|\right|^2$$
$$= \arg\min_{\alpha,a,\beta,b} ||\lambda||^2 - 2\Re\left\{y^H\lambda\right\}, \quad (4)$$

where $\lambda = \frac{Ag}{\sqrt{A^2|g|^2+1}}\sqrt{\frac{P_S}{2}}\left[h_\alpha x_\beta + jh_a x_b\right]$. It can be seen that optimal detection requires a joint detection of the antenna indices and symbols.

3 QSM-AF Performance Analysis

In this section, the ABEP is investigated, where a closed-form expression for the average PEP is derived. From which we derive a tight upper bound ABEP expression.

3.1 SNR statistics

Assuming $\lambda = \frac{Ag}{\sqrt{A^2|g|^2+1}}\sqrt{\frac{P_S}{2}}\left[h_\alpha x_\beta + jh_a x_b\right]$ is transmitted, the probability of deciding in favour of $\hat{\lambda} = \frac{Ag}{\sqrt{A^2|g|^2+1}}\sqrt{\frac{P_S}{2}}\left[\hat{h}_\alpha \hat{x}_\beta + j\hat{h}_a \hat{x}_b\right]$ is given as

$$\Pr\left(\lambda \to \hat{\lambda}|\mathbf{H}\right) = \Pr\left(d_\lambda > d_{\hat{\lambda}}|\mathbf{H}\right) = Q\left(\sqrt{\chi}\right) \quad (5)$$

where $d_\lambda = \| \lambda \|^2 - 2\Re\left\{y^H\lambda\right\}$.

We define χ as

$$\chi \equiv \frac{1}{2N_0} \parallel \lambda - \hat{\lambda} \parallel^2 = \frac{A^2|g|^2}{A^2|g|^2+1}\frac{P_S}{4N_0}|C+jD|^2, \quad (6)$$

where $j = \sqrt{-1}$ and

$$C = \left(h_\alpha^R x_\beta - h_a^I x_b - \hat{h}_\alpha^R \hat{x}_\beta + \hat{h}_a^I \hat{x}_b\right) \quad (7)$$
$$D = \left(h_\alpha^I x_\beta + h_a^R x_b - \hat{h}_\alpha^I \hat{x}_\beta - \hat{h}_a^R \hat{x}_b\right) \quad (8)$$

Since the symbols x_β and x_a are drawn from a real constellation, i.e., PAM, C and D are independent. Therefore, χ has the following mean (since h_a, h_α and g are independent)

$$\begin{aligned} Q\left(\sqrt{\chi}\right) &= Q\left(\sqrt{\frac{A^2|g|^2}{A^2|g|^2+1}\frac{P_S}{4N_0}|C+jD|^2}\right) \\ &= Q\left(\sqrt{\frac{\frac{P_R|g|^2}{N_0}}{\frac{P_R|g|^2}{N_0}+\frac{P_R}{N_0A^2}}\frac{P_S}{4N_0}|C+jD|^2}\right) \\ &= Q\left(\sqrt{\frac{\Lambda}{\Lambda+\Xi}\frac{P_S}{4N_0}|C+jD|^2}\right), \end{aligned} \quad (9)$$

where $\Lambda = \frac{P_R|g|^2}{N_0}$, and $\Xi = \frac{P_R}{N_0A^2}$.

Note that $\nu = \frac{P_S}{4N_0}|C+jD|^2$ is an exponential random variable with the following mean

$$\bar{\nu} = \begin{cases} \frac{P_S}{4N_0}\sigma_h^2\kappa_1 & \text{if } h_\alpha \neq \hat{h}_\alpha, h_a \neq \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\kappa_2 & \text{if } h_\alpha = \hat{h}_\alpha, h_a \neq \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\kappa_3 & \text{if } h_\alpha \neq \hat{h}_\alpha, h_a = \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\kappa_4 & \text{if } h_\alpha = \hat{h}_\alpha, h_a = \hat{h}_a, \end{cases} \quad (10)$$

where $\kappa_1 = \left(|x_\beta|^2 + |\hat{x}_\beta|^2 + |x_b|^2 + |\hat{x}_b|^2\right)$,
$\kappa_2 = \left(|x_\beta - \hat{x}_\beta|^2 + |x_b|^2 + |\hat{x}_b|^2\right)$,
$\kappa_3 = \left(|x_\beta|^2 + |\hat{x}_\beta|^2 + |x_b - \hat{x}_b|^2\right)$, and
$\kappa_4 = \left(|x_\beta - \hat{x}_\beta|^2 + |x_b - \hat{x}_b|^2\right)$. In the case that we have BPSK, i.e.± 1, $\bar{\zeta}$ can be greatly simplified to

$$\bar{\nu} = \begin{cases} \frac{P_S}{N_0}\sigma_h^2 & \text{if } h_\alpha \neq \hat{h}_\alpha, h_a \neq \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\left(|x_\beta - \hat{x}_\beta|^2 + 2\right) & \text{if } h_\alpha = \hat{h}_\alpha, h_a \neq \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\left(2 + |x_b - \hat{x}_b|^2\right) & \text{if } h_\alpha \neq \hat{h}_\alpha, h_a = \hat{h}_a, \\ \frac{P_S}{4N_0}\sigma_h^2\kappa_4 & \text{if } h_\alpha = \hat{h}_\alpha, h_a = \hat{h}_a, \end{cases} \quad (11)$$

and the terms $|x_b - \hat{x}_b|^2$ and $|x_\beta - \hat{x}_\beta|^2$ can be either 0 or 4.

Furthermore, if we assume Quadrature space shift keying (QSSk), i.e., $|x_\beta|^2 = |x_b|^2 = 1$, ζ can be simplified to

$$\bar{\nu} = \begin{cases} \frac{P_S}{N_0}\sigma_h^2 & \text{if } h_\alpha \neq \hat{h}_\alpha, h_a \neq \hat{h}_a, \\ \frac{P_S}{2N_0}\sigma_h^2 & \text{if } h_\alpha = \hat{h}_\alpha, h_a \neq \hat{h}_a, \text{or } h_\alpha \neq \hat{h}_\alpha, h_a = \hat{h}_a \end{cases} \quad (12)$$

To find the average error probability, we need to find the CDF of χ. This can be done as follows:

The PDF of Λ is $f_\Lambda(x) = \frac{1}{\bar{\Lambda}}\exp\left(-\frac{x}{\bar{\Lambda}}\right)$, where $\bar{\Lambda} = \sigma_g^2 P_R/N_0$, and the PDF of the term $\nu = \frac{P_S}{4N_0}|C+$

$jD|^2$ is $f_\nu(x) = \frac{1}{\bar{\nu}} \exp(-x/\bar{\nu})$. Therefor CDF of $\chi = \frac{\Lambda}{\Lambda+\Xi} \frac{P_S}{4N_0} |C + jD|^2$ is derived as

$$F_\chi(x) = \Pr(\chi < x) = \Pr\left(\frac{\Lambda}{\Lambda + \Xi} \frac{P_S}{4N_0} |C + jD|^2 < x\right)$$
$$= 1 - 2\sqrt{\frac{\Xi x}{\bar{\nu}\bar{\Lambda}}} \exp\left(-\frac{x}{\bar{\nu}}\right) K_1 \left(2\sqrt{\frac{\Xi x}{\bar{\nu}\bar{\Lambda}}}\right), \tag{13}$$

where $K_v(.)$ is the v^{th}-order modified Bessel function of the second kind.

3.2 Exact Average PEP

The alternative expression of the average pairwise error probability can be written as

$$\overline{\text{PEP}} = \frac{a}{2}\sqrt{\frac{b}{2\pi}} \int_0^\infty \frac{1}{\sqrt{x}} \exp\left(-\frac{bx}{2}\right) F_\chi(x) dx \tag{14}$$

Substituting (13) into (14) and solve the integral with the help of [9, 6.614.5,pp.698] we have $(a = 1, b = 1)$

$$\overline{\text{PEP}} = \frac{1}{2} - \frac{1}{2} \frac{\Xi}{\bar{\Lambda}(2 + \bar{\nu})} \sqrt{\frac{\bar{\nu}}{2 + \bar{\nu}}} \exp\left(\frac{\Xi}{\bar{\Lambda}(2 + \bar{\nu})}\right)$$
$$\times \left[K_1\left(\frac{\Xi}{\bar{\Lambda}(2 + \bar{\nu})}\right) - K_0\left(\frac{\Xi}{\bar{\Lambda}(2 + \bar{\nu})}\right)\right] \tag{15}$$

3.3 ABEP Performance

After the evaluation of the average PEP, the ABEP of the proposed scheme can be upper bounded by the following asymptotically tight union bound:

$$P_b = \frac{1}{2^k} \sum_{n=1}^{2^k} \sum_{m=1}^{2^k} \frac{1}{k} \Pr(\lambda_n \to \lambda_m) e_{n,m}, \tag{16}$$

where $\{\lambda_n\}_{n=1}^{2^k}$ is the set of all possible QSM symbols, $k = \log_2(MN_t^2)$ is the number of information bits per QSM symbol, and $e_{n,m}$ is the number of bit errors associated with the corresponding PEP event. Note that M represents the PAM constellation.

3.4 High SNR Analysis

Although the expressions for the average error probability in (15) enable numerical evaluation of the system performance and may not be computationally intensive, they do not offer insight into the effect of the system parameters. We now aim at expressing $F_\chi(x)$ and average PEP in simpler forms. This will ease the analysis of the optimization problems. The following steps will be done to simplify the expression of the error probability:

According to [10] and [11], the asymptotic error and outage probabilities can be derived based on the behavior of the CDF of χ around the origin. By using Taylor's series, $F_\chi(x)$ can be rewritten as

$$F_\chi(x) \approx \left(\frac{1}{\bar{\nu}} + \frac{\Xi}{\bar{\nu}\bar{\Lambda}}\left(\psi(1) - \log\left(\frac{\Xi}{\bar{\nu}\bar{\Lambda}}\right)\right)\right) x + \text{H.O.T} \tag{17}$$

where $\psi(.)$ is the digamma function, note that $\psi(1) = -0.57721$.

Substituting (17) into (14) and solve the integration we have

$$\text{PEP} \approx \frac{1}{2}\left(\frac{1}{\bar{\nu}} + \frac{\Xi}{\bar{\nu}\bar{\Lambda}}\left(\psi(1) - \log\left(\frac{\Xi}{\bar{\nu}\bar{\Lambda}}\right)\right)\right). \tag{18}$$

It is noted, in the Numerical Results Section, that this approximate error expression is in an excellent agreement with the exact expression especially in the pragmatic SNR values.

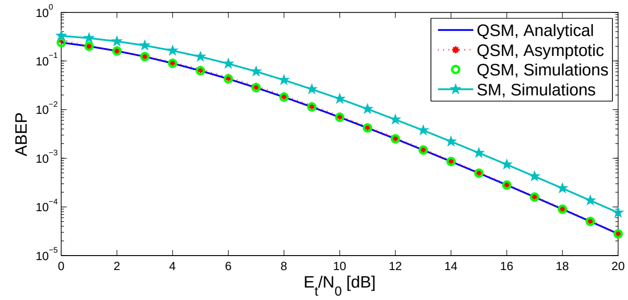

Figure 2: Analytical, simulation and asymptotic ABEP vs. $\frac{E_t}{N_0}$ (dB) with $N_t = 2$, and 4-QAM scheme.

4 Numerical analysis and Discussion

In this section, the performance of the QSM singe AF cooperative relaying system is evaluated via analytical results and validated through simulations for 4-QAM scheme. Unless otherwise stated, we assume $N_t = 2$, $\sigma_h^2 = 1, N_o = 1$, and $E_t = Es + E_r$. For comparison purposes, the performance of SM is included [8].

In Fig. 2, the ABEP versus $\frac{E_t}{N_0}$ is evaluated and simulated for $N_t = 2, 4$, respectively. It can be observed that the ABEP performance improves with increasing SNR. In the high $\frac{E_t}{N_0}$ region, the asymptotic

ABEP performance becomes in excellent agreement with exact one. The performance of conventional SM scheme with 8-QAM modulation achieving similar spectral efficiency 4-QAM in the considered QSM system is depicted in both figures. Obtained results demonstrate the significant enhancement of the QSM system over the SM system, where a gain of about 3 dB can be noticed in the figures. Note that this gain is attained at almost no cost. The receiver complexity of QSM and SM schemes are equivalent and depends on the considered spectral efficiency as reported in [1].

5 Conclusion

We analyzed the performance of the QSM-AF cooperative systems employing ML detector at the receiver. The tight upper bounded ABEP was derived using the closed-form average PEP. In addition, the asymptotic performance analysis was performed; a simple approximate error expression was derived at high SNR values. The QSM cooperative system outperforms the conventional SM cooperative system beside achieving higher spectral efficiency and maintaining most of its inherit advantages without any additional receiver complexity.

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