Adaptive Detection of Range-Spread Targets in Gaussian Noise Using the Generalized Detector

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Abstract: - In this paper, we address an adaptive detection of range-spread targets or targets embedded in Gaussian noise with unknown covariance matrix by the generalized detector (GD) based on the generalized approach to signal processing (GASP) in noise. We assume that cells or secondary data that are free of signal components are available. Those secondary data are supposed to process either the same covariance matrix or the same structure of the covariance matrix of the cells under test. In this context, under designing GD we use a two-step procedure. The criteria lead to receivers ensuring the constant false alarm rate (CFAR) property with respect to unknown quantities. A thorough performance assessment of the proposed detection strategies highlights that the two-step design procedure of decision-making rule in accordance with GASP is to be preferred with respect to the plain one. In fact, the proposed design procedure leads to GD that achieves significant improvement in detection performance under several situation of practical interest. For estimation purposes, we resort to a set of secondary data. In addition to the classical homogeneous scenario, we consider the case wherein the power value of primary and secondary data vectors is not the same. The design of adaptive detection algorithms based on GASP in the case of mismatch is a problem of primary concern for radar applications. We demonstrate that two-step design procedure based on GASP ensures minimal loss.

Key-Words: - Generalized detector, additive Gaussian noise, detection performance, constant false alarm rate (CFAR), generalized approach to signal processing (GASP), high resolution radar, signal-to-noise ratio (*SNR*).

1 Introduction

High-resolution radar (HRR) can resolve a target into a number of scattering centers, depending on the range extent of the target and the range resolution capabilities of the radar. In fact, measurements indicate that the radar properties of several targets, such as aircraft, boats, etc. are well modeled as being due primary to reflection from a few isolated points. These specular reflections match quite well with physical features on the target [1]–[3]. In the following, the discrete scattering centers of a target will be referred to as multiple dominant scattering centers. In particular, radar detection of distributed targets in white Gaussian noise of known spectral density level has been addressed in [4]. Therein, two detection structures have been proposed, and the results indicate that properly designed HRRs allow significant enhancement of the detection performance.

The possible improvement depends upon two factors, namely,

- Increasing the range resolution of the radar reduces the amount of energy per cell backscattered by distributed clutter;
- Resolved scatterers introduce less fluctuation than an unresolved point target.

However, this performance improvement is traded for a significant increase of the computational complexity. Therefore, a more general issue arises, i.e. the suitability of HRRs for operation in the scan mode, which is hard to implement at the current state of the art but may not be definitely ruled out in the near future. This explains the increasing academic and industry interest on the design and the assessment of new receivers for HRRs with a two-fold goal.

In fact, from one side, it is important to determine the maximum gain, in terms of achievable performance, with no complexity constraint, granted by HRRs on lower resolution radars. On the other side, one is interested to come up with suboptimum processors, representing a compromise between detection performance and complexity, demonstrating the applicability of HRRs in the scan mode. Several results, established with reference to HRRs can be easily imported in the general theory of range-spread target detection. It is needed well known that the point -target model may fail in many practical scenarios wherein a low/medium resolution radar is employed. For example, a detection of large ships with coastal radars and that of a cluster of point targets flying at the same velocity in close spatial proximity to one another.

The detection of the overall target set within the input data block is a way to combat signal combination of range cells in close spatial proximity with that under test and, hence, to reduce the corresponding detection loss. The effects of the clutter reduction in a single range cell and of a multiple dominant scattering target model on the target detection have been studied in [5], where was shown that the probability of detection of range-distributed targets depends on the signal bandwidth in the case of single pulse processing. The best performance is achieved when the radar bandwidth just resolves the individual scatterers. Resolving the dominant scatterers introduces less fluctuation, but when the signal bandwidth is further increased, the performance degrades as a consequence of the lack of knowledge about the position of the dominant scatterers within the extension of the target. This performance loss, which is referred to in the following as collapsing loss, is due to the presence of cells that contain mostly noise.

A possible way to circumvent this drawback is to resort to a Bayesian approach, namely, to assume some *a priori* statistical knowledge about the target [6]. Since the scattering geometry can differ significantly from target to target, the above approach is not always realistic. The constant false alarm rate (CFAR) detection of distributed targets in Gaussian noise with unknown covariance matrix, based on the generalized likelihood ratio test (GLRT), has been addressed in [7]–[9]. Returns from different range cells are modeled as independent and identically distributed (i.i.d.) Gaussian vectors with unknown covariance matrix. A set of independent secondary data that is free of signal components is available, and it is assumed that the covariance matrix is one and the same for all of the primary and secondary data vectors [10].

The above scenario will be referred to in the following as homogeneous environment. In [11], the modified GLRT for adaptive detection of a target distributed in range is derived, where the amplitudes of the desired target and the interference covariance matrix are modeled as unknown quantities, but the proposed strategy does not resort to secondary data. The distribution of the modified GLRT statistic, under the hypothesis H_0 – the noise only,– is dependent on the actual value of the covariance matrix and, hence, does not have the desirable quality of being CFAR processor. The proposed algorithm can be made bounded CFAR [11], thus being a viable means to adaptively detect the range-spread targets embedded in a highly nonstationary environment.

In [12] it is shown how additional data blocks that are free of signal components, can be used to construct a truly CFAR detector. GLRT for the adaptive detection of Doppler-shifted, and range-distributed targets embedded in noise with unknown, but structured, covariance matrix has been proposed in [13]. Such detector has been shown to be bounded CFAR via simulation.

In the present paper, we deal with the problem of detecting an extended target or targets (with unknown amplitudes) embedded in Gaussian noise with unknown covariance matrix across a number of adjacent range cells which are also referred to in the following as a primary data. For estimation purposees, we resort to a set of secondary data. We will consider the case wherein the power value of primary and secondary data vectors is not the same or more precisely, both groups of data separately satisfy the homogeneity condition, but the two covariance matrices coincide only up to a scaling factor. This scenario [14] is refereed as a partially homogeneous environment.

The design of the adaptive detection algorithms in the case of mismatch is a problem of primary concern for radar applications. Although most of the space-time adaptive processing detection schemes have been designed employing the assumption that interference returns were i.i.d. Gaussian vectors, experimental campaigns have demonstrated that such an assumption is not always verified [15]. In addition, the analysis of several space-time adaptive processing algorithms, mostly conducted assuming homogeneity of the secondary data, has shown that inhomogeneities magnify the loss between the adaptive implementation and optimum conditions [16]–[18].

Although other types of inhomogeneities are of interest, the design of GLRT-based detectors is not always feasible.Partially homogeneous environment is also a viable means to address a detection of signals buried in non-Gaussian disturbances. The Gaussian assumption is no longer met for modelling HRR clutter as viewed at low grazing angles. More specifically, the disturbance is better described as a compound-Gaussian process. It is the product of a temporary and spatially slowly varying texture component, accounting for the reflectivity of the illuminated patch, times a more rapidly varying process, the so-called speckle Gaussian distributed process, due to the local validity of the central limit theorem [19]–[21].

The spatial correlation of the texture is usually unknown and is thus a viable means to cope with this *a priori* uncertainty. It relies on modelling, at the design stage, and returns as independent Gaussian vectors with possibly different power values. This procedure has been followed in [22], where non-adaptive detectors for range-spread targets embedded in the compound-Gaussian noise with possibly varying texture from cell to cell have been introduced and assesses. With this model in mind, partially homogeneous scenarios fit in situations where the maximum spacing between any two primary range cells is small compared with the scale over which texture levels change, and the same holds true for secondary data. This case may apply, for instance, if the primary vectors are chosen from a set of adjacent range cells and similarly for the secondary data, but data under test are not in the immediate vicinity of secondary gates.

Analysis of clutter recordings which are collected to emulate airborne radars, have shown that the partially homogeneous model well describes clutter for moderately low values of the number of primary and secondary data [23]. We derive the GLRT based on the generalized approach to signal processing (GASP) in noise [24]–[29] for a partially homogeneous environment. We devise simplified detection strategies following [30]–[31].

This work is motivated by two main considerations. The GLRT-based generalized detector (GD) is very time consuming and, hence, difficult to implement for real-time applications. Additionally, the GLRT GD has no known optimality properties and, for homogeneous environment and point-like targets, simplified test statistics may achieve the higher detection probabilities [30]. In that case, the GLRT GD is not a uniformly most powerful (UMP) invariant one, and actually, a UMP-invariant test does not exist, as shown in [32].

In particular, in [30] the following two-step GLRT-based design procedure has been proposed. The first step is to derive the GLRT GD for the case where the covariance matrix of primary data $\mathbf{M}$ is known. The second step is to insert the sample covariance matrix based on the secondary data in place of the true covariance matrix into the test. A possible alternative has been conceived in [31], namely, the first step is to derive the GLRT for the case that only the structure Σ of the covariance matrix is known. A completely adaptive GD is obtained by plugging the sample covariance matrix, based upon secondary data in place of Σ into the previously derived test statistic.

The remainder of the present paper is organized as follows. The problem statement, brief description of GD functioning principles, simple GD flowchart, principles of designing the one-step and two-step GLRT GD are declared in Section 2. The probability of detection as a function of the target and clutter parameters is derived through the signal-to-noise ratio (*SNR)* in Section 3. Simulation results for the targets with non-random and random parameters are discussed in Section 4. Some conclusions are presented in Section 5.

2 Problem Statement and GD Design

We assume that data are collected from N sensors and deal with the problem of detecting the presence of a target across L range cells $\mathbf{z}_l, l = 1,\dots,L$. We suppose that the possible target is completely contained within those data and neglect range migration. As in [10], it is assumed that a secondary data set $\mathbf{z}_l, l = L+1,\dots,L(K+1)$ is available and that each of such snapshots does not contain any useful target echo and exhibits the same structure of the covariance matrix as the primary data.

The rationale of the assumed setup is to emphasize the existing relationship between the target extent and cell size, and, also, radar resolution. If the radar resolution is increased by a factor L, the cell size is reduced by the same factor, and the number of secondary date increases, correspondingly. Hence, the proposed arrangement of the data allows us to compare the performance at different resolutions, in particular, when the L range cells collapse into a larger one. The detection problem to be solved can be formulated in terms of the following binary hypotheses test:

$$\begin{cases} H_0: & \mathbf{z}_l = \mathbf{w}_l\,; \qquad l = 1,\dots,L(K+1) \\ H_1: & \begin{cases} \mathbf{z}_l = \alpha_l \mathbf{p} + \mathbf{w}_l\,; & l = 1,\dots,L \\ \mathbf{z}_l = \mathbf{w}_l\,, & l = L+1,\dots,L(K+1) \end{cases} \end{cases} \tag{1}$$

where $\mathbf{p}$ denotes the steering vector, and the α_l, $l = 1,\dots,L$, are unknown deterministic parameters accounting for both the target and the channel effects. As for the noise vectors, we assume that $\mathbf{w}_l$, $l = 1, \dots, L(K+1)$, are the independent zero mean Gaussian vectors with the covariance matrices given by

$$E[\mathbf{w}_l \mathbf{w}_l^*] = \mathbf{M}, \quad l = 1,\dots,L(K+1) \tag{2}$$

for the homogeneous environment and

$$\begin{cases} E[\mathbf{w}_l \mathbf{w}_l^*] = \mathbf{M}, & l = 1,\dots,L \\ E[\mathbf{w}_l \mathbf{w}_l^*] = \delta\mathbf{M} & l = L+1,\dots,L(K+1) \end{cases} \tag{3}$$

for the partially homogeneous environment with $\delta >0$, where $E[\cdots]$ denotes the mathematical expectation and * denotes the conjugate transpose. Moreover, we suppose that the noise vectors $\mathbf{w}_l$ possess the circular property usually associated with in-phase and quadrature pairs of a wide-sense stationary process [33].

According to the Neyman–Pearson criterion, the optimum solution to the hypotheses testing problem (1) is the likelihood ratio test based on GASP, but for the case at hand, it cannot be implemented since total ignorance of the parameters $\boldsymbol{\alpha}=(\alpha_1,\ldots,\alpha_L),\mathbf{M}$, and possibly δ is assumed. We resort to GLRT-based decision schemes based on GASP. Strictly speaking, the GLRT-based on GASP is tantamount to replace the unknown parameters with their maximum likelihood estimates under each hypothesis based on entirely of data [34]. Processors that implement the plain GLRT will be referred to in the following as the one-step GLRT GD. Receivers implementing modified GLRT statistics derived following the two-step design procedure will be referred to as the two-step GLRT GD.

Subsequent developments require specifying the complex multivariate probability density function (pdf) of the $L(K+1)$ vectors $\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}$ at both hypotheses. Previous assumptions imply that the joint pdf may be written in the following form:

$$f_{\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}}(\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}\,|\,\mathbf{M},\delta,H_0)$$
$$=\frac{1}{\delta^{NLK}[\pi^N\det(\mathbf{M})]^{L(K+1)}}\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_0)\}$$
$$\text{– the hypothesis } H_0; \tag{4}$$

$$f_{\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}}(\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}\,|\,\mathbf{M},\boldsymbol{\alpha},\delta,H_1)$$
$$=\frac{1}{\delta^{NLK}[\pi^N\det(\mathbf{M})]^{L(K+1)}}\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_1)\}$$
$$\text{– the hypothesis } H_1; \tag{5}$$

$$\mathbf{T}_0=-\sum_{l=1}^{L}\mathbf{w}_{l_{PF}}\mathbf{w}_{l_{PF}}^*+\sum_{l=1}^{L}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^*+\frac{1}{\delta}\sum_{l=L+1}^{L(K+1)}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^*$$
$$\text{– the hypothesis } H_0; \tag{6}$$

$$\mathbf{T}_1=\sum_{l=1}^{L}(\mathbf{z}_l-\alpha_l\mathbf{p})(\mathbf{z}_l-\alpha_l\mathbf{p})^*$$
$$+\sum_{l=1}^{L}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^*+\frac{1}{\delta}\sum_{l=L+1}^{L(K+1)}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^*$$
$$\text{– the hypothesis } H_1. \tag{7}$$

Description and explanation in detail of (6) and (7) and difference between the noise $\mathbf{w}_{l_{PF}}$ and $\mathbf{w}_{l_{AF}}$ are delivered below (see Subsection 2.1). Here $\det(\cdots)$ and $tr(\cdots)$ denote the determinant and the trace of a square matrix, respectively. Obviously δ is to be set equal to one under homogeneous environment.

2.1 Generalized detector (GD)

For better understanding (1)–(7), we recall the main GD functioning principles. The simple GD scheme is represented in Fig.1. In this model, we use the following notations: MSG is the model signal generator (the local oscillator), the AF is the additional filter (the linear system), and the PF is the preliminary filter (the linear system). A detailed discussion of the AF and PF can be found in [24] and [26].

Consider briefly the main statements regarding the AF and PF. There are two linear systems at the GD front-end that can be presented, for example, as the bandpass filters, namely, the PF with the impulse response $h_{PF}[m]$ and the AF with the impulse response $h_{AF}[m]$. For simplicity of analysis, we think that these filters have the same amplitude-frequency characteristics or impulse responses by shape. The PF bandwidth is matched with the target return signal bandwidth. A resonant or central frequency of the AF is detuned relative to the PF resonant or central frequency on such a value that the target return signal cannot pass through the AF. Thus, the PF and AF bandwidths are mismatched with respect to each other. The target return signal and noise can be appeared at the PF output and the only noise is appeared at the AF output. If a value of detuning between the AF and PF resonant or central frequencies or mismatching between the PF and AF bandwidths is more than $4\div 5\Delta f_a$, where Δf_a is the target return signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes and, in practice, under this condition, the coefficient of correlation between the PF and AF output processes is not more than 0.05 that was confirmed experimentally in [35] and [36].

In the case of target return signal absence at the GD input, the statistical parameters at the AF and PF outputs are the same, because the same noise is coming in at the AF and PF inputs and owing to the fact that the AF and PF are the linear systems, we can believe that the AF and PF do not change the statistical parameters of the input process. By this reason, the AF can be considered as a generator of the secondary data with a priori information a “no” target return signal.

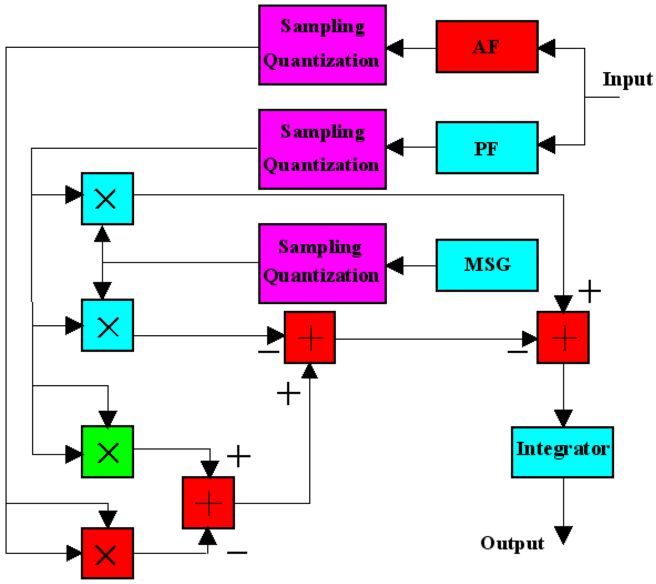

Figure 1. Principal flowchart of GD.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If the discrete-time Gaussian noise $w[n]$ comes in at the AF and PF inputs (the GD linear input system front-end), the noise forming at the AF and PF outputs is the discrete-time Gaussian, too, because the AF and PF are the linear systems. In a general case, this noise takes the following form

$$\begin{cases} w_{l_{PF}}[n] = \xi_l[n] = \sum_{m=-\infty}^{\infty} h_{PF}[m] w_l[n-m] \ ; \\ w_{l_{AF}}[n] = \eta_l[n] = \sum_{m=-\infty}^{\infty} h_{AF}[m] w_l[n-m] \ . \end{cases} \quad (8)$$

If the additive white Gaussian noise (AWGN) with zero mean and power spectral density $0.5N_0$ is coming in at the AF and PF inputs (the GD linear system front-end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by [26]

$$\sigma_w^2 = \frac{N_0 \omega_0^2}{8\Delta} \quad (9)$$

where in the case if AF (or PF) is the RLC oscillatory circuit, the AF (or PF) bandwidth $\Delta = \Delta f_a$ and resonance frequency ω_0 are defined in the following form:

$$\Delta = \pi\beta, \omega_0 = \frac{1}{\sqrt{LC}}, \ \beta = \frac{R}{2L} . \quad (10)$$

The main functioning condition of GD is an equality over the whole range of parameters between the model signal $\alpha_l \mathbf{p}^*$ at the GD MSG output for user l and the expected signal $\alpha_l \mathbf{p}$ forming at the GD input liner system (the PF) output, i.e. $\alpha_l \mathbf{p} = \alpha_l \mathbf{p}^*$. How we can satisfy this condition in practice is discussed in detail in [24] and [26, Chapter 6, pp. 611–621]. In the case of the target return signal presence (the hypothesis H_1) the PF output can be defined as

$$y_l^{H_1}[n] = \alpha_l p_l[n] + w_{l_{PF}}[n] = \alpha_l p_l[n] + \xi_l[n] \ , \quad (11)$$

Under the hypothesis H_0 the PF output defined as

$$y_l^{H_0}[n] = w_{l_{PF}}[n] = \xi_l[n] \quad (12)$$

is subjected to the Gaussian distribution with zero mean and variance σ_w^2. The AF output can be considered as the secondary data both under the hypothesis H_0 and under the hypothesis H_1. Taking into consideration (6), (7), (11), and (12) the decision statistics at the GD output can be presented in the following form [26, Chapter 3]:

$$Z_{GD}^{\text{out}}[n] = \begin{cases} \sum_{l=1}^{L} \eta_l^2[n] - \sum_{l=1}^{L} \xi_l^2[n] + \frac{1}{\delta} \sum_{l=LK}^{L(K+1)} \eta_l^2[n] \\ \qquad \Rightarrow \mathbf{T}_0 \Rightarrow H_0 \ ; \\ \sum_{l=1}^{L} \alpha_l^2 p_l^2[n] + \sum_{l=1}^{L} \eta_l^2[n] - \sum_{l=1}^{L} \xi_l^2[n] \\ + \frac{1}{\delta} \sum_{l=LK}^{L(K+1)} \eta_l^2[n] \qquad \Rightarrow \mathbf{T}_1 \Rightarrow H_1 \ . \end{cases}$$

(13)

As follows from (13), under the hypothesis H_0 the process forming at the GD output is the background noise only. Under the hypothesis H_1 the process forming at the GD output presents the target return signal energy plus the background noise. The background noise is a difference between the noise power forming at the PF output and the secondary data power forming at the AF output.

2.2 One-step GLRT GD

This subsection contains a derivation of the GLRT GD for partially homogeneous environment. According to the GLRT, we replace the unknown parameters with their maximum likelihood estimates and consider the following decision-making rule:

$$\frac{\max\limits_{\delta}\max\limits_{\boldsymbol{\alpha}}\max\limits_{\mathbf{M}} f_{\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}}(\mathbf{z}_1,\ldots,\mathbf{z}_{L(K+1)}\,|\,\mathbf{M},\boldsymbol{\alpha},\delta,H_1)}{\max\limits_{\delta}\max\limits_{\mathbf{M}} f_{\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L(K+1)_{AF}}}(\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L(K+1)_{AF}}\,|\,\mathbf{M},\delta,H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad . \tag{14}$$

Substituting the multivariate Gaussian pdf (4) and (5) into (14), we obtain

$$\frac{\max\limits_{\delta}\max\limits_{\boldsymbol{\alpha}}\max\limits_{\mathbf{M}}\left\{\dfrac{\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_1)\}}{\delta^{NLK}[\pi^N\det(\mathbf{M})]^{L(K+1)}}\right\}}{\max\limits_{\delta}\max\limits_{\mathbf{M}}\left\{\dfrac{\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_0)\}}{\delta^{NLK}[\pi^N\det(\mathbf{M})]^{L(K+1)}}\right\}} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad . \tag{15}$$

Assume that $L(K+1)\geq N$. Maximizing the numerator and denominator over **M**, (15) can be rewritten in the following form [10]:

$$\frac{\min\limits_{\delta}\left[\delta^{\frac{NK}{K+1}}\det(\mathbf{T}_0)\right]}{\min\limits_{\delta}\left[\delta^{\frac{NK}{K+1}}\min\limits_{\boldsymbol{\alpha}}\det(\mathbf{T}_1)\right]} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad . \tag{16}$$

As was shown in [37], it is possible to maximize (16) with respect to the complex vector $\boldsymbol{\alpha}$ assuming $LK\geq N$, i.e.

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}}\ \det(\mathbf{T}_1) = \left(\frac{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{z}_1}{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{p}},\ldots,\frac{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{z}_L}{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{p}}\right) , \tag{17}$$

where $\arg\ \min_{\boldsymbol{\alpha}}(\cdot)$ denotes the value of $\boldsymbol{\alpha}$ that minimizes the argument, and **S** is LK times the sample covariance matrix based on secondary data only, i.e.

$$\mathbf{S} = \frac{1}{\delta}\sum_{l=L+1}^{L(K+1)}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^* \quad . \tag{18}$$

Hereafter, we set $LK\geq N$. Direct substitution of $\hat{\boldsymbol{\alpha}}$ into (16) leads to the following expression for the GLRT-based GD decision-making rule:

$$\frac{\min\limits_{\delta}\left[\delta^{\frac{NK}{K+1}}\det\left(\mathbf{R}_0+\delta^{-1}\mathbf{S}\right)\right]}{\min\limits_{\delta}\left[\delta^{\frac{NK}{K+1}}\det\left(\mathbf{R}_1+\delta^{-1}\mathbf{S}\right)\right]} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad ; \tag{19}$$

$$\mathbf{R}_0 = \sum_{l=1}^{L}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^* - \sum_{l=1}^{L}\mathbf{w}_{l_{PF}}\mathbf{w}_{l_{PF}}^* \quad ; \tag{20}$$

$$\mathbf{R}_1 = \sum_{l=1}^{L}\left(\mathbf{z}_l - \frac{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{z}_l}{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{p}}\mathbf{p}\right)\times\left(\mathbf{z}_l - \frac{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{z}_l}{\mathbf{p}^*\mathbf{S}^{-1}\mathbf{p}}\mathbf{p}\right)^* . \tag{21}$$

Plugging $\delta=1$ in (19), we get the GLRT GD for homogeneous environment, i.e.

$$\frac{\det\left(\mathbf{R}_0+\mathbf{S}\right)}{\det\left(\mathbf{R}_1+\mathbf{S}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad . \tag{22}$$

The detector (22) has CFAR property with respect to **M**. Assuming that the radar is not able to resolve individual parts of a possible target and $L=1$, the detector (22) reduces to that proposed in [28] and [29]. It can be shown that the proposed test statistic (22) coincides with the GLRT with respect to the $\alpha_l,\ l=1,\ \ldots,L$, and **M** when the noise is modelled as a complex multivariate elliptically contoured distribution [38] and, in particular, as a spherically invariant random process [39] with all of the range cells sharing the same value of the texture. This result partially generalizes that derived in [38].

In order to come up with the GLRT GD for a partially homogeneous environment there is a need to minimize both the numerator and the denominator of (23) with respect to δ . As was shown in [37], if $LK\geq N$ and denoting $m=\frac{NK}{K+1}$ the one-step GLRT GD (19) can be presented in the following form:

$$\frac{\hat{\delta}_0^{\frac{NK}{K+1}}\det\left(\mathbf{R}_0+\hat{\delta}_0^{-1}\mathbf{S}\right)}{\hat{\delta}_1^{\frac{NK}{K+1}}\det\left(\mathbf{R}_1+\hat{\delta}_1^{-1}\mathbf{S}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad , \tag{23}$$

where $\hat{\delta}_j$, $(j=0,1)$, is the positive solution of equation

$$\sum_{l=1}^{v_j}\frac{\lambda_{l,j}\delta}{\lambda_{l,j}\delta+1} = \frac{N}{K+1} , \qquad j=0,1 \tag{24}$$

with $\nu_0 = \min\{L, N\}$ and the $\lambda_{l,0}, l = 1,\ldots,\nu_0$ denoting the nonzero eigenvalues of the matrix $\mathbf{S}^{-0.5}\mathbf{R}_0\mathbf{S}^{-0.5}$ under the hypothesis H_0, and $\nu_1 = \min\{L, N-1\}$ and the $\lambda_{l,1}, l = 1,\ldots,\nu_1$ denoting the nonzero eigenvalues of the matrix $\mathbf{S}^{-0.5}\mathbf{R}_1\mathbf{S}^{-0.5}$ under the hypothesis H_1.

It is important to point out that the newly introduced GLRT GD ensures the CFAR property with respect to the covariance matrix of both primary and secondary data. Due to complexity of the corresponding statistics, the real-time implementation of the above GLRT GD can be a formidable task, even for a high-performance computer. It is thus of relevant interest to investigate the suitability of simplified structures.

2.3 Two-step GLRT GD

We first derive the GLRT GD based on primary data assuming that the covariance matrix M or its structure Σ is known. Fully adaptive detectors are obtained by substituting the unknown matrix by the sample covariance matrix based on secondary data only.

2.3.1 Step 1

The pdf of the first L vectors under the hypothesis H_0 and H_1 is given by

$$f_{\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}}}(\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}} \mid \mathbf{M}, H_0) = \frac{1}{[\pi^N \det(\mathbf{M})]^L}\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_0')\}\,; \quad (25)$$

$$f_{\mathbf{z}_1,\ldots,\mathbf{z}_L}(\mathbf{z}_1,\ldots,\mathbf{z}_L \mid \mathbf{M}, \boldsymbol{\alpha}, H_1) = \frac{1}{[\pi^N \det(\mathbf{M})]^L}\exp\{-tr(\mathbf{M}^{-1}\mathbf{T}_1')\}\,; \quad (26)$$

$$\begin{cases} \mathbf{T}_0' = \sum_{l=1}^{L}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^* - \sum_{l=1}^{L}\mathbf{w}_{l_{PF}}\mathbf{w}_{l_{PF}}^* = \mathbf{R}_0\,; \\ \mathbf{T}_1' = \sum_{l=1}^{L}(\mathbf{z}_l - \alpha_l\mathbf{p})(\mathbf{z}_l - \alpha_l\mathbf{p})^* + \sum_{l=1}^{L}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^*\,. \end{cases} \quad (27)$$

Denote $\mathbf{M}$ by $4\sigma^4\Sigma$, where $4\sigma^4$ is the (1,1)-th component of the Toeplitz matrix $\mathbf{M}$. The derivation is begun by writing the GLRT under assumption that the covariance matrix or its structure only is known. It is given by for known $\mathbf{M}$

$$\frac{\max\limits_{\boldsymbol{\alpha}} f_{\mathbf{z}_1,\ldots,\mathbf{z}_L}(\mathbf{z}_1,\ldots,\mathbf{z}_L \mid \mathbf{M}, \boldsymbol{\alpha}, H_1)}{f_{\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}}}(\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}} \mid \mathbf{M}, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} C\,, \quad (28)$$

$$\frac{\max\limits_{\boldsymbol{\alpha}}\max\limits_{4\sigma^4} f_{\mathbf{z}_1,\ldots,\mathbf{z}_L}(\mathbf{z}_1,\ldots,\mathbf{z}_L \mid \boldsymbol{\alpha}, 4\sigma^4\Sigma, H_1)}{\max\limits_{4\sigma^4} f_{\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}}}(\mathbf{w}_{1_{AF}},\ldots,\mathbf{w}_{L_{AF}} \mid 4\sigma^4\Sigma, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad (29)$$

for known Σ, respectively. Substituting the multivariate Gaussian pdfs (25) and (26) in the previous formulas and performing required maximizations yields for known $\mathbf{M}$ and Σ, respectively

$$\sum_{l=1}^{L}\frac{|\mathbf{p}^*\mathbf{M}^{-1}\mathbf{z}_l|^2}{\mathbf{p}^*\mathbf{M}^{-1}\mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} C\,, \quad (30)$$

$$\sum_{l=1}^{L}\frac{|\mathbf{p}^*\boldsymbol{\Sigma}^{-1}\mathbf{z}_l|^2}{\mathbf{p}^*\boldsymbol{\Sigma}^{-1}\mathbf{p}\sum_{m=1}^{L}\mathbf{w}_{l_{AF}}^*\boldsymbol{\Sigma}^{-1}\mathbf{w}_{l_{AF}}} \underset{H_0}{\overset{H_1}{\gtrless}} C\,. \quad (31)$$

$|\cdot|$ denotes the modulus of a complex number. There is a need to distinguish $\mathbf{z}_l$ under the hypotheses H_0 and H_1 (1)–(3). Note that the left hand side of (30) is the sum of statistics corresponding to $L=1$ over the cells under test. Note also that when Σ is known, the denominator of the left hand side of (30) is independent of data. Construction of the left hand side of (31) is a bit different. To see the point, observe that it can be recast as

$$\sum_{l=1}^{L}\frac{|\boldsymbol{\upsilon}_l^*\mathbf{i}_{\mathbf{u}}|^2}{\frac{1}{2(N-1)L}\sum_{m=1}^{L}\|\mathbf{w}_{m_{AF}}^{\perp}\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} C\,, \quad (32)$$

$\|\cdot\|$ is the Euclidean norm of an N-dimensional vector over the complex field; $\boldsymbol{\upsilon}_l = \Sigma^{-0.5}\mathbf{z}_l$ is the "whitened" version of $\mathbf{z}_l$; $\mathbf{i}_{\mathbf{u}}$ is the unit vector parallel to the direction of the "whitened" version $\mathbf{u}$ of $\mathbf{p}$; $\mathbf{w}_{m_{AF}}^{\perp}$ is the component of $\mathbf{w}_{m_{AF}}$ orthogonal to the direction of $\mathbf{u}$.

Thus, for $L=1$, the test statistic is obtained by normalizing that corresponding to a conventional incoherent processing based on GASP to the estimated clutter power based on data from the cell under test [38]. If $L>1$, the left hand side of (31) is not exactly the sum of the statistics corresponding to $L=1$ over the cells under test since the normalization factor of each term of the sum is now an estimate of the clutter power based on all of the $\mathbf{w}_{m_{AF}}^{\perp}$, $m = 1,\ldots,L$.

2.3.2 Step 2

Remember that $LK \geq N$. We can make GD (30) and (31) fully adaptive by plugging the maximum likelihood estimate of $\delta\mathbf{M}$ based on the secondary date

$$\mathbf{z}_l = \mathbf{w}_{l_{AF}}\ ,\quad l = L+1,\ldots,L(K+1)\,, \tag{33}$$

i.e.

$$\hat{\delta\mathbf{M}} = \frac{1}{LK}\sum_{l=L+1}^{L(K+1)} \mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^{*} \tag{34}$$

in place of $\mathbf{M}$ in (30) and of Σ in (31). Equivalently, we can substitute $\mathbf{M}$ and Σ by $\mathbf{S}$. The resulting decision-making rules referred to in the following as the adaptive GD (AGD) and adaptive subspace GD (ASGD) are given by

$$\sum_{l=1}^{L}\frac{|\mathbf{p}^{*}\mathbf{S}^{-1}\mathbf{z}_l|^2}{\mathbf{p}^{*}\mathbf{S}^{-1}\mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} C \quad ; \tag{35}$$

$$\sum_{l=1}^{L}\frac{|\mathbf{p}^{*}\mathbf{S}^{-1}\mathbf{z}_l|^2}{\mathbf{p}^{*}\mathbf{S}^{-1}\mathbf{p}\sum_{m=1}^{L}\mathbf{w}_{m_{AF}}^{\perp *}\mathbf{S}^{-1}\mathbf{w}_{m_{AF}}^{\perp}} \underset{H_0}{\overset{H_1}{\gtrless}} C\,, \tag{36}$$

respectively. The above results deserve some comments.

Note that the AGD and ASGD reduce to the GD at $L = 1$. It is also worth noting that the left hand side of (31) is invariant with respect to multiplication of Σ by a real constant, but such an invariance property does not extend to the left hand side of (30). Because of this, AGD has the CFAR property in homogeneous environment only, i.e. with respect to $\mathbf{M}$, whereas the ASGD is a CFAR detector in both homogeneous and partially homogeneous environments, i.e. with respect to $\mathbf{M}$ and to δ and $\mathbf{M}$, respectively. We see that AGD and ASGD do not require the on-line inversion of the matrix. The ASGD is slightly more complex than the AGD since it requires evaluation of the trace of matrix. It is apparent that the two-step GLRT GD detectors are faster to implement than the one-step GLRT GD for homogeneous environment.

Finally, the one-step GLRT GD implementation for partially homogeneous environment requires to solve (24) under both hypotheses, and, hence, an additional computer cost with respect to the one-step GLRT GD for homogeneous environment.

3 Detection Performance

Probability of detection P_D is a function of the target and clutter parameters $\alpha_1,\ldots,\alpha_L,\mathbf{p},\mathbf{M}$ only through the signal-to-noise ratio (*SNR*) defined as

$$SNR = \frac{\sum_{l=1}^{L}|\alpha_l|^2}{N}\mathbf{p}^{*}\mathbf{M}^{-1}\mathbf{p}\quad . \tag{37}$$

Following [10], we recast (35) and (36) in a more convenient form. Denote by $\mathbf{U}$ the unitary transformation aimed at rotating the vector $\mathbf{M}^{-0.5}\mathbf{p}$ onto the direction of $\mathbf{e}_1 = (1,0,\ldots,0)$ by $\mathbf{x}_l, l = 1,\ldots,L(K+1)$, the transformed whitened data vectors, and by $\hat{\mathbf{C}}$ LK times the sample covariance matrix of the transformed secondary data, i.e.,

$$\begin{cases} \mathbf{U} : \mathbf{U}\mathbf{M}^{-0.5}\mathbf{p} = \sqrt{\mathbf{p}^{*}\mathbf{M}^{-1}\mathbf{p}}\mathbf{e}_1; \\ \mathbf{x}_l = \mathbf{U}\mathbf{M}^{-0.5}\mathbf{z}_l\ ,\quad l = 1,\ldots,L(K+1); \\ \hat{\mathbf{C}} = \sum_{l=L+1}^{L)K+1)}\mathbf{x}_l\mathbf{x}_l^{*}\ . \end{cases} \tag{38}$$

Then, (35) and (36) can be rewritten in the following form:

$$\frac{\sum_{l=1}^{L}\left|\mathbf{x}_l - \sum_{m=L+1}^{L(K+1)}\mathbf{x}_l q_{l,m}\right|^2}{\frac{1}{\delta}\sum_{m=L+1}^{L(K+1)}\left|\mathbf{x}_m - \sum_{i=L+1}^{L(K+1)}\mathbf{x}_i Q_{m,i}\right|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{C}{\sqrt{2}}\delta\ ; \tag{39}$$

$$\frac{\sum_{l=1}^{L}\left|\mathbf{x}_l - \sum_{m=L+1}^{L(K+1)}\mathbf{x}_l q_{l,m}\right|^2}{\frac{1}{\delta}\sum_{m=L+1}^{L(K+1)}\left|\mathbf{x}_m - \sum_{i=L+1}^{L(K+1)}\mathbf{x}_i Q_{m,i}\right|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{C}{\sqrt{2}(1-C)}\sum_{l=1}^{L}\mathbf{w}_{l_{AF}}^{*}(\delta\,\mathbf{S}_{\eta}^{-1})\mathbf{w}_{l_{AF}}\ , \tag{40}$$

where

$$\begin{cases} q_{l,m} = \mathbf{x}_m^{*}\mathbf{S}_{\eta}^{-1}\mathbf{x}_l; \\ Q_{m,i} = \mathbf{x}_i^{*}\mathbf{S}_{\eta}^{-1}\mathbf{x}_m; \\ \mathbf{S}_{\eta} = \sum_{l=L+!}^{L(K+1)}\mathbf{w}_{l_{AF}}\mathbf{w}_{l_{AF}}^{*}. \end{cases} \tag{41}$$

We see that owing to independence of the numerator and denominator in (39) and (40) (see Subsection 2.1), the denominator is a central chi-squared random variable with $2(LK+1-N)$ degrees of freedom [39] and the numerator is the Euclidean norm squared of the L-dimensional vector that under the hypothesis H_1 is a Gaussian vector with the mean vector $\sqrt{\mathbf{p}^*\mathbf{M}^{-1}\mathbf{p}}\boldsymbol{\alpha}$ and the covariance matrix $\mathbf{I}_L +$ $\boldsymbol{\eta}^*\delta(\mathbf{S}_\eta^{-1})^*\boldsymbol{\eta}$, where $\boldsymbol{\eta}=(\boldsymbol{\eta}_1^*,\ldots,\boldsymbol{\eta}_L^*)$ Introducing the unitary transformation $\mathbf{U}_1$ aimed at rotating the vector $\boldsymbol{\alpha}$ onto the direction of $\mathbf{e}_1$, it follows that under the hypothesis H_1 we have the Gaussian vector with the mean $\sqrt{\sum_{l=1}^{L}|\alpha_L|^2\,\mathbf{p}^*\mathbf{M}^{-1}\mathbf{p}}\mathbf{e}_1$ and the covariance matrix $\mathbf{I}_L+\mathbf{U}_1\boldsymbol{\eta}^*\delta(\mathbf{S}_\eta^{-1})^*\boldsymbol{\eta}\mathbf{U}_1^*$. Note that we can rewrite the right-hand side of (40) in the following form:

$$\sum_{l=1}^{L}\mathbf{w}_{l_{AF}}^*(\delta\mathbf{S}_\eta^{-1})\mathbf{w}_{l_{AF}}=tr\left[\boldsymbol{\eta}^*\delta(\mathbf{S}_\eta^{-1})^*\boldsymbol{\eta}\right]$$
$$=tr\left\{\mathbf{U}_1\boldsymbol{\eta}^*\left[\delta(\mathbf{S}_\eta^{-1})^*\right]\boldsymbol{\eta}\mathbf{U}_1^*\right\}. \quad (42)$$

The probability of detection P_D of GD (39) and (40) can be presented in the following form:

$$P_{D_{\boldsymbol{\eta}_1,\ldots,\boldsymbol{\eta}_{L(K+1)}}}=1-F_{|\mathbf{U}_1\boldsymbol{\eta}^*,\mathbf{S}_\eta}\left\{\frac{C}{\sqrt{2}}\delta,SNR\right\}; \quad (43)$$

$$P_{D_{\boldsymbol{\eta}_1,\ldots,\boldsymbol{\eta}_{L(K+1)}}}=1$$
$$-F_{|\mathbf{U}_1\boldsymbol{\eta}^*,\mathbf{S}_\eta}\left\{\frac{C}{\sqrt{2}(1-C)}\times tr\left[\mathbf{U}_1\boldsymbol{\eta}^*[\delta(\mathbf{S}_\eta^{-1})^*]\boldsymbol{\eta}\mathbf{U}_1^*\right],SNR\right\} \quad (44)$$

where $F_{|\mathbf{U}_1\boldsymbol{\eta}^*,\mathbf{S}_\eta}\{\cdot,SNR\}$ denotes the conditional cumulative distribution function (cdf) of the left-hand size of both tests.

In particular, previous notation highlights that the dependence of the conditional cdf on the $\boldsymbol{\eta}_l, l=1,\ldots$ $\ldots,L(K+1)$ is confined to $\mathbf{U}_1\boldsymbol{\eta}^*$ and $\mathbf{S}_\eta$. In order to determine the probability of detection P_D of GD, we can average out the $\boldsymbol{\eta}_l, l=1,\ldots,L$ and then the $\boldsymbol{\eta}_l, l$ $=L+1,\ldots,L(K+1)$. Following this guidance, we get for GD (39) and (40)

$$P_D=1-E_{\boldsymbol{\eta}_{L+1},\ldots,\boldsymbol{\eta}_{L(K+1)}}E_{\boldsymbol{\eta}_1,\ldots,\boldsymbol{\eta}_L}F_{|\mathbf{U}_1\boldsymbol{\eta}^*,\mathbf{S}_\eta}\left\{\frac{C}{\sqrt{2}}\delta,SNR\right\}, \quad (45)$$

$$P_D=1-E_{\boldsymbol{\eta}_{L+1},\ldots,\boldsymbol{\eta}_{L(K+1)}}E_{\boldsymbol{\eta}_1,\ldots,\boldsymbol{\eta}_L}F_{|\mathbf{U}_1\boldsymbol{\eta}^*,\mathbf{S}_\eta}$$
$$\times\left\{\frac{C}{\sqrt{2}(1-C)}\times tr\left[\mathbf{U}_1\boldsymbol{\eta}^*[\delta(\mathbf{S}_\eta^{-1})^*]\boldsymbol{\eta}\mathbf{U}_1^*\right],SNR\right\}, \quad (46)$$

respectively. We see that $\mathbf{U}_1\boldsymbol{\eta}^*$ is statistically equivalent to $\boldsymbol{\eta}^*$ and independent of $\mathbf{S}_\eta$. The similar result holds true for the detector (22). Moreover, simulation results indicate that the GLRT GD should also possess this property.

4 Numerical Results

The probability of false alarm P_{FA} and the probability of detection P_D of GLRT GD are estimated by Monte Carlo technique based on $P_{FA}^{-1}\times10^2$ and $P_D^{-1}\times10^2$ independent trials [40]. As a consequence, in order to limit the computational burden, we assume that the probability of false alarm is constrained, i.e., $P_{FA}=$ 10^{-4}. As for L, we observe that it is lower bounded by the ratio between the range extent of the target and the range resolution of the radar. We consider small values of L $(L\le 20)$ in order to save simulation time. Finally, we suppose that if the radar resolution is increased by a factor L, i.e. the cell size is reduced by L, the noise power per cell $2\sigma_1^2$ is decreased by the same factor, i.e., we set $\sigma_1^2=\sigma^2L^{-1}$.

4.1 Targets with nonrandom parameters

Before we discussed that the detection probabilities of the AGD and ASGD are independent of the actual multiple dominant scattering model being in force. This subsection is devoted to the detection performance evaluation of the GLRT GD (22) and (23), the AGD, and the ASGD.

In Figs. 2–4, we consider a homogeneous environment. In particular, in Fig.2, the probability of detection P_D of the GLRT GD (22), the AGD, and the ASGD are plotted versus SNR at $N=8, K=16$, and several values of L. Note that the case $L=1$ refers to unresolved targets. We see that increasing in the radar resolution capabilities and suitably exploiting them can produce a significant detection gain and the corresponding curves of the AGD and GLRT GD intersect, and, in particular, the AGD outperforms the one-step GLRT GD at high values of the probability of detection P_D. For example, at $L=4$, the AGD outperforms the GLRT GD for all values of the probability of detection P_D of practical interest $(P_D>0.5)$. The ASGD is poorer than the other two receivers, but the loss is less than 2.5 dB at the probability of detection less than 0.9, i.e., $P_D\le0.9$. This behaviour is valid if $LK>2N$.

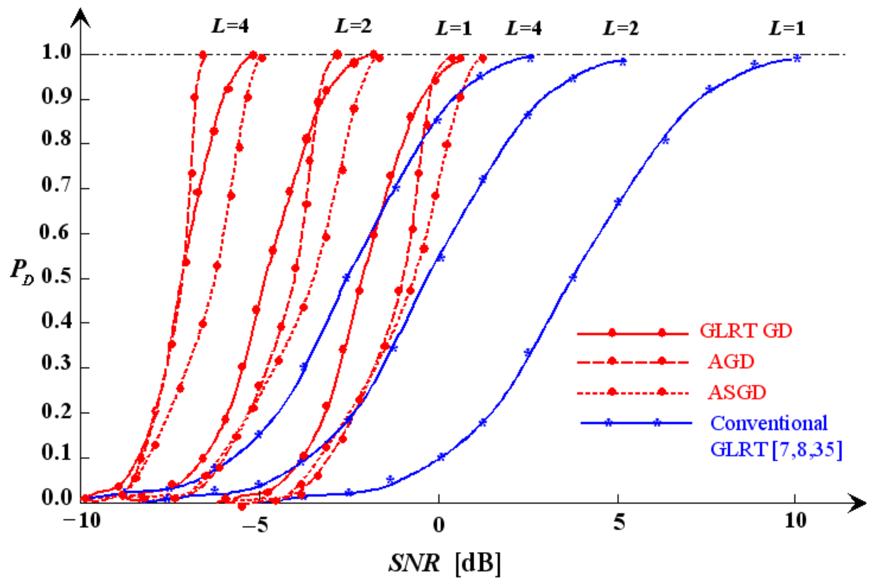

Figure 2. The probability of detection P_D versus SNR of the GLRT GD (22), AGD, and ASGD in homogeneous environment at $N = 8, K = 16,\ P_{FA} = 10^{-4}$, and L as a parameter.

Finally, the loss of the AGD and ASGD with respect to GLRT GD (25), namely, the one that possesses perfect knowledge of the covariance matrix $\mathbf{M}$ of the noise, can be read off Figs. 3 and 4 at $N = 8$, K as a parameter, $L = 2$ and $L = 4$, respectively. In Fig. 4 we plot the performance of the GLRT GD (18) and the ASGD in partially homogeneous environment at $N = 8, K = 16,$ and several values of L. In this case we do not consider AGD since it is not longer CFAR. We see that the one-step GLRT GD and the ASGD achieve approximately the same performance, but this is not true at $L > 1$ as can be shown by simulation for a properly reduced sample size. The ASGD performance and its loss with respect to

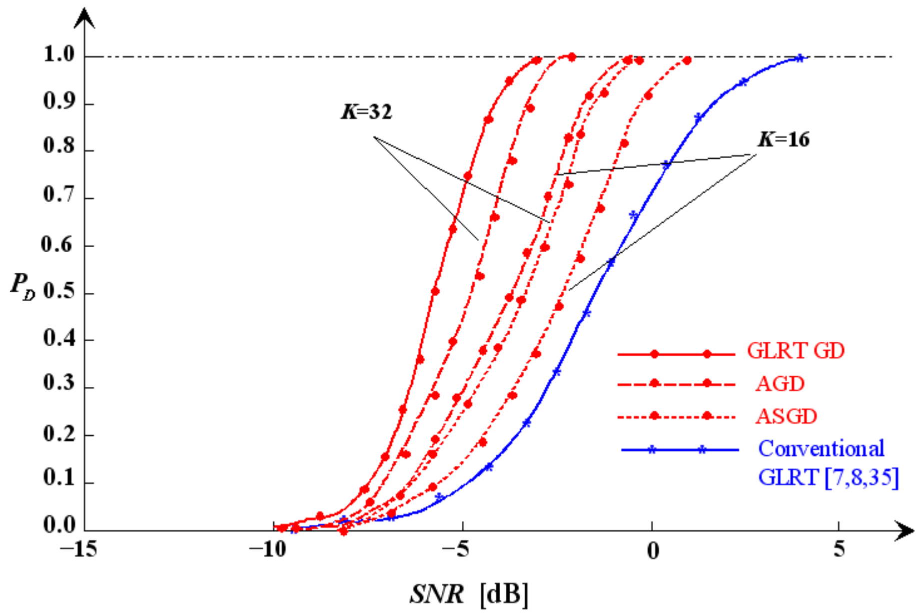

Figure 3. The probability of detection P_D versus SNR of the GLRT GD (22), AGD, and ASGD in homogeneous environment at $N = 8, L = 16,\ P_{FA} = 10^{-4}$, and K as a parameter.

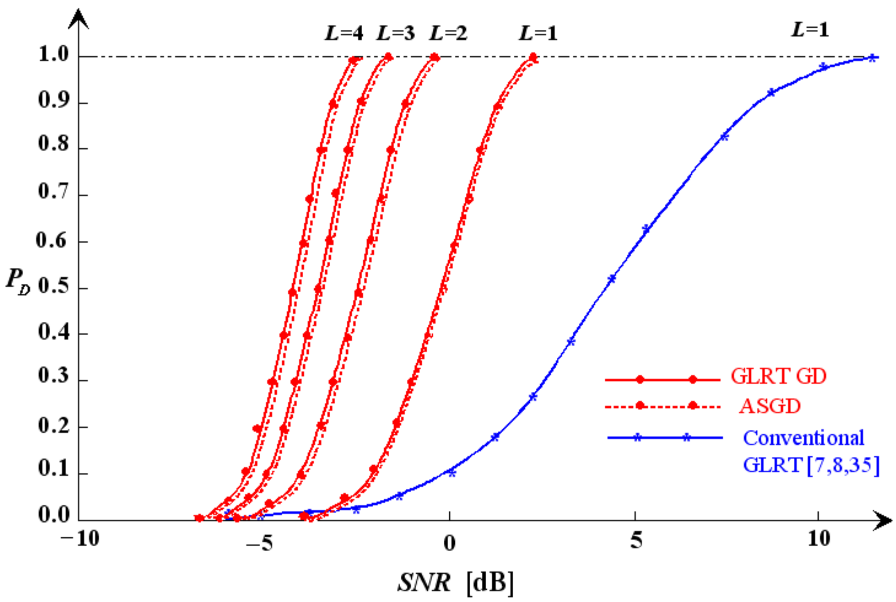

Figure 4. The probability of detection P_D versus *SNR* of the GLRT GD (23) and ASGD in partially homogeneous environment at $N = 8, K = 16,\ P_{FA} = 10^{-4}$, and L as a parameter.

the GLRT GD (30) can still be read of Figs. 2 and 3 since the ASGD is invariant under scaling of the secondary data. For all cases presented in Figs. 2–4 we observe a superiority of the GD over the conventional GLRT detector discussed in [7], [8], and [35].

4.2 Targets with random parameters

We assess the performance of the AGD and ASGD when the unknown deterministic parameters accounting for both the target and the channel $\alpha_l, l = 1,\ldots$ $\ldots, L$ are the random variables. Obviously, the probability of false alarm P_{FA} is unaffected by the actual characterization of the parameters α_l. Under the hypothesis H_1, the pdf of either statistics is independent of phase characterization of the parameters $\alpha_l, l = 1,$ $\ldots, L$. Thus, should only the phases are random, the probability of detection P_D of GD would not be changed and hence, the curves of Figs. 2–4 would still be valid. If the amplitudes are random variables, due to the dependence of *SNR* (37) on the parameters α_l, different statistical characterizations of the target can result in significantly different probabilities of detection. It is customary to model the $|\alpha_l|^2, l = 1,\ldots$ $\ldots, L$ as chi-squared random variables. It would be interesting to evaluate the impact on the performance of a degree of correlation among the scattering centers of the target. To this end, we assume that the $|\alpha_l|^2, l = 1,\ldots, L$ are drawn from an exponentially correlated random sequence with the one-lag correlation coefficient ρ. The procedure to generate $|\alpha_l|^2$ is discussed in [35] and we follow it.

In Fig. 5, we analyze an influence of the fluctuation law at $N = 8, K = 16, L = 4, \rho = 0$, i.e., the parameters $|\alpha_l|, l = 1,\ldots, L$ are independent of each other, the multiple dominant scattering target model 1 from Table 1 [35], and m as a parameter. Any permutation of scatterer positions among the cells under test does not influence the performance, also due to assumption that the parameters $|\alpha_l|, l = 1,\ldots, L$ are independent random variables. The AGD performance (35) operating in a homogeneous environment is shown in Fig. 5. The performance depends on the actual multiple dominant scattering target model being in force. The probability of detection P_D of GD can be obtained by averaging (45) and (46) with respect to the *SNR*, respectively, and the distribution of the *SNR* (37) depends on the multiple dominant scattering target model. Figures 2–5 show that the fluctuation law significantly affects the performance only for high values of the probability of detection P_D in the medium/high range.

In Fig. 6, we analyze the effect of correlation between the target amplitudes for the AGD (the dependences for the ASGD are little bit worse). We refer to $N = 8, K = 16, L = 4$, Rayleigh-fluctuating amplitudes, the multiple dominant scattering target model 1 from Table 1 [35], and several values of the one-lag correlation coefficient ρ. Figure 6 highlights that

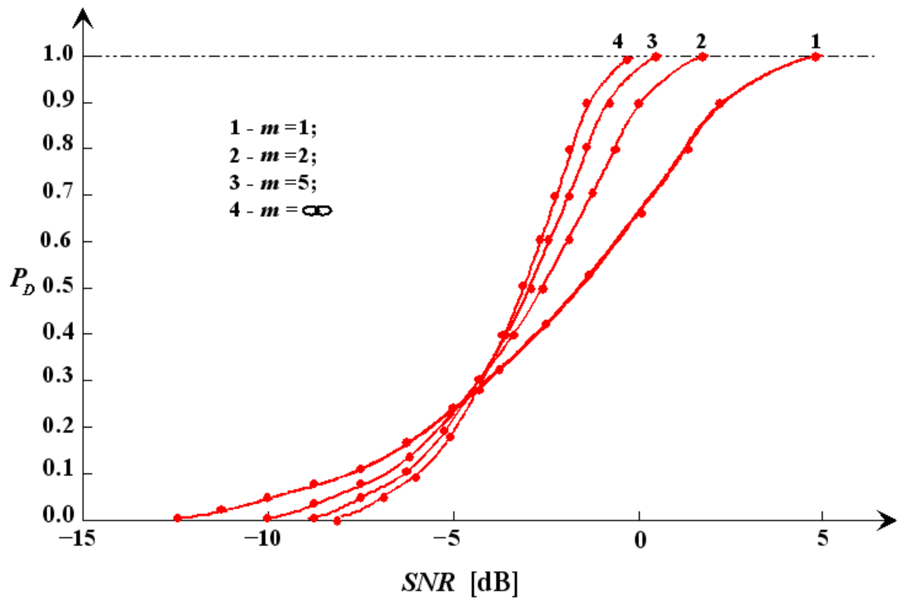

Figure 5. The probability of detection P_D versus *SNR* of the AGD in homogeneous environment at $N = 8, K = 16,\ P_{FA} = 10^{-4}$, and $L = 4$ with chi-fluctuating amplitudes and *m* as a parameter.

the correlation between the $|\alpha_l|, l = 1,\dots, L$ is responsible for an additional loss. This behaviour can be easily explained intuitively. In fact, when the received signals from target scatterers are significantly correlated it may happen that all of them "fade at the same time" and this may cause missing of the detection. We note that Figs. 2–6 highlight that the GLRT GD, the AGD, and the ASGD outperform the conventional GLRT detector discussed in [7], [8], and [35].

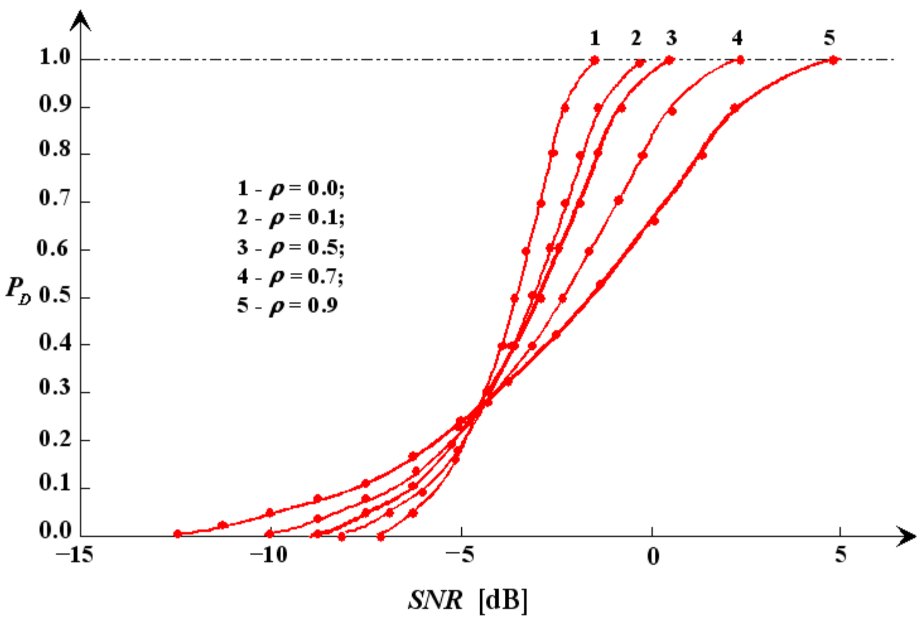

Figure 6. The probability of detection P_D versus *SNR* of the AGD in homogeneous environment at $N = 8, K = 16,\ P_{FA} = 10^{-4}$, and $L = 4$ with correlated Rayleigh-distributed amplitudes and ρ as a parameter.

5 Conclusions

In this paper, we have addressed the problem of adaptive detection of range-spread targets in homogeneous and partially homogeneous environment. We designed and assessed the one-step and two-step GLRT GD that possesses the CFAR properties. We have shown that the AGD and ASGD have the CFAR property under a homogeneous environment and the one-step GLRT GD (23) and ASGD have the CFAR property under a partially homogeneous environment.

As to computational complexity, we have shown that the two-step GLRT GD are faster to implement than the one-step one, and the amount of work required for their implementation grows linearly with the number of range cells L.

As to the detection performance, we have derived the analytical dependence of the probability of detection P_D of GD on the target and the noise parameters and estimated the probability of detection P_D of GD through the Monte Carlo simulations.

The cases of fluctuating and non-fluctuating targets are considered. We could find that:

- the GLRT GD do not suffer collapsing loss;
- the one-step GLRT GD (22) and AGD may have comparable detection performance under the homogeneous disturbance at high values of LK;
- the ASGD achieves the same performance of the one-step GLRT GD (23) in a partially homogeneous environment and has an aceptable loss with respect to the one-step GLRT GD (22) in the homogeneous disturbance.

In the latter case, we have focused on the AGD and ASGD and have found that the fluctuation law of the target amplitudes strongly affects the probability of detection P_D of GD in the medium/high range.

We have evaluated the impact on the performance of a degree of correlation between the scattering centers of the target. We have found that the correlation is responsible of an additional loss which is relevant for values of the probability of detection P_D of GD in the medium/high range.

In conclusion we state that

- increasing in the radar resolution capabilities and suitably exploiting them can produce a significant detection gain;
- the modified GLRT GD is superior to the plain GLRT, as it leads to superior detection performance;
- the ASGD is somewhat more robust than the AGD in that it guarantees CFARness under both scenarios.

We still must assess the capability of the proposed receivers based on GASP in detecting the slightly mismatched signals while rejecting the unwanted signals, i.e. the side-lobe signals. This is a problem of primary concern in a surveillance system and is the object of current and future investigations.

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References:

[1] Backman, C.G., ”Some recent developments in RCS measurements techniques,” *Proc. IEEE*, 1965, Vol. 53, No. 8, pp. 962–972.

[2] Li, Q., Rothwell, E.J., Chen, K.M., and Nyquist, D.P.,“Scattering center analysis of radar targets using fitting scheme and genetic algorithm,” *IEEE Transactions on Antennas Propagation*, 1996, Vol. 44, No. 2, pp. 198–207.

[3] Moon, T.T. and Bawden, P.J., “High resolution RCS measurements of boats,” in *Proc. Inst. Elect. Eng. F*, 1991, Vol. 138, No. 3, pp. 218–222.

[4] Hughes, P.K., “A high resolution radar detection strategy,” *IEEE Transactions on Aerospace and Electronic Systems*, 1983, Vol. AES-19, No. 9, pp. 663–667.

[5] Nitzberg, R., ”Effect of a few dominant specular reflectors target model upon target detection,” *IEEE Transactions of Aerospace and Electronic Systems*, 1978, Vol. AES-14, No. 7, pp. 670–673.

[6] Gerlach, K., Steiner, M.J., and Lin, F.C., “Detection of a spatially distributed target in white noise,” *IEEE Signal Processing Letters*, 1997, Vol. 4, No. 7, pp. 198–200.

[7] Kelly, E.J. and Forsythe, K., “Adaptive detection and parameter estimation for multidimensional signal models,” Lincoln Lab., Mass. Inst., Technol., Lexington, Tech. Rep. No. 848, 1989, April 19.

[8] Burgess, K. A. and Van Veen, B.D., “Subspace-based adaptive generalized likelihood ratio detection,” *IEEE Transactions on Signal Processing*, 1996, Vol. 44, No. 4, pp. 912–927.

[9] Bose, S. and Steinhardt, A.O., “Adaptive array

detection of uncertain risk one waveforms," *IEEE Transactions on Signal Processing*, 1996, Vol. 44, No. 11, pp. 2801–2809.

[10] Kelly, E.J.,"An adaptive detection algorithms," *IEEE Transactions of Aerospace and Electronic Systems*, 1986, Vol. AES-22, No. 3, pp. 115–127.

[11] Gerlach, K. and Steiner, M.J., "Adaptive detection of range distributed targets," *IEEE Transactions on Signal Processing*, 1999, Vol. 47, No. 7, pp. 1844–1851.

[12] Gerlach, K. and Steiner, M.J., "A study of adaptive detection of range-distributed targets," Naval Res. Lab., Tech. Rep. NRL/MR/5340-00-8438, March 2000.

[13] Gerlach, K. and Steiner, M.J., "Fast converging adaptive detection of Doppler-shifted range-distributed targets,"*IEEE Transactions on Signal Processing*, 2000, Vol. 48, No. 9, pp. 2686–2690.

[14] Kraut, S. and Scharf, L.L., "The CFAR adaptive subspace is a scale-invariant GLRT," *IEEE Transactions on Signal Processing*, 1999, Vol. 47, No. 9, pp. 2538–2541.

[15] Himed, B. and Melvin, W.L., "Analyzing space-time adaptive processors using measured data," in *Proc. 31st Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, November 6–9,1997, pp. 930–935.

[16] Blum, R.S. and McDonald, K.F., "Analysis of STAP algorithms for cases with mismatched steering and clutter statistics," *IEEE Transactions on Signal Processing*, 2000, Vol. 48, No.2, pp. 301–310.

[17] Richmond, C.D., "Performance of a class of adaptive detection algorithms in nonhomogeneous environments,"*IEEE Transactions on Signal Processing*, 2000, Vol. 48,No.5,pp.1248–1262.

[18] Melvin, W.L., "Space-time adaptive radar performance in heterogeneous clutter," *IEEE Transactions on Aerospace and Electronic Systems*, 2000, Vol. 36, No. 4, pp. 621–633.

[19] Ward, K.D., Baker, C.J., and Watts, S., "Maritime surveillance radar. Part 1:Radar scattering from the ocean surface," in *Proc. Inst., Elect. Eng. F*, 1990, Vol. 137, No. 2, pp. 51–62.

[20] Sangston, K.J. and Gerlach, K., "Coherent detection of radar targets in a non-Gaussian background," *IEEE Transactions on Aerospace and Electronic Systems*, 1994, Vol. 30, No. 4, pp. 330–340.

[21] Conte, E., Bisceglie. M.Di., Galdi, C., and Ricci, G. "A procedure for measuring the coherence length of the sea texture," *IEEE Transactions on Instrumental Measurements*, 1997, Vol. 46, No. 8, pp. 836–841.

[22] Gerlach, K., "Spatially distributed targets detection in non-Gaussian clutter," *IEEE Transactions on Aerospace and Electronic Systems*, 1999, Vol. 35, No. 7, pp. 926–934.

[23] Pulsone, N.B. and Raghavan, R.S., "Analysis of an adaptive CFAR detector in non-Gaussian interference," *IEEE Transactions of Aerospace and Electronic Systems*, 1999, Vol. 35, No. 7, pp. 903–916.

[24] Tuzlukov, V.P., "A new approach to signal detection theory," *Digital Signal Processing*, 1998, Vol. 8, No. 3, pp.166 –184.

[25] Tuzlukov, V.P., *Signal Processing in Noise*: *A New Methodology,* Minsk: IEC, 1998.

[26] Tuzlukov, V.P., *Signal Detection Theory*, New York: Springer-Verlag, 2001.

[27] Tuzlukov, V.P., *Signal Processing Noise*, Boca Raton, London, New York, Washington D.C.: CRC Press, Taylor & Francis Group, 2002.

[28] Tuzlukov, V.P., *Signal and Image Processing in Navigational Systems*, Boca Raton, London, New York, Washington D.C.: CRC Press, Taylor & Francis Group, 2005.

[29] Tuzlukov,V.P., *Signal Processing in Radar Systems*, Boca Raton, London, New York, Washington D.C. CRC Press, Taylor & Francis Group, 2012.

[30] Robey, F.C., Fuhrmann, D.R., Nitzberg, R., and Kelly, E.J., "A CFAR adaptive matched filter detector," *IEEE Transactions onAerospace and Electronic Systems*, 1992, Vol. 28, No. 1, pp. 208–216.

[31] Scharf, L.L. and McWhorter, L.T., "Adaptive matched subspace detectors and adaptive coherence estimators," in *Proc. 30th Annual Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Nov. 1996.

[32] Bose, S. and Steinhardt, A.O., "Optimum array detector for a weak signal in unknown noise," *IEEE Transactions on Aerospace and Electronic Systems*, 1996, Vol. 32, No.7, pp. 911–922.

[33] Picinbono, B., "Second-order complex random vectors and normal distribution," *IEEE Transactions on Signal Processing*, 1996, Vol. 44, No. 10, pp. 2637–2640.

[34] Van Trees, H.L., *Detection, Estimation, and Modulation Theory*. 2nd Ed. New York: Wiley & Sons Inc., 2001.

[35] Maximov, M., "Joint correlation of fluctuative noise at the outputs of frequency filters," *Radio Engineering and Telecommunications*, 1956, No. 9, pp. 28–38.

[36] Chernyak, Y., "Joint correlation of noise volta-

ge at the outputs of amplifiers with non-overlapping responses," *Radio Physics and Electronics*, 1960, No. 4, pp. 551–561.

[37] Conte, E., De Maio, A., and Ricci, G., "GLRT based adaptive detection algorithms for range-spread targets," *IEEE Transactions on Signal Processing*, 2001, Vol. 49, No. 7, pp. 1336–1348.

[38] Richmond, C.D., "A note on non-Gaussian adaptive array detection and signal parameter estimation,"*IEEE Signal Processing Letters*, 1996, Vol. 3, No. 8, pp. 251–252.

[39] Conte, E., Longo, M., and Lops, M., "Modeling and simulation of non-Rayleigh radar clutter," in *Proc. Inst., Elect. Eng. F*, 1991, Vol. 138, No. 2, pp. 121–130.

[40] Conte, E., Lops, M., and Ricci, G., "Asymptotically optimum radar detection in compound-Gaussian noise," *IEEE Transactions on Aerospace and Electronic Systems*, 1995, Vol. 31, No. 4, pp. 617–625.

[41] Scharf, L., *Statistical Signal Processing. Detecion, Estimation, and Time Series Analysis*. Reading, MA: Addison-Wesley, 1991.

[42] Echard, J., "Estimation of radar detection and false alarm probabilities," *IEEE Transactions on Aerospace and Electronic Systems*, 1995, Vol. 31, No. 4, pp. 617–625.

[43] Skolnik, M., *Introduction to Radar Systems*. 3rd Ed. New York: McGraw-Hill, 2001.

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