Accurate Closed-Form Approximations for the BER of Multi-Branch Amplify-and-Forward Cooperative Systems with MRC in Rayleigh Fading Channels

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Abstract: - Relay-based cooperative systems have recently attracted significant attention since they enable exploiting the inherent spatial diversity of wireless networks with single antenna terminals. In this paper, the authors address the error performance of a cooperative diversity network consisting of a source, a destination, and multiple dual-hop amplify-and-forward (AF) relays in Rayleigh fading channels, in which the source broadcasts the signal to the relays in the first time slot and the relays simultaneously forward signals to the destination in the second time slot. Analytically studying the error performance of multiple dual-hop AF cooperative networks with maximal ratio combining (MRC) receivers at the destination and deriving closed-form expressions has always been a difficult task. Considering an L-Relay nodes AF cooperative network in Rayleigh fading channels employing MRC, closed-form approximate expressions are derived for the bit error rate (BER) of a class of coherent modulation techniques that are easy to calculate, thus circumventing the computational inefficiency of the exact formulation. Exact results obtained using numerical integration are provided to validate the tightness of the proposed expressions. In addition, a slight modification for the amplification gain at the relay-node is proposed, which showed an improvement in the effective signal-to-noise ratio at the destination node.

Key-Words: - Cooperative communication, amplify-and-forward, Rayleigh fading, bit error rate, maximal ratio combining, and Prony approximation.

1 Introduction

Cooperative diversity is a promising technology that utilizes unused network resources to ensure high network reliability especially when the direct channel link between the two wireless communication ends is in severe conditions. Moreover, the cooperative wireless communications networks ensure low transmit RF power, which is an important factor in new wireless communication technologies. The basic idea of cooperative diversity can be briefly viewed by having one or more relays distributed in the channel between the two communication ends that can relay the transmitted information bearing signal to the destination side, as can be seen in Fig. 1.

There are mainly two types of cooperative relaying; the first one is called the regenerative cooperative relaying, a.k.a. decode-and-forward (DF), where the relay nodes receive the broadcasted signal in the first time slot, clear it from noise, regenerate it, and then retransmit it towards the destination node during the second time slot. The second type is called the non-regenerative cooperative relaying, a.k.a. amplify-and-forward (AF), where the relay nodes simply receive the broadcasted signal in the first time slot, amplify it, and then retransmit it towards the destination node during the second time slot. One drawback of cooperative communication is that it uses an extra time slot for transmitting a symbol and therefore the system throughput is reduced by half; yet it enhances the system performance in terms of bit error rates and outage probability.

In this paper, we consider the non-regenerative relaying type, in which the relay node just receives, amplifies, and forwards the signal to the destination node. We assume the general case where L-relay nodes are used between the source and the destination to relay the information bearing signal to the destination. At the destination node, the maximal ratio combining (MRC) scheme is assumed to be deployed as the diversity combining scheme to effectively maximize the overall signal-to-noise power ratio (SNR) at the input of the receiver. Rayleigh flat fading is assumed in both the broadcasting stage (source-relay link) and relaying stage (relay-destination link).

A wide range of research reported in literature has considered performance evaluation of cooperative diversity, both the regenerative and the non-regenerative types, under different system models [1]-[17]. Here, we provide a brief summary of what has been reported in the literature recently. Closed-form expressions for the bit error rate (BER) and outage probability were derived in [1] and [2] assuming a single branch multi-hop cooperative network under Rayleigh and Nakagami-*m* fading environments. The BER performance of non-coherent modulation techniques with maximum likelihood (ML) demodulate and forward cooperative communication was studied in [3]. In [4], the log-normal fading model was considered in the performance evaluation of cooperative diversity. In [5], a lower bound for the error rate performance for an amplify-and-forward cooperative network with equal gain combining (EGC) of L-branch dual-hop cooperative network under the Nakagami-m fading channel was derived, in which the BER result is expressed in terms of an integral form that was solved numerically (but no analytically closed-form solution was reported). In [6], the multi-branch amplify-and-forward relay-based cooperative diversity was considered to derive the BER with MRC combiner using the moment generating function (MGF) approach.

However, the BER expression has an integral-form term that was also solved numerically. The BER analysis of amplify-and-forward with L relay nodes cooperative diversity motivated the interest of many researchers since no closed-form expression has been found. In [7], the authors used different techniques to evaluate the BER of amplify-and-forward cooperative diversity, but the result was also given in terms of a cumbersome integral that was solved numerically as well. In [8], the statistical characteristics of the combined SNR were derived considering two branches dual-hop cooperative system without deriving the error rate. The multi-branch multi-hop amplify-and-forward cooperative diversity with variable amplification gain was studied in [9], in which the SNR at the output of the MRC combiner was approximated by a polynomial function that is valid only for a high range of SNR to simplify the BER analysis. The same system model described in [9] was also considered in [10] for a fixed gain amplify-and-forward cooperative system using the same polynomial approximation that is valid only for high SNR. In [11], the authors considered two branches dual-hop cooperative system and derived the optimal weighting vector for the transmitted signal to maximize the resultant SNR and then obtained an expression for the average symbol error rate (ASER), but the obtained ASER expression was also given in an integral-form that was solved numerically without finding a closed-form expression. In [12], the authors considered both amplify-and-forward and decode-and-forward relaying in OFDMA cooperative systems and analyzed the outage information rate and the diversity gain for the interference-limited environment. An optimal power allocation problem with delay constraint on data transmission using a cooperative relay network assuming Rayleigh and Rician fading channel models has been considered in [13] and solved with the objective of minimizing the outage probability.

In [14], the exact symbol error probability (SEP) of cooperative systems using amplify-and-forward relaying and partial relay selection was derived when the different links are independent and non-identically distributed. In [16], closed-form expressions for the bit error rate, the signal-to-noise ratio (SNR) outage probability and average achievable rate were derived for incremental-relaying cooperative-diversity networks over Rayleigh fading channels. In [16], only one dual-hop and the direct link were considered in the analysis. In [17], the authors investigated muti-branch adaptive decode-and-forward (DF) relaying over independent non-identical flat Nakagami-m fading channels and derived approximate expressions for the error and outage probabilities performance.

To the best of the authors' knowledge, there are no closed form expressions exist for the BER of a multi-branch dual-hop amplify-and-forward relay-based cooperative system with MRC combiner at the receiver. In [18], a novel highly accurate approximation for the Gaussian Q-function had been derived, which could be used to simplify the BER analysis in some scenarios of communication systems. In this paper, we show that the results in [18] can be used to greatly simplify the analysis of the BER performance of a multi-branch dual-hop amplify-and-forward relay-based cooperative network with an MRC combiner at the destination side. We derive closed-form expressions for the BER that are approximate, but are in high agreement with the exact results obtained from numerical integration. In addition, we propose a slight modification for the amplification gain at the relay-node showing an improvement in the system performance in the intermediate range and low values of the signal-to-noise ratio. Given this proposed gain modification, we compute the SNR gain as well as the amount of fading.

We outline the remainder of this paper as follows. The system model under consideration is introduced in Section 2. The preliminary mathematical analysis that is required in the performance evaluation of the system under study is presented in Section 3. Performance evaluation using different metrics for L-relay nodes amplify-and-forward Cooperative network with the MRC combiner is considered in Section 4. The system performance metrics considered are the BER evaluation for both coherent and non-coherent modulation techniques, and with the proposed relay gain we consider the SNR gain and the amount of fading metrics as well. Numerical results and discussions are provided in Section 5. Finally, some conclusions are drawn in Section 6.

2 System Model

The basic idea of cooperative diversity can be briefly viewed as having one or more relay nodes distributed in the channel between the two communication ends that can relay the transmitted signal to the destination. Fig. 1 depicts the cooperative diversity wireless network where S denotes the source node, D denotes the destination node, and R_l denotes the l^{th} relay node. In this cooperative system, we assume that the channel gain random variable between the source and the l^{th} relay, denoted by α_{Sl}, is independent from any other channel link between the source and any other relay, and a similar assumption is considered for the l^{th} channel gain random variable for the links between the relays and the destination, denoted by α_{Rl}. These are valid assumptions since the relay nodes' locations are assumed to be spaced far enough apart to ensure zero correlation between the links. Another independency assumption is also made between the broadcasting channel links (source-to-relays links) and the forward channel links (relays-to-destination links). This assumption is also valid since the two transmissions take place in different time slots.

The channel gain random variables for all the links in the two transmission hops (broadcasting and relaying) are assumed to be flat and follow Rayleigh distribution. With the wireless communication system shown in Fig. 1, the source is communicating with the destination through the relays. This can be done by broadcasting the transmitted signal by the source, and the relays pick this signal and transmit it to the destination. There are two different scenarios for the relay transmissions. The first scenario is called the regenerative transmission, in which the relay receives the signal broadcasted by the source, recovers the information bearing signal, and then retransmits it once again toward the destination. Whereas the second scenario is called the non-regenerative transmission, in which the relay receives the signal, amplifies it, and forwards it toward the destination without recovering the original data. In this paper, we consider the non-regenerative relays which just receive, amplify, and forward the data to the destination. This type of relaying is practically preferable since the relay complexity is a major issue especially in cellular systems. For instance, in cellular systems, the relay nodes can be simply other user equipments (UEs) that are neighbors to the source UE that would like to communicate with a base station or another remote UE. These neighboring UEs are assumed to experience good channel conditions in both the broadcasting and relaying stages or even with other relays if multiple hops are required to achieve better transmissions. Moreoever, these relays or UEs are assumed not to be using their resources at the time of transmitting, i.e., these relays or UEs are in idle states or another scenario these relaying UEs could divide their spectrum bandwidth into two portions; one used for transmitting its data and another for cooperation. This latter scenario can be used mutually among all users, which can be called as mutual cooperation (see e.g., [19]).

Now, assuming the source broadcasts a signal $s(t)$ with normalized power, then the received signal at the l^{th} relay node can be given as

$$r_{Rl}(t) = \alpha_{Sl} s(t) + n_{Sl}(t), \tag{1}$$

where $n_{Sl}(t)$ is the zero mean additive white Gaussian noise with a one-sided power spectral density of N_o associated with an l^{th} broadcasting link. For non-regenerative relaying, after amplification the relay forwards the received signal directly to the destination, assuming dual-hop relaying. Let the amplification gain associated with the l^{th} relay node be denoted by G_l, then the signal at the destination received from the l^{th} relay node can be written as

$$r_{D,l}(t) = \alpha_{Rl} G_l \left[\alpha_{Sl} s(t) + n_{Sl}(t)\right] + n_{Rl}(t), \tag{2}$$

and hence the equivalent SNR on the l^{th} link at the destination can be obtained as

$$\gamma_{eq,l} = \frac{\left(\alpha_{Rl} G_l \alpha_{Sl}\right)^2}{\left(\alpha_{Rl} G_l\right)^2 N_o + N_o}$$

$$= \frac{\frac{\alpha^2_{Rl}\alpha^2_{Sl}}{N_o\ N_o}}{\frac{\alpha^2_{Rl}}{N_o} + \frac{1}{G_l^2 N_o}} \cdot \tag{3}$$

One choice for the relay gain was considered in [1] to be $G^2 = \frac{1}{\alpha^2}$, which results in a new form for the equivalent SNR, $\gamma_{eq,l}$, that is more mathematically tractable in terms of finding the statistics associated with it (see e.g., expressions for PDF, CDF, and MGF derived in [1]). The simplified analysis presented in [1] was shown to lead to tight lower bound results for the average BER and outage probability performance metrics.

In this paper, we propose a slight modification for the relay node gain presented in [1] by introducing a weighting amplification factor, A, as follows:

$$G_l^2 = \frac{A^2}{\alpha^2_{Sl}} \cdot \tag{4}$$

where $A \geq 1$. The gain in (4) reduces to [1, Eq. (7)] for $A = 1$, as a special case. Then the equivalent SNR, from the source to the destination on the l^{th} link, becomes

$$\gamma_{eq,l} = A^2 \frac{\gamma_{Sl}\gamma_{Rl}}{\gamma_{Sl} + A^2\gamma_{Rl}}, \tag{5}$$

where $\gamma_{Sl} = \frac{|\alpha_{Sl}|^2}{N_o}$ and $\gamma_{Rl} = \frac{|\alpha_{Rl}|^2}{N_o}$ are the SNRs for the broadcasting and the relaying channel links, respectively. It is noteworthy that the equivalent SNR, $\gamma_{eq,l}$, given in (3) is simply a scaled version of $\mu_H(\gamma_{Sl}, \gamma_{Rl})$, where $\mu_H(x_1, x_2)$ is the harmonic mean function defined in [1, Eq. (8)].

In our model we consider that multiple relay nodes exist between the source and destination as shown in Fig. 1, where the destination node receives L signals. The overall received signal can then be written as

$$r_D(t) = \sum_{l=1}^{L} \alpha_{Rl} G_l \left[\alpha_{Sl} s(t) + n_{Sl}(t)\right] + n_{Rl}(t). \tag{6}$$

Assuming that MRC combiner is used at the receiver, then the combined received SNR can be given as [20]

$$\gamma_D = \sum_{l=1}^{L} \gamma_{eq,l} = \sum_{l=1}^{L} A^2 \frac{\gamma_{Sl}\gamma_{Rl}}{\gamma_{Sl} + A^2\gamma_{Rl}} \cdot \tag{7}$$

The probability density function (PDF) for each of the end-to-end l^{th} cooperative paths (the link from source to destination through the l^{th} relay node) can be obtained, with the help of [1, 1], as

$$f_{\gamma_{eq,l}}(\gamma) = \frac{2\gamma}{\overline{\gamma}_R\overline{\gamma}_S} e^{-\gamma\left(\frac{1}{\overline{\gamma}_R} + \frac{1}{\overline{\gamma}_S}\right)} \left[\left(\frac{\overline{\gamma}_R + \overline{\gamma}_S}{\sqrt{\overline{\gamma}_R\overline{\gamma}_S}}\right) K_1\left(\frac{2\gamma}{\sqrt{\overline{\gamma}_R\overline{\gamma}_S}}\right) + 2K_0\left(\frac{2\gamma}{\sqrt{\overline{\gamma}_R\overline{\gamma}_S}}\right)\right], \tag{8}$$

where $K_i(x)$ is the the i^{th} order modified Bessel function of the second kind, $\overline{\gamma}_S$ and $\overline{\gamma}_R$ are the average SNR in the broadcasting and the relaying stages, respectively.

3 Maximal Ratio Combining Reception (MRC)

In the model under study, we assume that multiple relays exist between the source and destination, as shown in Fig. 1 and that an MRC scheme is implemented on the L-relayed signals received at the destination node. As depicted in Fig. 1, the direct path between the source and the destination nodes is assumed to experience sever channel conditions, and hence, the signal on that path is negligible and will be ignored in the subsequent analysis. In MRC scheme, the destination node is assumed to have a complete knowledge of the channel state information (CSI) in order to combine all relayed paths coherently. It is well-known that if the channel gain random variable follows the Rayleigh distribution, then the SNR distribution will result in an Exponential distribution. As a result, the cooperative network with Rayleigh fading leads to having an equivalent SNR in the two hops (broadcasting and relaying), $\gamma_{eq,l}$, that follows the harmonic mean distribution of two Exponential random variables. Now, let's suppose the general case when the SNRs of the broadcasting stage and relaying stage satisfy $\overline{\gamma}_{Sl} = C\overline{\gamma}_{Rl} = C\overline{\gamma}$, where C is any positive number (both integers and non-integers) that includes both the balanced $(C = 1)$ and unbalanced channel links, then the PDF in (??) becomes

$$f_{\gamma_{eq,l}}(\gamma) = \left[\frac{A^2+C}{A\sqrt{C}} K_1\left(\frac{2\gamma}{A\sqrt{C}\overline{\gamma}}\right) + 2K_0\left(\frac{2\gamma}{A\sqrt{C}\overline{\gamma}}\right)\right] \frac{2\gamma}{A^2 C\overline{\gamma}^2} e^{-\frac{\gamma}{\overline{\gamma}}\left(\frac{A^2+C}{A^2 C}\right)}. \quad (9)$$

The expression in (9) is general (covers all cases of balanced and unbalanced channel links) and will be used in the subsequent BER analysis. The BER for an L-relay nodes cooperative communication system with MRC combining scheme can be found as

$$P_b = \underbrace{\int_0^\infty \cdots \int_0^\infty}_{L-foldintegral} P_b\left(e|\gamma_{eq,1}, \cdots, \gamma_{eq,L}\right) f_{\gamma_D}\left(\gamma_{eq,1}, \cdots \gamma_{eq,L}\right) d\gamma_{eq,1} \cdots d\gamma_{eq,L} \quad (10)$$

where $P_b\left(e|\gamma_{eq,1}, \cdots, \gamma_{eq,L}\right)$ is the conditional error probability in the AWGN channel, and $f_{\gamma_D}\left(\gamma_{eq,1}, \cdots \gamma_{eq,L}\right)$ is the joint PDF of the SNRs of the L complete branches. The joint PDF for the L cooperative complete paths is not easily determined unless the independency assumption between the cooperative links is adopted, which is indeed a valid assumption in cooperative communication systems since the relay nodes are assumed to be located far enough from each other which ensures independency. The other part of the integrand in (10), the conditional probability in the AWGN channel, is usually represented in terms of the Gaussian Q-function yielding an integral that can not be solved without considering some approximation assumptions. In the following we review the Prony approximation for the Gaussian Q −function that has been recently reported in [18], which will be used in the rest of the analysis in this paper to show that it leads to accurate BER expressions for the model under consideration.

3.1 The Q-Function Approximation

We present the Prony approximations for the Q-function that we will use to simplify the analysis of the system performance. The Prony approximation for the Gaussian Q −function is given as [18, Eq. (9)]

$$Q(x) \cong \sum_{i=1}^{p} \tilde{B}_i e^{-\tilde{b}_i x^q}, \quad (11)$$

where the parameters $\tilde{B}_i$, $\tilde{b}_i$, p, and q are non-negative real numbers. The q value is chosen in such a way that the resultant series can be further useful in subsequent analysis with a very good agreement with the exact Q-function. The values of both $\tilde{B}_i$ and $\tilde{b}_i$ are determined according to the Prony approximation algorithm to minimize the absolute error defined in [18, Eq. (10)]. It is worth mentioning here that the Exponential approximation for the Gaussian Q-function developed in [21, Eq. (14)] can be considered as a special case of the Prony approximation, for $p = 2$ and $q = 2$.

4 Performance Evaluation

4.1 Coherent Modulation Techniques

Assume a coherent modulation technique is used for the transmission, i.e.,

$P_b\left(e|\gamma_{eq,l}\right) = \psi_1 Q\left(\sqrt{2\psi_2\gamma_{eq,l}}\right)$, where ψ_1 and ψ_2 depend on the type of modulation used; e.g., for BPSK $(\psi_2 = 1)$, for BFSK $(\psi_2 = \frac{1}{2})$, and for minimum shift keying (MSK) $(\psi_2 = 0.715)$ and $\psi_1 = 1$ for all binary coherent modulation techniques [22]. In order to obtain a closed-form expression for the BER of the system model under consideration, we use the Prony approximation for the Gaussian Q −function given in (10) to simplify the analysis. Then, the conditional probability of error in the AWGN channel can be expressed as follows:

$$P_b(e|\gamma_D) = P_b(e|\gamma_{eq,1}, \gamma_{eq,2}, \cdots, \gamma_{eq,L})$$
$$= \psi_1 Q\left(\sqrt{2\psi_2 A^2 \sum_{l=1}^{L} \frac{\gamma_{Sl}\gamma_{Rl}}{\gamma_{Sl}+A^2\gamma_{Rl}}}\right) =$$
$$\psi_1 \prod_{k_1,\cdots,k_L} \sum_{i=1}^{p} \tilde{B}_i exp\left[-\tilde{b}_i(2\psi_2)^{\frac{q}{2}}\binom{\frac{q}{2}}{k_1,\cdots,k_L}\prod_{l=1}^{L}\gamma_l^{k_l}\right]. \quad (12)$$

where the multinomial expansion has been used and k_1, ⋯, k_L are non-negative integers such that the $\sum_{i=1}^{L} k_i = q/2$ [23]. The derivation of (11) and definition of γ_l are given in the Appendix of this paper (including the definition of the multinomial coefficients as well).

In the following we use (11) and (12) for evaluating the average BER assuming Rayleigh fading channels. Thus, the probability of error for the system model under consideration can be evaluated as

$$P_b = \underbrace{\int_0^\infty \cdots \int_0^\infty}_{L-fold} P_b(e|\gamma_{eq,1}, \cdots, \gamma_{eq,L})$$
$$\times f_{\gamma_D}(\gamma_{eq,1}, \cdots \gamma_{eq,L}) d\gamma_{eq,1} \cdots d\gamma_{eq,L} =$$
$$\psi_1 \prod_{k_1,\cdots,k_L} \sum_{i=1}^{p} \tilde{B}_i \underbrace{\int_0^\infty \cdots \int_0^\infty}_{L-fold} f_{\gamma_D}(\gamma_{eq,1}, \cdots \gamma_{eq,L})$$
$$\times exp\left[-\tilde{b}_i(2\psi_2)^{\frac{q}{2}} \sum_{k_1,k_2,\ldots,k_L}\binom{\frac{q}{2}}{k_1,\cdots,k_L}\prod_{l=1}^{L}\gamma_l^{k_l}\right]$$
$$\times d\gamma_{eq,1} \cdots d\gamma_{eq,L}. \quad (13)$$

The expression in (13) is a cumbersome L −fold integral that depends strongly on the approximation parameters, especially the value of q. This integral can not be solved for a general value of q. However, for $q = 2$ (which is a valid value for Prony approximating, see [18, Eq. (13c) and (13d)]), the integral in (13) becomes separable and can be solved by restating it into the following form

$$P_b = -\psi_1 \sum_{i=1}^{p} \tilde{B}_i \left\{\int_0^\infty e^{-2\tilde{b}_i\psi_2\gamma_l} f_{\gamma_{eq,1}}(\gamma_{eq,l}) d\gamma_{eq,l}\right\}^L \quad (14)$$

In [18], two sets of Prony parameters were found that approximate the Gaussian Q-function very well in the range of $\gamma \in [-5,15]$ dB. The first set is associated with $p = 2$ and the other one with $p = 3$. Now, substituting (9) in (14), the resultant integral can be evaluated with the help of [24, Eq. (6.621.3)] as

$$P_b = \psi_1 \sum_{i=1}^{p} \tilde{B}_i \left(\frac{16}{3A^2C\overline{\gamma}^2(\mu_i+\zeta)^2}\right)^L \times$$
$$\left[{}_2F_1\left(2,\frac{1}{2};\frac{5}{2};\frac{\mu_i-\zeta}{\mu_i+\zeta}\right) + \frac{4(A^2+C)}{A^2C\overline{\gamma}(\mu_i+\zeta)}\, {}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{\mu_i-\zeta}{\mu_i+\zeta}\right)\right]^L \quad (15)$$

where $\mu_i = \frac{2\tilde{b}_i\psi_2A^2C\overline{\gamma}+A^2+C}{A^2C\overline{\gamma}}$ and $\zeta = \frac{2}{A\sqrt{C\overline{\gamma}}}$, and ${}_2F_1(\alpha,\beta;\gamma,x)$ is the generalized Gauss hypergeometric function defined in [24, Section 9.10], which can be found in an integral-form as well as series-form that converges very fast and it is available in several software packages as a built-in function. Using the values of $\tilde{A}_1 = 0.208$, $\tilde{A}_2 = 0.147$, $\tilde{a}_1 = 0.971$, and $\tilde{a}_2 = 0.525$ (associated with $q = 2$, $p = 2$, and $q_1 = 1$) from Ref. [18], the BER of different modulation techniques can be determined using the expression in (14). Notice, the modulation technique is fully determined by the values of ψ_1 and ψ_2 not $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{A}_1$, or $\tilde{A}_2$.

4.2 Non-Coherent Modulation Techniques

The bit error rate for non-coherent modulation techniques can be evaluated using the same approach as before, but without the need for using approximation since the AWGN error rate for such modulation techniques is generally given as $P_b(e|\gamma) = \beta_2 e^{-\alpha\gamma}$, where α and β_2 depend on the type of the non-coherent detection. For instance, for non-coherent binary frequency shift keying (BFSK), both of the parameters β_2 and α share the value of $\frac{1}{2}$; whereas for non-coherent binary differential phase shift keying (BDPSK), the parameters are $\frac{1}{2}$ and 1 for β_2 and α, respectively. Using this general AWGN BER formula, the error rate considering our system model with MRC can be obtained as

$$P_b = \beta_2\left\{\int_0^\infty e^{-\alpha\gamma_{eq,l}} f_\Gamma(\gamma_{eq,l}) d\gamma_{eq,l}\right\}^L$$

$$= \beta \left(\frac{16}{3A^2\overline{\gamma}^2(\eta+\zeta)^2} \right)^L \left[{}_2F_1\left(2, \frac{1}{2}; \frac{5}{2}; \frac{\eta-\zeta}{\eta+\zeta}\right) + \frac{4(A^2+1)}{A^2\overline{\gamma}(\eta+\zeta)} \, {}_2F_1\left(3, \frac{3}{2}; \frac{5}{2}; \frac{\eta-\zeta}{\eta+\zeta}\right) \right]^L, \quad (16)$$

where $\eta = \frac{\alpha A^2\overline{\gamma}+A^2+1}{A^2\overline{\gamma}}$ and ζ is as defined earlier.

4.3 SNR Gain

It is well known that the SNR gain for the MRC diversity technique in non-cooperative communications equals to the number of branches (L). With the cooperative diversity, the SNR gain can be obtained as follows:

$$\overline{\gamma}^{MRC} = \sum_{l=1}^{L} \overline{\gamma}_{eq,l}$$

$$= \sum_{l=1}^{L} \frac{A^2\overline{\gamma}_R \overline{\gamma}_S}{A^2\overline{\gamma}_R + \overline{\gamma}_S} \cdot \quad (17)$$

For i.i.d. branches, i.e., $\overline{\gamma}_R = \overline{\gamma}_S = \overline{\gamma}$, the equivalent combined SNR becomes

$$\overline{\gamma}_{ModGain}^{MRC} = L \frac{A^2}{A^2+1} \overline{\gamma} \quad (18)$$

Clearly, the SNR gain with gain modification, A, defined in Section 2 is much better than that when $A = 1$ as in [1, Eq. (7)].

4.4 Amount of Fading in Multi-Branch Cooperative Network with MRC Scheme

The MGF of the harmonic mean distribution of two Exponential random variables is given as [1]:

$$\mathcal{M}_{\gamma_l}(s) = {}_2F_1\left(1,2; \frac{3}{2}; \frac{-s\overline{\gamma}}{2}\right) \cdot \quad (19)$$

Then, the combined SNR at the output of the MRC combiner can be given as

$$\mathcal{M}_{\gamma}^{MRC}(s) = \prod_{l=1}^{L} \mathcal{M}_{\gamma_l}(s)$$

$$= \left\{ {}_2F_1\left(1,2; \frac{3}{2}; \frac{-s\overline{\gamma}}{2}\right) \right\}^L \cdot \quad (20)$$

The amount of fading is defined as [25]

$$\mathcal{AF} = \frac{\mathbb{E}[\gamma^2] - \mathbb{E}^2[\gamma]}{\mathbb{E}^2[\gamma]} \cdot \quad (21)$$

The n^{th} moments can be found by taking the n^{th} derivative of the MGF with respect to s. Consequently, the first moment can be obtained as follows:

$$\mathbb{E}[\gamma] = \frac{d}{ds} \mathcal{M}_{\gamma}^{MRC}(s)|_{s=0}$$

$$= \frac{2}{3} L\gamma \left\{ {}_2F_1\left(1,2; \frac{3}{2}; \frac{-s\overline{\gamma}}{2}\right) \right\}^{L-1} \; {}_2F_1\left(2,3; \frac{5}{2}; \frac{-s\overline{\gamma}}{2}\right)|_{s=0}$$

$$= \frac{2\overline{\gamma}}{3} L, \quad (22)$$

where [23, Eq. (15.2.1)] has been used. Similarly, the second moment can be found as

$$\mathbb{E}[\gamma^2] = \frac{d^2}{ds^2} \mathcal{M}_{\gamma}^{MRC}|_{s=0}$$

$$= \frac{4L(5L+4)}{45} \overline{\gamma}^2, \quad (23)$$

and substituting (22) and (23) in (21) results in

$$\mathcal{AF}^{MRC} = \frac{4}{5L} \cdot \quad (24)$$

From the result in (24), as expected, it is clearly noticed that the $\mathcal{AF}$ of the Rayleigh channel in amplify-and-forward cooperative communication with MRC combining is less than the $\mathcal{AF}$ of the Rayleigh channel ($\mathcal{AF}_{Ray.} = 1$) in non-cooperative communication.

5 Numerical Results and Discussion

In this section, we provide numerical results for the expressions derived in this paper for different modulation schemes. Fig. 2 provides the BER versus the average SNR per bit per branch using coherent BPSK modulation for different number of relay nodes, L, in an amplify-and-forward cooperative communication system over flat Rayleigh fading channels employing MRC diversity combiner at the destination node computed using using the expression derived in (14). In this same figure exact results using numerical integration are also shown as a reference to show the accuracy of the derived closed-form expression in (14). It is seen from the figure that results obtained using the new BER expression in (14) provide good agreement with exact results. Fig. 3 shows BER performance results similar to those in Fig. 2 but for different values of the amplification gain factor, A. From the figure it is clearly observed that the error performance over the intermediate range and low values of the SNR increases as the gain factor A increases. This performance gain is more noticeable for larger values of the L (number of relays). Error performance results for different coherent modulation schemes, also using (14), are presented in Fig. 4 for different number of relays, L (assuming relay amplification gain $A = 2$). Finally, the SNR improvement versus L is presented in Fig. 5. It is clear from the figure that the SNR improvement is of noticeable amount when A is between 1.5 and 3 and slightly noticed when A is 4. In fact this improvement is becoming negligible when A is large, which is in agreement with the formula derived in (17).

6 Conclusion

In this paper, closed-form integral-free approximate expressions were derived for the BER of a class of coherent modulation schemes in multi-branch amplify-and-forward cooperative communication systems employing the MRC in Rayleigh fading channels. A gain modification at the relay node has also been proposed and the performance of an L-relay nodes cooperative communication system employing the MRC combiner and with proposed gain modification has been investigated. Using numerical results, the derived BER expressions for the prescribed system model has shown good agreement with the the exact ones obtained using numerical integration.

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Appindix

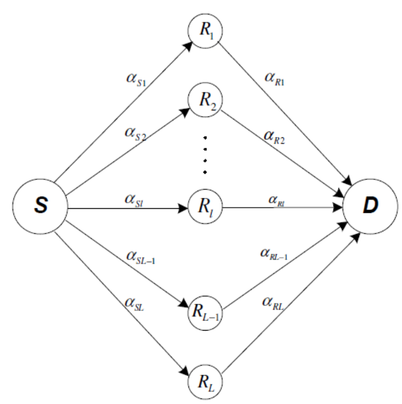

Fig. 1: Illustration of a cooperative diversity wireless network with L-relay nodes

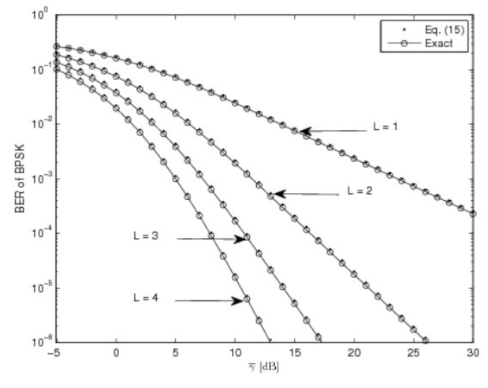

Fig.2: Comparison between the BER curves of the BPSK modulation technique using (15) and the exact BER curves (obtained by numerical integrations) versus the average SNR, $\overline{\gamma}$, for different number of relay nodes, L, in an amplify-and-forward cooperative communication system over Rayleigh fading channel model employing MRC diversity combiner at the destination node

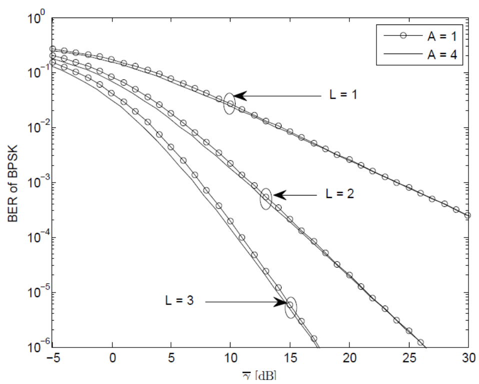

Fig. 3: BER of BPSK modulation technique versus the average SNR, $\overline{\gamma}$, considering two different values of the amplification gain factor, namely, A = 1 and A = 4, and for different number of relay nodes, L, in an amplify-and-forward cooperative network over Rayleigh fading channel model employing MRC diversity combiner at the destination node.

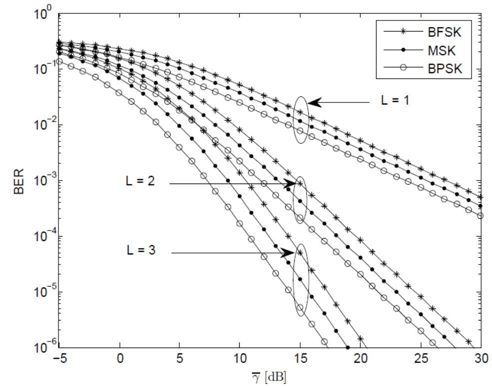

Fig.4: BER for different coherent modulation techniques versus the average SNR, $\overline{\gamma}$, for different number of relay nodes, L, in an amplify-and-forward cooperative network over Rayleigh fading channel model employing MRC diversity combiner at the destination node and for amplification gain factor of A = 2

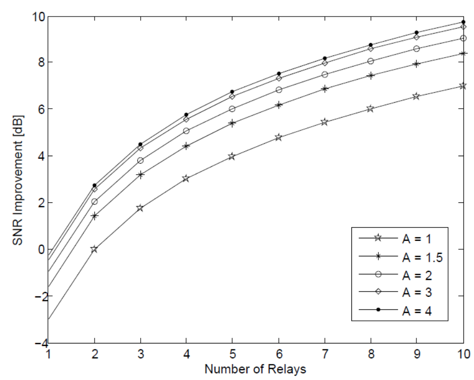

Fig. 5: Average SNR Improvement versus the number of relay nodes, L, for different values of the amplification gain factor, A, in an amplify-and-forward cooperative communication network employing MRC diversity combining scheme.