DGS Based Legendre Low-Pass Filters for RF/Microwave

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Abstract: - The Legendre group of lowpass filters are designed and compared against several known types of filters at f_c^{3dB} = 2.5GHz in lumped elements, microstrip and DGS environment. In microstrip format the Legendre Polynomial Product (LPP)-312 and 213 filters have reduced group delay variation in the passband, sharper cut-off and wider impedance matching BW. In the DGS format the measured selectivity of Legendre, LPP-312 and LPP-213 lowpass filters are 28.3dB/GHz, 18.8B/GHz and 18.2B/GHz. Their 20dB rejection BW (RBW) are 6.0 GHz, 7.7GHz and 7.9 GHz; it covers 20dB rejection up to $4f_c$-$5f_c$ in the stopband. The LPP-213 has better than 15dB impedance matching BW (94%) and more than 20dB rejection BW (7.3GHz).

Key-Words: - Group delay (GD), Defected Ground Structure (DGS), Low pass Filter (LPF), Legendre LPF

1 Introduction

The microwave high performance filter, like their counter part at lower frequency, should have high selectivity, low loss and small group delay variation in the passband. Normally these are contradictory requirements and a filter is optimized for one of these parameters giving Chebyshev, Butterworth and Bessel type all-pole filters. The lowpass filters (LPF) using the Chebyshev and elliptic functions have high selectivity with ripples in the response and large variations of group delay in the passband. The external delay equalizers are used with these filters to improve their group delay flatness [1-5]. However it increases the size and complexity of the circuit. The GBit/s digital signal transmission [6, 7] and WCDMA-UMTS mobile radio systems [8] needs LPF with reduced linear distortion and inter-symbol interference. Bessel filter is a suitable choice for such applications. However Bessel filters have poor selectivity. Use of the Butterworth filter is a compromise among conflicting factors. At 7.5 GHz cut-off frequency, Bessel filter is reported in the mixed microstrip - waveguide (MMW) structure [6]. However, it is a large and bulky filter. At lower frequency, its size becomes prohibitively large. The DGS based compact Bessel LPF at 2.5 GHz is also suggested with improved performance [9].

The researchers of circuit theory have proposed several exotic solutions, including transitional filters to meet the above mentioned conflicting requirements of LPF [10-16]. A transitional filter is a judicious combination of characteristics of any two filters; such as transitional Butterworth-Legendre [10], transitional Butterworth-Chebyshev [11] etc. Other filters used for achieving transitional behavior between the Chebyshev and Butterworth are- optimum L-type (L-opt) [12], Legendre filter [13], Legendre polynomial products (LPP) [15], Pascal filters [16] etc. Some of these lumped elements filters are compared for their relative performances [14]. However these are not compared in the microstrip and DGS environment. Unlike Chebyshev filter, the Legendre group of filters-Legendre and Legendre polynomial product filters, have significantly unequal ripples in the passband that improve return-loss and impedance matching BW in the passband. Both these parameters are important for a microwave filter. So combination of Legendre polynomial products, for a fixed order of LPF, is also used to control the ripple amplitude in the passband that also influences the return-loss, selectivity and group delay of a filter [15]. A microstrip LPP bandpass filter, at 1.6 GHz, is reported with improved performance [15]. However, to our knowledge, the LPP lowpass filters are not examined in microwave frequency ranges, using the microstrip and microstrip with defected ground structure (DGS). The DGS has been used to improve the selectivity i.e. the sharpness of cut-off of a filter and to enlarge rejection bandwidth of a microstrip LPF. It also reduces size of a filter significantly [17, 18]. A synthesis process has also been reported to realize the DGS based Butterworth and Chebyshev LPF from their prototype filter [19].

In this work we have designed and compared the performances of several kinds of LPF , at f_c = 2.5 GHz,; with respect to edge-ripple factor, sharpness of cut-off, group delay variation in the passband, % impedance matching in the passband, and rejection bandwidth in the stopband. These filters are considered in the lumped, microstrip and DGS environment. The lumped components based filters are simulated on a circuit simulator- Microwave Office [20]; microstrip and DGS based filters are evaluated on an EM-simulator- HFSS [21]. The investigation is intended to draw attention of the microwave system designers; as it widens their choice for the selection of microwave filters. The work is organized as follows: Section-II compares performances of the lumped elements all-pole LPF, using several kinds of polynomials, including Legendre polynomial and their products. It demonstrates control of the parameters of the LPF by proper choice of the products of Legendre polynomials. Section-III compares performances of several kinds of microstrip based lowpass filters. Section-IV presents comparative performance of DGS version of these filters. The experimental results of the fabricated LPF are also presented. Section-V concludes the investigations.

2 Comparison of performances of several lumped element LPF

The following generic equation expresses the amplitude response of a low-pass filter:

$$\left|S_{21}(j\omega)\right| = \frac{H}{\sqrt{1+\varepsilon^2 F_n^2(\omega)}} \qquad (1)$$

where ε is the band-edge ripple factor and function. $F_n(\omega)$ represents the filtering characteristics through several functions with $\varepsilon=1$; such as Butterworth maximally flat response [1-4], Papoulis Optimum L- type (L-opt) response [12], and Bessel maximally flat group delay response [4]. The L-opt filters have staircase type response in the passband. It is in between the ripple and flat response based filter. At $\omega_c=1$, it has the highest attenuation slope, more than the Butterworth LPF, for a monotonic magnitude response. The filters using these functions do not have ripple in the passband. So they could be a suitable candidate for the digital transmission [14]. The Bessel filters provide maximum flatness of the group delay in the passband without any ripple. Therefore, a Bessel filter, even with poor selectivity, could also be useful for the digital transmission [6-8]. Its matching BW in the passband is not good, that is important for the RF applications [3].

Other functions- Chebyshev [1-4], Legendre [13, 15] and Legendre polynomial products [15] have specified values of ε; $\varepsilon=\sqrt{10^{\frac{r}{10}}-1}$, where r is the band-edge ripple-factor in dB. The filters using these functions do have ripples in the passband- equal magnitude ripples for the Chebyshev and significantly unequal magnitude ripples for the Legendre group of filters. Practical consideration requires defining cut-off frequency f_c^{3dB} of a lowpass filter (LPF) at 3dB [3, 4]. However, in case of the Chebyshev and Legendre group of filters the components values are given only at the edge -ripple frequency f_c^r. Therefore, the specified f_c^{3dB} frequency should be converted to the f_c^r. For the Chebyshev filter it is carried out using the following expression [3]:

$$f_c^r = \frac{2f_c^{3dB}}{\left[x^{\frac{1}{n}} + x^{\frac{-1}{n}}\right]} \quad (a) \quad \text{where } x = \frac{1}{\varepsilon} + \sqrt{\frac{1}{\varepsilon^2}-1} \quad (b) \qquad (2)$$

In case of the Legendre group filters, we do not know any closed-form expression to convert the specified 3dB cut-off frequency f_c^{3dB} to the ripple edge frequency f_c^r. However, the following process can be used to obtain the ripple edge frequency; as the components values are given at f_c^r [15]. Initially we determine the components values of a LPF at the specified 3dB cut-off frequency $f_c^{3dB}(\text{specified})$, treating it as initial f_c^r, and simulate the LPF on a circuit simulator, Microwave Office [19]. The simulated 3dB cut-off frequency $f_c^{3dB}(\text{simulated})$ is higher than $f_c^{3dB}(\text{specified})$. We can define the frequency change ratio (FR) from these two frequencies. The FR helps to compute the correct edge -ripple frequency f_c^r for the specified 3dB cut-off frequency.

We compute the components values of the Legendre group of LPF at the correct f_c^r and simulate the filter on the circuit simulator to observe correct 3dB cut-off frequency f_c^{3dB}. The below expression is also valid for the Chebyshev LPF.

$$\begin{aligned} FR &= \frac{f_c^{3dB}(\text{simulated})}{f_c^{3dB}(\text{specified})} \quad (a) \\ f_c^{\,r} &= \frac{f_c^{3dB}(\text{specified})}{FR} \quad (b) \end{aligned} \qquad (3)$$

In case of the Legendre polynomials products (LPP) filters, the filtering characteristics function $F_n(\omega)$ are generated by the products of lower order Legendre polynomials while maintaining the same order of the Legendre polynomial. Thus for the 5th order case, we have following combinations of the Legendre polynomials products [15]:

$$F_5(\omega) = \begin{cases} P_4(\omega)P_1(\omega), P_3(\omega)P_2(\omega), \\ P_3(\omega)P_1^2(\omega) \text{ or } P_2(\omega)P_1^3(\omega) \end{cases} \qquad (4)$$

The above Legendre polynomials products (LPP) provide four LPP lowpass filters – LPP-41, LPP-32, LPP-312, and LPP-213 corresponding to $P_4(\omega)P_1(\omega)$, $P_3(\omega)P_2(\omega)$, $P_3(\omega)P_1^2(\omega) = P_3(\omega)P_1(\omega)P_1(\omega)$, and $P_2(\omega)P_1^3(\omega) = P_2(\omega)P_1(\omega)P_1(\omega)P_1(\omega)$ polynomial products. The ripple-factor (r) at the ripple-edge frequency (f_c^r) is in dB. It also decreases, like the original Legendre LPF, with decrease in frequency from ω_c^r to $\omega \rightarrow 0$ in the passband.

We compare below the performances of the above mentioned lumped elements 5-pole LPF at f_c^{3dB}=2.5 GHz with respect to several important parameters- insertion loss in the passband, % impedance matching bandwidths at, 15dB and 20dB return-loss, selectivity i.e. the sharpness of cut-off at 3dB cut-off frequency, and 15dB/ 20dB rejection bandwidth. The normalized group delay variations in the passband are also compared. The selectivity is defined as (20dB-3dB)/ (f_{20dB}-f_{3dB}) in dB/GHz.

The rejection bandwidth is not needed for a lumped element based LPF; as attenuation in the stopband increases monotonically with increase in frequency. However, it is required for the microstrip and DGS based LPF; as they have periodic and broken periodic response in the stopband.

- **5-pole Lumped Components LPF:**

The prototype 5-pole LPF is shown in Fig.1. The g-values and denormalized components values at f_c^{3dB}=2.5 GHz of L-optimum (L-opt) [12], Legendre (Leg) [15] , Legendre polynomial products (LPP) [15], Chebyshev (Cheb) [1,2], Butterworth (Butt) [1,2] and Bessel (Bess) LPF[2] are shown in table-1. For the ripple type LPF – Chebyshev, Legendre and LPP, the components values are obtained at the edge - ripple frequency for the ripple factor (r) = 0.5 dB.

Table: 1 Normalized and de-normalized component values of the L-opt , Legender (Leg) , Legendre polynomial products (LPP) , Chebyshev ,Butterworth and Bessel LPF

Filter **Type**	g_1 and L_1 (nH)	g_2 and C_2 (pF)	g_3 and L_3 (nH)	g_4 and C_4 (pF)	g_5 and L_5 (nH)	**Filter** **Type**	g_1 and L_1 (nH)	g_2 and C_2 (pF)	g_3 and L_3 (nH)	g_4 and C_4 (pF)	g_5 and L_5 (nH)
Cheb	1.7058 5.7517	1.2296 1.6585	2.5408 8.5675	1.2296 1.6585	1.7058 5.7517	**Leg**	1.1349 3.6125	1.4693 1.8708	1.9784 6.2974	1.4693 1.8708	1.1349 3.6125
Butt	0.6180 1.967	1.618 2.06	2.0 6.366	1.618 2.06	0.618 1.967	**LPP-41**	0.8505 2.7072	1.5731 2.0029	1.7072 5.4342	1.5731 2.0029	0.8505 2.7072
Bess	0.1743 0.55548	0.5072 0.6458	0.8040 2.559	1.1110 1.415	2.2582 7.188	**LPP-32**	0.8210 2.6133	1.4649 1.8652	1.8114 5.7658	1.4649 1.8652	0.8505 2.6133
L-opt	0.9512 3.0296	1.478 1.883	2.0673 6.5844	1.5395 1.9613	1.999 6.668	**LPP-312**	0.6823 2.1718	1.5075 1.9194	1.6506 5.2540	1.5075 1.9194	0.6823 2.1718

The edge- ripple frequencies of these filters are obtained as discussed above. These are 2.298 GHz, 2.232GHz, 2.196 GHz, 2.159 GHz and 2.083 GHz for Legendre (Leg), LPP-41, LPP-32, LPP- 312 and LPP- 213 respectively.

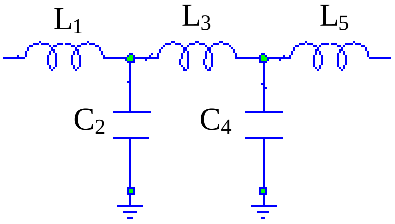
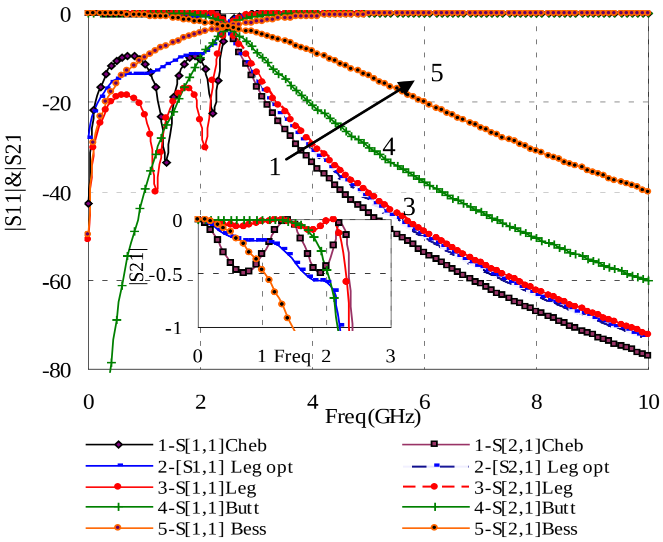


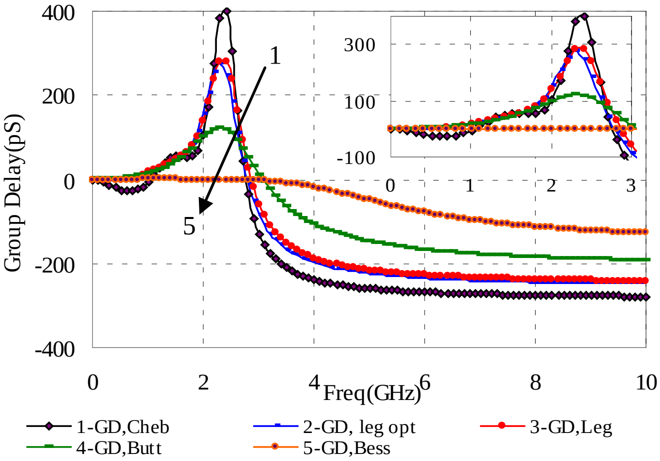
Fig.1: 5-pole prototype LPF.

Fig.2a - Fig.2b compare the frequency response- transmission response $|S_{21}|$ and reflection response $|S_{11}|$ and normalized group delay (GD) response, of the 5-pole Chebyshev and Legendre LPF at the 0.5dB edge-ripple factor. It also presents responses of the L-opt, Butterworth and Bessel LPF. The Chebyshev LPF has also 0.5dB equal magnitude ripples inside the passband. However, the Legendre LPF has unequal ripples within the passband with significantly reduced 0.07dB magnitude at frequency below 2.11 GHz.

The L- opt has monotonically increasing staircase type insertion loss response in the passband. At 1.6GHz, its passband attenuation even exceed 0.5dB ripple of the Chebyshev. Such response can cause distortion in the complex signals. As expected, the Butterworth LPF has flat response and its 0.07dB bandwidth is 1.7GHz that is much less than the 0.07 dB bandwidth of the Legendre LPF. Therefore at the insertion loss 0.07dB the passband BW of a Legendre LPF is more than that of a Butterworth. The corresponding BW for the Bessel LPF is only 0.41 GHz. We have also noted that at r=3dB, Legendre LPF has only 0.7dB ripple in the passband; as against 3dB ripple in the passband for the Chebyshev [15]. Fig.2a further shows that at r=0.5dB, the return-loss (RL) of a Chebyshev LPF is not satisfactory (<10dB). For the RF and microwave applications it should in the range 15 dB - 20dB [3].



(a):|S11| and |S21| response



(b) GD response

Fig.2: Frequency and group delay response of Chebyshev (Cheb and Legendre (Leg) (r=0.5 dB), Butterworth (Butt), Bessel (Bess) and L- opt. LPF at f_c^{3dB} =2.5 GHz.

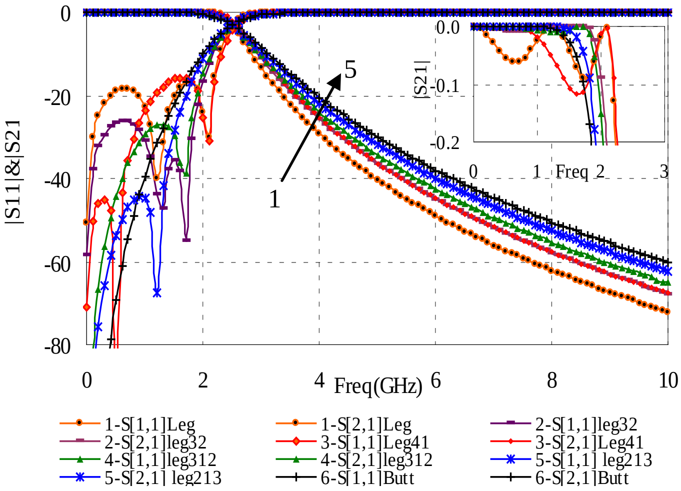
Table -2 shows that at r = 0.5dB, the Chebyshev LPF has very limited, 16.4% and 8.4% , impedance matching BW for 15dB and 20dB return-loss. Similarly the Bessel (24.4%, 12.4%) and L-opt (20.4%, 8.4%) LPF also have limited matching BW. The Butterworth LPF has 68.4% and 64.4% matching BW at 17dB and 20dB return loss. However, the Legendre LPF has good matching BW, 88.4%, at 15dB return loss; although at 20dB RL, it reduces to 16.4%. At r=0.5dB Legendre LPF appears to be a better candidate for the RF application. Its selectivity

(18.9dB/GHz) is somewhat less than that of the Chebyshev (24.63dB/GHz). However the selectivity of the Legendre LPF is much more than that of the Butterworth LPF (12.14dB/GHz). We note that the selectivity of the L-opt (21.25dB/GHz) is only a little less than that of the Chebyshev. The Bessel LPF has very poor selectivity, 4.84dB/GHz. The RL of the Chebyshev is improved at lower value of ripple factor (say r=0.1dB). In that case its selectivity degrades. Thus response of the Legendre filter is in between the Chebyshev and Butterworth - ripple in passband less than Chebyshev and selectivity more than Butterworth.

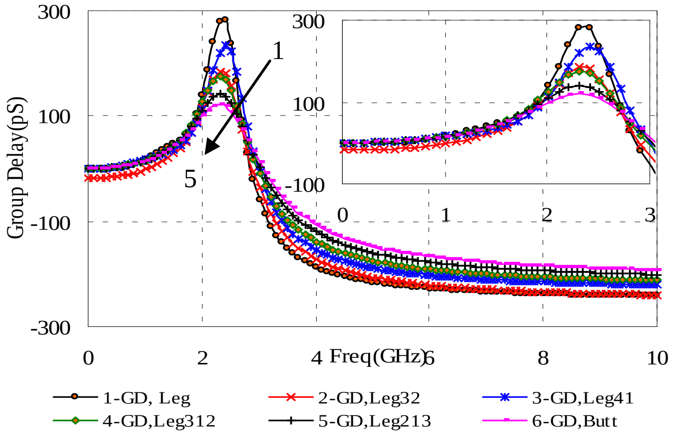
The group-delay performance of these filters is compared in Fig. (2b). Table-2 also summarizes the normalized group-delay performance at 2 GHz in the passband and also at $f_c^{3dB} = 2.5\,GHz$. At f_c^{3dB} the Chebyshev LPF has largest variation in GD (305.3pS) followed by the Legendre (236.5pS); whereas the Bessel has minimum (0.65pS).

The Butterworth LPF performance (108.3ps) is better than Chebyshev and Legendre filters. The L-opt GD at $f_c^{3dB} = 2.5\,GHz$ is also large, 186.1ps. The variation is reduced within 80% of passband, i.e. up to 2 GHz. We observe the following GD variations: Chebyshev (205.5pS), Legendre (186.3pS), Butterworth (100.4pS), L-opt (175.9pS), and Bessel (5.5pS). The Bessel filter has best performance for the GD variation in the passband. However its selectivity (4.84dB/GHz) is poor.

We have noted above better return-loss and impedance matching BW for a Legendre filter that is due to the its reduced ripple level 0.07dB in the passband. Other Legendre group of filters, i.e. the Legendre polynomial products (LPP) filters, provides even lesser ripple level in the passband on sacrificing selectivity. Of course, the selectivity is still more than that of a Butterworth LPF [15]. Fig.3 compares the responses of 4 LPP filters against the Legendre and Butterworth LPF. The performances are also summarized in table-2.



(a):|S11| and |S21| response



(b) GD response

Fig.3: Frequency and group delay response of Legendre, Legendre polynomial product (LPP) (r=0.5 dB), and Butterworth LPF at f_c^{3dB}=2.5 GHz.

We note two ripple peaks for LPP-32 and LPP-312, in the passband- 0.06dB and 0.08dB i.e. 0.07dB average ripple. The LPP-41 filter has nearly flat response up to 0.7GHz. However, it has larger ripple peak (0.12dB) at 1.5 GHz. The ripple level has adverse influence on the RL and impedance matching BW of a LPF. The Legendre LPF has the largest 15dB matching BW (88.4%) and minimum 20dB matching BW (16.4%). Its selectivity is 18.9dB/GHz. The Butterworth LPF has 68.4% and 64.4% matching BW at 15dB and 20dB with 12.14dB/GHz selectivity. The LPP-32 provides best compromise for the selectivity (16.19dB, GHz) and 15dB and 20dB return-loss impedance BW (80.4%, 76.4%). Both these parameters are better than that of the Butterworth LPF. However its GD performance (144 pS in passband) is inferior to the Butterworth (100pS). The LPP-312 has improved the GD performance, 149.7pS at 2.5 GHz and 145.4 pS in the passband by sacrificing the matching BW to 76.4% and selectivity to 14.16 dB/ GHz.

Table 2: Selectivity (Selct.), Impedance (Imp) matching BW and Group dealy (GD) Performance of several lumped LPF(Ripple Factor 0.5dB, $f_c^{3dB} = 2.5\text{GHz}$)

Filter Type	Selct. (dB/ GHz)	Imp. matching BW		GD in passband (pS)		**Filter Type**	Selct. (dB/ GHz)	Imp. matching BW		GD in passband (pS)	
		15dB GHz ; %	20dB GHz; %	<2GHz (pS)	2.5 GHz (pS)			15dB GHz ; %	20dB GHz; %	<2GHz (pS)	2.5 GHz (pS)
Cheb	24.63	0.41 16.4	0.21 8.4	205.5	305.4	**leg**	18.9	2.21 88.4	0.41 16.4	186.3	236.5
Butt	12.14	1.71 68.4	1.61 64.4	100.4	108.3	**LPP-41**	15.31	2.21 88.4	1.11 44.4	148.7	223.0
Bessel	4.84	0.61 24.4	0.31 12.4	5.5	0.65	**LPP-32**	16.19	2.01 80.4	1.91 76.4	144.1	157.1
L-opt	21.25	0.51 20.4	0.21 8.4	175.9	186.1	**LPP-312**	14.16	1.91 76.4	1.91 76.4	145.4	149.7
						LPP-213	13.07	1.81 72.4	1.71 68.4	116.08	124.7

Using more compromise on selectivity (13.07dB/GHz) and matching BW (72.2%, 68.4%); LPP-213 provides us better GD response (116pS) in the passband. The LPP-312 has demonstrated better bandpass filter, less ripple in passband and lesser GD variation, as compared to the Chebyshev BPF [15]. It appears that LPP-32 and LPP-312 could be considered for the RF and microwave applications; whereas the LPP-213 could be useful for the high speed digital transmission.

3 Low pass filters in microstrip

Normally we realize microwave planar filters in the microstrip environment [1-4]. So we focus our attention to the microstrip based 5th order $f_c^{3dB} = 2.5\text{GHz}$ Chebyshev and Legendre group LPF at the r=0.5dB. We also compare them against the Butterworth and Bessel LPF. The Legendre group of filters are compared in two steps- **i.** Legendre filters and **ii.** Legendre polynomial products (LPP) filters.

We have noted above that the Legendre groups of filters have unequal low magnitude within the passband. It results in better than 15dB-20dB return- loss. Table-1 shows values of the lumped elements of these filters. The following expressions are used to get the denormalized lumped component values and length of the inductive and capacitive sections of the microstrip sections. The lengths of microstrip sections are corrected for the step discontinuity [1-2].

$$L_i = \frac{g_i Z_0}{\omega_c}\ (a)\ C_{i+1} = \frac{g_{i+1}}{\omega_c Z_0}\ (b),\ i = 1,3,.5,...n \qquad (5)$$

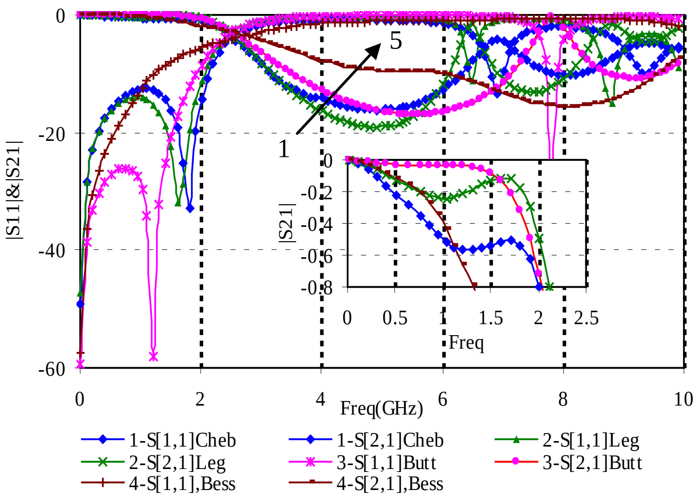
$$l_c = \frac{\lambda_{gL}}{2\pi}\sin^{-1}\left(\frac{\omega_c L}{Z_{oL}}\right)(a)\; l_c = \frac{\lambda_{gc}}{2\pi}\sin^{-1}(\omega_c C Z_{oc})\;(b) \qquad (6)$$

$$\lambda_{gi} = \frac{\lambda_c}{\sqrt{\varepsilon_{ri,eff}}} \quad (i = L, C) \quad (c)$$

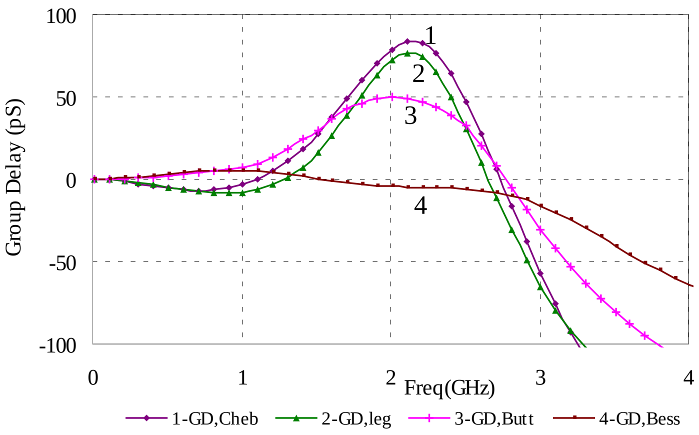
i. Microstrip Legendre LPF

Fig.4 compares amplitude and group delay (GD) performances of the microstrip based Chebyshev, Legendre, Butterworth and Bessel LPF. All filters exhibit the periodic response in the stopband; even 20dB rejection is not attained. The results are also summarized in table-3. We note that only Legendre and Butterworth LPF have meaningful 15dB impedance matching BW – 76.4% and 68.4% respectively. The 15dB impedance matching BW of the Legendre LPF in microstrip has reduced from its lumped element BW (88.4%). The Bessel and Chebyshev LPF have very limited impedance BW- 34.4% and 20.4%.

The selectivity of the filters, with some deviations, follows the lumped element format. However, the group delay performances of all LPF, except the Bessel, in microstrip are much improved; as compared to their counter parts in the lumped elements case. Fig.4b and table-3 show that at 2.5 GHz, the Chebyshev LPF has the largest GD variation - 47.1pS, followed by the Legendre and Butterworth LPF-32.2pS. The Bessel LPF has 5.83pS variation in the GD. At 0.5dB edge-ripple factor, the microstrip Legendre LPF appears a useful filter, due to its larger impedance matching BW.



(a):|S11| and |S21| response



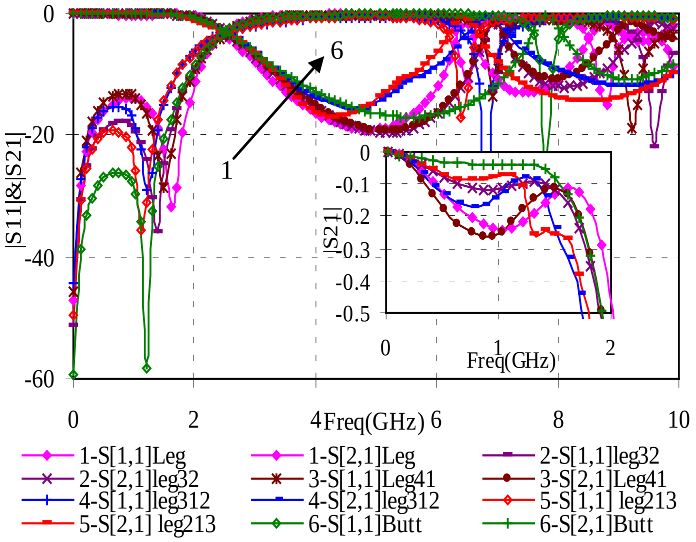
(b) GD response

Fig.4: Amplitude and group delay response of microstrip Chebyshev (r=0.5dB), Legendre (r=0.5dB), Butterworth and Bessel LPF at for $f_c^{3dB} = 2.5GHz$.

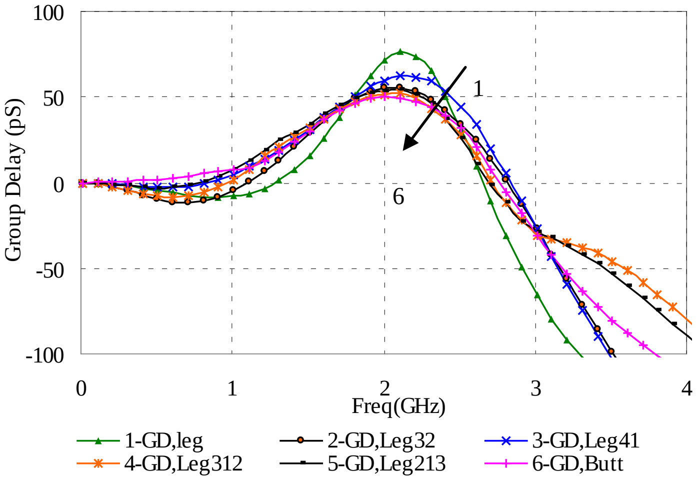
ii. Microstrip Legendre polynomial products (LPP) LPF:

Fig.5 compares performance of the LPP type filters against the Legendre and Butterworth. As noted earlier, performances of the LPP type filters are in between the Legendre and Butterworth. The selectivity and GD performance of the LPP-312 and LPP-213 filters - 16.16 dB/GHz and 15.07dB/GHz, are better than that of the Butterworth LPF- 11.14 dB/GHz. However, their 15dB impedance matching bandwidths are 200 MHz less.

Table-3 shows that with 3-4dB/GHz sacrifice in the selectivity and at 60.4% matching BW, we can get 29.7pS and 26.8pS GD variations for the LPP-312 and LPP-213 filters. It appears that the microstrip version of LPP-312 and LPP-213, with selectivity higher than Bessel one and better GD performance compared to the Butterworth LPF, could be useful for the high speed digital transmission [6-8]. The Legendre LPF with larger matching BW (76.6%), compared to the Chebyshev (20.4%) could be useful for the RF/microwave



(a):|S11| and |S21| response



(b): Group delay response

Fig.5: Amplitude and GD response of microstrip Legendre, LPP (r=0.5dB) and Butterworth LPF at $f_c^{3dB} = 2.5$GHz

applications as its GD performance are better. Selectivity of the Legendre LPF is 18.9dB/GHz; whereas for the Chebyshev it is 22.63dB/GHz. The matching BW of the Chebyshev LPF can be improved by reducing the edge-ripple factor. However, it also reduces its selectivity. Moreover, at the reduced edge-ripple factor, the return- loss and matching BW of the Legendre filter will be further improved. Both the 15dB-20dB matching BW and stopband performances of filters are improved using the DGS with microstrip [16-18]

Table 3: Amplitude and GD performance of several microstrip based LPF. (15dB RWBW: Return-loss BW at 15dB return-loss, GD within impedance BW)

Filter Type	Sharpness (dB/ GHz)	15dB RLBW GHz / %BW	GD at 2.5GHz pS	**Filter Type**	Sharpness (dB/ GHz)	15dB RLBW GHz / %BW	GD at 2.5 GHz pS
Cheb	22.63	0.51 20.4	47.1	**LPP-41**	15.31	1.71 68.4	44.6
Butt	11.14	1.71 68.4	32.2	**LPP-32**	15.19	1.81 72.4	34.5
Bessel	7.08	0.86 34.4	5.83	**LPP-312**	16.16	1.51 60.4	28.4
leg	18.9	1.91 76.4	32.2	**LPP-213**	15.07	1.51 60.4	25.7

4 Low pass filters in microstrip-DGS

In this section first we summarize a method to synthesize the DGS inductor. Next use it to replace the inductive sections of the microstrip LPF. Finally performances of the Legendre group of filters are validated by the experimental results.

i. DGS Inductor

The defected ground structures (DGS) in the microstrip and its equivalent circuit are shown in Fig.6a and Fig.6b. We can ignore resistance R. It does not contribute in the synthesis of a DGS based LPF. Fig.6c shows that below the pole, i.e. the resonance frequency, the DGS acts as an inductor [17-19]. The quasi lumped zero dimension DGS inductor is embedded in the microstrip and is located at the centre of the coupling gap g. The size of the twin heads of the DGS determines value of the inductor. Its geometrical shape influences the selectivity. The triangle shape, used in this work, has sharper cut-off as compared to that of the circular and square shape DGS [18].

The following closed-form expressions compute the twin slot-head area of a DGS, $S(\varepsilon_r, h)$ mm^2, in the $50\,\Omega$ microstrip line for the known inductance L in nH [19]:

$$S(\varepsilon_r, h) = M(\varepsilon_r, h)\ L + N(\varepsilon_r, h) \quad (7)$$

where $M(\varepsilon_r, h)$ and $N(\varepsilon_r, h)$ are substrate dependent intermediate parameters, obtained from the following curve-fitted expressions in the range- $2.2 \le \varepsilon_r \le 12.9$ *and* $0.6mm \le h \le 2.0\,mm$:

$$M(\varepsilon_r, h) = p_3 h^3 + p_2 h^2 + p_1 h + p_0 \quad (8)$$

The coefficients $p_0 - p_3$ of the expression for $M(\varepsilon_r, h)$, given below, are function of relative permittivity of a substrate:

$$p_i = f_i\, \varepsilon_r^2 + c_i\, \varepsilon_r + d_i \quad ; \; i = 0,1,2,3 \quad (9)$$

where co-efficient f_i , c_i and d_i are obtained from expressions given in table-4a.

Table 4a. co-efficient f_i, c_i, d_i

ε_r = 2.2- 4.4, $0.6 \le h \le 2$ mm											
c_0	c_1	c_2	c_3	d_0	d_1	d_2	d_3	f_0	f_1	f_2	f_3
0.647	-1.224	0.665	-0.161	6.963	0.813	2.109	-0.415	0.000	0.000	0.000	0.000
ε_r = 4.4 - 12.9, $0.6 \le h \le 2$ mm											
-1.591	4.161	-4.662	1.492	14.757	-17.378	19.382	-5.703	0.106	-0.284	0.318	-0.103

$$N(\varepsilon_r, h) = q_5 h^5 + q_4 h^4 + q_3 h^3 + q_2 h^2 + q_1 h + q_0 \quad (10)$$

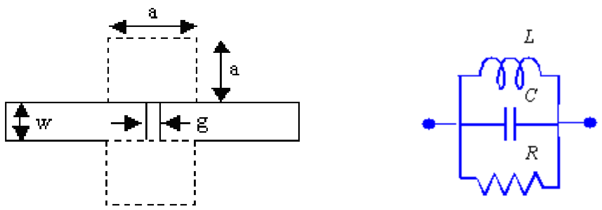
The coefficients $q_0 - q_5$ of the above expression are given by

$$q_i = l_i\, \varepsilon_r^2 + m_i\, \varepsilon_r + n_i \quad ; \; i = 0,1,2,3,4,5 \quad (11)$$

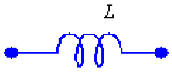
where coefficients l_i, m_i, and n_i are given in table-4b.

Table 4b. coefficients l_i, m_i, and n_i

ε_r = 2.2- 4.4, $0.6 \le h \le 2$ mm											
m_0	m_1	m_2	m_3	m_4	m_5	n_0	n_1	n_2	n_3	n_4	n_5
-48.03	230.5	-397.8	319.3	-120.9	17.48	191.9	-823.6	1369.3	-1074.4	399.8	-57.1
l_0	l_1	l_2	l_3	l_4	l_5						
0.000	0.000	0.000	0.000	0.000	0.000						
ε_r = 4.4 – 12.9, $0.6 \le h \le 2$ mm											
m_0	m_1	m_2	m_3	m_4	m_5	n_0	n_1	n_2	n_3	n_4	n_5
-89.71	366.1	-576.9	438.2	-162.3	23.70	242.1	-876.1	1296.9	-942.9	339.8	-49.1
l_0	l_1	l_2	l_3	l_4	l_5						
6.88	-28.10	44.44	-33.82	12.52	-1.82						



(a): DGS slot in ground plane (b) Equivalent circuit.



(c): DGS inductor below resonance frequency

Fig.6: DGS inductor.

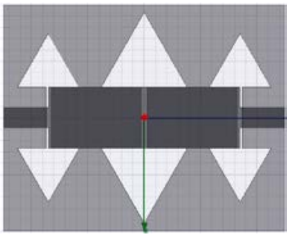


Fig.7: Layout of DGS based 5-pole Legendre and LPP lowpass filters at $f_c^{3dB} = 2.5\,\text{GHz}$.

Table- 5: Area of DGS of Legendre type LPF

Filter Type	Slot area	
	Area (mm^2)	Area (mm^2)
Leg	A_1=A_5=12.5	A_3=25
Leg P_4P_1	A_1=A_5=8.5	A_3=22
Leg P_3P_2	A_1=A_5=8.5	A_3=24
Leg $P_3P_1^2$	A_1=A_5=7	A_3=22
Leg $P_2P_1^3$	A_1=A_5=10	A_3=22.5

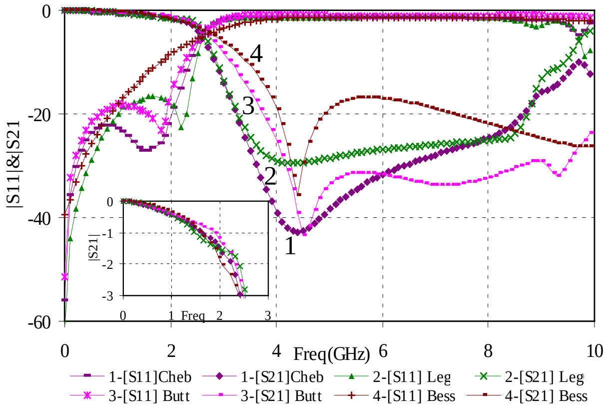
The prototype shunt capacitances of a LPF are realized by the low impedance, say 23Ω, microstrip [1]. The DGS embedded in a low impedance microstrip has lower inductance value, as compared to the DGS embedded in a 50Ω microstrip. In the low impedance microstrip the value of inductance is compensated by increasing the size of a DGS slot that is computed from the above expressions [19]. However, the compensated DGS area of a DGS based LPF can also be obtained using an EM-simulator.

ii. Microstrip - DGS LPF

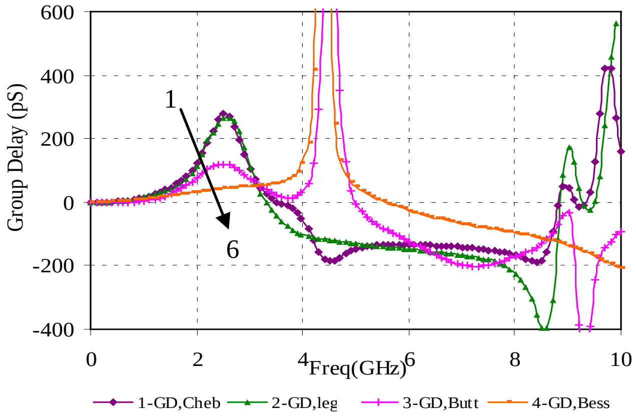
We designed the DGS based 5-pole Legendre group of filters at $f_c^{3dB} = 2.5GHz$. Fig.7 shows the layout of the DGS implementation of the 5-pole filters.The equal lengths of two capacitive 23Ω microstrip sections are computed using equation- 6b. Three inductances of the Legendre group LPF are computed using above expressions and the DGS slot sizes are summarised in table-5. Likewise, other DGS based LPF are also designed. We compare DGS based Legendre group of filters in two steps- i. Legendre and ii. Legendre polynomial products (LPP).

- **DGS Legendre LPF**

Fig.8a and Fig8b compare the amplitude and group delay performances of the DGS based 5-pole Legendre filters against the results of the DGS based Chebyshev, Butterworth and Bessel LPF at $f_c^{3dB} = 2.5\,\text{GHz}$.



(a):|S11|&|S21| response



(b) GD response

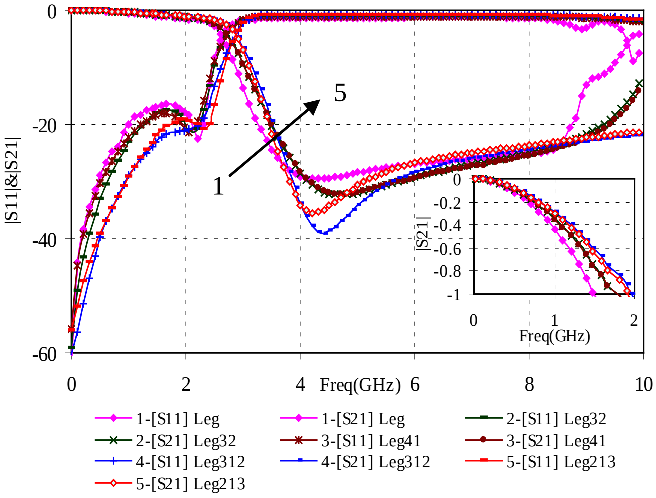
Fig.8: Responses of the DGS based Chebyshev, Butterworth, Bessel and Legendre lowpass filters $f_c^{3dB} = 2.5\,\text{GHz}$.

Table-6 shows about 1dB/GHz - 4dB/GHz improvement in the selectivity of the DGS based filters, as compared to the microstrip based filters. The Bessel filter shows maximum improvement in the selectivity, from 7.08dB/GHz to 10.96 dB/GHz. Return losses of the DGS based filters have also improved, providing 15dB -20dB impedance matching. It is not available for the microstrip based filters. The 15dB matching BW of the Legendre filter is maximum (91.5%); followed by the Chebyshev (88.4%), Butterworth (82.0%) and Bessel (52.4%). The Bessel filter has the least 15dB matching BW. However, it is still much more than that of the lumped element Bessel filter (24.4%).

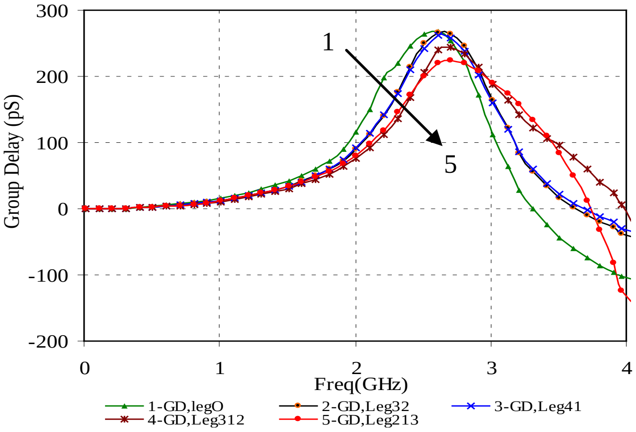
The simulated selectivity of the Legendre LPF is 19.8dB/GHz. It is less than that of the Chebyshev (24.66dB/GHz). The Legendre filter has the largest 20dB -28dB rejection BW (6 GHz); followed by the Chebyshev (5.46 GHz), Butterworth (5.3 GHz) and Bessel (0.86 GHz). Fig.8a shows more attenuation depth for the Chebyshev as compared to other filters.
Fig.8b compares variations in the GD of filters. The GD variation, within 15dB matching BW. is maximum (220pS) for the Legendre filter; followed by the Chebyshev (188pS), Butterworth (91 pS) and Bessel (16pS). The filter with larger matching BW has more variation in GD. We conclude that DGS based Legendre LPF has the largest 20dB impedance matching BW, with 4.86dB/GHz less selectivity. The Butterworth is a compromised DGS based LPF.

- **DGS based Legendre polynomial products (LPP) LPF:**

Fig.9a and Fig.9b compare the amplitude and group delay performances of the DGS based 5-pole Legendre and four types of LPP lowpass filters at $f_c^{3dB} = 2.5GHz$. The results are also summarized in table-6. The LPP-312 and LPP-213 filters have interesting results. The LPP-213 has wider 15dB matching BW (94%) and large 20dB rejection BW (7.3GHz); as compared to the matching BW (91.5%) and rejection BW (6.0GHz) of Legendre filter. Its GD within passband is 199pS, better than that of the Legendre. However, its selectivity is 17.89dB/GHz i.e. 2dB/GHz less compared to the Legendre and 4.29 dB/GHz more compared to the Butterworth filter (13.6 dB/GHz).The GD performance of LPP-312 filter is better (167pS) as compared to the LPP-213; although, shown in table-6, its selectivity, 15dB matching BW and 20dB rejection BW are a little less compared to the LPP-213.



(a):|S11|&|S21| response



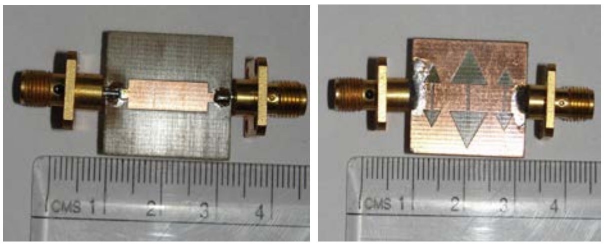
(b) GD response

Fig.9: Responses of the DGS based Legendre and LPP lowpass filters at $f_c^{3dB} = 2.5GHz$.

Both of these filters could be useful to the RF/microwave applications. The DGS based Bessel, with improved selectivity and matching BW, could be useful to the high speed digital applications [6-8].

iii. Experimental validation

In order to validate the performances of the Legendre group LPF, we have fabricated and measured Legendre, LPP-312 and LPP-213 lowpass filters. The fabricated filters are shown in Fig.10. Fig.11a and Fig.11b compare experimental performances of these filters. The experimental results are also presented in table-6. We note that measured selectivity of Legendre, LPP-312 and LPP-213 lowpass filters are 28.3dB/GHz, 18.8B/GHz and 18.2B/GHz. Their 20dB rejections BW (RBW) are 6.0 GHz, 7.7GHz and 7.9 GHz. It covers the 20dB rejection up to 4fc-5fc. Their 15dB impedance matching bandwidths are between 78% -80.8% and the GD variation in the passband is between 226pS-175pS.



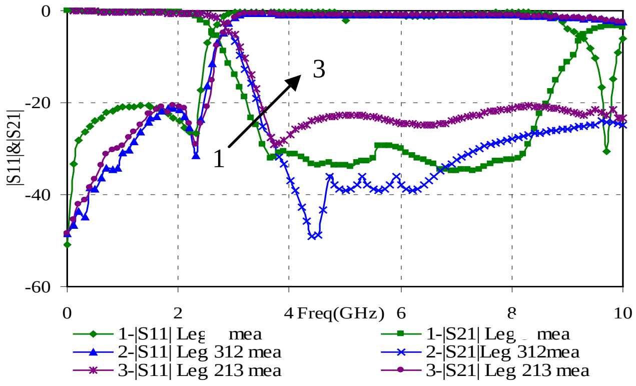
(a): Top side (b): Bottom side

Fig.10: Fabricated LPF

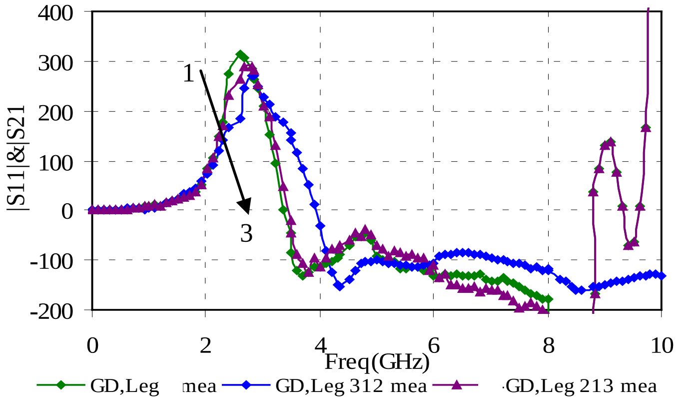
The selectivity of Legendre filter, in all three environments, is maximum as compared to the LPP-312 and LPP-213. In case of the DGS based Legendre filter, the simulated and experimental results are 19.8dB/GHz and 28.3 dB/GHz respectively.

Table -6: Amplitude and GD performances of several DGS based LPF.

Filter Type		Shrp dB/ GHz	20dB RBW GHz	15dB Imp. BW		20dB Imp. BW		**Filter Type**		Shrp dB/ GHz	20dB RBW GHz	15dB Imp. BW		20dB Imp. BW	
				BW GHz, %	GD (pS)	BW GHz, %	GD (pS)					BW GHz, %	GD (pS)	BW GHz, %	GD (pS)
Cheb.		24.66	5.46	2.21 88.4	188	2.08 83.2	145	**LPP-41**		16.83	5.9	2.38 91.5	200	2.25 86.5	160
Leg	**Sim**	19.8	6.0	2.38 91.5	220	2.31 88.8	197	**LPP-32**		16.83	5.8	2.38 91.5	200	2.25 86.5	160
	Exp	28.3	6.0	2.10	226	2.0	178	**LPP -312**	**S im**	16.66	7.2	2.45 90.4	167	2.25 83.3	111
									Exp	18.8	18.8	7.7	2.02	175	1.94
Butt		13.6	5.3	2.1 84.0	91	1.98 79.2	75	**LPP -213**	**Sim**	17.89	7.3	2.55 94.0	199	2.41 89.2	172
									Exp	18.2	7.9	1.95	230	1.81	176
Bessel		10.96	0.86	1.31 52.4	16	0.89 35.6	8	Shrp: Sharpness, Imp.BW: Impedance matching BW, RBW: Rejection BW							



(a):|S11|&|S21| response



(b) GD response

Fig.11: Experimental amplitude and group delay response of DGS based Legendre, LPP-312 and LPP-213 LPF

5 Conclusion

The Legendre group of filters have, unlike the Chebyshev, low unequal ripple level in the passband. So for an identical edge-ripple factor (r=0.5dB), the return-loss and corresponding impedance matching BW in the passband of the Legendre group of filters are more than that of a Chebyshev LPF; although selectivity is less. Its GD variation is also less compared to the Chebyshev LPF. The performance of the Legendre LPF is in between the Chebyshev and Butterworth; while performance of the LPP filters are in between the Legendre and Butterworth. The GD performance of the filters is improved in the microstrip compared to the lumped elements LPF. However, their impedance matching BW and rejection BW are reduced. The performance of filters; in respect of return-loss, impedance matching BW, selectivity and 20dB rejection BW, improves in the DGS environment. The measured selectivity of Legendre, LPP-312 and LPP-213 lowpass filters are 28.3dB/GHz, 18.8B/GHz and 18.2B/GHz. Their 20dB rejections BW (RBW) are 6.0 GHz, 7.7GHz and 7.9 GHz. It covers 20dB rejection up to 4fc-5fc. Their 15dB impedance matching bandwidths are between 78% -80.8%. So these filters, in the DGS format, can find application in microwave frequency range. The microstrip based LPP-312 filter; with better GD performance and sharper cut-off compared to Bessel, could be useful to the high speed digital applications.

Acknowledgement: Authors are thankful to UGC for the project grant.

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