

# Error Probability Performance of Quadriphase DS-CDMA Wireless Communication Systems Based on Generalized Approach to Signal Processing

VYACHESLAV TUZLUKOV

Department of Information and Communication Engineering,  
School of Electronics Engineering,  
College of IT Engineering  
Kyungpook National University  
1370 Sankyuk-dong Buk-gu Daegu 702-701  
SOUTH KOREA

[tuzlukov@ee.knu.ac.kr](mailto:tuzlukov@ee.knu.ac.kr) <http://spl.knu.ac.kr>

*Abstract:* - Probability of bit-error  $P_{er}$  performance of asynchronous direct-sequence code-division multiple-access (DS-CDMA) wireless communication systems employing the generalized detector (GD) constructed based on the generalized approach to signal processing in noise is analyzed. The effects of pulse shaping, quadriphase or direct sequence quadriphase shift keying (DS-QPSK) spreading, aperiodic spreading sequences are considered in DS-CDMA based on GD and compared with the coherent Neyman-Pearson receiver. An exact probability of error  $P_{er}$  expression and several approximations, namely, one using the characteristic function method, a simplified expression for the improved Gaussian approximation (IGA) and the simplified improved Gaussian approximation are derived. Under conditions typically satisfied in practice and even with a small number of interferers, the standard Gaussian approximation (SGA) for the multiple-access interference component of the GD test statistic and the probability of error  $P_{er}$  performance is shown to be accurate. Moreover, the IGA is shown to reduce to the SGA for pulses with zero excess bandwidth. Second, the GD probability of error  $P_{er}$  performance of quadriphase DS-CDMA is shown to be superior to that of bi-phase DS-CDMA. Numerical examples by Monte Carlo simulation are presented to illustrate the GD probability of error  $P_{er}$  performance for square-root raised-cosine pulses and spreading factors of moderate to large values. Superiority of GD employment in CDMA systems over the Neyman-Pearson receiver is demonstrated.

*Key-Words:* - Generalized detector, additive white Gaussian noise (AWGN), probability of bit-error, pulse shaping, generalized approach to signal processing (GASP), code division multiple-access, direct-sequence quadriphase shift-keying spreading.

## 1 Introduction

We study the probability of error  $P_{er}$  performance of band-limited quadriphase direct-sequence code-division multiple-access (QDS-CDMA) wireless communication systems. The considered wireless communication systems have asynchronous users, pulse shaping, direct-sequence quadriphase phase-shift-keying (DS-QPSK) spreading and aperiodic spreading sequences or random spreading [1]. The uplink of the wide-band-CDMA system [2] that uses the square-root raised cosine (Sqrt-RC) pulse of 22% excess bandwidth, DS-QPSK spreading and long pseudo noise sequences is an example of QDS-CDMA system.

The pseudo noise sequences are modeled as random, independent, and identically distributed (i.i.d.) binary sequences [1], [3]. We focus on the analysis of pulse shaping in wireless communication systems that is very important under designing process. The pulse shaping influences a spectral efficiency and reduces out-of-band radio emissions. Firstly, its relevance was mentioned in [4]. In the majority cases, the effect of bandwidth-efficient pulse shaping by using the pulses of one-chip period duration, in particular, the rectangular pulse, is neglected [5]–[9]. This approach simplifies the framework for the probability of error  $P_{er}$  analysis and can lead to overly optimistic estimates for the probability of error  $P_{er}$  performance of band-limited systems using the di-

rect-sequence binary phase shift keying (DS-BPSK) spreading [10].

DS-CDMA systems with band-limited pulse shapes and DS-BPSK spreading are referred to as B-CDMA. Previous study of the probability of error  $P_{er}$  for QDS-CDMA [11]–[13] assumes a simple and widely used standard Gaussian approximation [6], [7] that models the multiple-access interference (MAI) generated by the signals of many users as a Gaussian random process. The difficulty with the standard Gaussian approximation is its tendency to break down and return the over-optimistic probability of error  $P_{er}$  results for many systems with either a small number of users or a small number of high-powered users that was observed both for systems with DS-BPSK or DS-QPSK spreading using rectangular pulses [3], [5]–[8], [14] and both for B-CDMA [15]–[18].

In contrast to [1], [12], [13], an accurate analysis of the probability of error  $P_{er}$  performance of QDS-CDMA was done in [11]. Though its focus was on the design of optimum chip waveforms for QDS-CDMA, it used a more accurate method of the probability of error  $P_{er}$  analysis based on the improved Gaussian approximation or conditional Gaussian approximation for pulses time-limited to one-chip period [19]. We study the probability of error  $P_{er}$  performance for the case of employment of the generalized detector (GD) constructed based on the generalized approach to signal processing in noise [20]–[25].

We assume that the standard Gaussian approximation can serve as an accurate one under conditions typically satisfied in practice. The improved Gaussian approximation can be reduced to the standard one for pulses with zero excess bandwidth. It is shown that the probability of error  $P_{er}$  performance of QDS-CDMA with GD is superior or equivalent to the B-CDMA with GD case. This result is consistent with a similar finding in [9] which was restricted to pulses (such as the rectangular pulse) of one-chip period. An exact probability of error  $P_{er}$  formula for QDS-CDMA with GD is derived according to the characteristic function method described in [8] for pulses of one-chip period and in [15] and [16] for general pulse shapes with DS-BPSK spreading. More tractable formula for the improved Gaussian approximation that simplifies greatly for pulses with small excess bandwidth is presented.

The simplified improved Gaussian approximation that has a complexity on par with that of standard Gaussian approximation is derived. Numerical examples with Monte Carlo simulation show the accuracy of the standard Gaussian approximation for QDS-CDMA with GD wireless communication systems given pulse shapes of small excess bandwidth and spreading factors (the chip-rate to bit-rate ratio) of moderate to large values.

## 2 System Model

The QDS-CDMA system model consists of  $K + 1$  asynchronous users in the additive white Gaussian noise (AWGN) channel. This section describes the modelling of the received signal, coherent GD, and bit decision statistic based on DS-QPSK spreading with pulse shaping. An arbitrary user is designated as the desired user and indexed as user 0.

### 2.1 Received signal

The received signal can be presented in the following form

$$x(t) = \sum_{k=0}^K s^{(k)}(t) + w(t) \quad (1)$$

for  $-\infty < t < \infty$ . The second term  $w(t)$  is the additive white Gaussian noise (AWGN) with a two-sided power spectral density of  $0.5N_0$ . The first term is the sum of the received signals from each user.

The DS-QPSK signal of user  $k$  can be expressed as

$$s^{(k)}(t) = \sqrt{P_k} [s_I(t) \cos(\omega_c t + \theta_k) + s_Q \sin(\omega_c t + \theta_k)], \quad (2)$$

where  $\omega_c$  and  $\theta_k \in [0, 2\pi)$  represent, respectively, the carrier frequency in radians and carrier phase offset of user  $k$  relative to that of user 0; the in-phase ( $I$ ) and quadrature-phase ( $Q$ ) baseband signals are, respectively

$$x_I(t) = \sum_{n=-\infty}^{\infty} b_{\lfloor n/N^{(k)} \rfloor}^{(k)} \alpha_n^{(I,k)} q(t - \tau_k - nT_c) ; \quad (3)$$

$$x_Q(t) = \sum_{n=-\infty}^{\infty} b_{\lfloor n/N^{(k)} \rfloor}^{(k)} \alpha_n^{(Q,k)} q(t - \tau_k - nT_c) . \quad (4)$$

The bits  $b_i^{(k)} \in \{\pm 1\}$  of user  $k$  are transmitted at a rate of  $1/T_b^{(k)}$  over both the  $I$  and  $Q$  branches. The floor function  $\lfloor y \rfloor$  returns the integer portion of a real nu-

mber  $y$ . The spreading factor  $N^{(k)}$  is the chip-rate  $1/T_c$  to bit-rate ratio. The terms  $\alpha_n^{(I,k)}$  and  $\alpha_n^{(Q,k)}$  represent, respectively, the chips of long pseudo noise sequences used to direct-sequence spread the bits in the in-phase and quadrature branches. The pseudo noise sequence of each user and in each branch is modeled as i.i.d. random sequence with the probabilities [1], [6]

$$P[\alpha_n^{(I,k)} = 1] = P[\alpha_n^{(I,k)} = -1] = P[\alpha_n^{(Q,k)} = 1] = P[\alpha_n^{(Q,k)} = -1] = 0.5 . \quad (5)$$

The term  $P_k$  represents the received signal power for user  $k$ . The term  $\tau_k \in [0, T_b^{(k)}]$  represents the bit delay of user  $k$  relative to that of the receiver for user 0. For  $k \in [1, K]$ , the parameters  $\theta_k$  and  $\tau_k$  are modelled as independent and uniformly distributed random variables. At  $k = 0$ , the code and phase synchronization (for coherent detection) are assumed such that  $\tau_0 = \theta_0 = 0$ . The received baseband chip pulse  $q(t)$  is assumed to be a real, even symmetric pulse with an energy of

$$\int_{-\infty}^{\infty} |q^2(t)| dt = T_c \quad (6)$$

and with a time duration  $MT_c$ . The bandwidth of the pulse can be expressed as

$$W = \frac{1 + \alpha}{2T_c} , \quad (7)$$

where  $\alpha$  represents the excess bandwidth of the pulse. We consider pulses which either satisfy the Nyquist criterion for zero intersymbol interference (ISI) or introduce ISI at a level negligible with respect to that of MAI. Furthermore, the effect of ICI and ISI is negligible to large processing gains [26].

### 2.2 GD functioning principles

For better understanding, we recall the main functioning principles of GD. The simple model of GD in form of block diagram is represented in Fig.1. In this model, we use the following notations: MSG is the model signal generator (the local oscillator), the AF is the additional filter (the linear system) and the PF is the preliminary filter (the linear system). A detailed discussion of the AF and PF can be found in [20] and [22].

Consider briefly the main statements regarding the AF and PF. There are two linear systems at the GD

front end that can be presented, for example, as bandpass filters, namely, the PF with the impulse response  $h_{PF}(\tau)$  and the AF with the impulse response  $h_{AF}(\tau)$ . For simplicity of analysis, we think that these filters have the same amplitude-frequency responses and bandwidths. Moreover, AF resonant frequency is detuned relative to PF resonant frequency on such a value that signal cannot pass through the AF (on a value that is higher the signal bandwidth). Thus, the signal and noise can be appeared at the PF output and **the only noise** is appeared at the AF output.

It is well known, if a value of detuning between the AF and PF resonant frequencies is more than  $4 \div 5 \Delta f_a$ , where  $\Delta f_a$  is the signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes. In practice, the coefficient of correlation is not more than 0.05. In the case of a “no” signal in the input process, the statistical parameters at the AF and PF outputs will be the same, because the same noise is coming in at the AF and PF inputs, and we may think that the AF and PF do not change the statistical parameters of input process, since they are the linear GD front end systems. By this reason, the AF can be considered as a reference sample source with *a priori* information *a “no” signal is obtained in the additional reference noise* forming at the AF output.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If AWGN  $w(t)$  comes in at the AF and PF inputs (the GD linear system front end), the noise forming at the AF and PF outputs is Gaussian, too, because the AF and PF are the linear systems and, in a general case, take the following form:

$$\begin{cases} n_{l_{PF}}(t) = \int_{-\infty}^{\infty} h_{PF}(\tau) w_l(t - \tau) d\tau ; \\ n_{l_{AF}}(t) = \int_{-\infty}^{\infty} h_{AF}(\tau) w_l(t - \tau) d\tau ; \end{cases} \quad (8)$$

If AWGN with zero mean and two-sided power spectral density  $0.5N_0$  is coming in at the AF and PF inputs (the GD linear system front end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by [22]

$$\sigma_n^2 = \frac{N_0 \omega_0^2}{8\Delta_F} , \quad (9)$$

where in the case if AF (or PF) is the RLC oscillatory circuit, the AF (or PF) bandwidth  $\Delta_F$  and resona-

nance frequency  $\omega_0$  are defined in the following manner

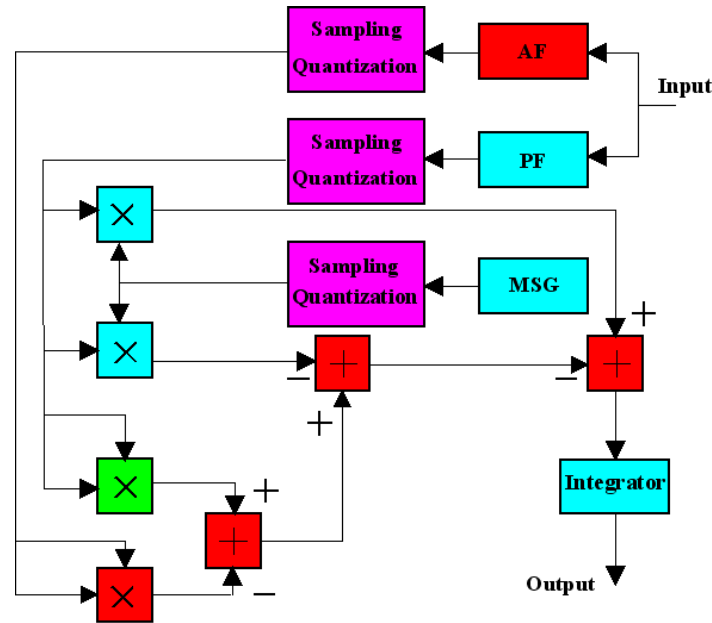


Figure 1. Principal flowchart of GD.

$$\Delta_F = \pi\beta, \omega_0 = \frac{1}{\sqrt{LC}}, \beta = \frac{R}{2L} . \quad (10)$$

The main functioning condition of GD is an equality over the whole range of parameters between the model signal forming at the GD MSG output for user  $l$  and the expected signal forming at the GD input linear system (the PF) output. How we can satisfy this condition in practice is discussed in detail in [20] and [22]. More detailed discussion about a choice of PF and AF and their amplitude-frequency responses is given in [23] (see also <http://www.sciencedirect.com/science/journal/10512004> , click “Volume 8, 1998”, “Volume 8, Issue 3”, and “A new approach to signal detection theory”).

### 2.3 Coherent GD

Let  $N = N^{(0)}, E_b = E_b^{(0)}$ . Henceforth,  $N$  and  $E_b$  will be used to denote, respectively, the spreading factor and bit energy for user 0 employed by QDS-CDMA wireless communication system is shown in Fig. 2. The received signal given by (1) is demodulated to form the  $I$  and  $Q$  base-band signals in the GD correlation channel.

The received signal in each branch of the GD correlation channel is passed through the receiver filter

$Q^*(f)$  matched to the received chip pulse shape  $Q(f)$  and direct-sequence spread. The outputs of two GD correlation channel branches are added and mixed with the GD autocorrelation channel owing to the GD compensation channel. The resulting process is passed to a summation device to generate the bit decision statistic as

$$Z_i^{(0)'} = \left\{ \sum_{l=iN}^{(i+1)N-1} \left\{ [\alpha_l^{(I,0)}]^2 + [\alpha_l^{(Q,0)}]^2 \right\} \times \left\{ \int_{-\infty}^{\infty} \sum_{k=0}^K [s^{(k)}(t)q(-u)]^2 du \right\}^2 + \left\{ \int_{-\infty}^{\infty} [n_{iAF}^2 - n_{iPF}^2] q^2(-u) du \right\}^2 \right\}^{\frac{1}{2}} . \quad (11)$$

This is then fed to a hard decision device to produce the bit decision

$$\tilde{b}_i^{(0)} = \text{sgn } Z_i^{(0)'} , \quad (12)$$

where  $\text{sgn}(\dots)$  is the *signum* function.

**2.4 Bit decision statistic**

Without loss of generality, the bit decision statistic  $Z_0^{(0)'}$  for bit 0 of user 0,  $b_0^{(0)}$  is considered. In anticipation of the characteristic function method of the

following section, the statistic  $Z_0^{(0)'}$  is rescaled and expressed as

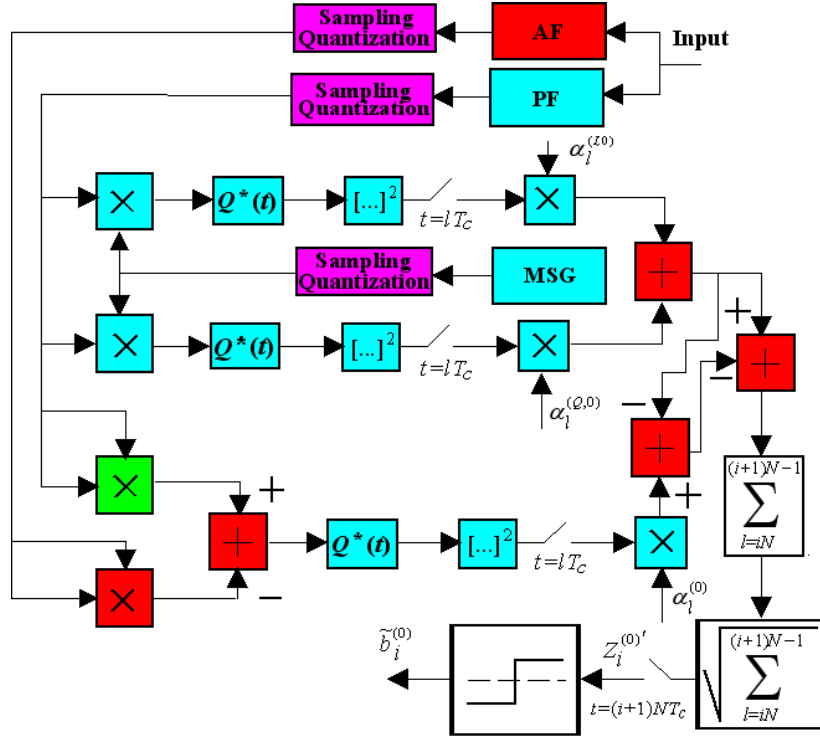


Figure 2. Coherent GD.

$$Z_0^{(0)} = b_0^{(0)} + M + \{n_{AF}^2 - n_{PF}^2\} \quad (13)$$

where

$$Z_0^{(0)} = \frac{Z_0^{(0)'}}{\sqrt{P_0 NT_c}} \quad (14)$$

The third term in (13) is the GD background noise defined in [22] and can be approximated by Gaussian pdf with zero mean and variance equal to  $4\sigma_n^4$ , where  $\sigma_n^2$  is given by (9). The received energy per bit for user 0 is defined in the following form:

$$E_b = P_0 NT_c \quad (15)$$

The contribution of the MAI takes the form:

$$M = \sum_{k=1}^K M_k \quad (16)$$

$$M_k = \sqrt{\frac{P_k}{P_0}} \frac{1}{2N} [X(\mathbf{a}^I, \mathbf{d}^{I,k}) \cos \theta_k$$

$$+ X(\mathbf{a}^I, \mathbf{d}^{Q,k}) \sin \theta_k + X(\mathbf{a}^Q, \mathbf{d}^{I,k}) \sin(-\theta_k) + X(\mathbf{a}^Q, \mathbf{d}^{Q,k}) \cos \theta_k] \quad (17)$$

where

$$X(\mathbf{a}^{\lambda_1}, \mathbf{d}^{\lambda_2,k}) = \sum_{l=0}^{N-1} \sum_{n=-\infty}^{\infty} \alpha_l^{(\lambda_1,0)} d_n^{(\lambda_2,k)} \rho[(l-n)T_c - T_k] \quad (18)$$

and  $\lambda_1, \lambda_2 \in \{I, Q\}$  and

$$T_k = \text{mod}\{\tau_k, T_c\} \quad (19)$$

is the chip delay of user  $k$  with a uniform distribution over  $T_k \in [0, T_c)$ . The function

$$\rho(\tau) = \mathbf{F}^{-1}(T_c^{-1} |Q(f)|^2) \quad (20)$$

is the inverse Fourier transform of  $T_c^{-1} |Q(f)|^2$ . It is the autocorrelation function of the chip pulse normalized by  $\sqrt{T_c}$ . For example, given a Sqrt-RC pulse for  $q(t)$ ,  $\rho(\tau)$  is the raised cosine pulse scaled by  $T_c^{-1}$ .

### 3 MAI Analysis

This section derives an expression for the probability of error  $P_{er}$  and presents an accurate approximation for the distribution of the MAI component  $M$  in QDS-CDMA.

#### 3.1 Characteristic function of $M$

The exact characteristic function for QDS-CDMA wireless communication system is derived according to the approach used in [16] and [27] for B-CDMA. First, the conditional characteristic function

$$\phi_{M|\mathbf{a}^I, \mathbf{a}^Q, \mathbf{T}, \Theta}(\omega) = E[e^{j\omega M} | \mathbf{a}^I, \mathbf{a}^Q, \mathbf{T}, \Theta] \quad (21)$$

is derived by substituting in  $M$  from (16) and (17). The terms  $\mathbf{T}$  and  $\Theta$  are defined as

$$\mathbf{T} = [T_1, \dots, T_K] \quad (22)$$

and

$$\Theta = [\theta_1, \dots, \theta_K] \quad (23)$$

respectively. Next, the conditioning of each random vector is removed step by step in the order of  $\mathbf{T}, \Theta$  and, finally, both  $\mathbf{a}^I$  and  $\mathbf{a}^Q$ . Doing so yields

$$\phi_M(\omega) = \frac{1}{2^{2N}} \sum_{p=1}^{2^N} \sum_{q=1}^{2^N} \phi_{M|\mathbf{a}^I, \mathbf{a}^Q}(\omega) \quad (24)$$

where  $M$  for  $p, q \in [1, 2^N]$  represent, respectively, the  $2^N$  distinct sequences both for  $\mathbf{a}^I$  and  $\mathbf{a}^Q$ . Each  $\{\pm 1\}$  sequence is of equal likelihood and of length  $N$ . The characteristic function of  $M$ , conditioned on  $\mathbf{a}^I$  and  $\mathbf{a}^Q$ , is given by the following equations

$$\phi_{M|\mathbf{a}^I, \mathbf{a}^Q}(\omega) = \prod_{k=1}^K \phi_{M_k|\mathbf{a}^I, \mathbf{a}^Q}(\omega) \quad (25)$$

$$\phi_{M_k|\mathbf{a}^I, \mathbf{a}^Q}(\omega) = \frac{1}{2\pi T_c} \int_0^{2\pi T_c} \int_0^{2\pi T_c} \phi_{M_k|\mathbf{a}^I, \mathbf{a}^Q, T_k, \theta_k}(\omega) dT_k d\theta_k \quad (26)$$

$$\begin{aligned} & \phi_{M_k|\mathbf{a}^I, \mathbf{a}^Q, T_k, \theta_k}(\omega) \\ &= \prod_{n=-M}^{N+M-1} \cos\left\{ \frac{\omega \zeta_k}{N} [A^I \cos \theta_k - A^Q \sin \theta_k] \right\} \\ & \times \prod_{n=-M}^{N+M-1} \cos\left\{ \frac{\omega \zeta_k}{N} [A^I \cos \theta_k + A^Q \sin \theta_k] \right\}, \end{aligned} \quad (27)$$

where

$$\zeta_k = \frac{1}{2} \sqrt{\frac{P_k}{P_0}} A^{\lambda_1} = \sum_{l=0}^{N-1} \alpha_l^{(0, \lambda_1)} \rho[(l-n)T_c - T_k]$$

and  $\lambda_1 \in \{I, Q\}$ . To evaluate (24), (25) must be averaged across  $2^N \times 2^N$  distinct sequences. Thus,  $2^{2N} K$  double integrations are required in general. This is a daunting task for moderate to large values of  $N$ . For example, if  $N = 10$ , roughly  $10^6 K$  double integrations are required. Thus, a more computationally efficient expression is needed for  $\phi_M(\omega)$  given moderate to large  $N$ , i.e.  $N > 10$ .

#### 3.2 Approximation for the distribution $M|_{\mathbf{T}, \Theta}$

The approach in deriving an approximation for the distribution of  $M|_{\mathbf{T}, \Theta}$  follows that of [15] and [16]. The approximation assumes large bit finite  $N$  where  $\gamma = K/N$  is constant. Under these conditions, it can be shown that

$$M|_{\mathbf{T}, \Theta} \approx N [0, N\text{Var}(X_0)] \quad (29)$$

where

$$\begin{aligned} & \text{Var}(X_0) \\ &= \frac{2}{N^2} \sum_{k=1}^K \zeta_k^2 \left\{ B(0) + \sum_{i=1}^{1+\lfloor \alpha \rfloor} 2B\left(\frac{i}{T_c}\right) \cos\left(\frac{2iT_k\pi}{T_c}\right) \right\}. \end{aligned} \quad (30)$$

Two key observations follow. First, in contrast to that for DS-BPSK in [16], the effect of interference phase offsets disappear as  $\text{Var}(X_0)$  in (30) is independent of  $\Theta$ . Hence,

$$\text{Var}(M|_{\mathbf{T}, \Theta}) = \text{Var}(M|_{\mathbf{T}}) \quad (31)$$

Second, if  $\alpha = 0$ ,  $\text{Var}(X_0)$  reduces to

$$\text{Var}(X_0) = \frac{2B(0)}{N^2} \sum_{k=1}^K \zeta_k^2 \quad (32)$$

and is independent of  $\mathbf{T}$  as well. Hence,

$$\begin{cases} \text{Var}(M|_{\mathbf{T}}) = \text{Var}(M) ; \\ M \approx N [0, N\text{Var}(X_0)] . \end{cases} \quad (33)$$

That is the standard Gaussian approximation for large  $K$  and  $N$  if  $\alpha = 0$ .

#### 3.3 Approximation of $\phi_M(\omega) : \hat{\phi}_M(\omega)$

An approximation for  $\phi_M(\omega) : \hat{\phi}_M(\omega)$ , following the same steps outlined in [15], can be derived based on

(29) and the simple form of the characteristic function of a normal random variable. The approximation can be expressed as

$$\hat{\phi}_M(\omega) = \prod_{k=1}^K \hat{\phi}_{M_k}(\omega) ; \quad (34)$$

$$\hat{\phi}_{M_k}(\omega) = \frac{1}{T_c} \int_0^{T_c} \hat{\phi}_{M_k|T_k}(\omega) dT_k ; \quad (35)$$

$$\hat{\phi}_{M_k|T_k}(\omega) = \exp\left\{-\xi_\omega^{(k)} B(0)\right\} \times \prod_{i=1}^{1+[\alpha]} \exp\left\{-2\xi_\omega^{(k)} B\left(\frac{i}{T_c}\right) \cos\left(\frac{2iT_k\pi}{T_c}\right)\right\}, \quad (36)$$

where

$$\xi_\omega^{(k)} = \frac{\omega^2 P_k}{4P_0 N}. \quad (37)$$

Two simplifications can be obtained for pulses with small  $\alpha$ . First, if  $\alpha < 1, [\alpha = 0]$  and (36) reduces to

$$\hat{\phi}_{M_k|T_k}(\omega) = \exp\left\{-\xi_\omega^{(k)} B(0)\right\} \times \exp\left\{-2\xi_\omega^{(k)} B\left(\frac{1}{T_c}\right) \cos\left(\frac{2iT_k\pi}{T_c}\right)\right\}. \quad (38)$$

Substituting (38) into (35) returns

$$\hat{\phi}_{M_k}(\omega) = \exp\left\{-\xi_\omega^{(k)} B(0)\right\} I_0\left\{2\xi_\omega^{(k)} B(T_c^{-1})\right\}, \quad (39)$$

where

$$I_0(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{x \cos \theta\} d\theta \quad (40)$$

is the modified Bessel function of the first kind of zero order [28]. Second, if  $\alpha = 0, \hat{\phi}_{M_k}(\omega)$  reduces to

$$\hat{\phi}_{M_k}(\omega) = \exp\left\{-\xi_\omega^{(k)} B(0)\right\}. \quad (41)$$

## 4 Bit Error Probability Analysis

This section presents the standard Gaussian approximation, an exact probability of error  $P_{er}$  expression via the characteristic function procedure, its approximation, the improved Gaussian approximation, simplified improved Gaussian approximation, a discussion comparing the performance of QDS-CDMA to that of B-CDMA and insights on the effect of DS-BPSK spreading, DS-QPSK spreading and pulse shaping on the accuracy of the standard Gaussian approximation.

### 4.1 Standard Gaussian approximation

The standard Gaussian approximation models  $M$  as a Gaussian random variable and approximates the probability of error  $P_{er}$  as

$$P_{er}^{SGA} = Q\left\{\sqrt{\frac{2N_0^2}{E_b^2} + \frac{B(0)}{2N} \sum_{k=1}^K \frac{P_k}{P_0}}\right\}, \quad (42)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\{-0.5y^2\} dy. \quad (43)$$

### 4.2 Characteristic function procedure

According to the characteristic function procedure [8], [15], [16], the probability of error  $P_{er}$  can be expressed as

$$P_{er}^{CF_{exact}} = \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} \hat{\phi}_M(\omega) \phi_{n_{AF}^2 - n_{PF}^2}(\omega) \frac{\sin \omega}{\omega} d\omega, \quad (44)$$

where

$$\phi_{n_{AF}^2 - n_{PF}^2}(\omega) = \exp\left\{-\frac{2\omega^2 N_0^2}{E_b^2}\right\} \quad (45)$$

and  $\hat{\phi}_M(\omega)$  is given by (24). This expression is exact. However, as pointed out earlier, the number of computations grows exponentially with  $N$ .

### 4.3 Characteristic function procedure approximation

An approximation for the probability of error  $P_{er}$ , denoted as  $P_{er}^{CF}$ , can be obtained by replacing  $\hat{\phi}_M(\omega)$  in (44) with  $\hat{\phi}_M(\omega)$ . This yields an expression with significantly less complexity

$$P_{er}^{CF} = \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} \hat{\phi}_M(\omega) \phi_{n_{AF}^2 - n_{PF}^2}(\omega) \frac{\sin \omega}{\omega} d\omega, \quad (46)$$

where the complexity in  $\hat{\phi}_M(\omega)$  is independent of  $N$ . The results of this procedure are equivalent to those obtained by the improved Gaussian approximation described below.

### 4.4 Improved Gaussian approximation

The improved Gaussian approximation can be expressed as the mean of the bit-error probability  $P_{er}$  conditioned on  $\mathbf{T}$  and evaluated by random sampling across  $\mathbf{T}$  [11]

$$P_{er}^{IGA} = E \left\{ Q \left[ \sqrt{\frac{E_b^2 + \Psi}{2N_0^2}} \right] \right\}, \quad (47)$$

where the conditional variance,  $\Psi = \text{Var}(M|_{T})$  can be expressed as

$$\Psi = \sum_{k=1}^K \Psi_k, \quad (48)$$

where

$$\Psi_k = \frac{P_k}{2NP_0} \left\{ B(0) + \sum_{i=1}^{1+|\alpha|} 2B\left(\frac{i}{T_c}\right) \cos\left(\frac{2iT_k\pi}{T_c}\right) \right\}. \quad (49)$$

A simplified expression is derived if  $\alpha < 100\%$  using the approach of [19]. According to  $M|_{T,\theta}$  in (29), (49) can be expressed as

$$\Psi_k = C_1 + C_2 U_k, \quad (50)$$

where

$$\begin{cases} C_1 = \frac{B(0)P_k}{2NP_0} & \text{and} & C_2 = \frac{B(T_c^{-1})P_k}{2NP_0}; \\ U_k = \cos(2\pi T_k T_c^{-1}). \end{cases} \quad (51)$$

The  $\Psi_k$  s are independent since the  $T_k$  s are independent. Since  $T_k \in [0, T_c)$  is uniformly distributed, the pdf of  $U_k$  can be presented in the following form:

$$f_{U_k}(u) = \frac{1}{\pi\sqrt{1-u^2}}, \quad (52)$$

where  $|u| \leq 1$  [29]. If  $C_2 > 0$ , the pdf of  $\Psi$  can be expressed as the  $K - 1$  convolutions

$$f_{\Psi}(z) = f_{\Psi_1}(z) \times f_{\Psi_2}(z) \times \dots \times f_{\Psi_K}(z), \quad (53)$$

where the pdf of  $\Psi_k$ , a linear function of  $U_k$  [29], takes the following form

$$f_{\Psi_k}(z) = \frac{1}{\pi\sqrt{C_2^2 - (z - C_1)^2}}, \quad |z - C_1| \leq C_2. \quad (54)$$

Thus, the improved Gaussian approximation can be expressed as

$$P_{er}^{IGA} = \int_0^{\infty} Q \left[ \sqrt{\frac{E_b^2 + \Psi}{2N_0^2}} \right] f_{\Psi}(z) dz. \quad (55)$$

If  $\alpha = 0$ ,  $C_2 = 0$

$$\Psi = \frac{B(0)}{2NP_0} \sum_{k=1}^K P_k \quad (56)$$

and the improved Gaussian approximation reduces to the standard Gaussian approximation given by (42).

### 4.5 Simplified improved Gaussian approximation

The simplified improved Gaussian approximation for QDS-CDMA is derived based on the simplified improved Gaussian approximation derived for B-CDMA [17] and [18]. It approximates the probability of error  $P_{er}^{IGA}$  in (55) by applying the procedure of expansion of differences as described in [14]. It can be expressed as the sum of three weighted  $Q$ -functions

$$P_{er}^{SIGA} = \frac{2}{3} P(\mu) + \frac{1}{6} P[\mu + \sqrt{3}\sigma] + \frac{1}{6} P\{\max[0, \mu - \sqrt{3}\sigma]\}, \quad (57)$$

where  $P(x) = Q(1/\sqrt{x})$ . The terms  $\mu$  and  $\sigma$  represent, respectively, the mean and standard deviation of the variance of the interference including AWGN. Two terms are defined as

$$\mu = \sum_{k=1}^K E[\Psi_k] + \frac{2N_0^2}{E_b^2}; \quad (58)$$

$$\sigma^2 = \sum_{k=1}^K E[\Psi_k^2] - \{E[\Psi_k]\}^2. \quad (59)$$

Due to the earlier normalization of the bit statistic,  $1/\sqrt{\mu}$  represents the SNR. It can be readily shown that two moments  $E[\Psi_k]$  and  $E[\Psi_k^2]$  for general pulses are, respectively,

$$E[\Psi_k] = \frac{B(0)P_k}{2NP_0}; \quad (60)$$

$$E[\Psi_k^2] = \frac{P_k^2}{4N^2P_0^2} \left\{ |B(0)|^2 + \sum_{i=1}^{1+|\alpha|} 2|B(iT_c^{-1})|^2 \right\}. \quad (61)$$

In the case of ideal Nyquist pulse, simplifications occur, i.e.  $B(0) = 1$ ,  $B(i/T_c) = 1$ ,  $E[\Psi_k^2] = \{E[\Psi_k]\}^2$ ,  $\sigma^2 = 0$ . Thus, consistent with the earlier results for



$P_{er}^{IGA}$ , the simplified improved Gaussian approximation expression reduces to  $P_{er}^{SIGA} = P(\mu)$  that is equivalent to  $P_{er}^{SGA}$ .

#### 4.6 QDS-CDMA and B-CDMA comparison

The probability of error  $P_{er}$  performance of QDS-CDMA is compared with that of B-CDMA. The comparison is made on their respective expressions for the variance of the MAI component conditioned on  $\Theta$  and  $\mathbf{T}$  and the probability of error  $P_{er}^{SIGA}$  at identical conditions. For convenience, the corresponding B-CDMA expressions for the QDS-CDMA expression of  $\Psi, \Psi^{(B)}$  and  $P_{er}^{SIGA}, P_{er}^B$  are presented from [17]. The variance of the conditional MAI component can be expressed as

$$\Psi^{(B)} = \sum_{k=1}^K \Psi_k^{(B)} \quad , \quad (62)$$

where

$$\Psi_k^{(B)} = \frac{P_k \cos^2 \theta_k}{NP_0} \times \left\{ B(0) + \sum_{i=1}^{1+|\alpha|} 2B\left(\frac{i}{T_c}\right) \cos\left(\frac{2iT_k\pi}{T_c}\right) \right\} \quad (63)$$

such that

$$\Psi_k^{(B)} = 2\Psi_k \cos^2 \theta_k \quad . \quad (64)$$

The corresponding simplified improved Gaussian approximation expression for B-CDMA [17], the probability of error  $P_{er}^B$  is identical to that for QDS-CDMA in (57), (58)–(61) except that (62) is replaced with

$$E\left\{\left[\Psi_k^{(B)}\right]^2\right\} = \frac{3P_k^2}{8N^2P_0^2} \left\{ |B(0)|^2 + \sum_{i=1}^{1+|\alpha|} 2\left|B(iT_c^{-1})\right|^2 \right\} \quad (65)$$

such that

$$E\left\{\left[\Psi_k^{(B)}\right]^2\right\} = 1.5E[\Psi_k^2] \quad . \quad (66)$$

The means of the two MAI components over  $\Theta$  and  $\mathbf{T}$  are equal:

$$E[\Psi] = E[\Psi^{(B)}] = \mu \quad . \quad (67)$$

However, the variance of  $\Psi^{(B)}, \sigma_B^2$  is greater than that of  $\Psi$  and

$$\sigma_B^2 - \sigma^2 = \frac{1}{2} \sum_{k=1}^K E[\Psi_k^2] > 0 \quad . \quad (68)$$

The larger variation in the conditional MAI component variance (or power) for B-CDMA is not unexpected since the power level of the  $k$ -th interferer in B-CDMA is scaled by a  $\cos^2 \theta_k$  factor while that in QDS-CDMA remains constant irrespective of  $\theta_k$ . The larger variance of the conditional MAI component in B-CDMA follows intuitively and was previously hinted at in [1]. For example, in a two user scenario of B-CDMA, the signal of an interferer and the conditional MAI component, in the best case, disappears if out-of-phase relative to other desired user at  $\theta_1 = \{\pi/2, 3\pi/2\}$ . In the worse case, it appears at its maximum power level if it is in phase at  $\theta_1 = \{0, \pi\}$ . In contrast, in QDS-CDMA, the variance of the conditional MAI component with respect to phase is constant. The probability of bit-error  $P_{er}$  for QDS-CDMA is shown to be less or equal to  $P_{er}^B, P_{er}$  for B-CDMA, in regions of large SNR and nearly identical to  $P_{er}^B$  in regions of small SNR and  $\sigma_B^2 \ll SNR$ . We can introduce the following difference

$$\begin{aligned} \Delta P_{er} &= P_{er}^B - P_{er}^{SIGA} \\ &= \frac{1}{6} \left\{ P[\mu + \sqrt{3}\sigma] + P[\mu - \sqrt{3}\sigma] \right. \\ &\quad \left. - P[\mu + \sqrt{3}\sigma] - P[\mu - \sqrt{3}\sigma] \right\} \quad . \quad (69) \end{aligned}$$

The function given by (58) is a monotonically increasing function of  $x$  for  $x > 0$  as it represents the area in the tail end of the standard Gaussian pdf over the interval  $[1/\sqrt{x}, \infty)$ . In regions of large  $\mu$ , small SNR, and  $\mu \gg \sigma_B, P(x)$  is a slowly increasing function of  $x$  for large  $x$ . This implies that the four  $P(\dots)$  functions in (69) are approximately equal and that  $\Delta P_{er} \approx 0$ . On other hand, in regions of small  $\mu$ , i.e., high SNR,  $P(x)$  is an exponentially increasing function with  $x$  for small  $x \ll 1$ . This implies that the first  $P(\dots)$  function in (69) dominates the sum and that  $\Delta P_{er} > 0$ . Thus, in regions where  $\mu$  is small,  $P_{er} > P_{er}^B$ .

#### 4.6 DS-CDMA versus DS-QPSK spreading. Pulse shaping effect on standard Gaussian approximation accuracy

The effect of DS-BPSK spreading, DS-QPSK spreading and pulse shaping on the accuracy of the stand-

ard Gaussian approximation is discussed. The inaccuracy of the standard Gaussian approximation for B-CDMA can be explained intuitively based on the dependence of the conditional MAI variance on interferer phase offset. Implicit in the standard Gaussian approximation is the assumption that the MAI, like AWGN, is circularly symmetric and not depend on the phase off-sets [2]. In B-CDMA, the effect of phase offsets on the conditional MAI variance implies that the MAI process is not circularly symmetric under DS-BPSK spreading. Indeed, this was observed previously in [30] as well as in [11].

Pulse shaping and, more specifically, the excess bandwidth  $\alpha$  of the pulse shape also play an important role on the accuracy of the standard Gaussian approximation. It is well-known that the standard Gaussian approximation can be fairly inaccurate for time-limited pulses like the rectangular pulse [3], [5]–[8], [14]. In contrast, as was shown in Section 3, the improved Gaussian approximation in QDS-CDMA is reduced to the standard Gaussian approximation if  $\alpha = 0$  indicating a greater accuracy of the standard Gaussian approximation for QDS-CDMA. This phenomenon can be explained by the effect of pulse

shaping on the wide-sense cyclostationarity of the MAI process conditioned on the interferer chip delays [31]. A wide-sense cyclostationary process  $X(t)$  with a period of  $T$  is a process whose autocorrelation function is periodic with  $T$  such that [31]–[33]

$$E[X(t+T)X(u+T)] = E[X(t)X(u)] \quad . \quad (70)$$

Implicit in the standard Gaussian approximation is the assumption that the conditional MAI, like AWGN, is wide-sense stationary (WSS). However, given  $\alpha > 0$ , the conditional MAI process is wide-sense cyclostationary (WSCS) with a period of  $T_c$  under aperiodic spreading [31]. The level of cyclostationarity is intimately related to the excess bandwidth  $\alpha$  while in contrast, as  $\alpha \rightarrow 0$ , the cyclostationarity disappears and the WSCS process reduces to a WSS process. This observation explains the larger deviation of the standard Gaussian approximation for time-limited pulses, like the rectangular pulse, with large  $\alpha$  and its smaller deviation for band-limited pulse shapes.

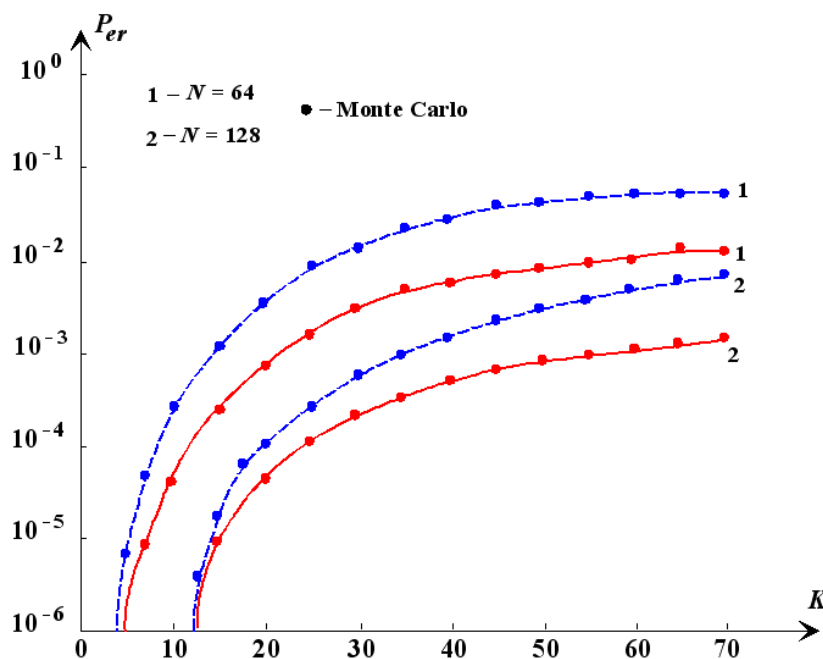


Figure 3.  $P_{er}$  versus  $K$  in QDS-CDMA.  $P_{er}^{IGA}$ ,  $P_{er}^{SIGA}$ ,  $P_{er}^{SGA}$  are matched and indistinguishable,  $\alpha = 22\%$  .

### 5 Simulation Results

Numerical examples are presented to illustrate the accuracy of the standard Gaussian approximation

with respect to the improved Gaussian approximation for performance analysis in QDS-CDMA. All the examples consider systems under uniform power co-

nditions and MAI-limited conditions where the effect of AWGN is neglected by setting  $N_0 = 0$ . Estimates of the bit-error probability  $P_{er}$  based on Monte Carlo simulations are also presented. These results

denoted by  $P_{er}^{Monte Carlo}$  requires of 100-1000 times the inverse of the estimated  $P_{er}$ . Figure 3 plots the bit-error probability  $P_{er}$  for QDS-CDMA systems employing the GD with Sqrt-RC  $\alpha = 22\%$  pulse where

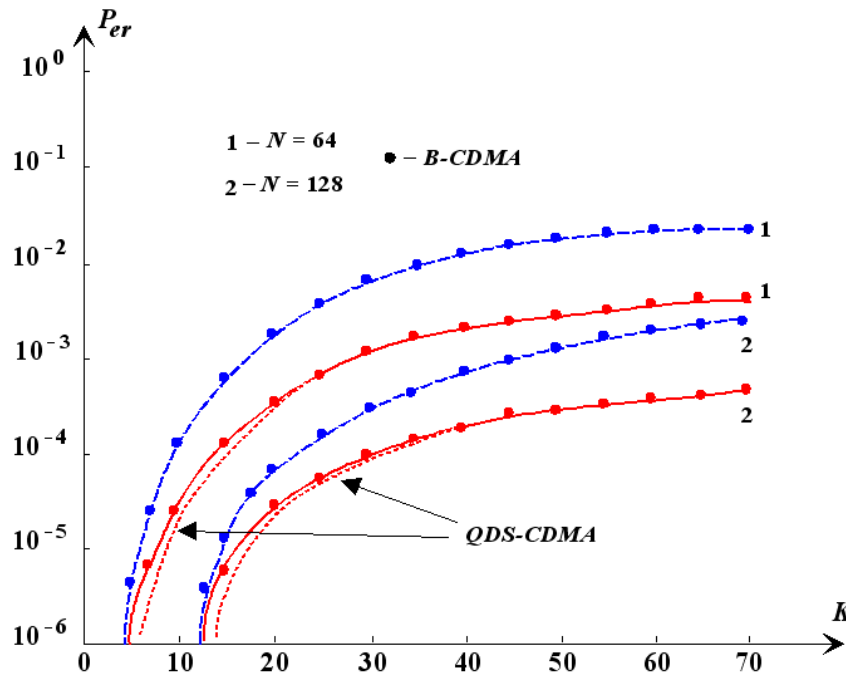


Figure 4.  $P_{er}$  versus  $K$  in QDS-CDMA.  $P_{er}^{IGA}$ ,  $P_{er}^{SIGA}$ ,  $P_{er}^{SGA}$  are very close to each other,  $\alpha = 100\%$ .

$N \in \{64,128\}$ . It shows that  $P_{er}^{IGA}$ ,  $P_{er}^{SIGA}$ ,  $P_{er}^{SGA}$ , and  $P_{er}^{Monte Carlo}$  are essentially identical. For example, with  $N = 128$ , the four estimates return  $P_{er} = 6.3 \times 10^{-5}$  at  $K = 20$ . Also, comparison with QDS-CDMA system employing the Neyman-Pearson receiver is presented. Comparative analysis demonstrates a superiority in favour of the GD.

Figure 4 plots the bit-error probability  $P_{er}$  versus  $K$  for the Sqrt-RC  $\alpha = 100\%$  pulse with  $N \in \{64,128\}$  and compares QDS-CDMA against B-CDMA. Again, it shows that  $P_{er}^{IGA}$ ,  $P_{er}^{SIGA}$ ,  $P_{er}^{SGA}$ , and  $P_{er}^{Monte Carlo}$  are essentially identical and that the standard Gaussian approximation is still a very good approximation for based on GD with  $\alpha$  as large as 100%. It also shows that as  $\alpha$  increases, the probability of bit-error  $P_{er}$  performance improves as would be expected. Figure 4 also shows that QDS-CDMA returns slightly superior the probability of bit-error  $P_{er}$  performance and capacity compared to that of B-CDMA in regions

where the number of users is small and the bit-error probability  $P_{er}$  is much less unit. On the other hand, in regions of large numbers of users and the high bit-error probability  $P_{er}$ , i.e.  $P_{er} \geq 10^{-4}$ , the two systems return indistinguishable the bit-error probability  $P_{er}$  performance. Again, we observe a superiority of GD employment in CDMA systems in comparison with the Neyman-Pearson receiver.

## 6 Conclusions

The bit-error probability  $P_{er}$  performance of QDS-CDMA was investigated. The exact bit-error probability  $P_{er}$  expression using the characteristic function procedure and several approximations, namely, the characteristic function procedure with an approximate characteristic function, the improved Gaussian approximation and the simplified improved Gaussian approximation were derived. Under conditions typically satisfied in practice, the MAI component of the GD statistic and the GD bit-error probability

$P_{er}$  performance can be accurately approximated by the standard Gaussian approximation. The accuracy of the standard Gaussian approximation in QDS-CDMA stems primarily from two sources: DS-QPSK spreading and bandwidth-efficient pulse shaping, in addition to moderate to large spreading factors and aperiodic spreading. The former makes the conditional MAI process, conditional on interferer chip delays and phase offsets, circularly symmetric while the latter improves the approximation of the wide-sense cyclostationary MAI process as a wide-sense stationary process. Second, the bit-error probability  $P_{er}$  performance of QDS-CDMA is shown to be superior or equivalent to that of B-CDMA. The performance difference diminishes as the number of users increase and as the bit-error probability  $P_{er}$  performance increases. Great superiority of the GD employment in comparison with the matched filter and Neyman-Pearson receiver is demonstrated.

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**Dr. Vyacheslav Tuzlukov** received the MSc and PhD degrees in radio physics from the Belorussian State University, Minsk, Belarus in 1976 and 1990, respectively. From 2000 to 2002 he was a Visiting Professor at the University of Aizu, Japan and from 2003 to 2007 served as an Invited Professor at the Ajou University, Suwon, South Korea, within the Department of Electrical and Computer Engineering. Since March 2008 to February 2009 he joined as Full Professor at the Yeungnam University, Gyeongsang, South Korea within the School of Electronic Engineering, Communication Engineering, and Computer Science. Currently he is a Full Professor of the Department of Communication and Information Technologies, School of Electronics Engineering, College of IT Engineering, Kyungpook National University, Daegu, South Korea. His research emphasis is on signal processing in radar, wireless communications, wireless sensor networks, remote sensing, sonar, satellite communications, mobile communications, and other signal processing systems. He is the author over 190 journal and conference papers, seven books in signal processing area published by Springer-Verlag and CRC Press, some of them are *Signal Detection Theory* (2001), *Signal Processing Noise* (2002), *Signal and Image Processing in Navigational Systems* (2005), *Signal Processing in Radar Systems* (2012), Editor of the forthcoming book *Communication Systems: New Research* (2013), Nova Science Publishers, Inc, USA, and has also contributed Chapters “Underwater Acoustical Signal Processing” and “Satellite Communications Systems: Applications” to *Electrical Engineering Handbook: 3<sup>rd</sup> Edition*, 2005, CRC Press; “Generalized Approach to Signal Processing in Wireless Communications: The Main Aspects and Some Examples” to *Wireless Communications and Networks: Recent Advances*, InTech, 2012; “Wireless Communications: Generalized Approach to Signal Processing”, to *Communication Systems: New Research*: Nova Science Publishers, Inc., USA, 2013, and “Radar Sensor Detectors for

Vehicle Safety Systems” to *Autonomous Vehicles: Intelligent Transport Systems and Automotive Technologies*, 2013. He participates as Keynote Speaker, Plenary Lecturer, Chair of Sessions, and Tutorial Instructor and organizes Special Sections at the major International Conferences and Symposia on signal processing.

Dr. Tuzlukov was highly recommended by U.S. experts of Defence Research and Engineering (DDR&E) of the United States Department of Defence as a recognized expert in the field of humanitarian demining and minefield sensing technologies and had been awarded by Special Prize of the United States Department of Defence in 1999. Dr. Tuzlukov is distinguished as one of the leading achievers from around the world by Marquis Who’s Who and his name and biography have been included in the *Who’s Who in the World, 2006-2012*; *Who’s Who in World, 25<sup>th</sup> Silver Anniversary Edition, 2008*, Marquis Publisher, NJ, USA; *Who’s Who in Science and Engineering, 2006-2012* and *Who’s Who in Science and Engineering, 10<sup>th</sup> Anniversary Edition, 2008-2009*, Marquis Publisher, NJ, USA;

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Phone: 053-950-5509

Email: [tuzlukov@ee.knu.ac.kr](mailto:tuzlukov@ee.knu.ac.kr)