# Efficient Algorithms for Noise Estimation in Electrical Power Line Communications

ABDULLAH ALWADIE
Department of Electrical Engineering
Najran University,
P. O. Box 1988, Najran,
SAUDI ARABIA
E-mail: alwadie@hotmail.com

*Abstract:* - Power Line Communication (PLC) has received much attention due to the wide connectivity and availability of power lines. Effective PLC must overcome the harsh and noisy environments inherent in PLC channels. Noise in power lines is modeled as a cyclostationary Gaussian process. In order to achieve reliable communication using power lines, effective measures including error control techniques need to be taken against this particular noise. Low-Density Parity-Check (LDPC) codes have excellent performance in power lines. This paper presents two new iterative algorithms for noise estimation on power lines based on Higher-order statistics and the Maximum-Likelihood (ML) estimation principle, respectively. The algorithm based on Higher-order statistics uses second, fourth, and sixth moments of the received noisy signal to provide a signal-to-noise ratio (SNR) estimate. For the ML estimation algorithm, a derivation of the ML estimate of the amplitude of a Binary Phase-Shift Keying (BPSK) modulated signal is presented. Then, the proposed iterative search algorithm is developed. The proposed algorithms are especially favorable in cases of low SNR values, e.g., the ML estimation algorithm can achieve as large as 7.5 dB and 11.7 dB gains over conventional estimators at an SNR of -5 dB and -10 dB, respectively. Furthermore, since accurate SNR estimation is required for "good" (in terms of bit-error rate (BER)) decoding performance of LDPC codes, the performance of the proposed schemes is compared to some of the previously suggested SNR estimation algorithms. Finally, simulation results show that the proposed estimators perform close-to-optimum at a significantly lower computational complexity.

*Key-Words:* - Power Line Communications (PLC), Low-Density Parity-Check (LDPC) Codes, signal-to-noise ratio (SNR) estimation.

## 1 Introduction

PLC [1−4] has recently been a subject for an important research work. The motivation behind exploiting the power grid for providing high speed multimedia communications reside in the vast infrastructure in place for power distribution, and the penetration of the service could be much higher than any other wireline alternative. Moreover, since devices that access the Internet are normally connected to an electrical outlet, the unification of the two networks seems a compelling option. Thus, PLC provides a convenient and cost-effective solution for data transmission. However, like many other technologies, PLC faces its own challenges.

The power distribution network has not been designed for communication purposes and does not present a favorable transmission medium. Unlike many other communication channels, power-lines do not represent an additive white Gaussian noise (AWGN) environment. Noise in power-lines has the cyclic nature for power voltage [5, 6]. Such cyclic nature of power line noise is called "*cyclo-stationary*" [6]. That is, a *periodically* time-varying behavior, where the frequency of the variation is typically twice the mains frequency (50 or 60 Hz). To cope with the impairments of such a *horrible* channel [7], PLC systems have to apply robust and efficient modulation and coding schemes.

To achieve reliable communications using power lines, it is natural to consider adopting an error control technique. LDPC Codes [5−7] are capable of operating near Shannon capacity on an AWGN channel by iterative decoding and, therefore, have enough capability to be candidates. The authors of [8] have shown that LDPC codes can perform better than Reed-Solomon or convolutional codes on PLC channel. In [12], it was found that the performance of LDPC codes is superior to that of the Turbo codes [10] under a cyclostationary Gaussian noise environment. The iterative decoders utilize "channel information" in their decoding process. The channel information is unchanged if the noise process is stationary, such as AWGN. However, the channel information on the power lines periodically changes because the noise process is modeled as cyclostationary process. Thus, one has to trace the change of the channel information and get accurate signal to noise ratio (SNR) estimates from the power lines in order to achieve a high decoding performance [11–13]. This paper shows an effect of LDPC codes on PLC in presence of noise.

The paper is organized as follows. Section 2 below describes PLC channel noise and its classification. LDPC codes and the encoding process are presented in section 3, for which the Sum-Product Decoding algorithm is summarized in section 4. Sections 5 and 6 investigate in detail the two new SNR estimation algorithms. Section 7 provides simulation results on the proposed methods. Finally, section 8 concludes the paper summarizing the main findings and results.

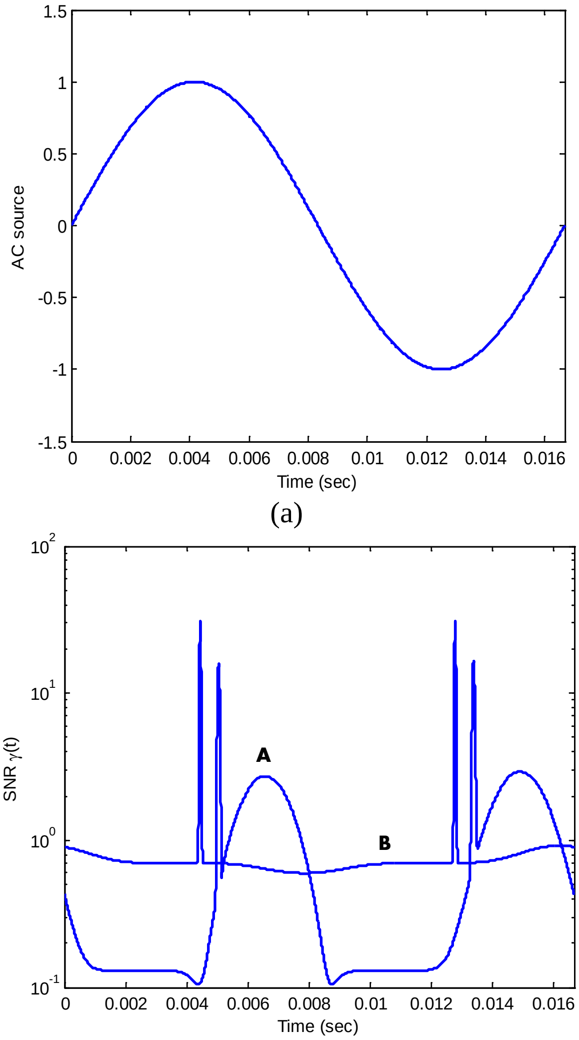
## 2 PLC Channel Noise

Noise in the PLC channel can be modeled as a cyclostationary process and its amplitude distribution at the same phase as the AC source can be modeled as Gaussian distribution. The variance is assumed to be the sum of three types of noise and it can be expressed as follows [12],

$$\sigma_{PL}^2 = \sum_{i=1}^{3} A_i |\sin(2\pi f t + \theta_i)|^{n_i}, \qquad (1)$$

where $f = 1/\mathrm{T}$ is the frequency of the AC voltage, typically 50 or 60 Hz. $A_i$ is the parameter of amplitude, $\theta_i$ is the parameter of phase and $n_i$ is the parameter for degree of impulsiveness. Equation (1) comprises three types of noise; stationary noise ($i = 1$), cyclical continuous noise ($i = 2$), and cyclical impulsive noise ($i = 3$). The first category is a time invariant noise component and parameters $\theta_1$ and $n_1$ have no meaning. The second and third categories are the periodic noise components. Table 1 has two examples of these parameters and Fig. 1 shows the behavior of these variances.

In this paper, two simple algorithms for the estimation of the variances are presented in sections 5 and 6 below. The authors of [11–13] asserted that accurate signal to noise ratio (SNR) estimates from the power lines are required in order to achieve a high decoding performance, however, they did not present any algorithm for such problem. The authors of [21] presented, among others, the SNV RXDA estimation method. But, it has a significant bias when the SNR is low. The author here develops iterative search algorithms to find an accurate SNR estimate in a few steps. Besides, a closed form solution based on statistical moments is also presented.




(a) (b)

**Fig. 1** (a) AC waveform and, (b) the variances of environments A and B given in Table 1.

| Parameter | $A_1$ | $A_2$ | $n_2$ | $\theta_2$ | $A_3$ | $n_3$ | $\theta_3$ |
|---|---|---|---|---|---|---|---|
| Environment A | 0.13 | 2.8 | 9.3 | $128^\circ$ | 16 | 5.3 | $161^\circ$ |
| Environment B | 0.70 | 0.22 | 5.3 | $100^\circ$ | 30 | 1.5 | $174^\circ$ |

**Table 1.** Examples of set of the parameters identifying the power lines noise.

# 3 LDPC Codes and Encoding

LDPC codes, discovered by Gallager in 1962 [5], were recently rediscovered and shown to form a class of Shannon-limit-approaching codes [6, 7], [14–16]. An LDPC code is specified by a sparse parity-check matrix **H**. In particular, an LDPC code is denoted $(n, t_c, t_r)$, where $n$ denotes the block length, $t_c$ denotes column weight of matrix **H**, and $t_r$ denotes row weight with $t_r > t_c$. The rate of such code is $(1 - t_c/t_r)$.

Further, if $k$ denotes the information sequence, and the dimension $m = n - k$, then, the generator matrix $\mathbf{G}_{k\times n}$ of the code can be found by performing Gauss-Jordan elimination on **H** to obtain it in the form,

$$\mathbf{H} = [\mathbf{I}_{n-k}, \mathbf{P}^{\mathbf{T}}], \tag{2}$$

where **P** is a $k \times (n-k)$ binary matrix and $\mathbf{I}_{n-k}$ is the $(n-k)\times(n-k)$ identity matrix. The generator matrix is then,

$$\mathbf{G} = [\mathbf{P}, \mathbf{I}_k]. \tag{3}$$

The row space of *G* is orthogonal to *H*. Thus, if *G* is the generator matrix for a code with parity-check matrix *H* then,

$$\mathbf{G}\cdot\mathbf{H}^{\mathrm{T}} = \mathbf{0}. \tag{4}$$

If **H** is rank deficient, $r = \text{rank}(\mathbf{H}) < m$, then the linear dependent rows of **H** should be truncated and the corresponding code has rate, $R = k/n > (n - m)/n$, so it is a higher rate code than the dimensions of **H** would suggest. It should be noted here that the codes generated by our algorithm (described in section 4 below) have full-rank parity-check matrices.

LDPC codes are well represented by bipartite graphs [6] in which a set of nodes, the variable nodes, corresponds to elements of the codeword and the other set of nodes, the check nodes, corresponds to the set of parity-check constraints, which define the code. Regular LDPC codes are those, for which all nodes of the same type have the same degree. Irregular LDPC codes were studied in [15], [17], and [18].

# 4 Sum-Product Decoding

LDPC codes can be specified by a sparse $\mathrm{M}\times\mathrm{N}$ parity-check matrix (here $\mathrm{M}, \mathrm{N}$ denote dimensions, but *m*, *n* denote indices) $\mathbf{H} = [\mathbf{H}_{\mathrm{M}\times\mathrm{N}}]$. Each row of **H** is referred to as a check. The set of bits participating in check $m$ is denoted by $\mathbf{N}(m) = \{n: \mathbf{H}_{mn} = 1\}$. Similarly, the set of checks in which bit n participates is denoted by $\mathbf{M}(n) = \{m: \mathbf{H}_{mn} = 1\}$. The sum-product algorithm (SPA) with respect to Log-likelihood-Ratio (LLR) is used for the decoding of LDPC codes. The algorithm using LLR consists of the following steps [5] [19] [20]:

**Step 1) Initialization:** For each $(m, n)$ satisfying $\mathbf{H}_{mn} = 1$, set $\boldsymbol{\beta}_{mn} = 0$.

**Step 2) Processing in check nodes:** For each $(m, n)$ satisfying $\mathbf{H}_{mn} = 1$, calculate

$$\begin{aligned}\boldsymbol{\alpha}_{mn} = &\prod_{n'\in N(m)\setminus n} \text{sgn}(\boldsymbol{\lambda}_{n'} + \boldsymbol{\beta}_{mn'}) \\ &\times f[\sum_{n'\in N(m)\setminus n} f(|\boldsymbol{\lambda}_{n'} + \boldsymbol{\beta}_{mn'}|)]\end{aligned} \tag{5}$$

where $f(x) = \ln((e^x + 1)/(e^x - 1))$. $\lambda_n$ is the LLR of bit $n$, i.e.,

$$\lambda_n = \ln \frac{P(y_n \mid x_n = 0)}{P(y_n \mid x_n = 1)}. \tag{6}$$

**Step 3) Processing in bit nodes: For each** $(m, n)$ satisfying $\mathbf{H}_{mn} = 1$, calculate

$$\boldsymbol{\beta}_{mn} = \sum_{m'\in M(n)\setminus m} \boldsymbol{\alpha}_{m'n}. \tag{7}$$

**Step 4) Hard decision and stopping criterion test:**

1. Create $\hat{\mathbf{c}} = [\hat{\mathbf{c}}_n]$ such that

$$\hat{\mathbf{c}}_n = \begin{cases} 1, & \lambda_n + \sum_{m\in M(n)} \boldsymbol{\alpha}_{mn} \geq 0 \\ 0, & \lambda_n + \sum_{m\in M(n)} \boldsymbol{\alpha}_{mn} < 0 \end{cases} \tag{8}$$

2. If $\hat{c}\,\mathbf{H}^{\mathbf{T}} = \mathbf{0}$, then $\hat{c}$ is considered as a valid decoded word and the decoding process ends; if the number of iterations exceeds some maximum number and $\hat{c}$ is not a valid codeword, a failure is declared and the process ends; otherwise the decoding repeats from Step 2.

# 5 Noise Estimation Based on Higher Order Statistics

A novel SNR estimator is proposed which makes use of the second, fourth, and sixth moments of the observations. This scheme is especially helpful at low SNRs. It is assumed that time and phase synchronization have been carried out for a binary phase-shift keying (BPSK) modulated signal, so the received samples can be expressed as [21, 26],

$$y_n = \sqrt{S}a_n + \sqrt{N}\omega_n, \tag{9}$$

where, $a_n$ is the transmitted signal taking values {–1,+1}, $\omega_n$ is a zero-mean real additive white Gaussian noise with unit variance, $S$ is a signal power scale factor, and $N$ is a noise power scale factor, so SNR can be expressed as $S/N$. Let $M_2$ denote the second moment of $y_n$ as,

$$M_2 = E\{y_n^{\;2}\} = S\,E\{|a_n|^2\} + 2\sqrt{S\,N}\,E\{a_n\omega_n\} + N\,E\{|\omega_n|^2\} \quad (10)$$

and let $M_4$ denote the fourth moment of $y_n$ as,

$$\begin{aligned} M_4 &= E\{y_n^{\;4}\} \\ &= S^2 E\{|a_n|^4\} + 4S\sqrt{SN}E\{|a_n|^2\, a_n\omega_n\} \\ &\quad + 6S\,N\,E\{|a_n|^2|\omega_n|^2\} + 4N\sqrt{S\,N}E\{|\omega_n|^2\, a_n\omega_n\} \\ &\quad + N^2 E\{|\omega_n|^4\}. \end{aligned} \quad (11)$$

Assuming the signal and noise are zero-mean independent random processes, Equations (10) and (11) reduce to,

$$M_2 = S + N\text{, and} \quad (12)$$

$$M_4 = k_a S^2 + 6\,S\,N + k_\omega N^2 \quad (13)$$

respectively, where $k_a = E\{|a_n|^4\}/E\{|a_n|^2\}^2$ and $k_\omega = E\{|\omega_n|^4\}/E\{|\omega_n|^2\}^2$ are the kurtosis of the signal and the kurtosis of the noise, respectively. Further, let $M_6$ denote the sixth moment of $y_n$ as,

$$\begin{aligned} M_6 = E\{y_n^{\;6}\} &= S^3E\{|a_n|^6\} + 6S^2\sqrt{S\,N}E\{|a_n|^4\, a_n\omega_n\} \\ &+ 15S^2\,N\,E\{|a_n|^4|\omega_n|^2\} \\ &+ 20\,S\,N\sqrt{S\,N}E\{|a_n|^2|\omega_n|^2\, a_n\omega_n\} \\ &+ 15\,S\,N^2E\{|a_n|^2|\omega_n|^4\} \\ &+ 6\,N^2\sqrt{S\,N}E\{|\omega_n|^4\, a_n\omega_n\} \\ &+ N^3E\{|\omega_n|^6\} \end{aligned} \quad (14)$$

With the same assumptions as for $M_2$ and $M_4$ stated above, this reduces to,

$$M_6 = k_a S^3 + 15 k_a S^2 N + 15 k_\omega S\,N^2 + k_\omega N^3 \quad (15)$$

For BPSK signals $k_a = 1$ and for real noise $k_\omega = 3$, so Equations (13) and (15) can now read,

$$M_4 = S^2 + 6\,S\,N + 3\,N^2 \quad (16)$$

$$M_6 = S^3 + 15\,S^2 N + 45\,S\,N^2 + 3\,N^3 \quad (17)$$

Multiplying Equations (12) and (16) results,

$$M_2 M_4 = S^3 + 7\,N\,S^2 + 9\,N^2 S + 3\,N^3 \quad (18)$$

Substituting $N = M_2 - S$ in Equations (17) and (18) and subtracting Eq. (18) from Eq. (17) gives,

$$M_6 - 33M_2M_4 = 28\,S^3 + 36M_2^{\;2}S - 96M_2^{\;3} \quad (19)$$

So the terms in $S^2$ cancel out. Further dividing Eq. (19) by $M_2^{\;3}$ yields,

$$28\gamma^3 + 36\gamma - (D + 96) = 0 \quad (20)$$

where $\gamma = S / (S + N)$ is the signal-to-total ratio and $D = (M_6 - 33M_2M_4)/M_2^{\;3}$.

An estimated $\gamma$ can be found by solving for the root in [0,1] of Eq. (20) by using Cardan's method, or alternatively, the following iterative formula will find such a root in a few steps:

$$\hat{\gamma}^{(n+1)} = \left(\frac{36\gamma^{(n)} + D - 96}{28}\right)^{1/3} \quad (21)$$

Either $\hat{\gamma} = 0$ or $1$ can be used as a starting point. Having obtained the estimated value for $\gamma$, the SNR estimate, $\rho$, can be calculated as follows:

$$\rho_{M_2M_4M_6} = \frac{\hat{\gamma}}{1-\hat{\gamma}} \quad (22)$$

And the noise power $\sigma^2$ (equal to $N$) can be calculated by.

$$\sigma^2 = \frac{M_2}{1-\hat{\gamma}} \quad (23)$$

In practice, the different moments are estimated by their respective time averages, *i.e.*, the $k$-th moment of $y_n$ with $K$ symbols is calculated as:

$$M_k = \frac{1}{K}\sum_{n=0}^{K}|y_n|^k \quad (24)$$

## 6 Noise Estimation Based on ML Estimation Principle

In this method, a maximum-likelihood (ML) estimate of the amplitude of a BPSK modulated signal is first derived (see [21, 26]), and then an iterative SNR estimation algorithm is developed. For the sake of simplicity, the received samples will be expressed as $r_k = s_k + n_k$. $s_k$ is the transmitted signal taking values from $\{-A, A\}$ with equal probability and $n_k$ is real additive white Gaussian noise with variance $\sigma^2$. The probability density function (pdf) of $r_k$ can be expressed as

$$f(r_k) = \frac{1}{2}\{f_+(r_k) + f_-(r_k)\} \quad (25)$$

where $f_+(r_k) = 1/(\sqrt{2\pi}\,\sigma)e^{-(r_k-A)^2/(2\sigma^2)}$ and $f_-(r_k) = 1/(\sqrt{2\pi}\,\sigma)e^{-(r_k+A)^2/(2\sigma^2)}$.The pdf of a received vector $(r_1, r_2, \ldots, r_N)$ can be expressed as:

$$f_N(r_1, r_2, \ldots, r_N) = \prod_{k=1}^{N} f(r_k)\,. \quad (26)$$

Letting $\partial f_N(r_1,r_2,\ldots,r_N)/\partial A=0$, one can implicitly obtain an ML estimate of $A$ as the solution to the equation

$$A=\frac{1}{N}\sum_{k=1}^{N} r_k \tanh\left(\frac{A r_k}{\sigma^2}\right) \tag{27}$$

When the SNR is high, the following approximation can be made

$$\tanh(A r_k/\sigma^2) \cong \begin{cases} +1, & r_k>0 \\ -1, & r_k<0 \end{cases}. \tag{28}$$

Then the amplitude estimate is,

$$\hat{A}=\frac{1}{N}\sum_{k=1}^{N}|r_k|. \tag{29}$$

This is the decision-directed estimate of $A$. The noise power can be estimated as total power minus the signal power and the SNR can, therefore, be estimated as:

$$SNR=\frac{\left(\frac{1}{N}\sum_{k=1}^{N}|r_k|\right)^2}{\frac{1}{N}\sum_{k=1}^{N} r_k^{\,2}-\left(\frac{1}{N}\sum_{k=1}^{N}|r_k|\right)^2}. \tag{30}$$

The above is essentially the same as the SNV RXDA estimation [21]. It has a significant bias when the SNR is low. The authors here develop an iterative search algorithm to find the amplitude that satisfies Eq. (27). Given a vector containing $N$ samples of $r_k$, define the function,

$$F(x)=x-\frac{1}{N}\sum_{k=1}^{N} r_k \tanh\left(\frac{x r_k}{\sigma^2}\right). \tag{31}$$

The root of this equation is the maximum-likelihood amplitude estimate, that is, $F(x)=0$ at $x=\hat{A}$. Let also $M_2$ denote the total power of the received vector calculated as:

$$M_2=\frac{1}{N}\sum_{k=1}^{N} r_k^{\,2} \tag{32}$$

Now the iterative algorithm is summarized as follows:

**Step 1)** Calculate the total power, $M_2$ given by Eq. (32). Select the minimum and maximum amplitudes of interest, $A_{\min}$, $A_{\max}$, and the number of iterations $I$. Initialize $A_1 = A_{\min}$ and $A_2 = A_{\max}$. Initialize $i = 0$.

**Step 2)** Calculate $A_m = (A_1 + A_2) / 2$ and $\sigma_m^{\,2}=M_2-A_m^{\,2}$.

**Step 3)** Calculate

$$F(A_m)=A_m-1/N\sum_{k=1}^{N} r_k \tanh(A_m r_k/\sigma_m^{\,2})$$

**Step 4)** If $F(A_m) > 0$, then update $A_2 = A_m$. Otherwise update $A_1 = A_m$.

**Step 5)** Increase $i$ by one. If $i = I$, then output $A_m = (A_1 + A_2) / 2$ as the estimated amplitude and $A_m^{\,2}/(M_2-A_m^{\,2})$ as the estimated SNR. Otherwise go to Step 2.

Now, let $\varphi$ and $\hat{\varphi}$ denote the true and estimated noise power respectively. To assess the performance of different estimation methods the Mean-Squared Error (MSE) of noise power estimation is defined as

$$\mathrm{MSE}(\hat{\varphi})=\frac{1}{N_t}\sum_{i=1}^{N_t}(\hat{\varphi}_i-\varphi)^2, \tag{33}$$

where $N_t$ is a largely chosen number of trials. Further, define the normalized MSE (NMSE) as:

$$\mathrm{NMSE}(\hat{\varphi})=\frac{\mathrm{MSE}(\hat{\varphi})}{\varphi^2} \tag{34}$$

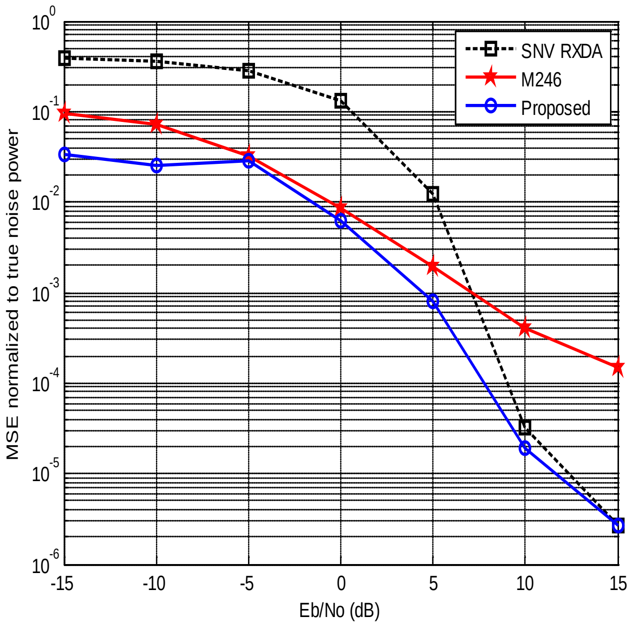



**Fig. 2** NMSE of the two proposed methods as compared to the conventional SNV RXDA method.

In Fig. 2, the NMSE of the two proposed methods is plotted against the SNV RXDA method [21]. A 1024-bit long block is used with BPSK modulation and the modulated signal is corrupted with AWGN noise of the required level. The number of iterations for the ML estimator is set to 50. As can be observed, the two algorithms work well especially at low SNR region when the SNR is less than 10 dB, the ML new estimator being better over a wider range of SNR values.

# 7 Simulation Results

## 7.1 Frequency-Domain Modeling of Power Line Channel

The work of Zimmermann and Dostert [22] is the most cited in the literature adopting the multipath model as an empirical approach for the PLC channel modeling. The advantage of the multipath model is that, once the measurements have been made and the modeling parameters have been derived, simulating the PLC channel requires little computation, and the algorithm implementation is relatively easy. Based on the analysis of multipath propagation, the transfer function of PLC channel can be represented by:

$$H(f) = \sum_{i=1}^{N} g_i \cdot A(f, d_i) \cdot e^{-j2\pi f \tau_i}, \qquad (35)$$

where, for $N$ relevant paths, $g_i$ represents the weighting factor that changes the amplitude due to the reflection and transmission, $A(f, d_i)$ is the attenuation term with dependence of the frequency and propagation length, and $e^{-j2\pi f \tau_i}$ refers to the phase difference due to the time delay of a path $i$. In turn, the time delay is defined by:

$$\tau_i = \frac{d_i}{v_p} = \frac{d_i \sqrt{\varepsilon_r}}{c_0}, \qquad (36)$$

where, $\varepsilon_r$ is the dielectric constant, $c_0$ is the speed of light in vacuum conditions, and $d_i$ is the length of the path $i$. After making extensive measurements in the MHz range, Zimmermann and Dostert modeled the attenuation term by the following expression:

$$A(f, d_i) = e^{-(a_0 + a_1 f^k) d_i}, \qquad (37)$$

where, $a_0$, $a_1$ are the attenuation parameters, and $k$ is the exponent of the attenuation factor with usual values between 0.2 and 1. Finally, using Equations (36) and (37) into (35), results:

$$H(f) = \sum_{i=1}^{N} g_i \cdot e^{-(a_0 + a_1 f^k) d_i} \cdot e^{-j2\pi f \left(\frac{d_i}{v_p}\right)} \qquad (38)$$

Equation (38) gives the frequency response of the adopted PLC channel model. In our simulations, a PLC channel with the parameters given in Table 2 and frequency response plotted in Fig. 3 is used.

## 7.2 Linear Equalization

Linear equalization is an efficient technique to suppress the ISI caused by the multipath environment and thereby improve the performance of the communication system. There are different kinds of linear equalization in frequency domain such as the linear minimum mean square (LMMSE) equalizer, the zero forcing (ZF) equalizer and the regularized zero forcing (RZF) equalizer. The ZF solution can be written as [23]:

$$\mathbf{W}_{ZF} = \left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \qquad (39)$$

where, **H** is the channel matrix.

| **attenuation parameters** | | | | | |
|---|---|---|---|---|---|
| $k = 1$ | | $a_0 = 0$ | | $a_1 = 2.5 \times 10^{-9}$ | |
| **path-parameters** | | | | | |
| $i$ | $g_i$ | $d_i$ (m) | $i$ | $g_i$ | $d_i$ (m) |
| 1 | 0.029 | 90 | 9 | 0.071 | 411 |
| 2 | 0.043 | 102 | 10 | -0.035 | 490 |
| 3 | 0.103 | 113 | 11 | 0.065 | 567 |
| 4 | -0.058 | 143 | 12 | -0.055 | 740 |
| 5 | -0.045 | 148 | 13 | 0.042 | 960 |
| 6 | -0.040 | 200 | 14 | -0.059 | 1130 |
| 7 | 0.038 | 260 | 15 | 0.049 | 1250 |
| 8 | -0.038 | 322 | | | |

**Table 2**. Parameters of the 15-path model.

The drawbacks of the frequency domain ZF equalizer are that, it causes noise enhancement and the computations needed for matrix inversion are time consuming. However, its advantage is that the statistics of the additive noise and source data are not required. To solve the problem of noise enhancement in the ZF equalizer, a new regularization term is added into (39) to give [24, 25]:

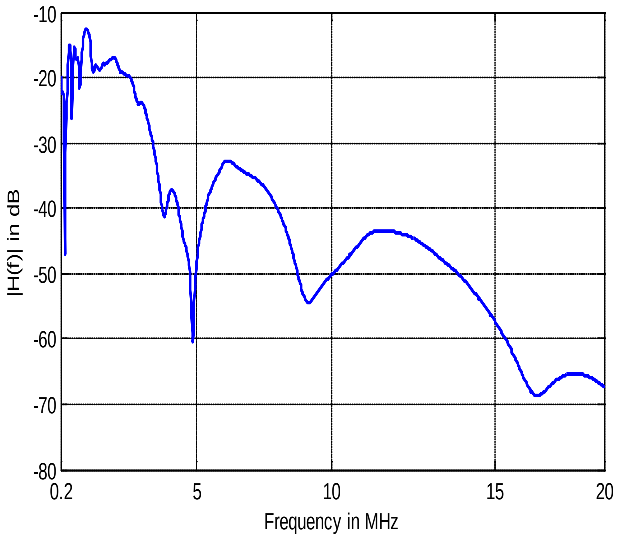



**Fig. 3** Frequency response of the 15-path reference model for PLC [22].

$$\mathbf{W}_{\mathrm{RZF}} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}\mathbf{H}^{\mathrm{H}} \tag{40}$$

where $\alpha$ is a regularization parameter. The resulting equalizer in (40) is called RZF equalizer. From this equation, it is clear that the statistics of the transmitted data and the additive noise are not required in the RZF equalizer. Given the statistics of the additive noise and the users' data, a better equalizer is a one that can minimize the mean square error (MSE) and partially remove the ISI. This equalizer is called the LMMSE equalizer. This can be achieved when $\alpha$=1/*SNR*. It is generally preferred to the ZF linear equalizer, because of its better treatment to noise. The LMMSE solution is given by [23]:

$$\mathbf{W}_{\mathrm{LMMSE}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right)^{-1}\mathbf{H}^{\mathrm{H}} \tag{41}$$

### 7.3 Impact of Noise Estimation

LDPC decoder accepts as an input the parity-check matrix of the code and the Log-Likelihood Ratio (LLR) of the received signal. As stated in previous sections, the power lines are modeled as a cyclo-stationary Gaussian noise environment. The Gaussian PDF (Probability Density Function) of the noise amplitude *z* assuming zero-mean and variance $\sigma^2$ is given by

$$p_G(z) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{z^2}{2\sigma^2}\right) \tag{42}$$

and the LLR assuming BPSK modulation is given by

$$\lambda_G(n) = \ln\left(\frac{y_n \mid x_n = +1}{y_n \mid x_n = -1}\right) = \ln\left(\frac{p_G(y_n - 1)}{p_G(y_n + 1)}\right) = \frac{2y_n}{\sigma^2}. \tag{43}$$

Therefore, the performance of LDPC decoders depends on the accuracy of estimation of noise variance $\sigma^2$. The authors have already introduced two new methods for SNR estimation or simply noise power estimation on PLC channel in sections 5 and 6 above. This subsection provides the simulation results for the proposed methods. In all simulations that follow, BPSK modulation is used with rate-1/2 LDPC codes designed using Modified Shortest-Path (MSP) Algorithm [9]. For decoding, log-domain sum-product iterative decoding summarized in section 4 above is used with a maximum of 50 decoding iterations. For each SNR point, the simulation continues until at least 50 code words are in error. The simulations were executed on a 3.1 GHz PC running Matlab R2011B. Figure 4 below shows the simulation system design blocks.

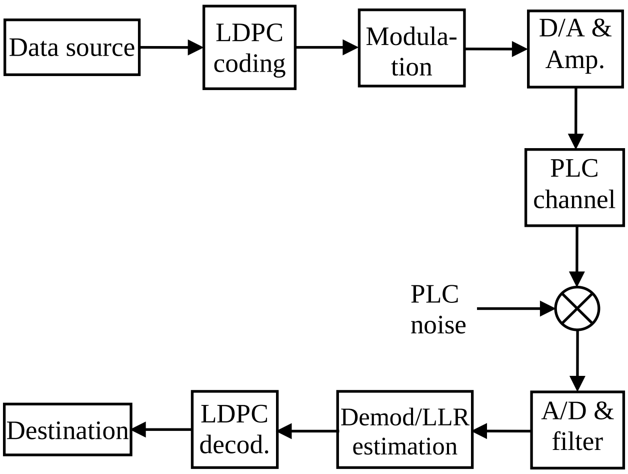



**Fig. 4** Simulation system arrangement

As can be observed from Fig. 2 in section 4, the ML iterative estimation algorithm is superior to the algorithm based on higher-order signal statistics. In Fig. 5 below, the estimated SNR is plotted against the number of ML estimation algorithm iterations. A data block of 10,000 bits is used and exact SNR is fixed at 5dB and 0dB, respectively. It is observed that the ML algorithm is fast converging to the correct SNR value, indicating a lower complexity. For even lower SNRs the new ML algorithm performs better. Figure 6 shows a plot of the estimated SNR against number of ML estimation algorithm iterations for exact SNR values of -5dB and -10dB, respectively. In this case, a gain of about 7.5dB and 11.7dB respectively has been achieved over the conventional method.

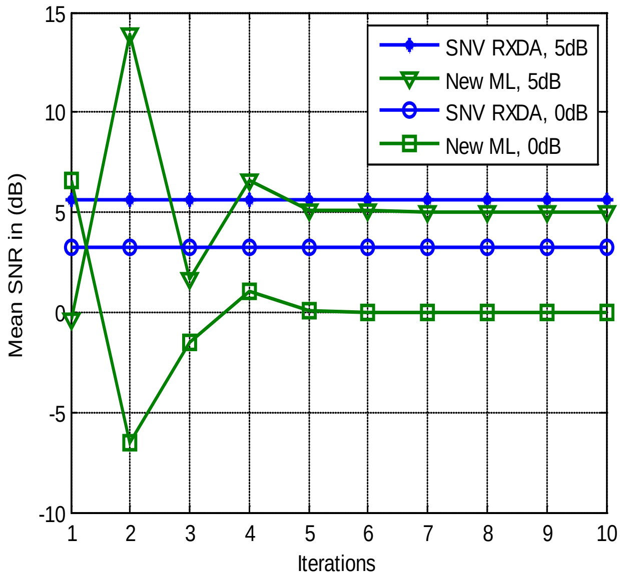



**Fig. 5** Mean estimated SNR vs. no. of iterations for ML estimation algorithm as compared to conventional SNV RXDA estimation [21].

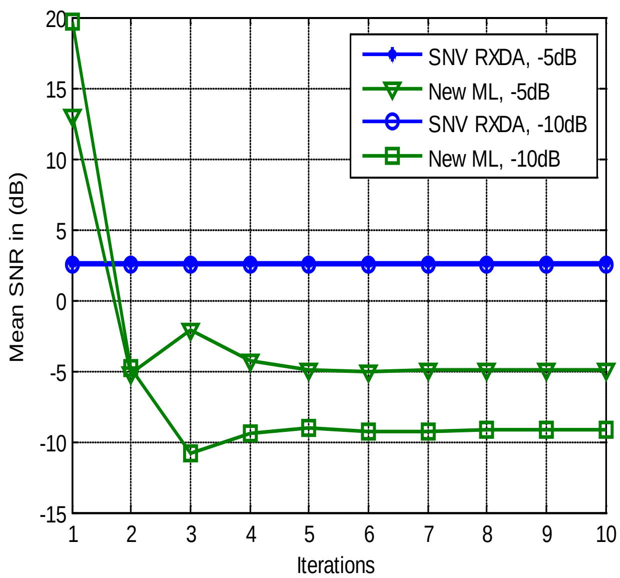



**Fig. 6** Mean estimated SNR vs. no. of iterations for ML estimation algorithm as compared to conventional SNV RXDA estimation [21] for negative SNRs.

Figure 7 shows the BER performance of a rate-1/2 regular LDPC code with block length n = 1024, column weight $t_c$=3, and maximum row weight $t_r$=7 in two cases in which $\sigma^2$ is perfectly known and calculated using Eq. (23). As can be seen from Fig. 7, the proposed SNR estimation method is close to perfect estimation, for example, at a BER of $10^{-5}$ the proposed method is only about 0.14 dB away from perfect estimation and hence, the proposed algorithms are shown to perform well over PLC channel. Similarly, Fig. 8 shows the BER performance of a rate-1/2 regular LDPC code with block length $n$ = 1024, column weight $t_c$=3, and maximum row weight $t_r$=7 in two cases in which $\sigma^2$ is perfectly known and calculated using the ML iterative algorithm.

As can be seen from Fig. 8, the proposed SNR estimation method is close to perfect estimation, and hence, the proposed algorithms are shown to perform well over PLC channel.

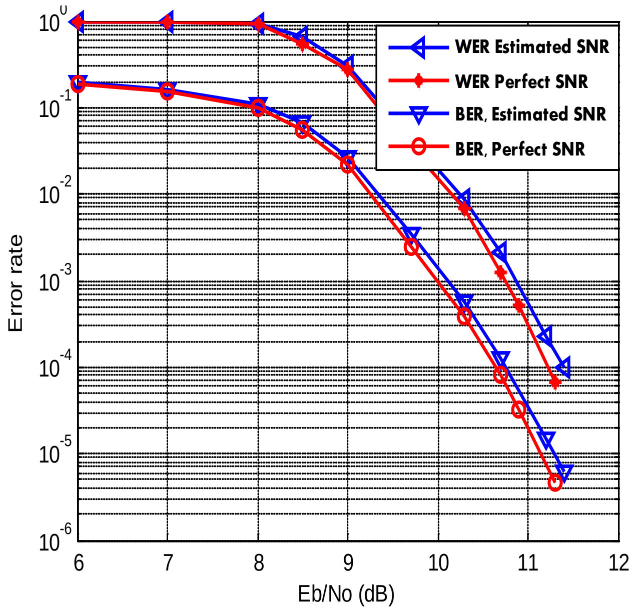



**Fig. 7** BER and WER of a rate-1/2 regular LDPC code with SNR perfectly known and estimated using the Higher-order statistics method.

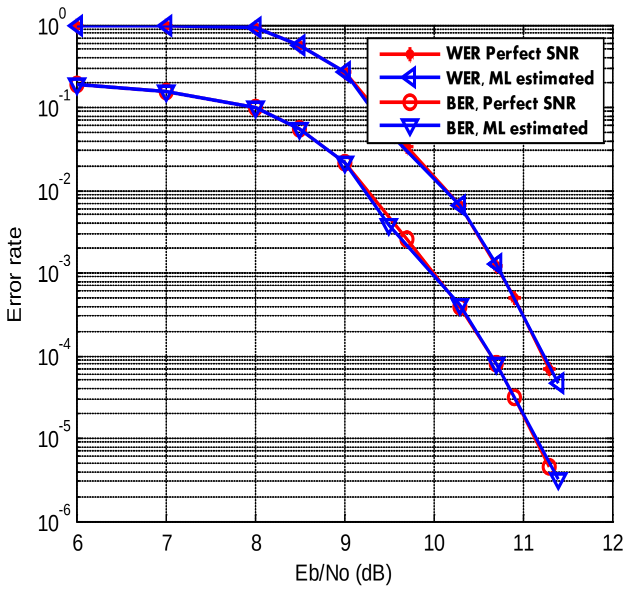



**Fig. 8** BER and WER of a rate-1/2 regular LDPC code with SNR perfectly known and estimated using ML estimation method.

# 8 Conclusions

The paper focused its attention on the effect of LDPC codes on PLC. Two new SNR estimation algorithms have been developed to evaluate the effect in terms of BER performance. The proposed algorithms are best tuned for low or negative SNR values, indicating that they are efficient for PLC.

The first algorithm used higher-order statistics, viz., second, fourth, and sixth moment of received noisy signal to provide an SNR estimate. And the second is an iterative algorithm of lower complexity based on the ML estimation principle. Derivation of ML estimate of the amplitude of a Binary Phase-Shift Keying (BPSK) modulated signal has been presented. Finally, using computer simulations, it has been found that the proposed algorithms perform close-to-optimum on PLC channel, the ML estimation algorithm being better on a wider range of SNR values.

*References*


[1] N. Pavlidou, A. J. Han Vinck, J. Yazdani, and B. Honary "Power line communications: state of the art and future trends," *IEEE Communications Magazine*, pp. 34–40, April 2003

[2] M. Gebhardt, F. Weinmann, and K. Dostert, "Physical and regulatory constraints for communication over the power supply grid," *IEEE Communications Magazine*, vol. 41, no. 5, pp. 84–90, May 2003.

[3] H. Hrasnica, A. Haidine, R. Lehnert, *Broadband Powerline Communications*: *Network Design*, John Wiley & Sons, 2005.

[4] Powerline Communications Network Technology eLibrary. [Online]. Available: http://plugtek.com/, accessed Mar. 2012.

[5] R. G. Gallager, "Low density parity check codes," *IRE Transactions on Information Theory*, vol. IT-8, pp. 21–28, Jan. 1962.

[6] M. Tanner, "A recursive approach to low complexity codes," *IRE Transactions on Information Theory Theory*, vol. IT-27, 1981.

[7] D. J. C. MacKay, "Good error-correcting codes based on very sparse matrices," *IRE Transactions on Information Theory*, 1999.

[8] N. Andreadou, C. Assimakopoulos, F.-N. Pavlidou, "Performance Evaluation of LDPC Codes on PLC Channel Compared to Other Coding Schemes", *IEEE International Symposium on Power Line Communications (ISPLC)*, pp. 296–301, 26–28 March 2007.

[9] Seif Shebl, Nawal El-Fishawy, Atef Abou Elazm, and Fathi Abd El-Samie, "A Random Construction of LDPC Codes Using a Sub-Optimal Search Algorithm", *Proceedings of The 26th National Radio Science Conference (NRSC)*, Faculty of Engineering, Future University, New Cairo. 17-19 March 2009.

[10] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo codes," *Proceedings of the IEEE International Conference on Communications*, Geneva, Switzerland, May 1993

[11] T. Wada, "An Effect of SNR Estimation Error on Iterative Decoding for Power Line Noise," in *International Symposium on Power Line Communications (ISPLC)*, Kyoto, Japan, March 26-28, 2003

[12] T. Wada, "A study on performance of LDPC codes on power line communications", *IEEE International Conference on Communications*, vol. 1, pp. 109–113, June 2004.

[13] T. Wada, "A study on performance of Turbo coding for noise environments in power lines," *IEEE International Conference on Communications*, Vol. 1, pp. 3071–3075, June 2003.

[14] D. J. C. MacKay and R. M. Neal, "Near-Shannon-limit performance of low density parity check codes," *Electronic Letters*, vol. 32, pp. 1645–1646, Aug. 1996.

[15] T. Richardson, M. A. Shokrollahi and R. Urbanke, "Design of capacity approaching irregular low-density parity-check codes," *IRE Transactions on Information Theory*, vol. 47, no. 2, pp. 619-637, Feb. 2001.

[16] Sae-Young Chung, G. David Forney, Thomas J. Richardson, and Rüdiger Urbanke, "On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit," *IEEE Communications Letters.*, vol. 5, no. 2, Feb. 2001.

[17] M. Luby, M. Mitzenmacher, A. Shokrollahi, and D. Spielman, "Analysis of low density codes and improved designs using irregular graphs," in *Proceedings of the 30th Annual ACM Symposium*. Theory of Computing, 1998

[18] D. MacKay, S. Wilson, and M. Davey, "Comparison of constructions of irregular Gallager codes," *IEEE Transactions on Communicatios*, vol. 47, pp. 1449–1454, Oct. 1999.

[19] T. Wadayama, "An iterative decoding algorithm of low density parity check codes for hidden Markov noise channels," in

*International Symposium on Information Theory and its Applications*, pp. 188–19, 5-8 Nov., 2000.

[20] J. Chen and M. P. Fossorier, "Near optimum universal belief propagation based decoding of low-density parity check codes," *IEEE Transaction on Communications*, vol. 50, no. 3, pp. 406–414, March 2002.

[21] D. R. Pauluzzi and N. C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Transactions on communications*, vol. 48, pp. 1681–1691, Oct. 2000.

[22] M. Zimmermann and K. Dostert, "A multipath model for the powerline channel," *IEEE Transactions on communications,*" vol. 50, pp. 553–559, Apr. 2002.

[23] A. Klein, "Data detection algorithms specially designed for the downlink of CDMA mobile radio systems," *IEEE VTC*, vol. 1, pp. 203–207, 4-7 May 1997.

[24] F. S. Al-kamali, M. I. Dessouky, B. M. Sallam, and F. E. El-Samie, "Frequency Domain interference cancellation for single carrier cyclic prefix CDMA systems," *Progress In Electromagnetics Research* B, PIERB 3, pp. 255-269, 2008.

[25] F. S. Al-kamali, M. I. Dessouky, B. M. Sallam, and F. E. El-Samie, "Performance Evaluation of Cyclic Prefix CDMA Systems with Frequency Domain Interference Cancellation," *Digital Signal Processing Journal*, *Elsevier Inc.*, Volume 19, Issue 1, pp. 2-13, 2009.

[26] F. Brannstrom, and L.K. Rasmussen, "Non-data-aided parameter estimation in an additive white gaussian noise channel," *Proceedings of International Symposium on Information Theory, (ISIT 2005),* pp. 1446 – 1450, Oct. 2005.