Adaptive Beamforming Algorithms for Smart Antenna Systems

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Abstract: In this paper, Various evolutionary algorithms are used to adapt the weights of the smart antenna arrays to maximise the output power of the signal in desired direction and minimise the power in the unwanted direction. Different types of arrays (i.e., Linear, Circular, and Planar) are considered. Numerical results are presented to verify the improved convergence of the algorithms. Simulation results using different sets of antenna elements for different geometries are provided. In Least Mean Square algorithm, the convergence speed of the algorithm depends on the step size, which depends on the correlation matrix. With large eigen values spread, it converges slowly in a dynamic channel environment. This problem is solved by normalised least mean square and recursive least square algorithms. Simulation results show that the better adaptive beamforming algorithms for smart antenna systems in mobile communications.

Key–Words: Constant Modulus Algorithm (CMA), Beamforming, Least Mean Square (LMS), Planar array geometry, Recursive Least Square (RLS), Smart antenna.

1 Introduction

Due to the globalization, the modern wireless communication services are spreading rapidly. This necessitates to improve the coverage area, quality of the signal, and capacity of present network by the service providers. The upcoming technologies (Third Generation-3G and Fourth Generation-4G) are adopting the Space Division Multiple Access (SDMA) technique with Smart Antenna System [7]. With this antenna architecture, the weights of the antennas are adapted to point the main beam in the desired direction and place nulls in the interference directions. Different algorithms are used to adjust the weights in Smart Antenna Systems [1]. A comparison of Least Mean Square (LMS) and Recursive Least Square (RLS) algorithms for smart antennas in a Code Division Multiple Access (CDMA) mobile communication environment has been presented in [2]. S.F.Shaukat et al.[3] present a performance comparison of non blind algorithms (LMS, RLS) and blind algorithm (CMA) for Smart Antenna System. In Normalized LMS with proper selection of μ value, the method converges the weights quickly than earlier methods [4, 8, 10, 11].

The paper is organized as follows. Section 2 describes the mathematical model for the problem and also presents the implementation of the different algorithms. Simulated results are discussed in Section 3 and conclusions are mentioned in Section 4.

2 Mathematical Model

A Smart antenna system consists of a number of elements which are arranged in different geometries (like Linear, Circular etc.,) and whose weights are adjusted with signal processing techniques and evolutionary algorithms to exploit the spatial parameters of wireless channel characteristics under noisy environment. Fig.1 shows the block diagram of smart antenna system.

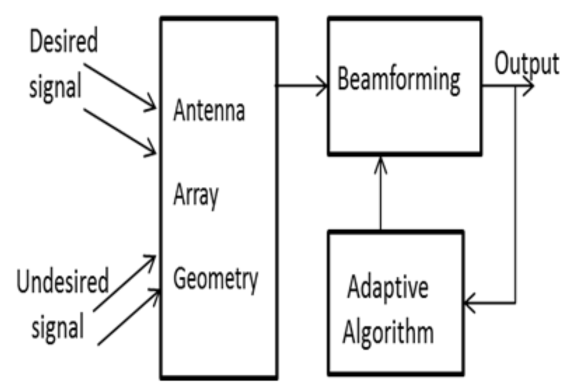

Figure 1: Block diagram of Smart antenna system

2.1 Linear Array Geometry

For the array shown in Fig.2, the array factor, for a linear array of N elements with an inter-element spacing d, is given by [5]:

$$AF(\theta) = \sum_{n=0}^{N-1} \omega_n e^{jnkdcos(\theta)} \quad (1)$$

Where

ω_n = complex array weight at element n,
θ = angle of incidence of electromagnetic plane wave from array axis,
k = wave number ($2\pi/\lambda$), and
λ = wavelength.

Let M be the number of plane waves, impinging on the array from directions (θ_1,θ_2,. . .,θ_m) , as shown in Fig.2. The received signal at the n^{th} element can be given as:

$$x_n(t) = \sum_{i}^{M} S_i(t) e^{-j(i-1)nkdsin(\theta_i)} + n_n(t) \quad (2)$$

Here $S_1(t)$ is the desired signal, $S_{2,M}(t)$ is the interference signal and $n_n(t)$ is the noise signal received at the n^{th} element.

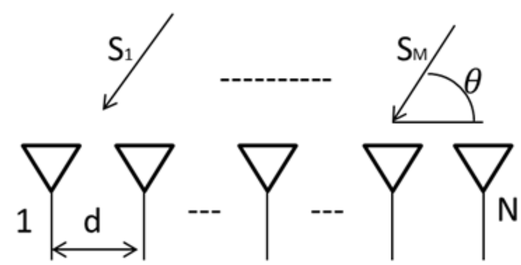

Figure 2: Linear array geometry

The total array output is expressed as:

$$Y(t) = W^H.X(t) \quad (3)$$

where

$W^H = [w_0 w_1 w_2 \ldots w_{n-1}]^T$ is matrix of weights, and

$X(t) = [x_1(t) x_2(t) \ldots x_n(t)]^T$ is matrix of signal vector.

In wireless mobile communication systems the desired signal arrival angle changes with time due to continuous modifications in channel environment. Hence continuous adaptation of weights in the array is required to get the desired signal which is done by optimization techniques. The least mean squares algorithm is a gradient based optimization technique. The reference signal used to update the weights at each iteration is given by [1]:

$$w(n+1) = w(n) + \mu x(n) e^*(n) \quad (4)$$

where $e(n)$ is error signal. The constant μ is called the step size. It determines how close the weights are moving to optimum value. The convergence of the algorithm depends on the step size. Typical values for the step size are $0 < \mu < Trace(R_{ss})$. The normalized LMS algorithm is a modified form of the standard LMS algorithm. It uses a time-varying adaptive step size $\mu(n)$. This step size can improve the convergence speed of the algorithm. At the n^{th} iteration, the step size is given by [6]:

$$\mu(n) = \alpha / (\gamma + X^H(n) X(n)) \quad (5)$$

Where α is a positive constant chosen to be between 0 and 2, while γ is a small positive term. It is simple to implement, but the drawbacks of the LMS algorithm are its slow convergence speed and getting struck at local minimum value when the weight value is close to optimal value rather than actual global minimum value. These problems motivate for another algorithm i.e. Recursive Least Squares (RLS). At every iteration, the LMS algorithm minimizing the estimation error, where as the RLS algorithm minimizing the errors up to and including the current iteration. The auto correlation matrix (R_{ss}) and the cross-correlation (P_{ss}) vectors of the desired signals are updated and then used to compute weight vector (W_k). The following steps are involved to compute optimal weights [6]:

Step 1: Update R_{ss} through

$$R_{ss,k+1} = R_{ss,k} + X(k) X^T(k) \quad (6)$$

Step 2: Update P_{ss} through

$$P_{ss,k+1} = P_{ss,k} + d(k) X(k) \quad (7)$$

Step 3:

$$Invert R_{ss,k+1} \quad (8)$$

Step 4: Compute W_{K+1} through

$$W_{k+1} = (R_{ss,k+1})^{-1} P_{ss,k+1} \quad (9)$$

To reduce the computational load of the algorithm, the matrix inversion lemma technique is applied to find the inversion of R_{ss}. Let ABCD be four matrices. The lemma of ABCD is [6]:

$$(A+BCD)^{-1} = A^{-1}A^{-1}B(DA^{-1}B+C^{-1})^{-1}DA^{-1} \quad (10)$$

In the present case, $A = R_k$, $B = X(k)$, $C = 1$, and $D = X^T(k)$, then

$$R_{k+1}^{-1} = R_k^{-1} - \frac{R_k^{-1}X(k)X^T(k)R_k^{-1}}{1+X^T(k)R_k^{-1}X(k)} \quad (11)$$

The above two methods need the reference signal or training signal to find the weights of the array. But practically, most of the time reference signal is not available in wireless communication. Generally angle modulation (FM, PM, FSK, etc.,) which has constant amplitude is applied to send the signals in mobile communication. Simple Constant Modulus Algorithm (CMA) is not used in dynamic environment due to slow convergence. So the dynamic least square Constant Modulus algorithm is used to adapt the weights of the array for each iteration. The new updated weight vector is given by [7]:

$$\overline{w}(n+1) = R_{xx}^{-1}(n)p_{xr}(n) \quad (12)$$

Where

$$R_{xx}(n) = (X(n)X^H(n))/K \quad (13)$$

$$p_{xr}(n) = X(n)r(n)/K \quad (14)$$

2.2 Circular Array Geometry

Fig.3 shows a circular array of N elements in the x-y plane. The n^{th} array element is located at the radius $'a'$ with the phase angle ϕ_n. To direct the peak of the main beam in the (θ_0,ϕ_0) direction, the array factor is given by [5]:

$$AF(\theta,\phi) = \sum_{n=1}^{N} w_n e^{-jka[sin(\theta)cos(\phi-\phi_n)-sin(\theta_0)cos(\phi_0-\phi_n)]} \quad (15)$$

Where

w_n = excitation coefficients (amplitude and phase) of n^{th} element,

$\phi_n = 2\pi(n/N)$ = angular position of n^{th} element on x-y plane.

2.3 Planar Array Geometry

Fig.4 depicts a rectangular array in the x-y plane. The planar array can be viewed as M linear arrays of N elements or N linear arrays of M elements each. The pattern multiplication principle is used to find the pattern of the entire geometry.

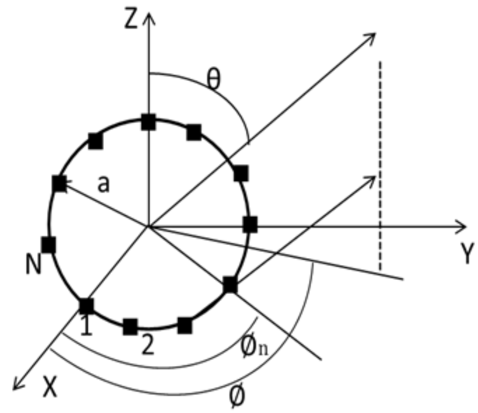

Figure 3: Circular array geometry of N elements

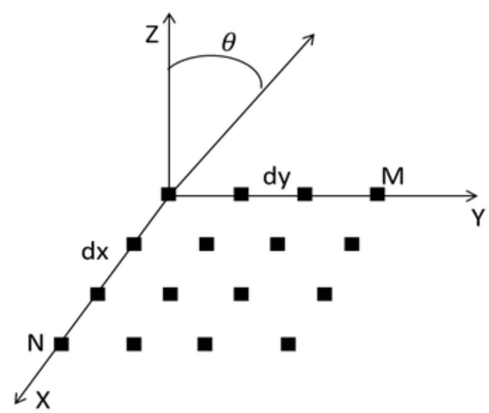

Figure 4: Planar array geometry

The array factor is given by [9]:

$$AF = AF_x AF_y = \sum_{m=1}^{M}\sum_{n=1}^{N} w_{mn} e^{j[(m-1)(\psi x+\beta x)+(n-1)(\psi y+\beta y)]} \quad (16)$$

where

$\psi x = kdxsin\theta cos\varphi$

$\psi y = kdysin\theta sin\varphi$

$\beta x = -kdxsin\theta_0 cos\varphi_0$ and

$\beta y = -kdysin\theta_0 sin\varphi_0$ are phase delays, which are used to steer the beam in desired direction.

3 Simulated Results

3.1 Linear Array

In this simulation, the adaptive algorithm is based on LMS. To establish the correctness of the proposed method, the main beam is steered in the desired signal direction at $+60^0$ and null is placed at -30^0 in the direction of undesired signal arrival. Fig.5 indicates the radiation patterns of two linear arrays with number of elements in array as 21 and 51. The inter-element spacing is 0.5λ.

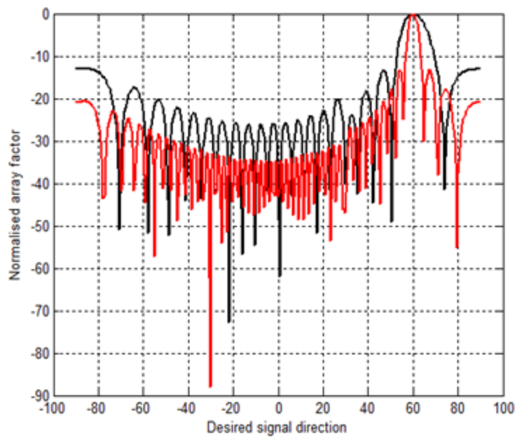

Figure 5: Radiation pattern of linear array with wider beam (N=21) and narrow beam (N=51) using Standard LMS algorithm

Figs.5 and 6 are presenting the results of the radiation pattern of the linear array by using LMS (Fig.5) and NLMS (Fig.6) generated according to (1). To improve the convergence of the algorithm normalized LMS is used. Radiation pattern of linear array with normalized LMS is presented in Fig.6. It can be seen that deep null is placed at the unwanted signal direction -30^0 with $-90dB$. The ratio between the powers of the main lobe and the first side lobe is -13.3 dB for both cases. It is observed that as the number of elements in the array increases, the directivity increases with less side lobe powers.

Figs.7 and 8 depict the Mean Square Error (MSE) plot or convergence plot for the standard LMS and the normalized LMS algorithm respectively. It is observed that error function value is reduced quickly. Fig.9 illustrates the radiation pattern of 21 elements beamforming array with different inter-element spacing d = 0.5λ and 1.5λ, multiple narrow beams are increased as d is increased. Figs.10 and 11 indicate how fast the algorithm is converged. In this section also, the error is reduced and converged quickly using NLMS, when inter-element distance is changed.

In Fig.10, it is observed that the standard LMS algorithm requires around 50 iterations to converge the weights when d = 0.5λ. Whereas NLMS algorithm requires around 20 iterations as shown in Fig.11. Therefore it is proved that Normalised LMS algorithm converged faster than standard LMS algorithm. Figs.12 and 13 show the response of array when RLS and CMA algorithms are used respectively.

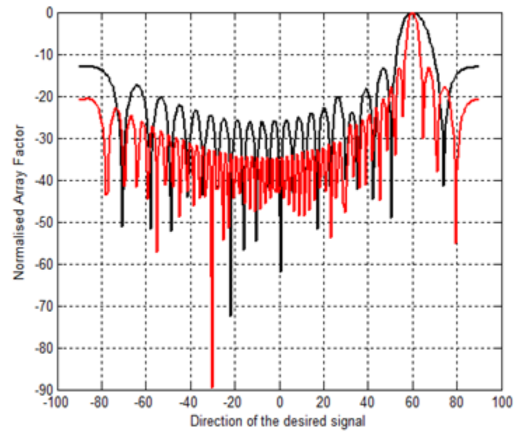

Figure 6: Radiation pattern of linear array for number of elements N=21(wider main beam curve) and N=51(narrow main beam curve) with direction arrival at 60^0 using Normalized LMS algorithm

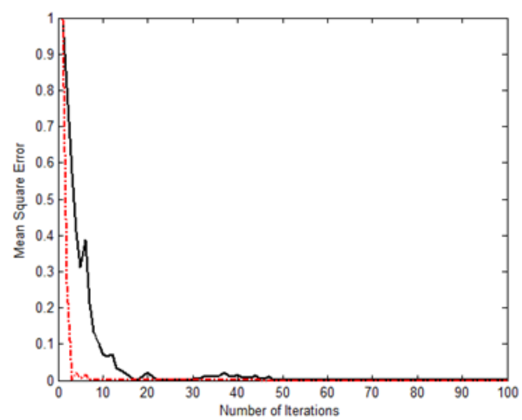

Figure 7: Number of iterations versus mean square error for LMS for linear array of N = 21 elements (solid curve) and N = 51 elements (dotted curve)

3.2 Circular Array Geometry

In this case circular array geometry is considered with number of elements 21 and radius of 1λ. Fig.14 shows the array factor response with maximum in the direction of the desired signal at 20^0 and placing null at unwanted signal direction at 50^0 using LMS algorithm. RLS algorithm is applied on this array. The response of the array factor can be seen in the Fig.15.

3.3 Planar Array Geometry

In this case 21X21 planar array geometry is considered. The radiation patterns are presented according

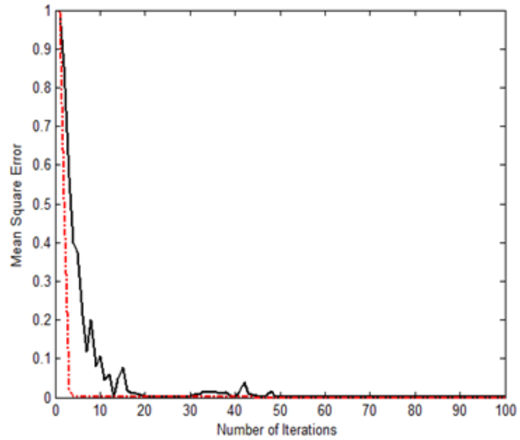

Figure 8: Number of iterations versus mean square error for NLMS for linear array of N = 21 elements (solid curve) and N = 51 elements (dotted curve)

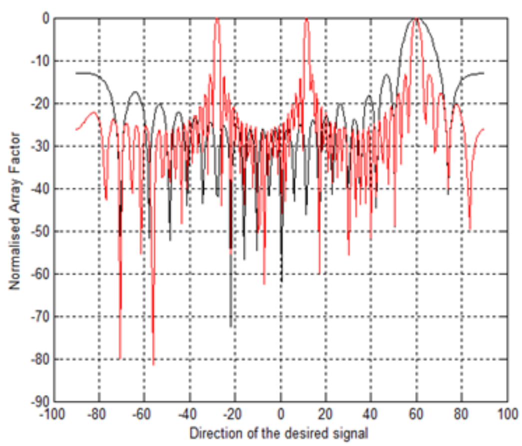

Figure 9: Desired signal direction at 60^0 and Interference signal direction at 30^0 with inter element spacing d = 1.5λ (multiple beams curve) and d = 0.5λ (one main beam curve)

to (16). Figs.16, 17, and 18 are indicate the response of the array factor with main lobe in direction of the desired signal and placing deep null in the direction of the undesired signal using adaptive methods LMS, RLS and CMA respectively. Fig.16 Radiation pattern of 21X21 Planar array with desired signal arrival at 40^0 and deep null is placed with $-50dB$ in the undesired signal direction at -60^0 using LMS algorithm. In Fig.18, the radiation pattern of planar array using CMA algorithm with direction of the desired signal at 0^0, and two deep nulls are placed in the directions of interference signals coming to the array at -60^0 and 60^0 with $-60dB$ and $-68dB$ respectively.

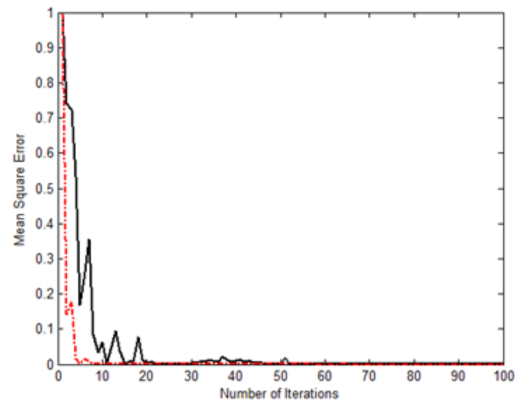

Figure 10: Number of iterations versus mean square error for LMS with d = 0.5λ (solid curve) and 1.5λ (dotted curve) for the linear array of 21 elements

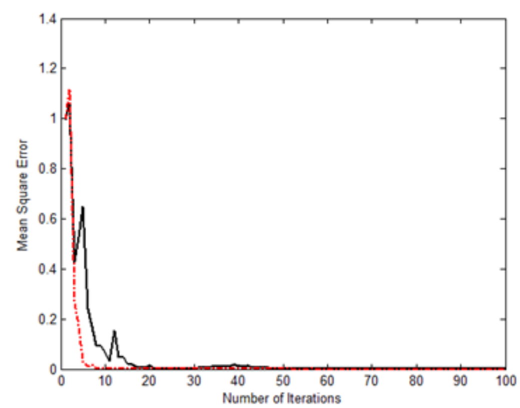

Figure 11: Number of iterations versus mean square error for NLMS with d = 0.5λ (solid curve) and 1.5λ (dotted curve) for the linear array of 21 elements

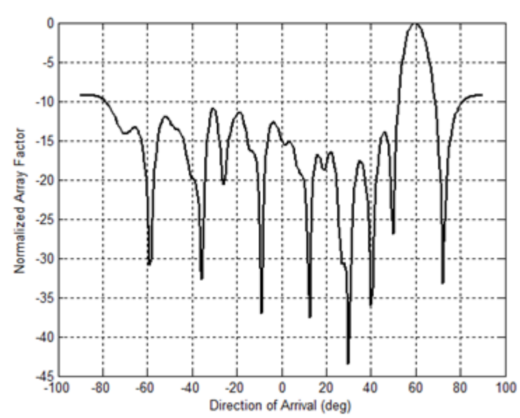

Figure 12: Radiation pattern of linear array of 21 elements using RLS with desired signal direction at 60^0 and undesired signal direction at 30^0

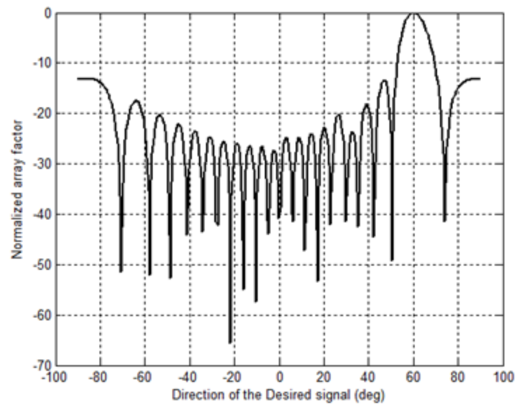

Figure 13: Radiation pattern of the linear array of 21 elements with the direction of arrival at 60^0 using CMA

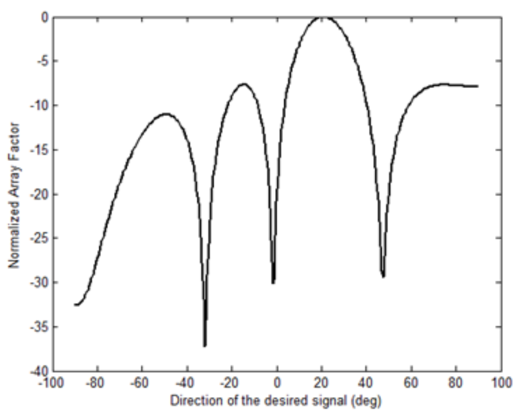

Figure 14: Radiation pattern of circular array with the direction of desired signal arrival at 20^0 and undesired signal direction at 50^0 using LMS

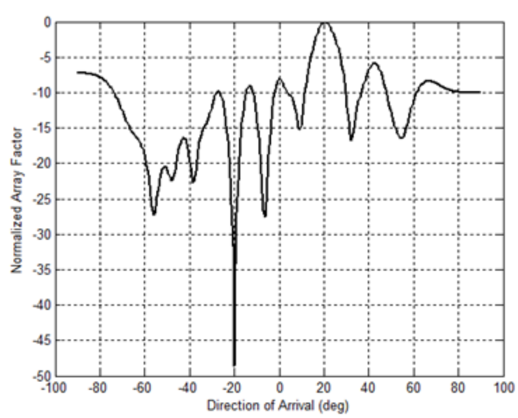

Figure 15: Radiation pattern of circular array with the direction of desired signal arrival at 20^0 and undesired signal direction at -20^0 using RLS algorithm

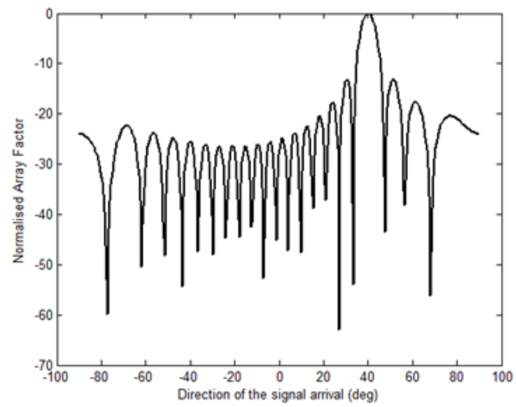

Figure 16: Radiation pattern of 21X21 Planar array with desired signal arrival at 40^0 and undesired signal direction at -60^0 using LMS

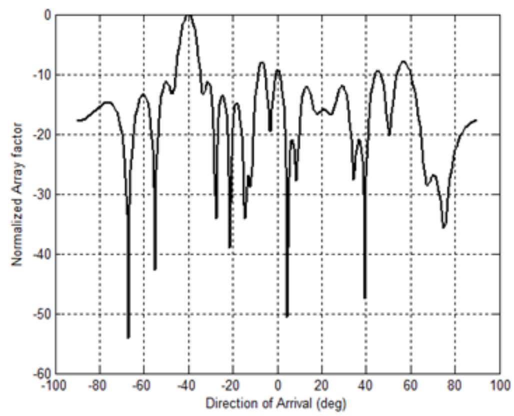

Figure 17: Radiation pattern of 21X21 Planar array using RLS algorithm with DOA -40^0 and Undesired signal direction at 40^0

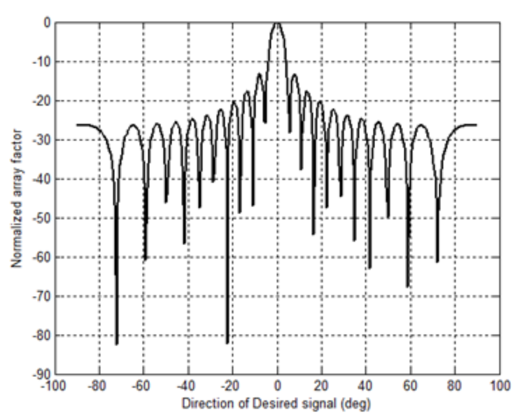

Figure 18: Radiation pattern of planar array using CMA algorithm with direction of arrival at 0^0, and interference signals at -60^0 and 60^0

4 Conclusion

Here three different geometries are considered in smart antenna system, whose weights are updated with the help of adaptive algorithms to get the desired signal patterns in dynamic channel condition. The convergence speed of the LMS algorithm depends on the eigen values of the correlation matrix. With large eigen values spread it converges slowly in a dynamic channel environment. This problem is solved by RLS algorithm. Both the cases need the reference signal.

Most of the times reference is not available, in that situation constant modulus algorithm is used. And also noticed that as the number of elements increases, it converges rapidly. The convergence rates for LMS and NLMS are 0.0052 and 0.0041 seconds respectively when 100 iterations are taken for simulation. The simulations are carried on Intel(R) Core(TM) i5 CPU M460, @2.53 GHz, 6 GB of RAM hardware, using MATLAB (R2009b) software. The computation time can further be reduced, if higher end sophisticated signal processor is used for spatial processing in smart antenna system. These algorithms are used to adapt the weights of the array, realizing the desired parameters (i.e., main beam steering, deep null placement in the undesired signal direction, etc.,) under noise environment.

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