Performance Evaluation of Cooperative Versus Receiver Coded Diversity

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Abstract: The amplify-and-forward and the decode-and-forward cooperative coded diversity and is compared with the conventional receiver coded diversity in terms of the pairwise error probability and the overall bit error rate. The diversity systems under consideration can achieve the diversity order at most two. The performance comparison assumes channel coding with non-binary linear modulations, independent fading channels with path-loss attenuations proportional to the distances between the communicating nodes, and the diversity combining at the destination receiver. The expressions for the pairwise error probabilities are obtained analytically and verified by computer simulations. The performance of the cooperative diversity is found to be strongly relay location dependent. Hence, using the analytical as well as extensive numerical results, the geographical areas of the relay locations are obtained for small to medium signal-to-noise ratio values, such that the cooperative coded diversity outperforms the receiver coded diversity. On the other hand, for sufficiently large signal-to-noise ratio values, or if the path-loss attenuations are not considered, then the receiver coded diversity always outperforms the cooperative coded diversity. The obtained results have important implications on the deployment of the next generation cellular systems supporting the cooperative as well as the receiver diversity.

Key–Words: Channel coding, communication system performance, cooperative systems, diversity methods, fading channels.

1 Introduction

The roll-out of the 4G cellular systems is expected to commence in the near future. Various forms of the transmission diversity are one of the key technical enablers of the 4G systems. The relays deployed about the 4G base stations will provide the improved coverage and enable higher data rates services by realizing the distributed transmission diversity. The existence of relays, however, also significantly complicates the deployment of the 4G networks, for example, due to the increased capital and operational expenditures, and the need to allocate additional communication channels within the cell. It is therefore vital to investigate the conditions when the cooperative diversity realized by the relays can bring the antennas closer to the user terminals, and thus, outperform the conventional receiver diversity realized by the multiple antennas at the receiver. Such comparison can be done in terms of the transmission reliabilities represented by the pairwise error probabilities (PEPs) and the bit error rates (BERs).The uncoded cooperative diversity techniques were studied in [1] and in [2]. The multiuser cooperative protocols are proposed in [3, 4]. An overview of the coded cooperation schemes is given in [5]. The performance of conventional coded antenna diversity techniques is investigated in [6]. The performance of coded systems over block fading channels is analyzed in [7]. An upper-bound of the transmission error probability for binary block codes over slow and fast fading channels is obtained in [8]. A specific two-user coded cooperative scheme is proposed and analyzed [9,10]. General analytical expressions for the error performance of the amplify-and-forward (AF) and the decode-and-forward (DF) relaying employing the turbo codes are obtained in [11]. The coded cooperation is also studies in [12].

In this paper, a comparison is carried out between the transmission reliabilities of a cooperative diversity system employing a single relay and a system employing the conventional receiver diversity with the two receiver antennas. Thus, both systems can achieve the diversity order of at most two. We formulate the research problem such that the source and the destination are stationary, and the task is to find the relay locations, so that the cooperative diversity can outperform the receiver diversity. This is a dual problem to the scenario where the destination (source) and the relay are stationary, and the task is to find the source (destination) locations, so that the cooperative diversity can outperform the receiver diversity. The locations of network nodes are taken into account through the path-loss attenuations. The results indicate that, if the path-loss attenuations, and thus, the mutual nodes locations are not considered, then the conventional receiver diversity always outperforms the cooperative diversity. On the other hand, the path-loss attenuations may cause the system with the cooperative diversity to outperform the system with the receiver diversity, particularly at smaller values of the signal-to-noise ratio (SNR). All the channels between network nodes are assumed to be independent. In both systems, the destination coherently combines the received signals using the maximum ratio combining (MRC) or the equal gain combining (EGC) [13].

More importantly, we assume encoding of the packets using a simple binary linear block coding and mapping to non-binary linear modulation constellations prior to their transmission. For the cooperative diversity, assuming that time division channel orthogonalization and a usual two time-slot relaying protocol are used in order to avoid the interference of the transmitted packets. In case of the DF relaying, assuming that the relay uses the same encoder as the source and the same decoder as the destination. For the decoding of short length binary linear block codes, we employ the soft-decision decoding techniques developed in [14] that are referred to as the partial-order statistics decoding (POSD). These techniques achieve a good BER performance versus the implementation complexity trade-off, and, in some cases, the POSD techniques can even closely approach the performance of the maximum-likelihood (ML) decoder [14, 15].

The rest of this paper is organized as follows. Section II describes the system models including the modulation and channel coding and decoding for two systems employing the receiver and the cooperative diversity, respectively. The PEP as a key measure of the transmission reliability for the two systems under consideration is analyzed in Section III. The performance of the two systems are compared in Section IV the optimum relay locations for the system with the cooperative diversity are determined, so that it outperforms the system with the receiver diversity. Finally, conclusions are given in Section V.

2 System Model

We compare the BER performance of two communication systems. System I uses a single relay 'R' to realize a distributed diversity in order to improve the transmission reliability from a source 'S' to a destination 'D'. All nodes in System I are equipped with a single transmitting and a single receiving antenna. On the other hand, System II achieves the transmission reliability by exploiting the receiver diversity. In System II, a source 'S' with one transmitting antenna transmits information to a destination 'D' having two receiving antennas. Hence, both systems can achieve the transmission diversity of order at most two. We assume a flat fading channel model with an additive white Gaussian noise (AWGN) between any pair of network nodes, and also, that all channels are mutually independent. Without any loss of generality, we omit symbol-time indices in the expressions.For System I using the cooperative diversity, we use the following notation to describe the transmission from a node $\mathrm{X} \in \{\mathrm{S}, \mathrm{R}\}$ to a node $\mathrm{Y} \in \{\mathrm{R}, \mathrm{D}\}$, i.e.,

$d_{\mathrm{XY}} > 0$	distance between X and Y
$\alpha_{\mathrm{XY}} > 0$	path-loss coefficient
$h_{\mathrm{XY}} \in \mathcal{C}$	channel fading coefficient
$\gamma_{\mathrm{XY}} \geq 0$	instantaneous SNR at node Y
$w_{\mathrm{XY}} \in \mathcal{C}$	AWGN
$y_{\mathrm{XY}} \in \mathcal{C}$	received signal at node Y

where $\mathcal{C}$ denotes the set of complex numbers. For System II using the receiver diversity with the receiver antenna $i = 1, 2$, we use the notation,

$d > 0$	distance between S and D
$\alpha > 0$	path-loss coefficient
$h_{(i)} \in \mathcal{C}$	channel fading coefficient
$\gamma_{(i)} \geq 0$	instantaneous SNR at node D
$w_{(i)} \in \mathcal{C}$	AWGN
$y_{(i)} \in \mathcal{C}$	received signal at node D.

Furthermore, we make the following assumptions common to both systems. The channel fading coefficients h are complex-valued wide-sense stationary jointly Gaussian random processes having zero-mean and unit-variance. Thus, the channel fading amplitudes $|h|$ are Rayleigh distributed, and $\mathrm{E}[h] = 0$ and $\mathrm{E}\big[|h|^2\big] = 1$, where $\mathrm{E}[\cdot]$ is expectation, and $|\cdot|$ is the absolute value. The channel fading coefficients are either assumed to be constant and then change independently during the transmission of one codeword (corresponding to a slow block fading channel model), or they change independently for every transmitted symbol (i.e., a fast fading channel model with ideal interleaving and deinterleaving of symbols). All coefficients of AWGNs w are uncorrelated zero-mean complex-valued jointly Gaussian random processes having the *equal* variance $\sigma_{\rm w}^2 = \mathrm{E}\left[|w|^2\right] = N_0$ where N_0 is a constant one-sided power spectral density of the AWGNs.

In general, the signal amplitude attenuation due to a path-loss at distance d from the transmitter antenna is proportional to $\mathrm{const} \times d^{-\mu/2}$ where the constant is a function of the carrier frequency, and $\mu > 0$ is the path-loss exponent. Let d_0 be the reference distance at which the path-loss is equal to unity. Then, the path-loss coefficient $\alpha_{\rm XY}$ and α at the distance $d_{\rm XY}$ and d, respectively, from the transmitter antenna can be expressed as,

$$\alpha_{\rm XY} = \left(\frac{d_{\rm XY}}{d_0}\right)^{-\mu/2} \qquad \alpha = \left(\frac{d}{d_0}\right)^{-\mu/2}.$$

Since the nodes S and D are common to both systems under consideration, in the sequel, we assume that the path-loss between S and D in both systems is unity, i.e., $d_0 = d_{\rm SD} = d$. Hence, the path-loss coefficients at distances greater (smaller) than the reference distance d_0 are smaller (larger) than unity. Note that the choice of the reference distance shifts the SNR values of all links equally. Thus, one can choose an arbitrary common reference distance d_0 without biasing the BER comparisons of the two systems.

Let x denote a modulation symbol in the transmitted codeword. The modulation symbols have zero-mean and are normalized, so that the average energy per symbol $\mathrm{E}\left[|x|^2\right]$ is equal to a constant $E_s > 0$. For the cooperative diversity system with the AF relaying, the received signals at two consecutive time slots corresponding to the transmitted symbol x can be written as,

$$\begin{aligned} y_{\rm SD} &= \alpha_{\rm SD} h_{\rm SD}\, x + w_{\rm SD} \\ y_{\rm SR} &= \alpha_{\rm SR} h_{\rm SR}\, x + w_{\rm SR} \\ y_{\rm RD} &= \beta_{\rm AF}\alpha_{\rm RD} h_{\rm RD} y_{\rm SR} + w_{\rm RD} \end{aligned}$$

where $\beta_{\rm AF}$ is the amplification factor used at the relay. The amplification factor $\beta_{\rm AF}$ normalizes the average energy of the signal transmitted from the relay to be equal to E_s, i.e., [11, 18],

$$\beta_{\rm AF} = \frac{\sqrt{E_s}}{\sqrt{\mathrm{E}[|y_{\rm SR}|^2]}} = \sqrt{\frac{E_s}{\alpha_{\rm SR}^2 |h_{\rm SR}|^2 E_s + \sigma_{\rm w}^2}}$$

where expectation in the denominator is conditioned on the amplitude $|h_{\rm SR}|$. For the cooperative diversity system with the DF relaying, the received signal at the destination at the second time slot corresponding to the transmitted symbol x can be written as,

$$y_{\rm RD} = \beta_{\rm DF}\alpha_{\rm RD} h_{\rm RD}\, \hat{x} + w_{\rm RD}$$

where the relay amplification factor $\beta_{\rm DF} = 1$ and $\hat{x}$ is a re-encoded symbol at the relay. We assume that the symbol $\hat{x}$ is from the same modulation constellation as the symbol x; if $\hat{x} \neq x$, then a decoding error occurred at the relay.

For the receiver diversity system, the received signals at the two receiver antennas corresponding to the transmitted symbol x can be written as,

$$\begin{aligned} y_{(1)} &= \alpha\, h_{(1)}\, x + w_{(1)} \\ y_{(2)} &= \alpha\, h_{(2)}\, x + w_{(2)}. \end{aligned}$$

At the destination, the received signals are coherently combined using MRC or EGC. In particular, the MRC output signals are written as,

$$\begin{aligned} y &\overset{\text{System I}}{=} \frac{\beta\, \alpha_{\rm RD}\alpha_{\rm SR} h_{\rm RD}^* h_{\rm SR}^*}{\beta^2\alpha_{\rm RD}^2 |h_{\rm RD}|^2 + 1} y_{\rm RD} + \alpha_{\rm SD} h_{\rm SD}^* y_{\rm SD} \\ y &\overset{\text{System II}}{=} h_{(1)}^* y_{(1)} + h_{(2)}^* y_{(2)} \end{aligned}$$

and for EGC, the output signals are written as,

$$\begin{aligned} y &\overset{\text{System I}}{=} \frac{\mathrm{e}^{-\mathrm{j}(\angle h_{\rm RD} + \angle h_{\rm SR})}}{\sqrt{\beta^2\alpha_{\rm RD}^2 |h_{\rm RD}|^2 + 1}} y_{\rm RD} + \mathrm{e}^{-\mathrm{j} h_{\rm SD}} y_{\rm SD} \\ y &\overset{\text{System II}}{=} \mathrm{e}^{-\mathrm{j}\angle h_{(1)}} y_{(1)} + \mathrm{e}^{-\mathrm{j}\angle h_{(2)}} y_{(2)} \end{aligned}$$

where $\mathrm{j} = \sqrt{-1}$ is the imaginary unit, and $\angle(\cdot)$ denotes the phase of a complex number. Note that, since the path-loss coefficients are time-invariant, they can be used as the weighting factors of the EGC; however, in this paper, only the phase-compensating weighting factors are considered in the EGC combiner.

Recall that all the receivers in the network are assumed to have the identical time-invariant power spectral densities of the background AWGNs. The instantaneous SNR of the communication link between a pair of nodes for the system with the cooperative and the receiver diversity, respectively, is defined as,

$$\gamma_{\rm XY} = \alpha_{\rm XY}^2\, |h_{\rm XY}|^2 \gamma_b \qquad \gamma_{(i)} = \alpha^2\, |h_{(i)}|^2\, \gamma_b$$

where $\gamma_b = E_s/(N_0 \log_2 M)$ is the SNR per transmitted bit assuming an M-ary modulation constellation. In this paper, we assume that all links are subject to independent and identically distributed Rayleigh fading, and thus, the SNR of each link is exponentially distributed [13]. Provided that a channel coding of rate $R < 1$ is used at the source, the AWGNs at the relay and destination receivers have the equal variance $\sigma_{\rm w}^2 = \mathrm{E}\left[|w|^2\right] = N_0 = E_s/(R\gamma_b \log_2 M)$. Then, the instantaneous SNR at the output of the MRC combiner at the destination for the system with the cooperative and the receiver diversity, respectively, can be expressed as,

$$\gamma \overset{\text{System I}}{=} \gamma_{\text{SD}} + \frac{\gamma_{\text{SR}}\,\gamma_{\text{RD}}}{\gamma_{\text{SR}} + \gamma_{\text{RD}} + 1}$$

$$\gamma \overset{\text{System II}}{=} \gamma_{(1)} + \gamma_{(2)}.$$

In general, depending on the relay location, the average SNR at the combiner output at the destination can be larger or smaller for the cooperative diversity than for the case of the receiver diversity. However and importantly, if the path-loss is not considered (i.e., the average SNR values are location-invariant), then the average SNR of the receiver diversity is always larger than the average SNR of the cooperative diversity. In addition, note that, for a fair comparison, we assume that both the source and the relay transmits with the average energy per symbol E_s, so that the total average energy per transmitted symbol is $2E_s$ over the two time-slots whereas the total average energy per transmitted symbol for the system with the receiver diversity is E_s.

2.1 Modulation and Channel Coding and Decoding

We assume that the transmissions between nodes are realized using a linear memoryless modulation and using a linear binary block code of short block length. The encoding of information bits by a binary channel code is performed by multiplying the vector of K information bits by a binary generator matrix in order to produce a binary codeword of N encoded bits. The binary channel coding C is denoted as a triplet $(N, K, d_{\min})$ where $d_{\min}$ is the minimum Hamming distance between any two codewords, and $R = K/N$ is the code rate. The codewords are possibly interleaved and mapped to either binary phase shift keying (BPSK) symbols or to 16 quadrature amplitude modulation (QAM) symbols. For the 16QAM modulation, we assume a natural mapping of the consecutive sequences of 4 encoded bits (c_1, c_2, c_3, c_4) to the modulation symbols $x = x_I + \mathrm{j}x_Q$ such that the encoded bits (c_1, c_3) are mapped to $x_I \in \{\pm 1, \pm 3\}$, and the encoded bits (c_2, c_4) are mapped to $x_Q \in \{\pm 1, \pm 3\}$, as in paper 4 and [19].

3 Analysis of Transmission Reliability

The theoretical analysis is mathematically tractable provided that we assume a block fading channel model, i.e., the channel fading coefficients are generated independently and held constant for the transmission of each codeword. Recall that the channel fading coefficients between the network nodes are assumed to be mutually independent, and they are perfectly known at the receivers. For notational simplicity, the path-loss coefficients α are merged into the channel fading coefficients h, so that the variances $\mathrm{E}\big[|h|^2\big]$ are scaled by α^2. We denote as $g = |h|$ the amplitudes of the channel fading coefficients h. In our analysis, we consider the performance of the EGC at the destination receiver for the case of BPSK modulation. For BPSK signaling, we denote the codewords $\mathbf{0} = (0, \cdots, 0)$, $\mathbf{a} = (a_1, \cdots, a_N)$ and $\mathbf{b} = (b_1, \cdots, b_N)$ corresponding to the transmitted sequences $\mathbf{x}^{(0)} = (1, \cdots, 1)$, $\mathbf{x}^{(a)} = (x_1^{(a)}, \cdots, x_N^{(a)})$ and $\mathbf{x}^{(b)} = (x_1^{(b)}, \cdots, x_N^{(b)})$, respectively. We assume that all codewords are equally likely to be transmitted and that an all-zero codeword has been transmitted. Note that the latter assumption may slightly bias the analysis for System II due to non-linearity of the DF relaying. The ML detector at the destination receiver selects the most likely codeword corresponding to the transmitted sequence with the smallest Euclidean distance from the received sequence $\mathbf{y}$.

In general, the probability of transmission error for coded systems can be upper-bounded using a union-bound [21]. Thus, the BER of coded systems can be upper-bounded as [22],

$$\text{BER} \leq \sum_{\substack{\mathbf{a} \in C \\ \mathbf{a} \neq \mathbf{0}}} \frac{w_{\text{H}}[\mathbf{u}]}{K} \Pr(\mathbf{0} \to \mathbf{a}) \qquad (1)$$

where $w_{\text{H}}[\mathbf{u}]$ is the Hamming weight of the information vector $\mathbf{u}$ corresponding to the codeword $\mathbf{a}$ of a binary linear block code $C = (N, K, d_{\min})$. The PEP $\Pr(\mathbf{0} \to \mathbf{a})$ is the probability that the all-zero codeword $\mathbf{0}$ was transmitted, and the receiver decides between the codewords $\mathbf{0}$ and $\mathbf{a}$ that $\mathbf{a}$ has been transmitted. Provided that the PEP $\Pr(\mathbf{0} \to \mathbf{a})$ can be expressed as a function of the Hamming weight $w_{\text{H}}[\mathbf{a}]$, the union bound (1) can be evaluated more effectively using a weight enumerator of the code C [20]. More importantly, note that the union bound (1) is dominated by the largest PEP $\Pr(\mathbf{0} \to \mathbf{a})$. Thus, in the sequel, we evaluate the PEP $\Pr(\mathbf{0} \to \mathbf{a})$ rather than the overall union bound (1) as a key measure of the transmission reliability for the coded communication systems.

3.1 System I with the AF Diversity

Assuming System I with the AF relaying, the output signal of the EGC at the destination receiver can be written as,

$$\begin{aligned} y_i &= \Re\left\{ \left(g_{\mathrm{SD}} + \frac{\beta_{\mathrm{AF}} g_{\mathrm{RD}} g_{\mathrm{SR}}}{\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}} \right) x_i^{(0)} + \frac{\beta_{\mathrm{AF}} g_{\mathrm{RD}} w_{\mathrm{SR}i} + w_{\mathrm{RD}i}}{\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}} + w_{\mathrm{SD}i} \right\} \\ &= \left(g_{\mathrm{SD}} + \frac{g_{\mathrm{RD}} g_{\mathrm{SR}}}{\sqrt{g_{\mathrm{RD}}^2 + g_{\mathrm{SR}}^2 + c_1}} \right) x_i^{(0)} + w_{\mathrm{AF}i} \\ &= g_{\mathrm{AF}} x_i^{(0)} + w_{\mathrm{AF}i} \end{aligned}$$

where $i = 1, 2, \cdots, N$ is the symbol index in the transmitted codeword, $\Re\{\cdot\}$ is the real part of a complex number, $w_{\mathrm{AF}i}$ is an equivalent zero-mean AWGN having the variance $\mathrm{E}\big[|w_{\mathrm{AF}i}|^2\big] = \sigma_{\mathrm{w}}^2 = N_0$, and $c_1 = N_0/E_s$ is the inverse of the SNR per transmitted symbol. Note that the signal received from the relay is normalized by the factor $\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}$ in order to make the AWGN variances of the two diversity signals before combining equal. Given the value of g_{AF}, the conditional PEP of System II with the AF relaying is calculated as the probability that the Euclidean distance $w_{\mathrm{E},0}$ from the received sequence $\mathbf{y}$ for the codeword $\mathbf{0}$ is greater than the Euclidean distance from $\mathbf{y}$ for the codeword $\mathbf{a}$, i.e.,

$$\begin{aligned} \Pr(\mathbf{0} \to \mathbf{a} | g_{\mathrm{AF}}) &= \Pr\big(w_{\mathrm{E},0}^2 > w_{\mathrm{E},a}^2\big) \\ &= \Pr\left(\sum_{i=1}^{N} \left(y_i - g_{\mathrm{AF}} x_i^{(0)} \right)^2 > \sum_{i=1}^{N} \left(y_i - g_{\mathrm{AF}} x_i^{(a)} \right)^2 \right) \\ &= \Pr\left(\sum_{i=1}^{N} -g_{\mathrm{AF}}^2 (x_i^{(a)} - x_i^{(0)})^2 + 2 g_{\mathrm{AF}} (x_i^{(a)} - x_i^{(0)}) w_{\mathrm{AF}i} > 0 \right). \end{aligned}$$

Since, for a zero mean unit variance Gaussian random variable W, the probability $\Pr(W > w) = Q(w)$ where $Q(\cdot)$ is the Q-function [13], we have that,

$$\begin{aligned} \Pr(\mathbf{0} \to \mathbf{a} | g_{\mathrm{AF}}) &= \Pr\left(W > g_{\mathrm{AF}} \frac{\sqrt{\sum_{i=1}^{N} (x_i^{(a)} - x_i^{(0)})^2}}{2\sqrt{N_0}} \right) \\ &= Q\left(g_{\mathrm{AF}} \frac{w_{\mathrm{E}}\big[\mathbf{x}^{(a)}, \mathbf{x}^{(0)}\big]}{2\sqrt{N_0}} \right) = Q\Big(g_{\mathrm{AF}} \sqrt{w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b} \Big) \end{aligned}$$

where $w_{\mathrm{E}}\big[\mathbf{x}^{(a)}, \mathbf{x}^{(0)}\big]$ is the Euclidean distance between the vectors $\mathbf{x}^{(a)}$ and $\mathbf{x}^{(0)}$. Then, the PEP is evaluated as,

$$\Pr(\mathbf{0} \to \mathbf{a}) = \int_0^\infty \Pr(\mathbf{0} \to \mathbf{a} | z)\, f_{g_{\mathrm{AF}}}(z) \mathrm{d}z \tag{2}$$

where $f_{g_{\mathrm{AF}}}(z)$ is the probability density function (PDF) of g_{AF}. In general, a closed form expression for $f_{g_{\mathrm{AF}}}(z)$ is difficult to obtain. However, since, always, $g_{\mathrm{AF}} \leq g_{\mathrm{SD}} + \min(g_{\mathrm{RD}}, g_{\mathrm{SR}}) = \tilde{g}_{\mathrm{AF}}$, and the channel fading amplitudes g_{SD}, g_{SR} and g_{RD} are independent and have the variances σ_{SD}^2, σ_{SR}^2 and σ_{RD}^2, respectively, we can lower-bound the PEP (2), i.e.,

$$\Pr(\mathbf{0} \to \mathbf{a}) \geq \int_0^\infty \Pr(\mathbf{0} \to \mathbf{a} | z)\, f_{\tilde{g}_{\mathrm{AF}}}(z) \mathrm{d}z$$

where, after lengthy manipulations, the closed form expression of the PDF $f_{\tilde{g}_{\mathrm{AF}}}(z)$ is shown at the top of this page, $c_2 = \sigma_{\mathrm{SD}}^2 \sigma_{\mathrm{SR}}^2 + \sigma_{\mathrm{RD}}^2 (\sigma_{\mathrm{SD}}^2 + \sigma_{\mathrm{SR}}^2)$, $c_3 = \log(\mathrm{e})$ and the function $\mathrm{erf}(x) = 1 - 2Q\big(\sqrt{2}x\big)$.

$$
\begin{aligned}
f_{\tilde{g}_{\mathrm{AF}}}(z) = \frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2c_2^{5/2}c_3^{3/2}} \mathrm{e}^{-(\sigma_{\mathrm{RD}}^{-2}+\sigma_{\mathrm{SD}}^{-2}+\sigma_{\mathrm{SR}}^{-2})z^2/2} \Big(- 2c_2^2\sigma_{\mathrm{SR}}^2\sigma_{\mathrm{SD}}^2\sqrt{c_3}\; z\mathrm{e}^{\frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}z^2} - \\
2c_2^2\sqrt{c_3}\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2 z\mathrm{e}^{\frac{1}{2\sigma_{\mathrm{SD}}^2}z^2} - 2c_2^2\sqrt{c_3}\sigma_{\mathrm{SD}}^2\sigma_{\mathrm{SR}}^2 z\mathrm{e}^{\frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}z^2} + \\
\sqrt{2\pi}\sigma_{\mathrm{SD}}\sigma_{\mathrm{SR}}\sigma_{\mathrm{RD}}\mathrm{e}^{\left(\frac{1}{\sigma_{\mathrm{RD}}^2}+\frac{1}{\sigma_{\mathrm{SR}}^2}+\frac{\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}{\sigma_{\mathrm{SD}}^2c_2}\right)\frac{z^2}{2}} \times \\
(\sigma_{\mathrm{SD}}^2\sigma_{\mathrm{SR}}^2 + \sigma_{\mathrm{RD}}^2(\sigma_{\mathrm{SD}}^2+\sigma_{\mathrm{SR}}^2)\; c_3z^2) \left(\mathrm{erf}\left(\frac{\sigma_{\mathrm{RD}}\sigma_{\mathrm{SR}}\sqrt{c_3}z}{4\sigma_{\mathrm{SD}}v}\right) + \mathrm{erf}\left(\frac{\sigma_{\mathrm{SD}}(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)\sqrt{c_3}z}{4\sigma_{\mathrm{RD}}\sigma_{\mathrm{SR}}\sqrt{c_2}}\right)\right)
\end{aligned}
$$

3.2 System I with the DF Diversity

In order to analyze the PEP of the DF relaying, we assume that the source transmits the all-zero codeword $\mathbf{0}$, however, the relay decodes and forwards a codeword $\mathbf{b}$. In this case, the EGC output signal at the destination receiver is written as,

$$
\begin{aligned}
y_i &= \Re\Big\{ g_{\mathrm{SD}}x_i^{(0)} + w_{\mathrm{SD}i} + g_{\mathrm{RD}}x_i^{(b)} + w_{\mathrm{RD}i} \Big\} \\
&= g_{\mathrm{SD}}x_i^{(0)} + g_{\mathrm{RD}}x_i^{(b)} + w_{\mathrm{DF}i}
\end{aligned}
$$

where $w_{\mathrm{DF}i}$ is an equivalent zero-mean AWGN with the variance $\mathrm{E}\big[|w_{\mathrm{DF}i}|^2\big] = \sigma_{\mathrm{w}}^2 = N_0$. The PEP of the destination receiver conditioned on the values of the channel fading amplitudes g_{SD}, g_{SR} and g_{RD} is then calculated as,

$$
\Pr(\mathbf{0} \to \mathbf{a}|g_{\mathrm{SD}}, g_{\mathrm{SR}}, g_{\mathrm{RD}}) = \sum_{\mathbf{b}\in C} \Pr(\mathbf{0} \to \mathbf{a}|\mathbf{b}, g_{\mathrm{SD}}, g_{\mathrm{RD}}) \Pr(\mathbf{0} \to \mathbf{b}|g_{\mathrm{SR}}) \tag{3}
$$

where $\Pr(\mathbf{0} \to \mathbf{b}|g_{\mathrm{SR}})$ is the conditional PEP that the relay decodes the codeword $\mathbf{b}$. The first conditional PEP in (3) is again equal to the probability that the Euclidean distance $w_{\mathrm{E},0}$ from the received sequence $\mathbf{y}$ for the all-zero codeword is greater than the Euclidean distance $w_{\mathrm{E},a}$ corresponding to the codeword $\mathbf{a}$, i.e.,

$$
\begin{aligned}
&\Pr(\mathbf{0} \to \mathbf{a}|\mathbf{b}, g_{\mathrm{SD}}, g_{\mathrm{RD}}) = \Pr\big(w_{\mathrm{E},0}^2 > w_{\mathrm{E},a}^2\big) \\
&= \Pr\left(\sum_{i=1}^{N} \Big(y_i - (g_{\mathrm{SD}} + g_{\mathrm{RD}})x_i^{(0)} \Big)^2 > \sum_{i=1}^{N} \Big(y_i - (g_{\mathrm{SD}} + g_{\mathrm{RD}})x_i^{(a)} \Big)^2 \right) \\
&= \Pr\left(\sum_{i=1}^{N} (g_{\mathrm{RD}}t_i - g_{\mathrm{SD}}s_i)s_i + 2s_i w_{\mathrm{DF}i} > 0 \right) \\
&= Q\left(\frac{\sum_{i=1}^{N}(g_{\mathrm{SD}}s_i - g_{\mathrm{RD}}t_i)s_i}{2\sqrt{N_0\sum_{i=1}^{N} s_i^2}} \right),
\end{aligned} \tag{4}
$$

where we defined, $s_i = x_i^{(a)} - x_i^{(0)}$ and $t_i = 2x_i^{(b)} - x_i^{(a)} - x_i^{(0)}$. Assuming that $x_i^{(0)} = 1$ for $\forall i$, we can show that, for any values of g_{SD} and g_{RD}, the argument of the Q-function in (4) is, in general, increasing with the Hamming distance between the codewords $\mathbf{a}$ and $\mathbf{b}$. The argument of the Q-function in (4) is minimized for $\mathbf{a} = \mathbf{b}$ (i.e., the vectors are component-wise identical) while $\mathbf{0} \neq \mathbf{a}$ which corresponds to the worst case scenario when the value of the PEP defined in (4) is maximized. On the other hand, we can show that, for any values of g_{SD} and g_{RD}, the value of the PEP (4) is minimized provided that $\mathbf{b} = \mathbf{0}$ (i.e., the relay correctly decodes the codeword transmitted from the source). This also indicate that the ability of the relay to correctly decode the transmitted codeword from the source has a major effect upon the overall probability of transmission error of the cooperative system.

Denote as $w_{\mathrm{E},b}$ the Euclidean distance from the received sequence $\mathbf{y}_{\mathrm{SR}}$ for the codeword $\mathbf{b}$ at the relay receiver. Then, the PEP $\Pr(\mathbf{0} \to \mathbf{b}|g_{\mathrm{SR}})$ for the link from the source to the relay can be expressed as [7],

$$\begin{aligned}\Pr(\mathbf{0}\to\mathbf{b}|g_{\rm SR}) &= \Pr\big(w_{{\rm E},0}^2 > w_{{\rm E},b}^2\big)\\ &= \Pr\left(\sum_{i=1}^{N}\Big(y_{{\rm SR},i}-g_{\rm SR}x_i^{(0)}\Big)^2 > \sum_{i=1}^{N}\Big(y_{{\rm SR},i}-g_{\rm SR}x_i^{(b)}\Big)^2\right)\\ &= Q\left(\frac{g_{\rm SR}w_{\rm E}\big[\mathbf{x}^{(0)},\mathbf{x}^{(b)}\big]}{2\sqrt{N_0}}\right) = Q\Big(g_{\rm SR}\sqrt{w_{\rm H}[\mathbf{b}]\,\gamma_b}\Big)\end{aligned}$$

where $y_{{\rm SR},i}$ is the received signal at the relay, $w_{\rm H}[\mathbf{b}]$ is the Hamming weight of the codeword $\mathbf{b}$, and $w_{\rm E}\big[\mathbf{x}^{(0)},\mathbf{x}^{(b)}\big]$ is the Euclidean distance between the modulated sequences corresponding to the vectors $\mathbf{0}$ and $\mathbf{b}$.

Using (3), the PEP averaged over the independent Rayleigh distributed channel fading amplitudes $g_{\rm SD}$, $g_{\rm SR}$ and $g_{\rm RD}$ is expressed as,

$$\begin{aligned}\Pr(\mathbf{0}\to\mathbf{a}) &= \iiint_0^\infty \Pr(\mathbf{0}\to\mathbf{a}|u,v,r)\, f_{g_{\rm SD}}(u)f_{g_{\rm SR}}(v)f_{g_{\rm RD}}(r){\rm d}u{\rm d}v{\rm d}r\\ &= \sum_{\mathbf{b}\in C}\iint_0^\infty \Pr(\mathbf{0}\to\mathbf{a}|\mathbf{b},u,v)\, f_{g_{\rm SD}}(u)f_{g_{\rm SR}}(v){\rm d}u{\rm d}v \int_0^\infty \Pr(\mathbf{0}\to\mathbf{b}|r)\, f_{g_{\rm RD}}(r){\rm d}r\\ &= \sum_{\mathbf{b}\in C}\Pr(\mathbf{0}\to\mathbf{a}|\mathbf{b})\Pr(\mathbf{0}\to\mathbf{b})\end{aligned}$$

Let the argument of the Q-function in (4) be a random variable,

$$Z = C_1 g_{\rm SD} - C_2 g_{\rm RD}$$

where the constants,

$$C_1 = \frac{\sqrt{\sum_{i=1}^N s_i^2}}{2\sqrt{N_0}} \quad \text{and} \quad C_2 = \frac{\sum_{i=1}^N s_i t_i}{2\sqrt{N_0\sum_{i=1}^N s_i^2}}.$$

Then, the average PEP $\Pr(\mathbf{0}\to\mathbf{a}|\mathbf{b})$ can be evaluated as,

$$\Pr(\mathbf{0}\to\mathbf{a}|\mathbf{b}) = \int_{-\infty}^{\infty}\Pr(\mathbf{0}\to\mathbf{a}|\mathbf{b},z)\, f_Z(z){\rm d}z.$$

The PDF of the random variable Z can be obtained by conditioning and integration [23], i.e.,

$$f_Z(z) = \begin{cases} \frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1/2}(k_1+k_2)^{-3} f_1(z) & z\geq 0\\ \frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1/2}(k_1+k_2)^{-3}\left(\frac{k_1}{k_5+k_4}\right)^{-1/2} f_2(z) & z<0\end{cases}$$

where $k_1 = C_2^2\sigma_1^2$, $k_2 = C_2^2\sigma_2^2$, $k_3 = C_1^4\sigma_1^4$, $k_4 = C_2^4\sigma_2^4$, $k_5 = C_1^2C_2^2\sigma_1^2\sigma_2^2$, and,

$$\begin{aligned}f_1(z) = {\rm e}^{-(\frac{1}{k_1}+\frac{2}{k_2})\frac{z}{2}}\Bigg(&2k_3\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}{\rm e}^{\frac{z^2}{k_2}}z + 2k_5\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}{\rm e}^{\frac{z^2}{k_2}}z\\ &+\sqrt{2\pi}(k_1+k_2)(k_1+k_2-z^2){\rm e}^{\frac{(2k_3+2k_5k_4)z^2}{2k_5(k_1+k_2)}}\\ &-\sqrt{\frac{2\pi k_2}{k_1}+2\pi}\sqrt{k_1^2+k_1k_2}(k_1+k_2-z^2)\,{\rm erf}\left(\frac{\sqrt{k_2}z}{\sqrt{2k_3+2k_5}}\right)\Bigg)\end{aligned}$$

$$\begin{aligned}f_2(z) = {\rm e}^{\frac{-z^2}{k_2}}(k_1+k_2)\sqrt{\frac{k_1}{k_5+k_4}}\Bigg(&-2k_2\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}z + \sqrt{2\pi}(k_1+k_2-z^2)e^{\frac{(2k_1+k_2)z^2}{2k_5+2k_4}}\Bigg)\\ &+\sqrt{\frac{2\pi}{k_1}+\frac{\pi}{k_2}}(k_1^2+k_1k_2-k_1z^2){\rm e}^{\frac{(2k_1+k_2)z^2}{2k_5+2k_4}}\,{\rm erf}\left(\frac{\sqrt{k_1}z}{\sqrt{2k_5+2k_4}}\right).\end{aligned}$$

The average PEP $\Pr(\mathbf{0} \to \mathbf{b})$ can be obtained by using the Chernoff bound $Q(x) \leq \frac{1}{2}\mathrm{e}^{-x^2/2}$, for example, as in [8], or by using the Prony approximation $Q(x) \doteq 0.208\mathrm{e}^{-0.971x^2} + 0.147\mathrm{e}^{-0.525x^2}$ as in [22]. Assuming the latter expression, the average PEP is approximately equal to,

$$\Pr(\mathbf{0} \to \mathbf{b}) \doteq \frac{0.208}{1.942\, w_{\mathrm{H}}[\mathbf{b}]\, \sigma^2_{\mathrm{SR}}\gamma_b + 1} + \frac{0.147}{1.050\, w_{\mathrm{H}}[\mathbf{b}]\, \sigma^2_{\mathrm{SR}}\gamma_b + 1}$$

where γ_b is the SNR per encoded binary symbol.

3.3 System II with the Rx Diversity

Assuming the receiver diversity without relay, the output signal of the EGC at the destination receiver can be written as,

$$\begin{aligned} y_i &= \Re\Big\{ g_{(1)} x_i^{(0)} + g_{(2)} x_i^{(0)} + w_{(1)_i} + w_{(2)_i} \Big\} \\ &= (g_{(1)} + g_{(2)}) x_i^{(0)} + w_{\mathrm{Rx}i} = g_{\mathrm{Rx}} x_i^{(0)} + w_{\mathrm{Rx}i} \end{aligned}$$

where $w_{\mathrm{Rx}i}$ is an equivalent zero-mean AWGN with the variance $\mathrm{E}\big[|w_{\mathrm{Rx}i}|^2\big] = \sigma^2_{\mathrm{w}} = N_0$. The PEP of System II is obtained similarly as for the source to relay link in System I. Thus, conditioned on the channel fading amplitude g_{Rx}, and BPSK signaling, the PEP is evaluated as,

$$\begin{aligned} \Pr(\mathbf{0} \to \mathbf{a} | g_{\mathrm{Rx}}) &= \Pr\big(w^2_{\mathrm{E},0} > w^2_{\mathrm{E},a}\big) \\ &= \Pr\left(\sum_{i=1}^{N} \Big(y_i - g_{\mathrm{Rx}} x_i^{(0)} \Big)^2 > \sum_{i=1}^{N} \Big(y_i - g_{\mathrm{Rx}} x_i^{(a)} \Big)^2 \right) \\ &= Q\Big(g_{\mathrm{Rx}} \sqrt{w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b}\, \Big). \end{aligned}$$

Consequently, the average PEP is calculated using the integration,

$$\Pr(\mathbf{0} \to \mathbf{a}) = \int_0^\infty \Pr(\mathbf{0} \to \mathbf{a} | z)\, f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z.$$

The integration to obtain the average PEP $\Pr(\mathbf{0} \to \mathbf{a})$ can be carried out using the Prony approximation method [22]. In particular, the conditional PEP is approximately equal to,

$$Q\Big(g_{\mathrm{Rx}} \sqrt{w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b} \Big) \doteq 0.208\mathrm{e}^{-0.971 g^2_{\mathrm{Rx}} w_{\mathrm{H}}[\mathbf{a}]\gamma_b} + 0.147\mathrm{e}^{-0.525 g^2_{\mathrm{Rx}} w_{\mathrm{H}}[\mathbf{a}]\gamma_b}$$

so that the average PEP is calculated as,

$$\Pr(\mathbf{0} \to \mathbf{a}) = 0.208 \int_0^\infty \mathrm{e}^{-A_1^2 z^2} f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z + 0.147 \int_0^\infty \mathrm{e}^{-A_2^2 z^2} f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z \qquad (5)$$

where $A_1 = 0.971\ \ w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b$ and $A_2 = 0.525\ w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b$.

The PDF $f_{g_{\mathrm{Rx}}}(z)$ of the channel fading amplitude g_{Rx} is again obtained by conditioning and integration. Thus, assuming the independent Rayleigh distributed channel fading amplitudes $g_{(1)}$ and $g_{(2)}$ of the variances $\sigma^2_{(1)}$ and $\sigma^2_{(2)}$, respectively, we obtain the PDF,

$$f_{g_{\mathrm{Rx}}}(z) = \frac{z \sigma^2_{(1)}}{V^2}\, \mathrm{e}^{-\frac{z^2}{2\sigma^2_{(1)}}} + \sqrt{\frac{\pi}{2}} \frac{z^2 - V}{V^{5/2}} r \sigma_{(1)} \mathrm{e}^{-\frac{z^2}{2V}} \left(1 + \mathrm{erf}\left(\frac{\sigma_{(2)} z}{\sqrt{2V}\sigma_{(1)}} \right) \right)$$

where $V = \sigma^2_{(1)} + \sigma^2_{(2)}$ is the variance of the EGC amplitude g_{Rx}. Finally, a closed form expression for the average PEP (5) based on the Prony approximation method is obtained using the following integration, i.e.,

$$\begin{aligned} I_{\sigma_{(1)},\sigma_{(2)}}(a) &= \int_0^\infty \mathrm{e}^{-a\,z^2} f_{g_{\mathrm{Rx}}}(z)\mathrm{d}z \\ &= \frac{2\sigma_{(1)}V^{3/2}}{1+2aV} + \frac{4a\sigma_{(2)}V^{5/2}}{(1+2aV)^{3/2}}\left(\arctan\left(\sqrt{1+2aV}\frac{\sigma_{(1)}}{\sigma_{(2)}}\right)-\pi\right) \end{aligned}$$

where $a > 0$ is a real constant, and V was defined previously. The PEP (5) is then computed as,

$$\Pr(\mathbf{0}\to\mathbf{a}) = 0.208\ I_{\sigma_{(1)},\sigma_{(2)}}(0.971\ w_{\mathrm{H}}[\mathbf{a}]\,\gamma_b) + 0.147\ I_{\sigma_{(1)},\sigma_{(2)}}(0.525\ w_{\mathrm{H}}[\mathbf{a}]\,\gamma_b)\,.$$

4 Performance Comparison of System I and System II

We use the PEP expressions obtained in the previous section to compare the error rate performances of System I and System II with the cooperative and the receiver diversity is investigated, respectively. In particular, the effect of the relay location on the performance of the cooperative diversity, and determine geographical areas for positioning the relay in which the relaying can outperform the conventional receiver diversity. Recall that the upper-bound of the BER (1) is dominated by the largest PEP $\Pr(\mathbf{0}\to\mathbf{a})$, so that we can consider the PEP $\Pr(\mathbf{0}\to\mathbf{a})$ to be the key performance metric of the system. More importantly, assuming our analysis in Section 6.3, it can be shown that, for System I as well as System II, the largest PEP $\Pr(\mathbf{0}\to\mathbf{a})$ corresponds to the codeword $\mathbf{a}$ of the minimum Hamming weight $w_{\mathrm{H}}[\mathbf{a}] = d_{\min}$.

Denote as $\mathrm{PEP}_{\mathrm{AF}}$, $\mathrm{PEP}_{\mathrm{DF}}$ and $\mathrm{PEP}_{\mathrm{Rx}}$ the PEPs $\Pr(\mathbf{0}\to\mathbf{a})$ of System I with the AF relaying, System I with the DF relaying and System II with the receiver diversity, respectively. The PEPs $\mathrm{PEP}_{\mathrm{AF}}$ and $\mathrm{PEP}_{\mathrm{DF}}$ are the relay location dependent. The relay location is denoted as a triplet $(d_{\mathrm{SR}}/d_0, d_{\mathrm{RD}}/d_0, d_{\mathrm{SD}}/d_0)$ where d_0 is the reference distance. Recall that, without loss of generality, we assume $d_0 = d_{\mathrm{SD}}$, i.e., the relay location is given by the triplet $(d_{\mathrm{SR}}/d_{\mathrm{SD}}, d_{\mathrm{RD}}/d_{\mathrm{SD}}, 1)$. For System I, the distance between the source and the destination is a scalar variable d; we assume that $d/d_{\mathrm{SD}} = 1$. Thus, for System I as well as System II, the path-loss between the source and the destination is unity.

Fig. 1 shows an excellent agreement between the mathematical expressions obtained in Section 6.3 and the computer simulations for the PEP $\Pr(\mathbf{0}\to\mathbf{a})$ of System I with the DF relaying assuming independent slow Rayleigh fading channels, BPSK modulation, and a codeword $\mathbf{a}$ of the Hamming weight $d_{\min}$ for the BCH codes $(31, 16, 7)$ and $(32, 26, 4)$. Fig. 2 compares the PEPs $\Pr(\mathbf{0}\to\mathbf{a})$ of System II with the two receiver antennas and System I with the DF relaying assuming again independent slow Rayleigh fading channels, BPSK modulation, and a codeword $\mathbf{a}$ of the Hamming weight $d_{\min}$ for the BCH code $(31, 16, 7)$. Note that the distance between the source and the destination is normalized to 1. The relay location denoted as $(1, 1, 1)$ corresponds to the case when the path-loss is not considered. Provided that the path-loss is not considered, the receiver diversity always outperforms the DF diversity as one may intuitively expect. Relaying outperforms the receiver diversity, particularly at smaller values of the SNR. This is further confirmed by the PEP values in Fig. 3 versus the relay location $(d_{\mathrm{SR}}/d_{\mathrm{SD}}, 1 - d_{\mathrm{SR}}/d_{\mathrm{SD}}, 1)$ at a constant SNR $\gamma_b = 9$dB. More importantly, we observe from Fig. 3 that the relay located closer to the source achieves a better PEP performance than the relay located at the center between the source and the destination (cf. Fig. 5). Thus, the optimum relay location has to trade-off the error propagation due to the DF relaying and the path-loss attenuations between the nodes, and it is also influenced by the particular channel code used. Assuming the same parameters and settings as in Fig. 2 and Fig. 3, the PEP performance of System I with the AF relaying is shown in Fig. 4.

Also a numerical examples are presented for the overall BER performances of System I and System II. We consider uncoded as well as coded transmissions from the source to the destination using the BCH systematic codes $(31, 16, 7)$ and $(32, 16, 8)$ and BPSK and 16QAM modulations. We employ the POSD decoder at the destination and also at the relay provided that the DF relaying is used. The POSD is optimized to achieve the best possible BER performance for the given decoding complexity [14]. In particular, for both BCH codes considered, the POSD searches two disjoint segments of 6 and 10 ordered information bits assuming at most 1 and 3 errors in each segment, respectively. We use the notation '2Rx' to denote the two antenna receiver diversity, and the notation '1Rx' to refer to the scenario where the destination is equipped with a single receiving antenna.

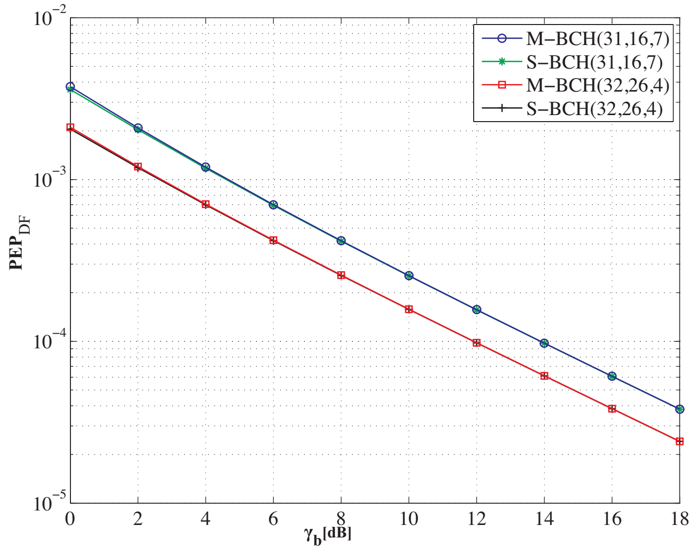

Figure 1: The PEP $\Pr(\mathbf{0} \to \mathbf{a})$ for System I with the DF relaying, the BCH $(31, 16, 7)$ and $(32, 26, 4)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination (M-mathematical expression, S-simulation).

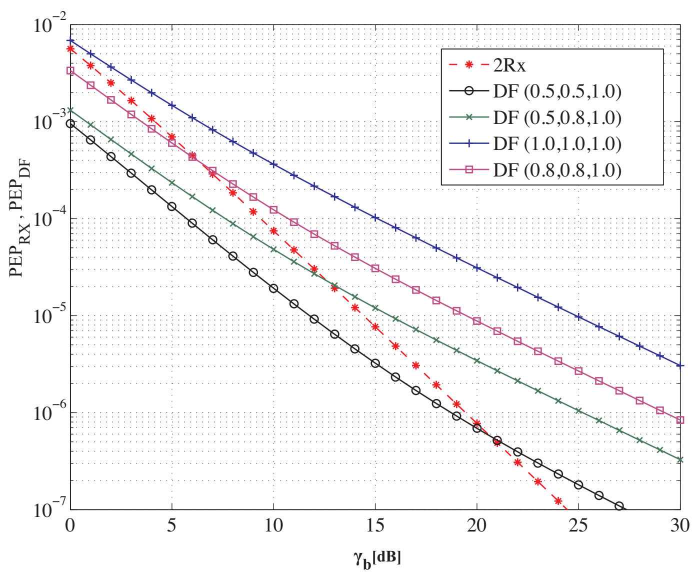

Figure 2: The PEP $\Pr(\mathbf{0} \to \mathbf{a})$ for System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination.

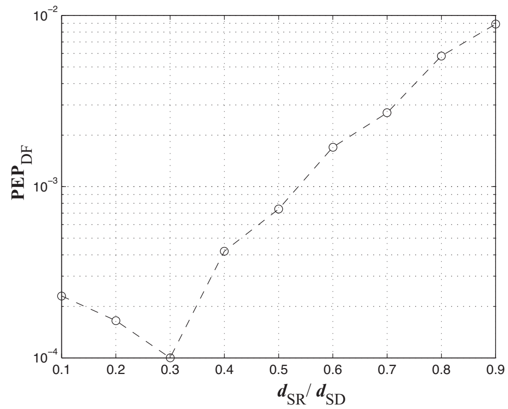

Figure 3: The PEP $\Pr(\mathbf{0} \to \mathbf{a})$ for System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, the EGC at the destination, the normalized distance $d_{\mathrm{RD}}/d_{\mathrm{SD}} = 1 - d_{\mathrm{SR}}/d_{\mathrm{SD}}$, and the SNR $\gamma_b = 9$dB.

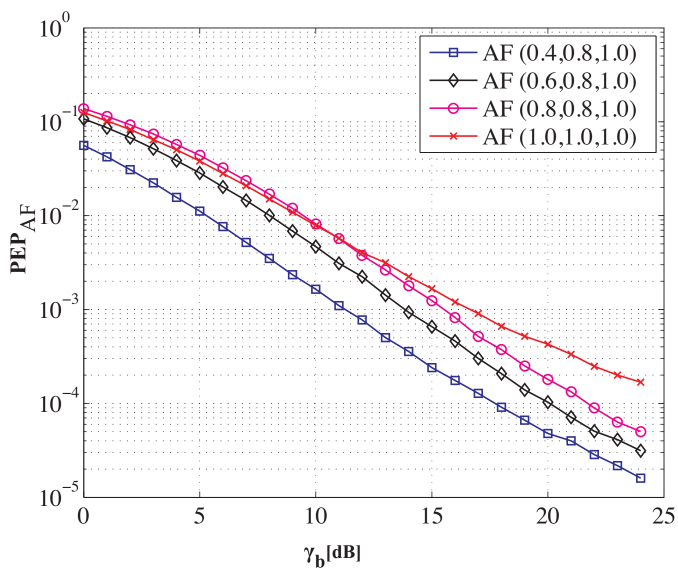

Figure 4: The PEP $\Pr(\mathbf{0} \to \mathbf{a})$ for System I with the AF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination.

Fig. 5 compares the BER performances of System I with the AF relaying and the conventional receiver diversity assuming MRC at the destination. We observe that, for some relay locations, the AF relaying outperforms the conventional receiver diversity. The best BER performance of the AF relaying is achieved when the relay is located in the center between the source and the destination. On the other hand, as intuitively expected, the BER performance of the AF relaying deteriorates significantly when the relay is located at larger distances away from the source and the destination. In addition, we observe that the channel coding benefits significantly from the available diversity gain due to the relaying and all relay locations or due to the multiple receiver antennas.

The BER performance of the DF relaying is shown in Fig. 6 assuming the same parameters and relay locations as in Fig. 5. Unlike for the AF relaying in Fig. 6, we observe from Fig. 6 that the BER performance of the DF relaying is much more relay location dependent than the BER performance of the AF relaying, and such dependence is even more pronounced for higher order modulations. In addition, as already indicated in Fig. 3, the optimum relay location for the DF relaying is found, in general, closer to the source than to the destination in order to suppress the detrimental effect of error propagation due to erroneous decoding at the relay. Further examples of the BER for the DF relaying over fast and slow Rayleigh fading channels are shown in Fig. 7 and Fig. 8. We can again observe that there exist geographical areas of the relay locations where the conventional receiver diversity outperforms the DF relaying for all SNR values. On the other hand, also it is observed that, for sufficiently large SNR values, the conventional receiver diversity outperforms the DF relaying for all relay locations considered. Furthermore, we observe from Fig. 2 and Fig. 5–Fig. 8 that, particularly for higher order modulations and the DF relaying, System I does not achieve the diversity order of System II.

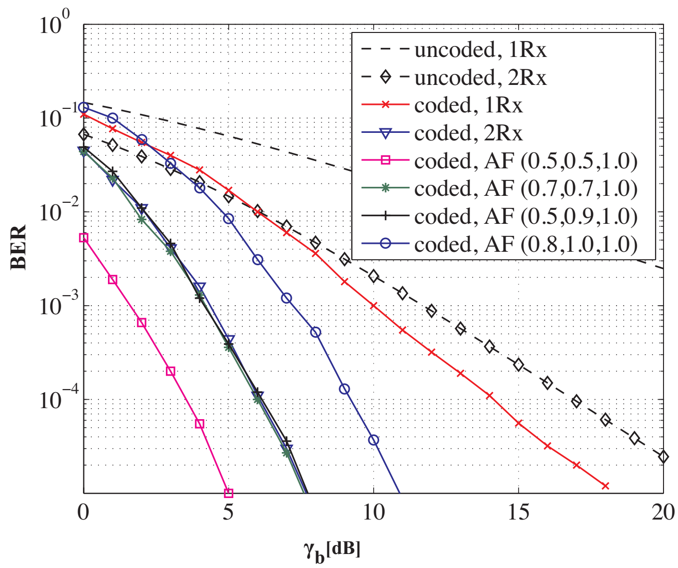

Figure 5: The BER of the BCH $(31, 16, 7)$ coded BPSK and the AF relaying and the receiver diversity with the MRC at the destination for several relay locations for fast Rayleigh fading channels.

4.1 Optimum Relay Locations

The performance results in Fig. 1–Fig. 8 indicate that the relay location significantly affects the BER performance of System I with the cooperative diversity. We determine the optimum relay locations in the sense that System I with the cooperative diversity outperforms System II with the receiver diversity. In particular, we evaluate the PEP differences,

$$\Delta\mathrm{PEP}_{\mathrm{Rx-AF}} = \mathrm{PEP}_{\mathrm{Rx}} - \mathrm{PEP}_{\mathrm{AF}} \qquad (6a)$$

$$\Delta\mathrm{PEP}_{\mathrm{Rx-DF}} = \mathrm{PEP}_{\mathrm{Rx}} - \mathrm{PEP}_{\mathrm{DF}}. \qquad (6b)$$

Hence, if $\Delta\mathrm{PEP}_{\mathrm{Rx-AF}} > 0$ or $\Delta\mathrm{PEP}_{\mathrm{Rx-DF}} > 0$, then the cooperative diversity with the AF or the DF relaying, respectively, outperforms the second order receiver diversity. The relay positions for which the PEP differences (6a) and (6b) are greater than zero are obtained numerically by sampling the two-dimensional space of all possible relay locations. Examples of the PEP differences (6a) and (6b) versus the relay locations $(d_{\mathrm{SR}}/d_{\mathrm{SD}}, d_{\mathrm{RD}}/d_{\mathrm{SD}}, d_{\mathrm{SD}})$ for the SNR $\gamma_b = 9$dB are shown in Fig. 9 and Fig. 10, respectively. More importantly, if the SNR exceeds a certain threshold value, then, for any relay location, the PEP differences (6a) and (6b) will always be negative, i.e., the receiver diversity will outperform the cooperative diversity.

In general, determination of the exact boundaries of the geographical areas of the relay locations where System I outperforms System II appears to be mathematically intractable, particularly, when the channel coding is employed. However, by evaluation of our extensive numerical results including those that are not presented in this paper, we make the following proposition.

Proposition 1. *Assuming path-loss attenuations of the transmitted signals and independent channel fadings between the transmitter and the receiver antennas, the*

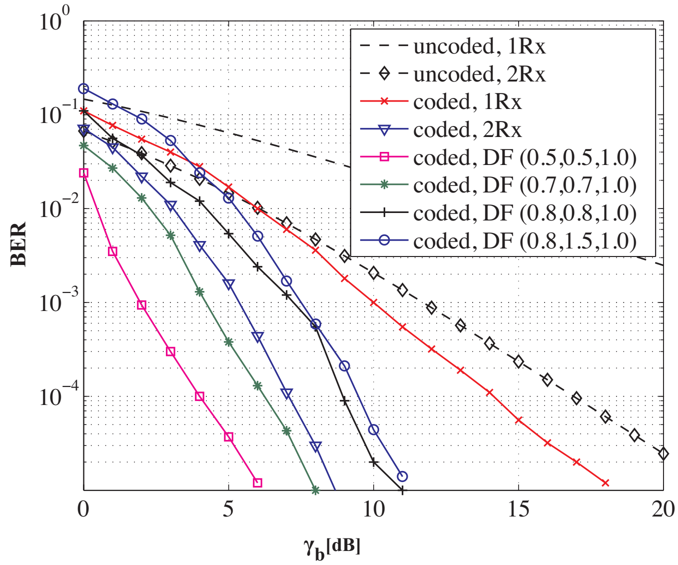

Figure 6: The BER of the BCH $(31, 16, 7)$ coded BPSK and the DF relaying and the receiver diversity with the MRC at the destination for several relay locations for fast Rayleigh fading channels.

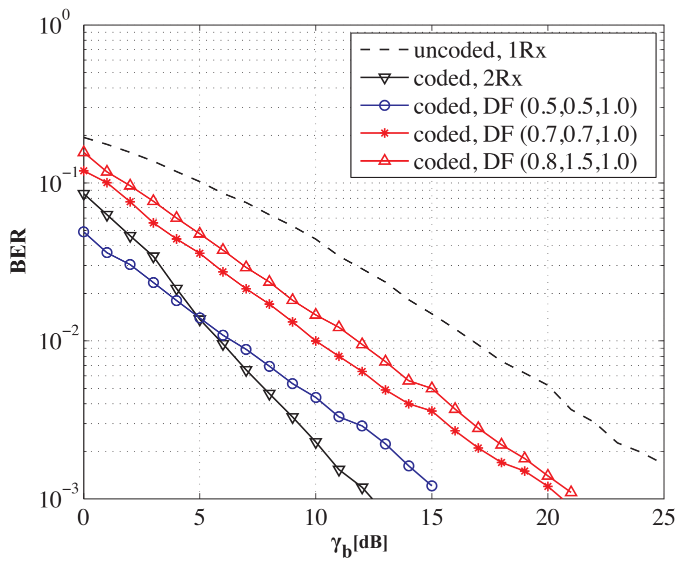

Figure 7: The BER of the BCH $(32, 16, 8)$ coded 16QAM and the DF relaying and the receiver diversity with the EGC at the destination for several relay locations for fast Rayleigh fading channels.

cooperative diversity with a single relay outperforms the two antenna receiver diversity provided that the relay location $(d_{\mathrm{SR}}/d_{\mathrm{SD}}, d_{\mathrm{RD}}/d_{\mathrm{SD}}, 1)$ *is constrained as,*

$$\begin{gathered} d_{\mathrm{SR}}/d_{\mathrm{SD}} < 1.0 \\ d_{\mathrm{RD}}/d_{\mathrm{SD}} < 1.0 \\ d_{\mathrm{SR}}/d_{\mathrm{SD}} + d_{\mathrm{RD}}/d_{\mathrm{SD}} < \mathcal{A}_{\gamma,C} \end{gathered}$$

where the parameter $\mathcal{A}_{\gamma,C} > 0$ *upper-bounding the path-length from the source to the destination via the relay is a decreasing function of the SNR and a function of the channel coding* C*. Specifically, for small to medium SNR values and the path-loss exponent* $\mu = 2$*, and binary linear block codes of* $d_{\min} < 10$*,* $\mathcal{A}_{\gamma,C} \approx 2.0$ *for the AF relaying, and* $\mathcal{A}_{\gamma,C} \approx 1.5$ *for the DF relaying. In addition, for sufficiently large SNR or when the path-loss attenuations are not considered,*

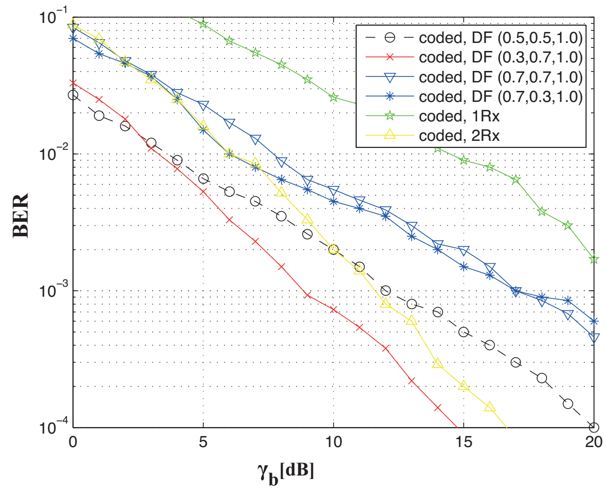

Figure 8: The BER of the BCH $(31, 16, 7)$ coded BPSK and the DF relaying and the receiver diversity with the EGC at the destination for several relay locations for slow Rayleigh fading channels.

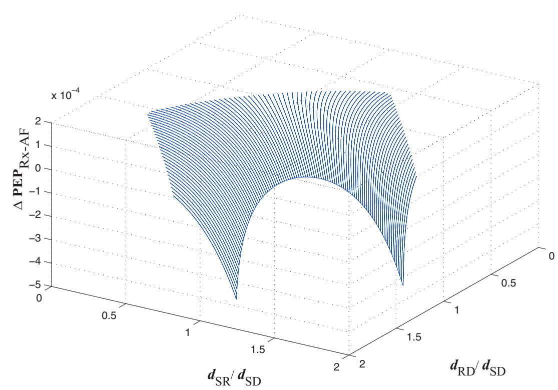

Figure 9: The PEPs difference $\Delta\text{PEP}_{\text{Rx}-\text{AF}}$ of System II and System I with the AF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination for the SNR $\gamma_b = 9$dB.

the parameter $\mathcal{A}_{\gamma,C} < 1.0$ and the receiver diversity always outperforms the cooperative diversity.

Note that Proposition 1 implicitly assumes the triangle inequality constraint, $d_{\text{SR}}/d_{\text{SD}} + d_{\text{RD}}/d_{\text{SD}} \geq 1.0$. Thus, if the parameter $\mathcal{A}_{\gamma,C}$ becomes smaller than 1, then, for no relay location can the cooperative diversity outperform the receiver diversity. A sub-optimum decoding scheme that is used in our numerical examples, and subsequently, used to formulate Proposition 1 appears to influence the threshold SNR value when the parameter $\mathcal{A}_{\gamma,C}$ becomes smaller than 1.0. Finally, it is straightforward to show that if the path-loss attenuations are not considered, then the receiver with K independent receiver antennas will always outperform a cooperative system with $(K-1)$ relays.

5 Conclusion

The transmission reliabilities of System I with the receiver diversity and System II with the cooperative diversity were investigated. Both systems can theoretically achieve the maximum diversity order of two. However, particularly the performance of System II suffers from the error propagation due to signal processing at the relay. Path-loss attenuations of the transmitted signals, independence of the channel fading coefficients and the use of channel coding with non-binary linear mcitepodulations were the main assump-

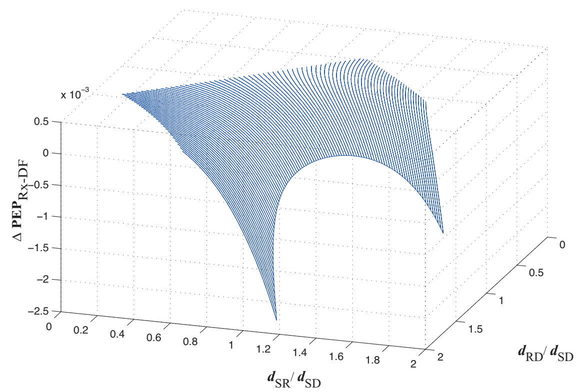

Figure 10: The PEPs difference $\Delta\mathrm{PEP}_{\mathrm{Rx-AF}}$ of System II and System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination for the SNR $\gamma_b = 9$dB.

tions adopted in the system modeling. At the destination receiver, the diversity signals were combined using either MRC or EGC. A low- complexity soft-decision POSD was extended for the decoding of binary linear block codes used with non-binary modulations. The PEP was investigated as the key performance measure of the system transmission reliability. In particular, assuming channel coding and BPSK signaling, the PEP expressions were derived analytically for System I as well as for System II. The obtained PEP expressions were verified by computer simulations. The performance of System II was found to be strongly dependent on the relay location as expected. More importantly, it was found that, for some relay locations and SNR values, System II with the cooperative diversity may outperform System I with the receiver diversity. The approximate boundaries of such geographical areas of relay locations when System II outperforms System I were formulated in Proposition 1 using both the obtained mathematical analysis of the PEPs as well as using extensive computer simulations. The DF relaying was found to be more sensitive to and more restrictive about the relay location than the AF relaying. More importantly, if the path-loss attenuations are not considered, then the receiver diversity always outperform the cooperative diversity. These results have significant implications for the deployment and design of the current cellular systems supporting both the receiver as well as cooperative diversity.

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