Linear Multiuser Detectors using Linear, Circular and Concentric Circular Antenna Arrays in CDMA Communication Systems

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Abstract: - The suitable use of an antenna array at the base station of a communication system results in improved Signal to Interference Ratio (SIR). In this paper, Linear Array (LA), Circular Array (CA) and Concentric Circular Antenna Array (CCAA) are used at the base station of a direct sequence code division multiple access (CDMA) systems. To suppress multiple access interference (MAI), the proposed scheme utilizes the spatial distribution of users as well as the time domain processing. The results show the significant improvement in capacity of CDMA system employing joint use of antenna array and multiuser detectors.

Key-Words: - Code Division Multiple Access, Bit error rate, Signal to Noise ratio, Multiuser detector, Antenna array

1 Introduction

The promising approach to increase the spectrum efficiency in 3rd generation cellular system is the use of CDMA systems [1]. In CDMA systems, all users communicate simultaneously in the same frequency band, and hence the MAI is one of the major causes of transmission impairments. The SIR improvement capability is, therefore an important measure of performance for CDMA based cellular systems. Smart Antenna Systems, Multiuser Detection methods and Power Control Techniques are some of the ways to improve SIR. Therefore, advanced receiver schemes are currently subject of an extensive research activity in order to ensure remarkable performance improvement for future 3G systems: in particular, main research areas can be identified in Multi User Detection (MUD) and adaptive antennas techniques. Conventional DS-CDMA mobile receivers are generally based on rake diversity principle, thus considering other user signals as pure interference. When these receivers are employed, the MAI limits the number of active users in relation to a specified Bit error probability. Conversely, in a receiver based on the MUD approach [2-4] all interfering signals are considered as additional information sources in order to perform an optimum joint decision so that negative effects that each users has on others can be eliminated. In particular, this approach permits to alleviate the near far problem and to increase system capacity. Unfortunately, due to their excessive implementation complexity, optimum MUD schemes have not been considered in practical applications. Hence, research efforts have been focused on suboptimal multiuser detection approach. These suboptimal detectors are further classified into linear and nonlinear type detectors. The linear detectors include decorrelator and Minimum Mean Square Error (MMSE) detectors [3,4]. These detectors need to compute the inverse of cross-correlation matrix [4], the complexity of which is linear. The decorrelator tries to completely remove the MAI for all users. The MMSE instead tries to minimize the square if the residual noise plus interference. Hence the decorrelator is simply a special case of the MMSE detector, when the noise is zero [4]. The decorrelator often results in unacceptable noise enhancement.

In addition to MUD schemes, DS-CDMA communication systems' performance can take benefit from adaptive antenna array, [5-7] since filtering in the spatial domain can separate spectrally and temporally overlapping signals from multiple mobile users, and hence the performance of system can be significantly improved. This discrimination ability is in general a function of array geometry (e.g. linear or circular), the number of antenna elements, spacing and the direction of signal arrival of the desired user and the interferers. Antenna arrays may be linear, two-dimensional, circular and spherical in element arrangement. The circular antenna array (CA) provides uniform coverage in terms of gain or pattern compared to linear array (LA). It means uniform SIR improvement and less mutual coupling due to array curvature. The number of antenna elements in a circular array may be arranged in many concentric circular antenna array (CCAA) of different radii [8-10]. This CCAA arrangement further has several advantages over CA including the flexibility in array pattern synthesis and provides almost invariant azimuth angle coverage.

This paper presents joint use of antenna array systems (LA, CA and CCAA) at the base station and deploying linear multiuser detection schemes at the CDMA receiver. The MAI is reduced in time domain and the residual interference is further reduced by antenna array. The proposed detector uses the modified conjugate gradient method (MCGM) [7] for weight calculations in antenna array, which increases the signal to noise /signal to interference ratio (SNR/SIR) combining beam forming with multiuser detection techniques to increase the channel capacity.

The aim of this paper is to present comparison in the performance of multiuser detectors in CDMA systems, one employing single antenna and other with antenna array structures i.e. linear, circular and concentric circular. We provide a detailed presentation of SNR/SIR improvement capability of linear array, circular array and CCAA combined with multiuser detection techniques. It is found that for full angular range, circular array provides more uniform improvement in SIR that linear array. Further, CCAA that contains many concentric circular rings of different radii has advantage of including flexibility in array pattern synthesis and provides almost invariant azimuth angle coverage. The remainder of the paper is organized as follows. In section 2, we present basic model of a CDMA system with antenna array. In section 3, we analyze SIR improvement with different types of antenna arrays and in section 4, the performance of different types of linear multiuser detectors is presented. The simulation results are shown and analyzed in section 5 and in section 6 conclusions is done.

2 System Model

In a typical synchronous DS-CDMA system having K active users sending data over the same channel, the received baseband signal over one data interval can be expressed as

$$s(t) = \sum_{i=1}^{K} A_i d_i(t) b_i(t) + n(t) \qquad (1)$$

where A_i, $d_i(t)$and $b_i(t)$ are the received amplitude, signature code waveform, and data symbol (-1 or +1 for the duration of data interval) of the i^{th} user, respectively and n(t) is additive white Gaussian noise with variance σ^2 and power density No. Now consider a linear array of N antenna elements receiving signals from K mobile users. Let, we take uniform linear array (ULA) of N antenna elements, spaced at $d=\lambda/2$ distance along x axis and the direction of arrival (DOA) of sources is ϕ. This array receives K narrowband signals from mobile users which are randomly distributed in xy plane. We assume that with regard to this array the users are located in the far-field region. The received signal now at the m^{th} antenna element can be expressed as [8]

$$X_m(t) = \sum_{k=1}^{K} s_k(t) e^{-j\frac{2\pi}{\lambda} d_m \sin\phi_k} + n_m(t) \qquad (2)$$

where, $s_k(t)$ is the signal transmitted by k^{th} user as received by reference antenna element, ϕ_k is angle of arrival of the k^{th} source as measured from array broadside. λ is carrier wavelength of signals, d_m is the distance between m^{th} antenna element and reference antenna element, $n_m(t)$ is additive white Gaussian noise with zero mean and variance σ^2 at the antenna arrays. We can also expressed (2) in vector form as

$$x(t) = \sum_{k=1}^{K} a(\phi_k) s_k(t) + n(t) \qquad (3)$$

$$x(t) = A(\phi_k) s(t) + n(t) \qquad (4)$$

where $\mathbf{x}(t)$ is Nx1 vector of signals i.e. $\mathbf{x}(t)=[x_1(t), x_2(t), \ldots, x_N(t)]^T$, s(t) is Mx1 vector from impinging signals at reference antenna i.e. $\mathbf{s}(t)=[s_1(t), s_2(t), \ldots, s_M(t)]^T$ and $\mathbf{A}(\boldsymbol{\phi})$ is NxM matrix describing steering vectors i.e. $\mathbf{A}(\boldsymbol{\phi}) = [\mathbf{a}(\phi_1), \mathbf{a}(\phi_2), \ldots, \mathbf{a}(\phi_M)]$. Here $\mathbf{a}(\phi_k)$ is again described as

$$\mathbf{a}(\phi_k) = \begin{bmatrix} H_1(\phi_k) \;\; H_2(\phi_k) e^{-jkd\sin\phi_k}, \ldots, \\ \qquad\qquad H_N(\phi_k) e^{-j(N-1)kd\sin\phi_k} \end{bmatrix}^T \qquad (5)$$

where $(.)^T$ denotes transpose operation, and $H_N(\phi_k)$ is the response of n^{th} antenna element. If individual antenna element patterns are identical and mutual coupling between antenna elements is neglected, then (5) is reduced to

$$\mathbf{a}(\phi_k) = \left[1, e^{-jkd\sin\phi_k}, e^{-j2kd\sin\phi_k}, .., e^{-j(N-1)kd\sin\phi_k}\right]^T \quad (6)$$

The eq. (6) can be further reduced by substituting inter antenna element spacing d=λ/2 and wave no. k with 2π/λ to

$$\mathbf{a}(\phi_k) = \left[1, e^{-j\pi\sin\phi_k}, e^{-j2\pi\sin\phi_k}, .., e^{-j(N-1)\pi\sin\phi_k}\right]^T \quad (7)$$

Next, we consider a circular array of N identical omni directional antenna elements evenly spaced in a circle of radius R= Nλ/4π in the xy plane. This radius is chosen to obtain approximately λ/2 spacing of the elements, equivalent to that used for linear array. For plane wave incident in the xy plane, the relative phase at the pth element with respect to the centre of array is -(2π/λ)R cos(φ-Δφ) where Δφ =(2πp/N). The array steering vector for circular array can then be written as

$$\mathbf{a}(\phi_k) = \left[e^{j\pi\cos\phi_k} \quad e^{j\pi\cos(\phi_k-\Delta\phi)} \cdots e^{-j\pi\cos(\phi_k-(N-1)\Delta\phi)}\right]^T \quad (8)$$

The elements of circular antenna array can further be arranged in form of concentric circles. The m^{th} ring has a radius r_m and number of elements N_m where m = 1,2,... ,M. Assuming that the elements are uniformly spaced within the ring so it has an element angular separation given by

$$\psi_m = \frac{2\pi}{N} \quad (9)$$

and the elements in this ring are therefore located with an angle measured from the x-axis given by

$$\phi_{mn} = n\psi_m, \qquad n = 1,2,\ldots\ldots,N_m \quad (10)$$

An expression for the array steering vector can be deduced by defining the array steering vector for a single ring and extending the analysis for the whole array. For the m^{th} ring, the array steering vector has elements given by

$$a_{mn}(\theta,\phi) = e^{j\kappa r_m \sin(\theta)\cos(\phi-\phi_{mn})} \quad (11)$$

therefore the array steering vector for such ring will be

$$\boldsymbol{a}_{mn}(\theta,\phi) = \begin{bmatrix} e^{j\kappa r_m \sin(\theta)\cos(\phi-\phi_{m1})} & e^{j\kappa r_m \sin(\theta)\cos(\phi-\phi_{m2})} \\ \ldots e^{j\kappa r_m \sin(\theta)\cos(\phi-\phi_{mn})} & \ldots e^{j\kappa r_m \sin(\theta)\cos(\phi-\phi_{mN_m})} \end{bmatrix}^T \quad (12)$$

3 SIR Improvement in CDMA Receiver using different types of Antenna Arrays

The SIR at the output of antenna array (SIR_o) can be written as [10] which is a function of angle ϕ_1

$$SIR_o = \frac{E\left[\left|s_1(t)^2\right|\right]}{\sum_{k=2}^{N} \alpha_k(\phi_1,\phi_k) E\left[\left|s_k(t)^2\right|\right]} \quad (13)$$

The mean SIR at the array output can be written as

$$G_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \alpha_k(\phi_1,\phi_k)\,d\phi_k \quad (14)$$

$$SIR_o = \frac{SIR_{in}}{G_{avg}(\phi_1)} \quad (15)$$

where, $G_{avg}(\phi_1)$ is known as spatial interference suppression coefficient. For linear antenna array, let we assume that incident angles for interferers ϕ_j (j=2, 3, …, N) are uniformly distributed in range from –90° to 90°. Variation of Spatial Interference Coefficient, $G_{avg}(\phi_1)$ for different no. of antenna elements has been plotted in [8]. It is observed that interference reduction is maximum over a certain range of ϕ_1, centered at ϕ_1 =0° (broadside). Further, the improvement in SIR versus direction of arrival (DOA) has also been plotted in [9] with different number of antenna elements with and without MC. It is shown that the maximum improvement in SIR is 11 dB achieved with N=8 and DOA = 0°. Therefore, in this paper a general case with N=8 is taken for analyzing the performance in different scenario.

Table 1: Improvement in SIR with different array configurations

Array Configuration (N=8)		Improvement in SIR (dB)	
Name	Spacing/radius	Without MC	With MC
Linear	d = 0.5 λ	7.78	6.25
Circular	r = 0.7 λ	7.67	6.90
CCAA (4+4)	r_1 = 0.7 λ, r_2 = λ	7.96	8.41

The Table gives the average improvement in SIR for different types of array configuration (N=8) with and without mutual coupling (MC). The linear array provides average improvement of 7.7dB but does not provide uniform coverage in terms of gain or pattern, and gain degrades in the end-fire direction giving way to interference coming from other directions. The circular array provides more range and average gain of 7.67 dB uniformly in all directions. The CCAA provides flexibility in array pattern synthesis and favored in both broadside and endfire applications. In addition to the above, it gives more improvement in SIR than linear or circular array without MC. In contrast, the SIR further increases in CCAA including effect of MC. Summarizing the above discussion regarding SIR improvement in antenna array, we see that SIR improvement at the output of antenna array provides more input SIR to CDMA detector than using one antenna system. This improved SIR reduces BER output which increases the capacity of CDMA system as shown in Table 2.

Table 2: BER output using antenna array

SIR (dB)	BER output with single antenna and antenna array			
	N=1	N=8 Linear array ($x10^{-5}$)	N=8 Circular array ($x10^{-5}$)	N=4+4 CCAA ($x10^{-5}$)
0	.5	4.12	3.303	3.269
1	.07865	1.43	1.151	1.139
2	.02275	.503	.4037	.3995
3	.00715	.176	.1421	.1406
4	.00233	.06245	.05021	.04969
5	.00078	.02221	.01779	.01761
6	.00026	.00785	.0063	.00635
7	$9.14x10^{-5}$	.00289	.0023 3	.00223
8	$3.16\ x10^{-5}$	.00099	.0008	.0008
9	$1.10\ x10^{-5}$	.00036	.00029	.00028
10	$3.87\ x10^{-6}$	.00013	.0001	.0001

It is clear from the Table (2), that the increase in SIR using antenna array compared to single antenna reduces bit error rate. It transforms into increase in the capacity.

4 Linear Multiuser Detectors

BER is one of the main performance measures in the most communication systems. SIR is useful in the assessment of the quality of multiuser detectors. SIR gives the ratio of powers due to the desired user and due to all other components at a soft decision variable such as the output of the single user matched filter. In the absence of interfering users, the output of , the output signal to interference (SIR) ratio of the k^{th} matched filter is A_k^2/σ^2 and single user performance is achieved i.e. probability of error is given by $P_k(\sigma)=Q(A_k/\sigma)$, where $Q(.)$ is the complementary Gaussian error function. In the presence of synchronous interferers, SIR equals [1-2].

$$\mathrm{SIR} = \frac{A_k^2}{\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2} \tag{16}$$

The presence of other users on the channel increases the BER due to MAI. This BER reduces with the increase in SIR. By using antenna array at the input of detector, SIR input to the detector is improved which leads to reduction in BER and transforms into increased capacity.

The bit error probability for matched filter detector with one antenna can be written as [1]

$$P_b = Q\left(\sqrt{\frac{1}{\frac{1}{2\frac{E_b}{N_o}} + \frac{(K-1)}{G}}} \right) \tag{17}$$

where, E_b/N_o is signal to noise ratio (SNR). It is assumed that multiple access is Gaussian distributed with zero mean and variance proportional to the number of users and Q(.) is the complementary Gaussian error function. It is clear from (17) that the increase in SNR i.e. E_b/N_o will reduce bit error rate. Therefore, using antenna array at the input of matched filter detector will reduce BER.

The result can also be applied to sub-optimal multiuser detectors i.e. decorrelator and MMSE detector. The bit error probability P_b for the decorrelator with perfect power control in a synchronous system on AWGN channel with single antenna, can be given by [3]

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}\left(1 - \frac{K-1}{G}\right)} \right) \tag{18}$$

It is clear from (18) that the bit error probability P_b for the decorrelator with one antenna will reduce with increase in SNR. Therefore, by using antenna array the input of decorrelator the capacity is increased.

The bit error probability for MMSE detector in term of number of users, signal to noise ratio and processing gain with one antenna can be calculated as

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o} - \frac{1}{4}F\left(\frac{2E_b}{N_o}, \frac{K-1}{G}\right)}\right) \qquad (19)$$

Where,

$$F(x,z) \overset{\text{def}}{=} \left(\sqrt{x(1+\sqrt{z})^2+1} - \sqrt{x(1-\sqrt{z})^2+1}\right)^2 \qquad (20)$$

The equations (19) and (20) give the bit error probability of linear multiuser detectors with single antenna. It is clear that bit error rate of decorrelator and MMSE detector reduces with increase in SIR. Therefore by using antenna array, effective SIR at the input of multiuser detectors will be increased which will result in less BER. This reduction in interference transforms into increased capacity.

5 Results

A synchronous CDMA system with processing gain 31, Gold Sequences and the rectangular chip waveform is assumed. The uniform linear antenna array, in which the spacing between antenna elements is one-half wavelength, is considered for results. Similarly a circular array of N identical omni directional antenna elements evenly spaced in a circle of radius R= $N\lambda/4\pi$ in the xy plane is taken. This radius is chosen to obtain approximately 0.7λ spacing of the elements, approximately equivalent to that used for linear array.

In Fig. 1, the use of antenna array on BER output is plotted in CDMA detector. It is clear that the use of antenna array (all types) drastically reduces BER which results in increase in the capacity. It is clear that BER output of circular array is less than linear array which is further improved in CCAA. This reduced BER will enhance the capacity in circular antenna array system than linear antenna array system. Therefore, use of circular array offers better SIR performance than linear arrays. Also, circular array at the base station provides more uniform improvement in SIR than linear array as well as additional advantage of less severe mutual coupling effects due to the array curvature.

Fig. 2 shows the variation in BER with SNR of linear detectors i.e. matched filter, decorrelator and MMSE detector for single antenna and linear antenna array with N=8 elements. The results are plotted according to (17), (18) and (19), where SNR is increased N times by using antenna array. In all the relations BER reduces due to increase in SNR. It is to be noted that the antenna array provides excellent improvement in BER for the linear multiuser detectors i.e. decorrelator and MMSE detector, while that of conventional detector the improvement is not much i.e. BER>10^{-2}. This again supports the need of multiuser detection to improve the capacity of system. Further, the performance of decorrelator and MMSE detector is almost same and touches the lower bound when there is no noise. Therefore in presence of noise, performance of conventional detector with single antenna will be much less than with multiple antenna system.

Fig. 3 shows the variation in BER with SNR of linear detectors i.e. matched filter, decorrelator and MMSE detector for single antenna and circular antenna array with N=8 elements. The performance of multiuser detectors with circular antenna array is better than the linear array. The same analysis is done with CCAA in Fig.4 and it is found that the capacity improvement is maximum with CCAA than linear or circular antenna array.

In Fig. 5 and 6, the performance of decorrelator and MMSE detector respectively is analyzed with three different antenna array configurations. The CCAA gives the best performance in any type of antenna arrays.

6 Conclusion

We have discussed a simple scheme to illustrate the increase in capacity of CDMA system employing linear multiuser detector with the use of different array configurations at the base. Linear multiuser detectors reduce MAI, meaning thereby increase in capacity. Use of antenna array further increases SIR, which is exploited to show the reduction in BER or increase in the capacity. It is shown that concentric circular antenna array is the best among all configurations. It gives the maximum capacity with and without considering the MC. In addition to this, it provides uniform coverage across all angles and suitable for narrowband and wideband applications.

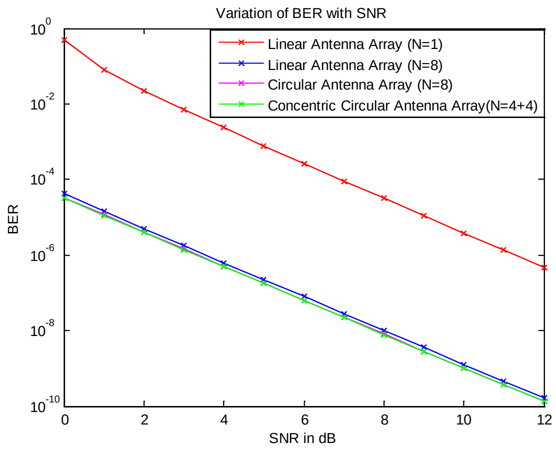

Fig. 1: Variation in BER with SNR for different antenna array configurations

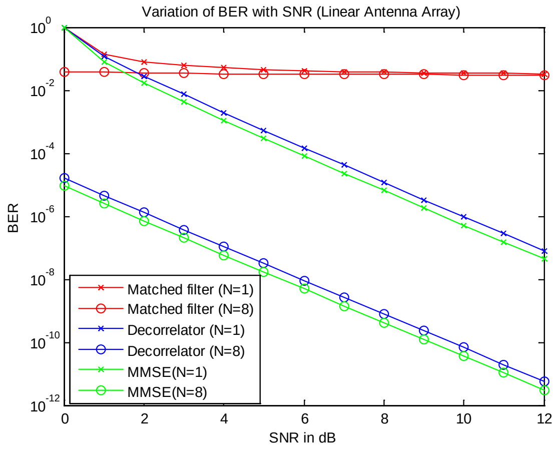

Fig. 2: Performance of Linear Multiuser Detectors with Linear Antenna Array

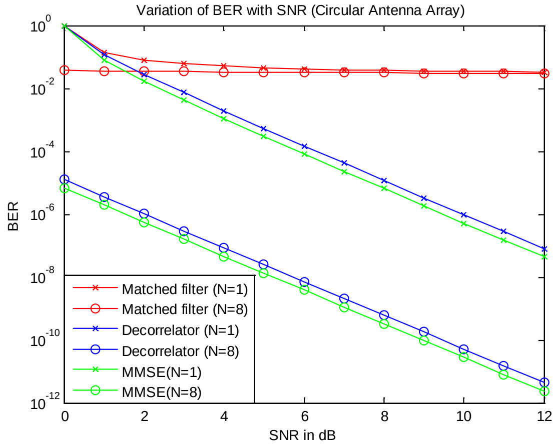

Fig. 3: Performance of Linear Multiuser Detectors with Circular Antenna Array

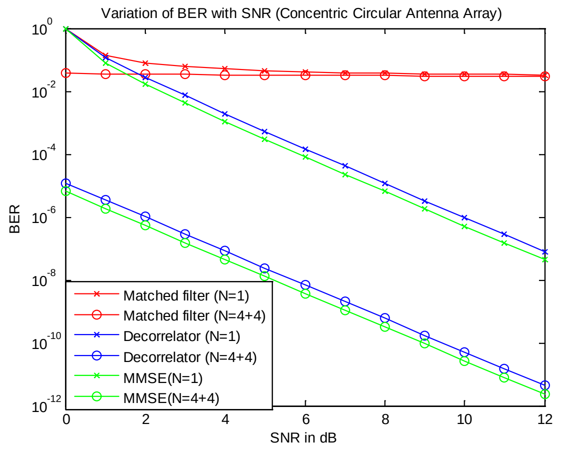

Fig. 4: Performance of Linear Multiuser Detectors with Concentric Circular Antenna Array

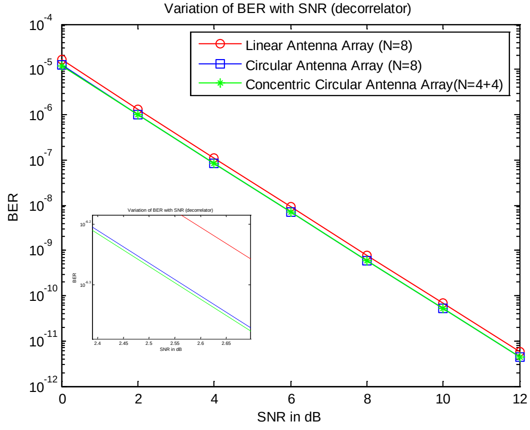

Fig. 5: Performance of Decorrelator with Linear, Circular and Concentric Circular Antenna Array

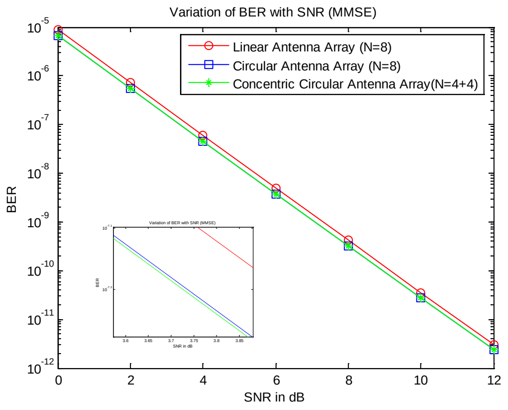

Fig. 6: Performance of MMSE Detector with Linear, Circular and Concentric Circular Antenna Array

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