The Counterchanged Crossed Cube Interconnection Network and Its Topology Properties

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Abstract: In this paper, by combining with the network structures of the twisted *n* cube and the crossed cube, the counterchanged crossed cube network is proposed, a rigorous recursive definition is made, and the network topology structure graph is offered. Basing on the definition of the counterchanged crossed cube, this paper also analyzes the basic properties of the network, proves that the network is *n*-regular and its connectivity is *n*, illustrates the recursion characteristics of the network and two important corollaries are obtained. Through the recursive properties of the counterchanged crossed cube, then we discuss the relationship between any two vertexes in the network, and finally prove that the network diameter is $D(CCQ_n) = \lceil (n+1)/2 \rceil$.

Key words: the counterchanged crossed cube; interconnection network; network diameter; connectivity; network topology

1. Introduction

In a parallel processing system, the network interconnection structure often determines the the system performance. Among many of the existing interconnection network structures, the hyper cube[1] is one of the most classic and most widely applied network. The hypercube network has very excellent structure properties and is a hotspot of research on network topology. However, with the development of practical application and needs, it has been found that the structure of the hypercube is not always optimal. In order to give full play to the hypercube its superior properties, and to avoid the defects in the structure of its own, a variety of variant structures basing on the hypercube is proposed, i.e. *the crossed cube* [2-4], *the twisted cube*[5, 6], *the Möbius cube*[7, 8], *the twisted n-cube*[9], *the locally twisted cube*[10, 11], etc. These variations optimize the performances of a certain aspects on the basis of the original hypercube, and were applied to solve practical problems.

These network variations always achieve the optimization about some parameters on the basis of the original network model, such as network diameter, connectivity, etc. For example, among the variations of the hypercube, the network diameters of the crossed cube and the twisted *n*-cube are smaller than that in the hypercube. And they themselves also have some excellent properties, for example, the twist *n*-cube reduces the network communication diameter by removing disjoint edges in the hypercube and crossing them to form the twisted edges in the network. In document [9], Esfahanian and others have proven that there are two disjoint n-1 dimensional hypercubes in the twisted n-cube, the twisted n-cube is n-regular, and routing time can be reduced from n to n-1 in the worst case; In addition, Kemal Efe in [2] introduced the routing algorithm and the broadcasting algorithm in the crossed cube, obtained the crossed cube network diameter $\lceil (n+1)/2 \rceil$, approximately half of the hypercube, and proved the embedding properties of basic networks into the crossed cube. Combining with the natures of the above two networks, a new network structure, the counterchanged crossed cube, is proposed and its primary network properties are carefully researched in this paper.

This paper is organized as follows: section 2 introduces the related basic concepts of the graph theory and interconnection network; Section 3 defines the counterchanged crossed cube network structure and proves its edge connecting property; Section 4 presents the main topology properties of the counterchanged crossed cube; Section 5 proves the diameter of the counterchanged crossed cube; Section 6 carries on a comparative analysis on the natures of the counterchanged crossed cube.

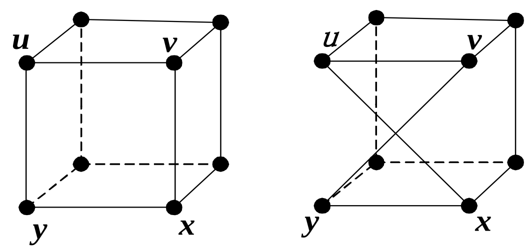
2. Relative Definitions

This paper uses normative terminologies and notations in the graph theory to represent relative concepts and formulas. Let G be an undirected graph where $V(G)$ and $E(G)$ represent the vertex set and edge set of graph G, respectively. Vertices in graph G are denoted by binary numbers. $e(u, v)$ denotes the edge that connects two adjacent vertices u and v; we call the length of the shortest path from vertex u to v the distance between u and v, denoted by $d(u, v)$, and call $max\{d(u,v) \mid u, v \in V(G)\}$ the diameter of graph G, denoted by $D(G)$; $deg(u)$ denotes the degree of vertex u; $\delta(G)$ and $\Delta(G)$ represent the maximum degree and minimum degree of graph G; $\kappa(G)$ and $\lambda(G)$ represent the vertex connectivity and edge connectivity of graph G, respectively; $\sigma(u)=i$ denotes vertices u that $u_n=u_{n-1}=\ldots u_{i+1}=0$ and $u_i=1$; for a binary string $u=u_n u_{n-1}...u_2 u_1$, $\overline{u_i}$ represents the i-th complementary bit of u_i.

Let $x=x_2x_1$, $y=y_2y_1$ be two binary strings, and we say x and y are pair-related if and only if $(x, y) \in \{(00,00),(10,10),(01,11),(11,01)\}$, denoted by $x \sim y$. If x and y are not pair-related, it is denoted by $x \not\sim y$.

Definition 1[9] For a 4-length cycle <u, v, y, x, u> in n-dimensional hypercube Q_n, if delete the edges (u, y) and (v, x), and add edges (u, x) and (v, y), the obtained network structure is called *twisted n-cube*, denoted as TQ_n.

The network structures of Q_3 and TQ_3 are shown in Fig 1.



Q_3 TQ_3

Fig 1 Q_3 and TQ_3

Definition 2[2] The n-dimensional *crossed cube* (CQ_n) is a n-label graph, it can be defined inductively as follows: CQ_1 is K_2, the complete graph of two vertices with labels 0 and 1; for $n>1$, CQ_n consists of two (n-1)-dimensional *crossed cube* $CQ_{n-1}^{(0)}$ and $CQ_{n-1}^{(1)}$, where

$V(CQ_{n-1}^{(i)})=\{x_n x_{n-1}\ldots x_1 \mid x_n=i\},(i=0,1)$

The vertex $x=0x_{n-1}x_{n-2}\ldots x_1$ in $CQ_{n-1}^{(0)}$ and the vertex $y=1y_{n-1}y_{n-2}\ldots y_1$ in $CQ_{n-1}^{(1)}$ are adjacent in CQ_n if and only if:

(1) $x_{n-1}=y_{n-1}$ if n is even, and

(2) For $1\le i\le \lfloor (n-1)/2 \rfloor$, $x_{2i}x_{2i-1} \sim y_{2i}y_{2i-1}$.

The network structures of CQ_3 and CQ_4 are shown in Fig 2.

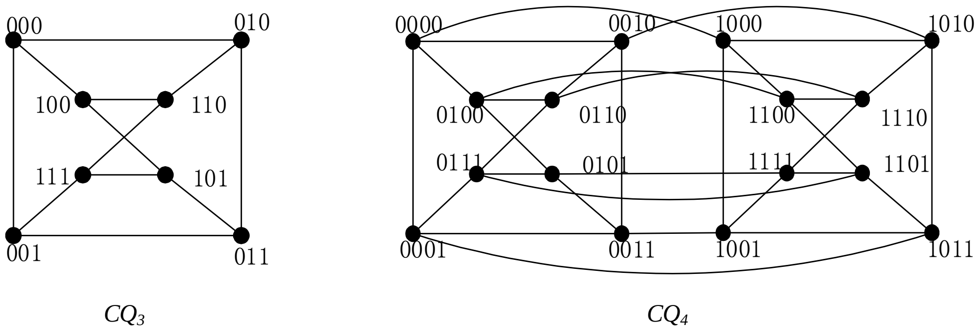


Fig 2 CQ_3 and CQ_4

Definition 3 $\forall u, v\in V(CQ_n)$, if u and v are adjacent and $\sigma(u+v)=i$, then we say u and v are adjacent in i-th dimension.

Definition 4[12] In graph G, two edges without public vertex are called independent edges. For two independent edges (u, v) and (x, y) in edge set E, delete the connections between the two edges and add new edges (u, y) and (v, x), then call the operation an X-counterchange.

The process of X-counterchange is shown in Fig 3.

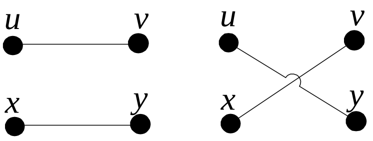


Fig 3 X-counterchange

It is observed that the degree of vertexes in the graph remains unchanged after X-counterchange.

3. Definition of the Network

The definition and network structure of the counterchanged crossed cube is as follows.

Definition 5 The n-dimensional *counterchanged crossed cube* (CCQ_n) is a n-label graph, CCQ_1 is K_2, the complete graph of two vertices with labels 0 and 1; CCQ_2 is a 4-length cycle <00, 10, 01, 11, 00>; for $n \ge 3$, CCQ_n consists of two (n-1)-dimensional *counterchanged crossed cube* $CCQ_{n-1}^{(0)}$ and $CCQ_{n-1}^{(1)}$, where

$V(CCQ_{n-1}^{(i)})=\{x_n x_{n-1}\ldots x_1 \mid x_n=i\},(i=0,1)$

The vertex $x=0x_{n-1}x_{n-2}\ldots x_1$ in $CCQ_{n-1}^{(0)}$ and the vertex $y=1y_{n-1}y_{n-2}\ldots y_1$ in $CCQ_{n-1}^{(1)}$ are adjacent in CCQ_n if and only if:

(1) $x_{n-1}=y_{n-1}$ if n is even, and

(2) for $1\le i\le \lfloor (n-1)/2 \rfloor$, $x_{2i}x_{2i-1}\sim y_{2i}y_{2i-1}$.

Fig 4 shows the network structures of 1-4 dimension CCQ_n.

According to Definition 5, the following theorem holds.

Theorem 1 For $n \ge 2$, any edge $e=(x,y)$ in CCQ_n, where $x=x_n x_{n-1}...x_2 x_1$ ($x_i \in \{0,1\}$, i=1, 2, … , n-1), $y=y_n y_{n-1}...y_2 y_1$ ($y_i \in \{0,1\}$, i=1, 2, … n-1), there exists m($2\le m\le n$) such that at least one of the following situations holds:

(1) If $x_n = y_n$, then $x_n x_{n-1}...x_{m+1} = y_n y_{n-1}...y_{m+1}$ and $x_m = \overline{y}_m$;

(2) When $1\le i\le \lfloor (m-1)/2 \rfloor$, there is $x_{2i}x_{2i-1} \sim y_{2i}y_{2i-1}$.

Proof: we make an inductive proof on n.

(1) For situation (1), when n=2, m=1,

the theorem is true.

Suppose the theorem is true for $n=k$, then when $n=k+1$, there is $x_{k+1}=y_{k+1}$, if $\sigma(x+y)=L$ and let $m=k+1-L+1$, apparently $x_m=\overline{y}_m$ and $x_{k+1-1}x_{k+1-1}...x_{m+1}=y_{k+1-1}y_{k+1-1}...y_{m+1}$, i.e. situation (1) is established;

(2) For situation (2),

①if $x_n \neq y_n$, then $m=n$, according to Definition 5, the conclusion holds;

②if $x_n = y_n$, from situation (1) we know there exists the positive integer m ($2 \le m \le n$) such that $x_n x_{n-1}...x_{m+1}=y_n y_{n-1}...y_{m+1}=\alpha$, and $x_m=\overline{y}_m$, it is easy to know that $x^{(m)}=\alpha x_m x_{m-1}...x_2 x_1$ and $y^{(m)}=\alpha y_m y_{m-1}...y_2 y_1$ are two vertexes of the edge ($x^{(m)}, y^{(m)}$) in the counterchanged crossed cube. According to the conclusion in①, when $1 \le i \le \lfloor (m-1)/2 \rfloor$, $x_{2i}x_{2i-1} \sim y_{2i}y_{2i-1}$ is true. In conclusion, the theorem is proven. ■

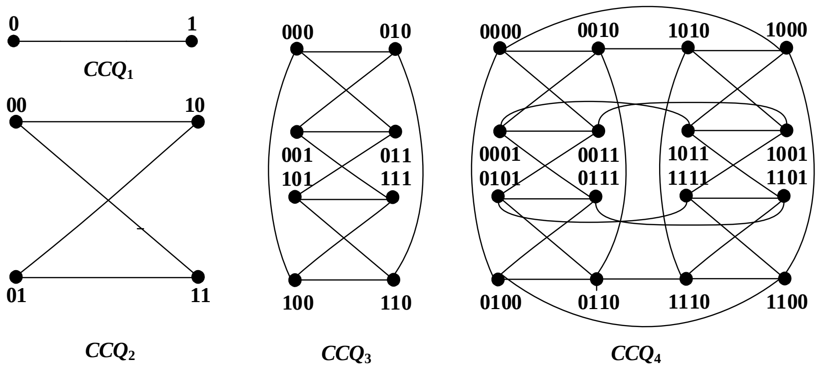


Fig 4 1-4 dimension CCQ_n

From the proof of the above theorem, if vertexes x and y differs from the n-m+1-th bit from the left, then call the m-th bit the leftmost different bit between x and y, say u and v are adjacent in m-th dimension, denoted by $u=N_m(v)$, and call the edge the i-dimensional edge between u and v, denoted by $e_m(u,v)$.

4. Structure Properties

The network properties are determined by its structure, and these properties have important impacts on the performance of the network. This section will study properties such as the regularity, recursiveness, and connectivity.

4.1.Regularity

In a regular graph, each vertex has the same degree. If the degree of every vertex is k, the graph is a k regular graph. A regular network has a good symmetry nature, the transmission speed and network loads are balanced. From the definition of CCQ_n, it is observed that the degrees of vertexes in the network remain unchanged after the X-counterchange. The proof of CCQ_n regularity is shown as follows.

Theorem 2 CCQ_n is a n-regular graph.

Proof: According to the definition of CCQ_n and Theorem 1, vertexes in the network have one and only one i-dimensional neighbor vertex, where i=1, 2, 3…n. Apparently, the degree of any vertex u in CCQ_n is $deg(u)=n$. On the definition of regular graph, CCQ_n is n-regular.

4.2.Recursiveness

From the CCQ_n network definition, CCQ_n is formed of lower dimensional subnets recursively, and the network structure is of recursiveness. The following will further research the subnet characteristics of the counterchanged crossed cube.

$\Gamma_{\alpha,\beta}(G)$ denote the subnets which consist of all the vertices that prefix with α

or β in G, $\Gamma_{\alpha,\alpha}(G)$ is abbreviated as $\Gamma_{\alpha}(G)$. These isomorphic subnets constitute higher dimensional CCQ_n subnets.

Theorem 3 For $n \geq 2$, there are $\Gamma_0(CCQ_n) \cong CCQ_{n-1}$ and $\Gamma_1(CCQ_n) \cong CCQ_{n-1}$.

Proof: According to the definition of CCQ_n, n-dimensional CCQ_n consist of two (n-1)-dimensional $CCQ_{n-1}^{(0)}$ and $CCQ_{n-1}^{(1)}$, and $CCQ_{n-1}^{(0)} \cong CCQ_{n-1}$, $CCQ_{n-1}^{(1)} \cong CCQ_{n-1}$, the theorem is true. ■

Let α, β be two binary strings of the same length, if there exists a positive integer l such that $\forall u \in \Gamma_{\alpha}(CCQ_n)$, $N_l(u) \in \Gamma_{\beta}(CCQ_n)$, and $\forall v \in \Gamma_{\beta}(CCQ_n)$, $N_l(v) \in \Gamma_{\alpha}(CCQ_n)$, then call $\Gamma_{\alpha}(CCQ_n)$ and $\Gamma_{\beta}(CCQ_n)$ two l-dimensional adjacent subnets, denoted by $\Gamma_{\alpha}(CCQ_n) = N_l(\Gamma_{\beta}(CCQ_n))$.

Theorem 4 Let α and β be two binary strings of the same length, and $|\alpha| = |\beta| = s$, if two subnets $\Gamma_{\alpha}(CCQ_n)$ and $\Gamma_{\beta}(CCQ_n)$ in CCQ_n satisfy $\Gamma_{\alpha}(CCQ_n) = N_l(\Gamma_{\beta}(CCQ_n))$, then

(1)For $l \leq n - s$, $\alpha = \beta$ and $\Gamma_{\alpha,\beta}(CCQ_n) \cong CCQ_l$;

(2)For $l > n - s$, $\alpha \neq \beta$ and $\Gamma_{\alpha,\beta}(CCQ_n) \cong CCQ_{n-s+1}$.

Proof: Since $\Gamma_{\alpha}(CCQ_n) = N_l(\Gamma_{\beta}(CCQ_n))$,

If $l \leq n - s$, according to the adjacent subnet definition, for $\forall u \in \Gamma_{\alpha}(CCQ_n)$ and $\forall v \in \Gamma_{\beta}(CCQ_n)$, there is $\sigma(u+v) = l$, so $\alpha = \beta$ and $u_n \cdots u_{n-s+1} u_{n-s} \cdots u_{l+1} = v_n \cdots v_{n-s+1} v_{n-s} \cdots v_{l+1}$, then in terms of the definition of CCQ_n, we have $\Gamma_{\alpha,\beta}(CCQ_n) \cong CCQ_l$;

If $l > n - s$, let the mapping $\varphi : \Gamma_{\alpha,\beta}(CCQ_n) \to CCQ_{n-s+1}$, such that $\forall u \in \Gamma_{\alpha,\beta}(CCQ_n)$,

$$\varphi(u) = u_l u_{n-s} \cdots u_1 \qquad \text{(I)}$$

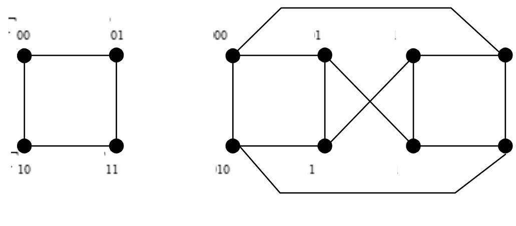
If $\forall u, v \in \Gamma_{\alpha,\beta}(CCQ_n)$, and u=$N_j(v)$, then for $j \leq n$-s, $u, v \in \Gamma_{\alpha}(CCQ_n)$ or $u, v \in \Gamma_{\beta}(CCQ_n)$, and hence $\varphi(u) = N_j(\varphi(v))$; for $j > n$-s, let $u \in \Gamma_{\alpha}(CCQ_n)$, $v \in \Gamma_{\beta}(CCQ_n)$, according to formula (I), we have $\varphi(u) = N_{n-s+1}(\varphi(v))$, i.e. $\varphi(u)$ and $\varphi(v)$ are adjacent in CCQ_{n-s+1}, so $\Gamma_{\alpha,\beta}(CCQ_n) \cong CCQ_{n-s+1}$. ■

The following corollaries can be easily obtained from Theorem 4.

Corollary 1 If n (≥ 4) is even, then $\Gamma_{00,10}(CCQ_n) \cong CCQ_{n-1}$, $\Gamma_{11,01}(CCQ_n) \cong CCQ_{n-1}$.

Corollary 2 If n (≥ 5) is odd, then $\Gamma_{000,100}(CCQ_n) \cong CCQ_{n-2}$, $\Gamma_{001,111}(CCQ_n) \cong CCQ_{n-2}$, $\Gamma_{011,101}(CCQ_n) \cong CCQ_{n-2}$.

We can see the recursive characters of CCQ_n from Corollary 1 and 2: when n (≥ 4) is even, CCQ_n is formed of four subnets prefixing with 00, 10, 01 and 11; when n ($\geq$ 5) is odd, CCQ_n is formed of eight subnets prefixing with 000, 100, 010, 110, 001, 111, 011, 101. If we imagine these subnets as a node, then for n (≥ 4) is even and n (≥ 5) is odd, the connection methods of these subnet are show in Fig 5.



(a) n ($\geq$4) is even (b) n ($\geq$5) is odd

Fig 5 the recursive connection methods of CCQ_n subnets

4.3.Vertex/edge Connectivity

In the actual network, there always exist failure nodes or edges due to various reasons. Connectivity is the minimum number of vertices or edges that need to be removed to make a graph unconnected. Connectivity reflects the tolerance degree towards fault, namely the ability of normal communication when there are failures in the network, which thus is a very important property.

Definition 6[9]. Let G_1 and G_2 be two graphs with the same vertex number: $V(G_1)=\{u_1,u_2,\ldots u_p\}$, $V(G_2)=\{v_1,v_2,\ldots v_p\}$. Let $H=G_1 \odot G_2$, where $V(H)=V(G_1) \cup V(G_2)$, $E(H)=E(G_1) \cup E(G_2) \cup \{(u_i,v_i) \mid u_i \in V(G_1), v_i \in V(G_2), 1 \le i \le p\}$.

Lemma 1[9]. Let G_1 and G_2 be two connected graphs defined in Definition 6, and $H=G_1 \odot G_2$, then:

$$\kappa(H) \ge 1+\min(\kappa(G_1),\kappa(G_2))$$

Lemma 2[13] For any graph G, $\kappa(G) \le \lambda(G) \le \delta(G)$.

Theorem 5 $\kappa(CCQ_n)=\lambda(CCQ_n)=n$.

Proof: According to Definition 5, CCQ_n constitutes of two subnets $CCQ_{n-1}^{(0)}$ and $CCQ_{n-1}^{(1)}$, then from Definition 6 and Lemma 1, we have $CCQ_n = CCQ_{n-1}^{(0)} \odot CCQ_{n-1}^{(1)}$, and $\kappa(CCQ_n) \ge 1+\kappa(CCQ_{n-1}) \ge 1+(1+\kappa(CCQ_{n-2}))\cdots \ge (n-1)+\kappa(CCQ_1)$. Since $\kappa(CCQ_1)=1$, then $\kappa(CCQ_n) \ge n$. And by Theorem 2 and Lemma 2,we know $\kappa(CCQ_n) \le \lambda(CCQ_n) \le \delta(CCQ_n)=n$. In conclusion, $\kappa(CCQ_n)=\lambda(CCQ_n)=n$, the theorem is proven. ■

5. Network Diameter

Network diameter embodies the network communication delay, therefore a good network should keep the diameter as small as possible on the premise of connectivity. The following will study the diameter of CCQ_n network.

Theorem 6 When $n \ge 2$, for $\forall x$, $y \in V(CCQ_n)$, the relationship between x and y must satisfy one of the following situations:

(1) When n is even, x and y exist in the same CCQ_n subnet G, where $G \cong CCQ_{n-1}$ or $G \cong CCQ_{n-2}$;

(2) When n is odd, x and y exist in the same CCQ_n subnet G, where $G \cong CCQ_{n-2}$ or $G \cong CCQ_{n-3}$;

(3) The vertex x (or y) has an adjacent vertex x' (or y'), such that x' (or y') and y (or x) exist in the same CCQ_n subnet G, where when n is even, $G \cong CCQ_{n-1}$; when n is odd, $G \cong CCQ_{n-2}$.

Proof: According to the recursiveness of CCQ_n and Corollaries 1 and 2, when n is even, CCQ_n is formed of four interconnected subnets which are isomorphic to CCQ_{n-2}; when n is odd, CCQ_n is formed of eight interconnected subnets which are isomorphic to CCQ_{n-3}. The following will make corresponding discusses about the above situations:

(1) When n is even, according to Fig 5(a), if x and y exist in the same subnet G, then $G \cong CCQ_{n-2}$; if x and y exist in two adjacent subnets, then these two subnets constitute a new subset G by interconnecting, and it is easy to know $G \cong CCQ_{n-1}$, situation (1) is true;

(2) When n is odd, according to Fig 5(b), if x and y exist in the same subnet G, then $G \cong CCQ_{n-3}$; if x and y exist in two adjacent subnets, then these two subnets constitute a new subset G by interconnecting, and it is easy to know $G \cong CCQ_{n-2}$, situation (2) is true;

(3) From Fig 5, we can see that the maximum distance among subnets is 2, i.e. any two non-adjacent subnets need at most one intermediate subnet to connect with each other. If x and y exist in two non-adjacent subnets, there must be a neighbor vertex x' (or y'), such that x' (or y') and y (or x) are in the same subnet G. When n is even, from Fig 5(a), we have $G \cong CCQ_{n-1}$; when n is odd, from Fig 5(b),

we have $G \cong CCQ_{n-2}$, situation (3) is true.

To sum up, the theorem holds. ■

Theorem 7 $D(CCQ_n) = \left\lceil \frac{n+1}{2} \right\rceil$.

Proof: According to Fig 4, we have $D(CCQ_1) = 1$, $D(CCQ_2) = D(CCQ_3) = 2$, it meets the theorem. We make an inductive proof on k. Suppose when $k \le m (n \le 2k+1)$, the theorem is true, then $D(CCQ_{2m}) = D(CCQ_{2m+1}) = m+1$. The next will discuss the situations when $k=m+1$.

(1) We first discuss the diameter of CCQ_{2m+2}. If vertexes a and b exist in the same subnet, it conforms to situation (1) in Theorem 6, then from the assumption, we know $d(a,b) = m+1$; if a and b exist in two non-adjacent subnets, then a must has a neighbor vertex a' , such that a' and b are in the same subnet, which conforms to situation (3) in Theorem 6, then similarly from the assumption, we have $d(a',b) = m+1$, and $d(a,a') = 1$. According to the definition of network diameter, we have $D(CCQ_{2m+2}) = m+2$;

(2) For the diameter of CCQ_{2m+3}, like the proof process in (1), we similarly have $D(CCQ_{2m+3}) = m+2$.

In conclusion, the assumption stands, i.e. $D(CCQ_{2k}) = D(CCQ_{2k+1}) = k+1$. By transforming the express, we obtain the equivalent formula $D(CCQ_n) = \left\lceil \frac{n+1}{2} \right\rceil$, the theorem holds. ■

6. Comparison and Analysis

Through the researches on CCQ_n network structure and properties, we can see that CCQ_n has the following superiorities than the twisted n-cube:

(1) $\kappa(CCQ_n) = \lambda(CCQ_n) = \delta(CCQ_n) = n$, compared to the twisted n-cube, CCQ_n has better connectivity and fault tolerance;

(2) $D(CCQ_n) = \left\lceil \frac{n+1}{2} \right\rceil$, the diameter of CCQ_n is almost half of those of the hypercube and the twisted n-cube, which greatly reduces the network communication diameter and increases the network communication efficiency.

7. Conclusion

In this paper, by combining with the twisted n cube and the crossed cube, we propose a new network structure - the counterchanged crossed cube (CCQ_n) network, and make detailed discussions about the network structure characteristics and basic properties of CCQ_n, for example, CCQ_n is n-regular, it has excellent recursiveness and its connectivity is n. Through the recursive properties of the counterchanged crossed cube, we prove that the network diameter is $D(CCQ_n) = \lceil (n+1)/2 \rceil$, which is nearly half of that of the hypercube. Through comparison, it is found that CCQ_n is also a favorable network structure. The subsequent researches will focus on problems such as the network embedding properties, the routing algorithm, the fault-tolerance strategies and the optimal paths.

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