Iterative Multicarrier Detector and LDPC Decoder for OFDM Systems

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Abstract:-In this paper, we propose iterative soft-input soft-output (SISO) multi-carrier detection (inter-carrier interference cancellation) and LDPC decoding of the OFDM system in the mobile channel. The proposed SISO multi-carrier detection and sum-product decoding of the LDPC-coded OFDM system can achieve better error rate performance than previous SISO multi-carrier detection and BCJR decoding of convolutional coded (CC)-OFDM systems in mobile channels. The simulation results show that the second iteration improves the performance significantly and the gain of the third iteration is less. The advantage of LDPC over CC is more significant when the codeword is longer. With channel estimation errors, the proposed scheme degrades by 0.5 dB for the 3rd iteration. In addition, the proposed LDPC-OFDM scheme has lower computational complexity than the CC-OFDM one.

Key-Words:- inter-carrier interference, turbo principle, OFDM, LDPC, computational complexity

1 Introduction

Orthogonal frequency division multiplexing (OFDM) [1] is popular for the very high-data-rate wireless transmission. Therefore, OFDM is one kind of key technologies and selected by many systems, for example, Digital Audio Broadcasting (DAB) [2], Digital Video Broadcasting (DVB) [3], IEEE 802.11 [4], IEEE 802.16 [5] and so on. The basic principle of OFDM is to transform a frequency selective fading channel into multipath flat fading channels. In the other words, OFDM is to split a high-data-rate stream into several lower-data-rate streams that are transmitted synchronously over several subcarriers.

In the past years, OFDM systems only consider the low mobility condition channel. However, OFDM has two problems. First, the signal may deeply fade through some subcarriers of OFDM in the frequency selective fading channels [6][7] and errors occur. It is essential to use forward-error correction coding in OFDM systems to combat fading, such as Reed-Solomon codes [8], convolutional codes, turbo codes, etc. Recently, low-density parity-check (LDPC) codes [9][10] got much attention. LDPC codes with large codeword sizes are found to approach the channel capacity and have lower decoding complexity [11].

LDPC codes have been applied to OFDM systems without iterations between LDPC decoder block and another functional block [12][13][14]. Iteration between LDPC decoder and soft demodulator has been proposed for LDPC coded OFDM systems [15][16]. Iteration between the LDPC decoder and soft carrier-frequency offset (CFO) estimator has been proposed for LDPC coded OFDM systems [17]. Iteration between LDPC decoder and channel estimator has been proposed for LDPC coded OFDM systems [18] and MIMO OFDM systems [17]. Iteration between LDPC decoder and channel/CFO estimator has been proposed for LDPC coded MIMO OFDM systems [19]. However, no iterative structure between LDPC decoder and turbo ICI (caused by Doppler effects) cancellation block has been proposed for OFDM systems in the literatures.

Second, the Doppler spreading in the mobile channels would destroy orthogonality of subcarriers and then result in ICI, leading to an irreducible error floor. Some methods are developed against ICI, for instance, frequency-domain equalization [20], time-domain windowing [21], and ICI self-cancellation [22].

Another viewpoint is to view ICI in OFDM systems as special form of Multiple Access Interference (MAI) in synchronous code-division multiple access (SCDMA) systems [23]. Following this viewpoint, we apply the turbo principle [24] to concatenate the ICI cancellation and an LDPC decoder which are both soft-in soft-out (SISO) modules in mobile channels. The received symbol is more reliable after more iterations, and thus the system performance is improved. Furthermore, a simple channel estimation scheme is used in the simulation to test the proposed scheme at the presence of channel estimation errors. Finally, we compared the proposed SISO ICI cancellation/LDPC decoding scheme with the SISO ICI cancellation/BCJR decoding of convolutional coded OFDM system in [25]. The simulation results show that our system can achieve the comparable error rate performance at lower complexity.

This paper is organized as follows. In Section 2, we describe the OFDM system model, LDPC channel coding, and ICI due to time-varying fading channels. The proposed turbo partial parallel interference cancellation(PIC)/MMSE multicarrier detection and LDPC decoding scheme is described in Section 3. The channel estimation is described in Section 4. The simulation results are given in Section 5. The comparison of decoding complexity is given in Section 6. Section 7 is the conclusion.

2 System Model and Interference Analysis

2.1 System Model

Fig. 1 shows the model of the LDPC-OFDM system. At the transmitter, information bits are encoded by the LDPC encoder. Each coded bit is randomly interleaved, and mapped into one BPSK symbol. The OFDM sub-channel modulated follows the serial-to-parallel conversion and then is implemented by using an inverse fast Fourier transform (IFFT). By N-size IFFT processing, the frequency selective channel in the frequency domain is divided into N parallel flat faded independent sub-channels and is inserted the guard interval (GI) in order to eliminate the ISI. At the receiver, the guard interval (GI) is removed. After the serial-to-parallel conversion, the OFDM sub-channel demodulation is implemented by using fast Fourier transform (FFT). The demodulated bits are decoded by LDPC decoder.

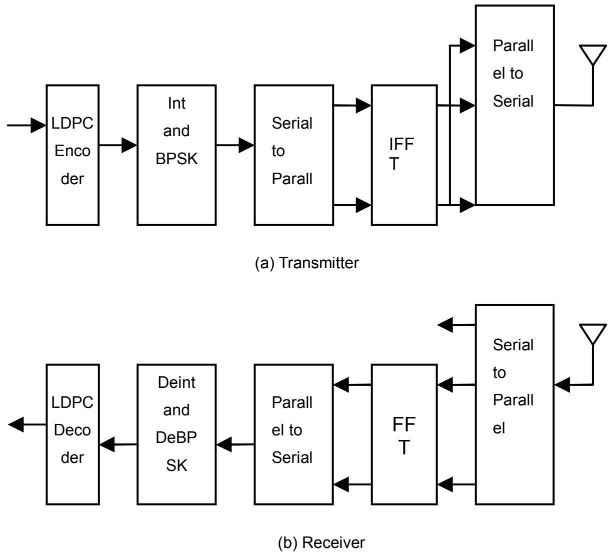

Fig.1 OFDM system model

2.2 LDPC Codes

Low Density Parity Check (LDPC) codes proposed by Gallager [3], but the codes have almost forgotten because VLSI technology was not mature enough at that time. After thirty years, LDPC codes are recognized since good error-correcting codes perform well near Shannon limit. In fact, LDPC codes are linear block codes using a sparse parity-check matrix with a very small number of 1's per column and row. LDPC codes classify into two groups, regular and irregular LDPC codes. Regular LDPC codes have a uniform column and row weight, while irregular LDPC codes have a nonuniform column and row weight.

A regular (N, K) LDPC code has the design code rate R = K/N (true code rate may be different). An LDPC code is defined by an M×N parity-check matrix V, where K = N-M and that matrix is linearly independent. At the receiver LDPC codes can be decoded by sum-product algorithm, which is represented by a factor graph and simpler than BCJR decoding algorithm [12] of convolutional codes (CC).

In Fig. 2, the LDPC code contains two types of nodes: "bit nodes" and "check nodes". Each column of parity-check matrix corresponds to bit nodes (bits of codeword) and each row of parity-check matrix corresponds to check nodes. Each edge can be connected between the check node and bit node according to 1's in the parity-check matrix.

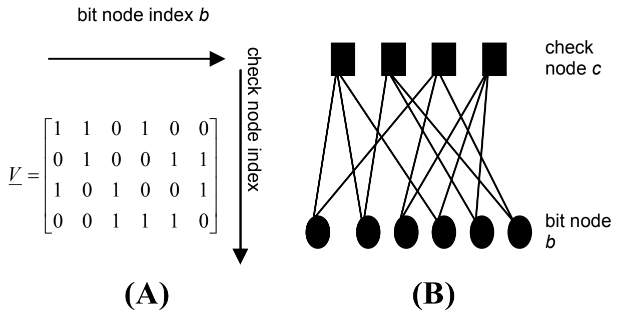

Fig.2 (A) parity-check matrix (B) factor graph

2.3 ICI from Time-Varying Fading Channel (Doppler Effect)

We modeled the frequency selective channel as a tapped delay line with L+1 time-varying coefficients. The channel impulse response is described by $h(n,l)$, $0 \le n \le N-1$, which describe as the tap gain of the l^{th} tap at the time n. Each gain is independent of other gains. The intersymbol interference (ISI) results from the delay spread. In OFDM system, the guard interval (GI) usually equal or larger than the maximum delay spread of channel is inserted to eliminate ISI. So here the ISI can be neglected.

On the other hand, a time-varying fading channel leads to the loss of subcarrier orthogonality and produces intercarrier interference. When data sequence is fed to an IFFF and output OFDM sequence {s(n)} with

$$s(n) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m e^{j2\pi mn/N} \quad ,0 \le n \le N-1 \qquad (1)$$

where x_m represents the symbol on the m^{th} subcarrier before IFFT transformation. At the receiver, the signal over multipath channel can be represented

$$y(n) = \sum_{l=0}^{L} h(n,l) s(n-l) + w(n)$$
$$= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m H_m(n) e^{j2\pi mn/N} + w(n) \qquad (2)$$

where $L+1$ is a total number of paths, w(n) is the zero mean AWGN noise with variance σ^2 and $H_m(n) = \sum_{l=0}^{L} h(n,l) e^{-j2\pi lm/N}$ is the channel impulse response on the m^{th} subcarrier at the time n after the Fourier transform. At the receiver the k^{th} subcarrierr output from the FFT can be written as

$$Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) e^{-j2\pi kn/N}$$
$$= x_k A_k + ICI_k + W_k \qquad (3)$$

where A_k is the complex channel reduction for the k^{th} subcarrier, ICI_k is the interference on the k^{th} subcarrier, and W_k represents the complex Gaussian noise after the Fourier transform. There are expressed as

$$A_k = \frac{1}{N} \sum_{n=0}^{N-1} H_k(n) \qquad (4)$$

$$ICI_K = \frac{1}{N} \sum_{m=0, m\neq k}^{N-1} x_m \sum_{n=0}^{N-1} H_m(n) e^{j2\pi n(m-k)/N} \qquad (5)$$

$$W_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w(n) e^{-j2\pi nk/N} \qquad (6)$$

According to [9], the ICI in OFDM systems are similar to the MAI in CDMA systems. Hence, the MAI cancellation method in CDMA systems can be used to cancel the ICI in OFDM systems. The received signal can be represented in vector form:

Y=Hx+W (7)

where Y = $[Y_1,Y_2,\ldots,Y_N]^T$ is the received symbols vector, x = $[x_1,x_2,\ldots,x_N]^T$ is the transmitted symbols vector. W = $[W_1,W_2,\ldots,W_N]^T$ is the Gaussian noise vector after FFT, $(\cdot)$ denotes the vector transpose, and H is channel matrix described as

$$\underline{H} = \frac{1}{N} \begin{bmatrix} A_1 & ICI_{12} & \cdots & ICI_{1N} \\ ICI_{21} & A_2 & \cdots & ICI_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ ICI_{N1} & ICI_{N2} & \cdots & A_N \end{bmatrix} \qquad (8)$$

where

$$ICI_{Km} = \sum_{n=0}^{N-1} H_m(n) e^{j2\pi n(m-k)/N}, m \neq k$$

3 Proposed Turbo Partial PIC/MMSE Multicarrier Detection and Channel

The proposed turbo ICI cancellation and decoding scheme for LDPC coded OFDM system is shown in Fig 3. This structure concatenates the SISO signal detection and SISO channel decoding and it can provides significantly gain through iterations. This allows for the soft information exchange between detector and decoder.

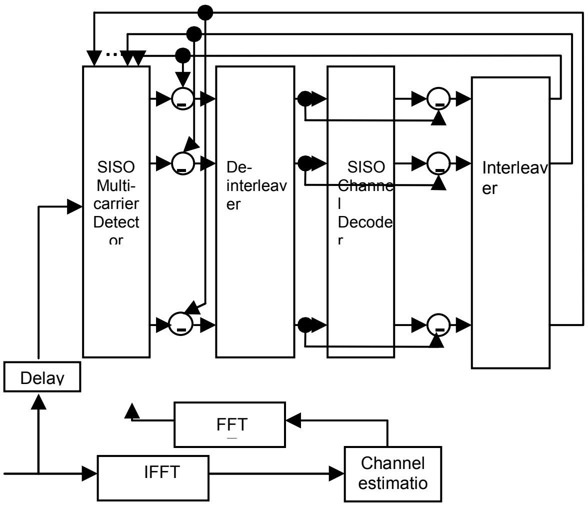

Fig. 3 The proposed turbo receiver with SISO partial PIC/MMSE multicarrier channel detector and SISO Sum-Product algorithm channel decoder

The SISO multicarrier detector for ICI cancellation in OFDM systems in this paper is described in [11] where BCJR decoding of convolutional codes is used instead of LDPC codes in this paper. Here we briefly describe it. For details, the authors are referred to [11].

Each bit's a posteriori probability (APP) log-likelihood ratio (LLR) can be calculated by the SISO multicarrier detector. The LLR value is given by:

$$\Lambda_1[x_k(m)] = \log\frac{P[x_k(m)=+1\mid Y_k(m)]}{p[x_k(m)=-1\mid Y_k(m)]} \tag{9}$$

k is subcarrier index, m is OFDM symbol index, and $Y_k(m)$ is the receive signal outputs of the FFT. $P[x_k(m)=+1\mid Y_k(m)]$ is the APP of input bit $x_k(m)$.
According to the Bayes' rule, the above equation can be rewritten as

$$\Lambda_1[x_k(m)] = \lambda_1[x_k(m)] + \lambda_2^p[x_k(m)]$$
$$= \log\frac{p[Y_k(m)\mid x_k(m)=+1]}{P[Y_k(m)\mid x_k(m)=-1]} + \log\frac{P[x_k(m)=+1]}{p[x_k(m)=-1]}$$

(10)

where $\lambda_1[x_k(m)]$ is the extrinsic information delivered form the SISO multicarrier detector to the SISO channel decoder and $\lambda_2^p[x_k(m)]$ is the extrinsic information deliver from SISO channel decoder at the previous iteration, which is represents *a priori* (the superscript emphasize a priori) information ratio $P[x_k(m)=+1]/P[x_k(m)=-1]$. At the first iteration, no a priori information is available, so a priori is set to zero.

3.1 SISO Multicarrier Detector

The SISO multicarrier detector is a concatenation of soft-input partial PIC and soft-output MMSE filter. A reason is the extrinsic information delivered by channel decoder is soft could be used to estimate the mean value of the code bit (similar to partial PIC cancellation weight multiplied by hard decision). Another reason is approximately Gaussian and thus soft output can be calculated [10].

In the following, describe how to combine soft-input partial PIC and soft- output MMSE detector. To mitigate interference, the coded bits soft estimates of each subcarrier can be obtained by calculating their expectation. The means of the coded bits can expressed as

$$\tilde{x}_k(m) = \sum_{x_k\in\{+1,-1\}} x_k P[x_k(m)]$$
$$= P(x_k(m)=+1) - P(x_k(m)=-1)$$
$$= \frac{P(x_k(m)=+1) - P(x_k(m)=-1)}{P(x_k(m)=+1) + P(x_k(m)=-1)}$$
$$= \frac{1-\dfrac{P(x_k(m)=-1)}{P(x_k(m)=+1)}}{1+\dfrac{P(x_k(m)=-1)}{P(x_k(m)=+1)}}$$
$$= \frac{1-\exp(-\lambda_2^p[x_k(m)])}{1+\exp(-\lambda_2^p[x_k(m)])}$$
$$= \frac{\exp(\frac{1}{2}\lambda_2^p[x_k(m)]) - \exp(-\frac{1}{2}\lambda_2^p[x_k(m)])}{\exp(\frac{1}{2}\lambda_2^p[x_k(m)]) + \exp(-\frac{1}{2}\lambda_2^p[x_k(m)])}$$
$$= \tanh(\frac{1}{2}\lambda_2^p[x_k(m)]) \tag{11}$$

where $P[x_k(m)]$ comes from a priori information
delivered by the channel decoder, and

$$\lambda_2^p[x_k(m)] = \log\frac{P[x_k(m)=+1]}{P[x_k(m)=-1]}.$$

By some manipulations,

$$P[x_k(m)] = \frac{\exp(x_k \lambda_2^p [x_k(m)])}{1+\exp(x_k \lambda_2^p [x_k(m)])}$$
$$= \frac{1}{2}\left[1 + x_k \tanh(\frac{1}{2}\lambda_2^p [x_k(m)])\right]$$
(12)

Define two vectors

$$\underline{\tilde{x}}(m) = [\tilde{x}_1 \cdots \tilde{x}_N]^T$$
(13)

$$\underline{\tilde{x}}_k(m) = \underline{\tilde{x}}(m) - \tilde{x}_k(m)\underline{e}_k$$
$$= [\tilde{x}_1(m), \cdots, \tilde{x}_{k-1}(m), 0, \tilde{x}_{k+1}(m), \cdots, \tilde{x}_N(m)]^T$$
(14)

where $\underline{e}_k$ is only the *k*th element is one for all zeros N-vector. For each subcarrier *k*, a soft-input partial PIC scheme can be performed by regenerating ICI signals using $\underline{\tilde{x}}_k(m)$ and subtracting interference from the outputs of FFT.

$$\underline{Y}_k(m) = \underline{Y}(m) - \underline{H}\underline{\tilde{x}}_k(m) + \underline{W}(m)$$
$$= \underline{H}[\underline{x}(m) - \underline{\tilde{x}}_k(m)] + \underline{W}(m) \quad (15)$$

On other hand, in order to provide soft output and further reduce the remaining interference and noise in $\underline{Y}_k$, it is processed by a linear MMSE filter.

$$z_k(m) = \underline{w}_k(m)^H \underline{Y}_k(m)$$
(16)

where $\underline{w}_k(m)$ satisfies

$$\underline{w}_k(m) = \underset{\underline{w}\in C}{\operatorname{argmin}} E\{\left\| x_k(m) - \underline{w}^H \underline{Y}_k(m) \right\|^2\}$$
$$= \underset{\underline{w}\in C}{\operatorname{argmin}} \underline{w}^H E\{\underline{Y}_k(m)\underline{Y}_k(m)^H\}\underline{w}$$
$$- \underline{w}^H E\{x_k(m)\underline{Y}_k(m)\} - E\{x_k(m)\underline{Y}_k(m)\}^H \underline{w} \quad (17)$$

$$E\{\underline{Y}_k(m)\underline{Y}_k(m)^H\} = \underline{H}\Delta_k(m)\underline{H}^H + \sigma^2 I \quad (18)$$

$$E\{x_k(m)\underline{Y}_k(m)\} = \underline{H}\underline{e}_k = \underline{h}_k \quad (19)$$

$$\underline{\Delta}_k(m) = \operatorname{cov}\{\underline{x}(m) - \underline{\tilde{x}}_k(m)\}$$
$$= diag[1-\tilde{x}_1(m)^2, \cdots 1-\tilde{x}_{k-1}(m)^2,$$
$$1, \cdots, 1-\tilde{x}_N(m)^2] \quad (20)$$

The solution to (17) is given by

$$\underline{w}_k(m) = [\underline{H}\Delta_k(m)\underline{H}^H + \sigma^2 I_N]^{-1}\underline{h}_k \quad (21)$$

According to [15], the outputs $z_k(m)$ of the MMSE for each carrier can be approximated to a Gaussian distribution. And it is modeled as

$$z_k(m) = \mu_k(m)x_k(m) + \eta_k(m) \quad (22)$$

where $\mu_k(m)$ is the amplitude of the *k*th subcarrier and $\eta_k(m) \sim N(0, \upsilon_k^2(m))$ is the Gaussian noise. $\mu_k(m)$ and $\upsilon_k^2(m)$ are given by as

$$\mu_k(m) = E\{z_k(m)x_k(m)\}$$
$$= \underline{h}_k^H [\underline{H}\Delta_k(m)\underline{H}^H + \sigma^2 I]^{-1}\underline{h}_k \quad (23)$$

$$\upsilon_k^2(m) = \operatorname{var}\{z_k(m)\}$$
$$= \underline{h}_k^H [\underline{H}\Delta_k(m)\underline{H}^H + \sigma^2 I]^{-1}\underline{h}_k - \mu_k(m)^2$$
$$= \mu_k(m) - \mu_k(m)^2 \quad (24)$$

Using conditional Gaussian approximation, the extrinsic information $\lambda_1[x_k(i)]$ delivered by MMSE is expressed

$$\lambda_1[x_k(m)] = \log\frac{P[z_k(m) \mid x_k(m) = +1]}{P[z_k(m) \mid x_k(m) = -1]}$$
$$= -\frac{|z_k(m) - \mu_k(m)|^2}{\upsilon_k^2(m)} + \frac{|z_k(m) + \mu_k(m)|^2}{\upsilon_k^2(m)}$$
$$= \frac{4\operatorname{Re}\{\mu_k(m)z_k(m)\}}{\upsilon_k^2(m)} = \frac{4\operatorname{Re}\{z_k(m)\}}{1-\mu_k(m)}$$
(25)

3.2 SISO Channel Decoder

In this paper, the sum-product algorithm [15] is used to decode the LDPC code. Here we give a brief description. *C*(*b*) denotes the set of check nodes connected to the bit node *n*. *B*(*c*) denotes the set of bit nodes connected to the check node *m*. *C*(*b*)*c* represents the *c*th check node excluded from the set *C*(*b*) and *B*(*c*)*b* represents the *b*th bit node excluded from the set *B*(*c*). $q_{b\to c}^i$, where i = 1,0, denotes the probability information from the bit node *b* to the check node *c*. $r_{c\to b}^i$,where i = 1,0, denotes the probability information from the check node *c* to the bit node *b*, and we can call the $r_{c\to b}^i$ is the extrinsic information for the *b*th bit node from the *c*th check node. There for a posteriori probability for bit node is calculated by collecting all the extrinsic information from the check nodes that connect to it, and we can operate in iterative steps

Then we discuss when to stop the iterative process. BER in AWGN channel could perform to the best with 100-iteration, and the result shows BER can approach to the maximum performance with only 50-iteration [16]. Therefore, we set the maximum number iteration to 50 in order to speed up the system. If the number of iterations becomes the maximum number of iterations, the decoder stops and outputs the results.

4 Channel Estimation

In the Section 3, the channel response is assumed perfectly known. Here, the channel estimation technique used in the simulation results is described as follows.

4.1 Pilot symbol pattern

For channel estimation, insertion of pilot symbols is a practical solution. There are many pilot symbol patterns in OFDM systems. For example, continual pilot, scattered pilot, and pilot tones are used in different OFDM standards. Here we use continual pilot defined in Digital Video Broadcasting-Terrestrial (DVB-T) standard. The continual pilot symbol pattern we used in the simulation results is shown in Fig. 4. All subcarriers in a given time slot are dedicated to pilot symbols. The received pilot signals to be used by the estimator are now defined as a vector

$$\mathbf{y}_p = [\mathbf{y}_n^T \ \mathbf{y}_{n+K}^T \cdots \mathbf{y}_{n+(M-2)K}^T \ \mathbf{y}_{n+(M-1)K}^T]^T \tag{26}$$

where K is pilot spacing and M is number of pilot symbols.

Then estimate channel can be known by $\tilde{h}_p = y_p / d_p = h_p + w_p / d_p$, estimate channel are defined as a vector

$$\tilde{\mathbf{h}}_p = [\tilde{\mathbf{h}}_n^T \ \tilde{\mathbf{h}}_{n+K}^T \quad \cdots \quad \tilde{\mathbf{h}}_{n+(M-2)K}^T \ \tilde{\mathbf{h}}_{n+(M-1)K}^T]^T \tag{27}$$

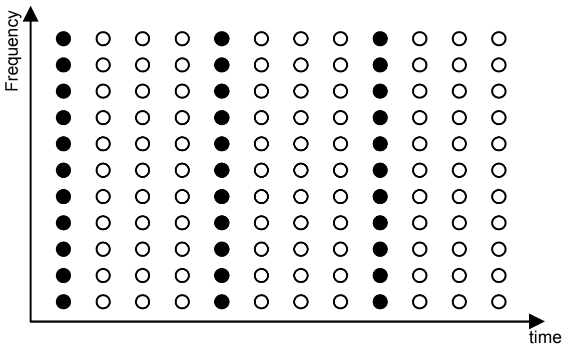

Fig. 4 OFDM pilot symbol pattern

4.2 Interpolation

We can estimation the channel coefficients for pilot symbols by the above method. But the channel coefficients for data symbols need interpolation between adjacent pilot symbols' channel estimates. There are 1st, 2nd, and 3rd order interpolation methods. Here we adopt the simplest linear interpolation. Thus the channel impulse response can be estimated using

$$\tilde{\mathbf{h}}(m) = \tilde{\mathbf{h}}(iK+c) = \tilde{\mathbf{h}}_p(i) + \frac{c}{K}[\tilde{\mathbf{h}}_p(i+1) - \tilde{\mathbf{h}}_p(i)] \tag{28}$$

$$0 \le c < K, 0 \le i < M$$

5 Simulation Results

In the simulations, we consider an OFDM system using BPSK modulation. The transmitter encodes data bits using (204,102), (408,204), and (816,408) Gallager encoders with rate 1/2 and column weight 3 in [Mac]. At the end of encoder every codeword block are randomly interleaved. The total number of subcarriers is N=32. The channel is modeled as the frequency selective fading Rayleigh channel with multiple paths (L=3). Each path gain is generated using Jake's model [17] where channel gain is independent and varies in time. The normalized Doppler frequency $f_d T=0.1$, where f_d represents the maximum Doppler frequency shift and T is one OFDM symbol duration.

Fig. 5-7 show the BER performance comparison without channel estimation errors for (204,102), (408,204), (816,408) LDPC codes, respectively. A rate 1/2 convolutional encoder (CC) with constraint length 3 and the same codeword length are used for comparison [11]. Total three iterations are performed. The proposed SISO multi-carrier detection and sum-product decoding of the LDPC-coded OFDM system can achieve better error rate performance than SISO multi-carrier detection and BCJR decoding of convolutional coded-OFDM systems in mobile channels. The performance advantage is more significant as the codeword length increases. The simulation results show that the second iteration improves the performance significantly and the gain of the third iteration is smaller.

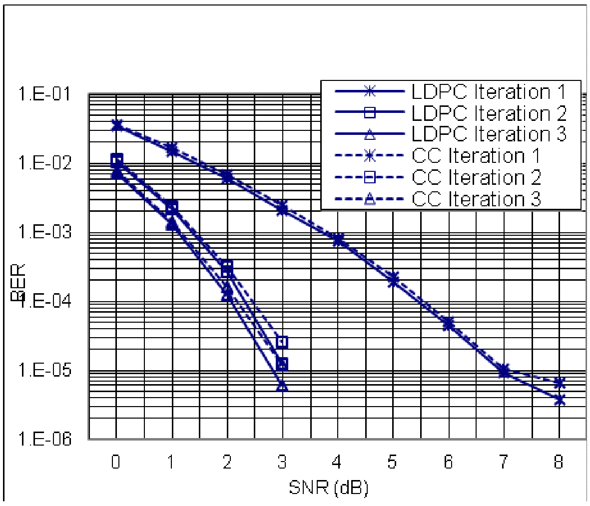

Fig. 5. The performance comparison of the proposed turbo partial PIC/MMSE receiver of various iterations with (204, 102) LDPC codes (proposed scheme) and CC codes (previous scheme [25]).

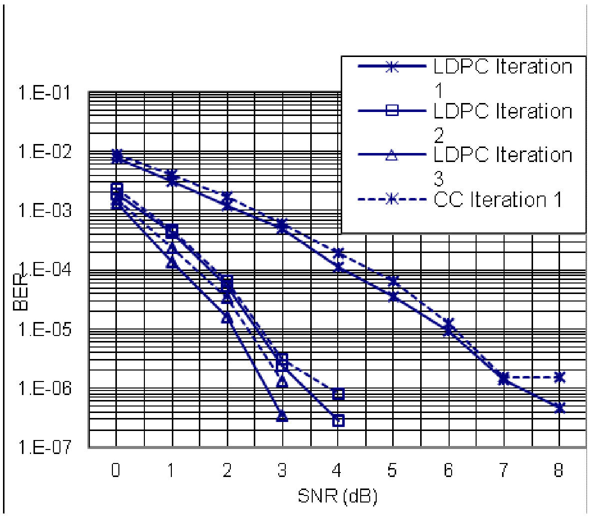

Fig. 6. The performance comparison of the proposed turbo partial PIC/MMSE receiver of various iterations with (408, 204) LDPC codes (proposed scheme) and CC codes (previous scheme [25]).

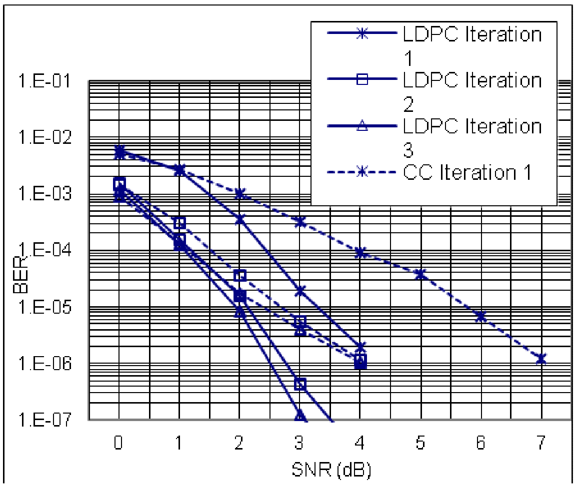

Fig. 7. The performance comparison of the proposed turbo partial PIC/MMSE receiver of various iterations with (816, 408) LDPC codes (proposed scheme) and CC codes (previous scheme [25]).

In Fig. 8, we consider channel estimation error case where the pilot spacing K is 10 and the number of pilot symbols M is 4. Fig. 8 shows the non-perfected channel estimation performs worse than perfect channel estimation case by about 0.5 dB for the 3rd iteration.

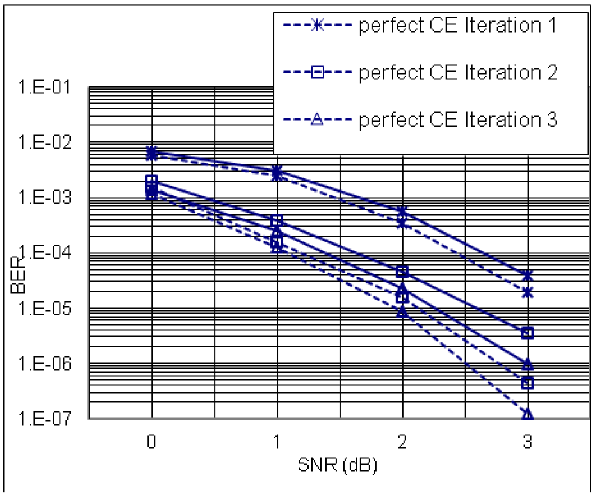

Fig. 8. The performance of the perfected channel estimation and non-perfected channel estimation (K=10, M=4).

6 Decoding Complexity

We compare the complexity of the LDPC-OFDM (sum-product algorithm) decoding and CC-OFDM (BCJR algorithm) decoding using the results in [18].

6.1 CC_OFDM decoding complexity

First, we consider the decoding complexity of convolutional code scheme using the binary Log-MAP decoder. The information bit u_t, $t\in\{0,1,\ldots,N-1\}$, and given the received coded be sequence of $y=\{y_0,y_1,\ldots,y_{Nn-1}\}$,where N is the number of n-bit coded symbols. The *a posteriori* probability can be computed as follows:

$$P_r\{u_t=\pm 1\mid y\}=\sum_{\substack{S^+(u_t=1)\\ S^-(u_t=-1)}}\alpha_t(s')\cdot\gamma_t(s',s)\cdot\beta_t(s) \tag{29}$$

, where S^+ is the set of ordered pairs (s`, s) corresponding to all state transition ($s_{t-1}$=s`) => (s_t=s) caused by data input u_t=+1, and S^- is similarly defined for u_t=-1. The Log Likelihood Ratio (LLR) of $P_r\{u_t=\pm1|y\}$ can be computed as:

$$L(u_t\mid y)=\log\left(\frac{P_r\{u_t=+1\mid y\}}{P_r\{u_t=-1\mid y\}}\right)=\log\left(\frac{\sum_{S^+(u_t=1)}\alpha_t(s')\cdot\gamma_t(s',s)\cdot\beta_t(s)/p(y)}{\sum_{S^-(u_t=-1)}\alpha_t(s')\cdot\gamma_t(s',s)\cdot\beta_t(s)/p(y)}\right) \tag{30}$$

, where we have

$$\alpha_t(s)=\sum_{s'\in S}\alpha_{t-1}(s')\cdot\gamma_t(s',s)$$

$$\beta_{t-1}(s')=\sum_{s'\in S}\beta_t(s)\cdot\gamma_t(s',s)$$

$$\gamma_t(s',s)=C_t(u_t)\cdot\gamma_t^e(s',s)$$

Let us now determine the complexity of decoder: $\gamma_t(s',s)$ need n+1 multiplications and n-1 additions, where n is legitimate transmitted code bits corresponding to the information bit. As for $\alpha_t(s)$ and $\beta_{t-1}(s')$ each term require S number of multiplications and S-1 number of additions, where $S=2^{Q-1}$ and Q is the constraint length of the code. Final the $P_r\{u_t=+1\mid y\}$ require 2S number of multiplications and S-1 number of additions. Therefore, the total of 4S+n+2 number of multiplications and 3S+n-4 number of additions are require to computing. However, we also have $P_r\{u_t=-1\mid y\}$ needed to calculate. By [18], the LLR in (30) need total of 2(4S+n+2)+1 = 8S+2n+5 number of multiplications/divisions and 2(3S+n-4) = 6S+2n-8 number of additions are required by decoder. But the multiplication is replaced by addition/subtraction in logarithmic domain,, and the addition/subtraction is replaced by addition, subtraction, table lookup, and maximum operations in logarithmic domain [Rob95]. Suppose we ignore table lookup and maximum operation, one addition/subtraction is equal to two addition/subtraction operations in logarithmic domain. Thus the number of addition/subtraction in logarithmic domain is given by

$$\begin{aligned}\text{comp}\{\text{BCJR}\} &= 8S+2n+5+2\times(6S+2n-8)\\ &=20S+6n-11\end{aligned} \tag{31}$$

6.2 LDPC_OFDM decoding complexity

The decoding complexity per iteration of LDPC codes in conjunction with parity check matrix having a column weight of *j* and a row weight of *k* can be approximated in terms of additions and subtractions.

$$\prod_{i=1}^{j}L(S\mid\{x_i\})=\log\left(\frac{P(S\mid x_d=1,y)}{P(S\mid x_d=0,y)}\right)=\log\left(\prod_{i=1}^{j}\left[\frac{1-\prod_{l=1}^{k}(1-2P_{il})}{1+\prod_{l=1}^{k}(1-2P_{il})}\right]\right) \tag{32}$$

where P_{il} is the probability the l^{th} bits in the i^{th} parity check set being a 1, $\prod_{l=1}^{k}(1-2P_{il})$ need (2k-3) number of multiplications and (k-1) additions, so we need (2k-3)×j number of multiplications and (k-1) ×j additions. The above equation will require another j+j-1 multiplications and 2×j additions. So the total required of (k+1)×j additions and (2k-2+j)×j multiplications.

The number of addition/subtraction in logarithmic domain for LDPC decoding is given by [18]

$$\text{comp}\{\text{LDPC}\}=(4k+j)\times j \tag{33}$$

The convolutional decoding have n = 816, S = 2^{Q-1}, where Q = 3, then S = 4. So the comp{CC}is calculated to be 4965. The max comp{LDPC} ≤ 4050 (j=3, k=6, low SNR iteration≤50) and the min comp{LDPC}≤810 (j=3, k=6, high SNR iteration≤10) . We can see that decoding complexity of LDPC_OFDM is lower than that of CC_OFDM.

7 Conclusions

The iterative ICI cancellation and LDPC decoding has not been proposed for OFDM systems. In this paper, we propose to combine two SISO modules: partial PIC/MMSE multicarrier detector (ICI cancellation) [11] and sum-product decoding of LDPC code (instead of BCJR decoding of convolutional codes in [11]). The proposed scheme has reduced decoding complexity (in terms of the number of addition/subtraction operations in Log domain) and the performance in fast-varying fading channels is better than [11], especially when the codeword length is longer.

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References:

[1] X1. Cimini, L. J. Jr., Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing, *IEEE Trans. Commun.*, vol.33, 1985, pp. 665-765.

[2] X2. ETS 300 401, Radio Broadcasting Systems: Digital Audio Broadcasting to mobile portable and fixed receivers, *ETSI*, 1996

[3] X3. EN 300 744, Digital Video Broadcasting (DVB); Frame structure, channel coding and modulation for digital terrestrial television, *ETSI*, 2004

[4] X4. IEEE 802.11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, *IEEE-SA*, 2007, doi:10.1109/IEEESTD.2007.373646

[5] X5. IEEE 802.16, *IEEE-SA*, (http://standards.ieee.org/about/get/802/802.16.html)

[6] X6. Salous, S., Feeney, S., Khokhar, K., Lewenz, R., Measurements of radio channels and bit error rate estimation of IEEE802.16 standard in semi-rural environment at three frequencies in the 2-6GHz frequency band, *IET Communications*, Vol.2, Issue 7, 2008, pp. 878-885.

[7] X7. Seo, J.-W., Yeo, S.-S., Park, K.S., Chang, H.B., Cho, S.-E., Kim, D.-K., A robust and secure time-domain ICI canceller for OFDM based ubiquitous systems in time-varying multipath channels, *Journal of Supercomputing*, Vol.45, Issue 1, 2008, pp. 29-43.

[8] X8. Wicker, S. B., Error Control Systems for Digital Communication and Storage, *Prentice Hall*, 1995

[9] X9. Gallager, R. G., Low Density Parity Check Codes, *Research Monograph Series. Cambridge, MA: MIT Press*, no.21, 1963.

[10] X10. Shu-Ming Tseng, Chih-Hung Hsieh, Yung-Chung Wang, Fu-Hsiang Tsai, and Kuen-Cherng Lin, Performance of LDPC-coded optical on-off keying communication systems, *Microwave and Optical Technology Letters*, vol.49, no.4, 2007, pp. 976-979.

[11] X11. Gruenbacher, D. M. and Serener, A., Performance of coded OFDM in a fading environment using high rate low-density parity-check codes, *in Proc. GLOBECOM '01*, vol.1., 2001, pp. 504-508.

[12] X12. H. Futaki, T. Ohtsuki, Low-density parity-check (LDPC) coded OFDM systems, *in Proc. 54-th IEEE Vehicular Technology Conference (VTC 2001-Fall)*, vol.1., pp. 82-86.

[13] X13. Yoo, H., Guilloud, F., Pyndiah, R, PAPR reduction for LDPC coded OFDM systems using binary masks and optimal LLR estimation, *Signal Processing*, Vol.91, Issue 11, 2011, pp. 2606-2614.

[14] X14. Zou, D., Djordjevic, I.B., Multi-Tb/s optical transmission based on polarization-multiplexed LDPC-coded multi-band OFDM, *International Conference on Transparent Optical Networks*, Article number 5970876, 2011

[15] X15. B. Lu, Guosen Yue, and Xiaodong Wang, Performance analysis and design optimization of LDPC-coded MIMO OFDM systems, *IEEE Trans. Signal Processing*, vol.52, no.2, 2004, pp. 348-361.

[16] X16. Hyoung Soon Kim and Sin Chong Park, Iteration based performance evaluation of LDPC coded MIMO-OFDM, *in Proc. the 8th International Conference Advanced Communication Technology, (ICACT 2006)*, vol.3, 2006, pp. 2103-2105.

[17] X17. S. Salari, M. Ahmadian, M. Ardebilipour, V. Meghdadi, and J.P. Cances, Carrier-Frequency Offset Estimation via the EM Algorithm for MIMO-OFDM Iterative Receivers, *in Proc. Fifth Annual Conference on Communication Networks and Services Research, (CNSR '07)*, 2007, pp. 101-106.

[18] X18. Xu, X., Mathar, R., Low complexity joint channel estimation and decoding for LDPC coded MIMO-OFDM systems, *2011*

IEEE 73rd Vehicular Technology Conference, VTC2011-Spring; Budapest, 2011

[19] X19. S. Salari, Iterative Receiver Design with Joint Carrier-Frequency Offset and Channel Estimation for LDPC-Coded MIMO-OFDM Systems, *Wireless Pers Commun.*, vol.62, 2012, pp. 127-138.

[20] X20. Jeon, W. G. et al., An equalization technique for orthogonal frequency-division multiplexing systems in time-variant multipath channels, *IEEE Trans. Commun.*, vol.47, no.1, 2001, pp. 27-32.

[21] X21. Muschallik, C., Improving an OFDM reception using an adaptive Nyquist windowing, *IEEE Transactions on Consumer Electronics*, vol.42, no.3, 1996, pp. 259-269.

[22] X22. Zhao, Y. and Häggman, S., Intercarrier interference self-cancellation scheme for OFDM mobile communication systems, *IEEE Trans. Commun.*, vol.49, no.7, 2001, pp. 1185-1191.

[23] X23. Cai, Jun, Mark, Jon W., and Shen, Xuemin, ICI cancellation in OFDM wireless communication systems, *in Proc. IEEE GLOBECOM '02*, 2002, pp. 17-21.

[24] X24. Wang, X. and Poor, H. V., Iterative (turbo) soft interference cancellation and decoding for coded CDMA, *IEEE Trans. Commun.*, vol. 47, 1999

[25] X25. Shu-Ming Tseng, An Iterative ICI Cancellation and Decoding Scheme for Coded OFDM Systems in Mobile Channels, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol.E88-A, 2005, pp. 3085-3091.

[26] X26. Bahl, L. R. et. Al, Optimal decoding of linear codes for minimizing symbol error rate, *IEEE Trans. Inform. Theory*, vol.IT-20, 1974, pp. 248-287.

[27] X27. F. R. Kschischang, B. J. Frey and H. A. Loeliger, Factor graphs and the sum-product algorithm, *IEEE Trans. Inform. Theory*, vol.45, 1999, pp. 399-431.

[28] X28. H. Futski, T. Ohtsuki, Performance of Low-Density Parity-Check (LDPC) coded OFDM systems, *in Proc. IEEE International Conference ICC 2002*, vol.3, 2002, pp. 1696-1700.

[29] X29. D.J.C. Mackay, Encyclopedia of Sparse Graph Codes, *(http://www.inference.phy.cam.ac.uk/mackay/codes/data.html)*

[30] X30. P. Dent, G. E. Bottomley, and T. Croft, Jake fading model revisited, *Electronics Letters*, vol.29, 1993, pp. 1162-1163.

[31] X31. M.Y. Alias, F. Guo, S. X. Ng, T. H. Liew, L. Hanzo, LDPC and turbo coding assisted space-time block coded OFDM, *in Proc. the 57th IEEE Semiannual Vehicular Technology Conference, 2003. VTC 2003-Spring*, Vol.4, 2003, pp. 2309-2313.

[32] X32. P. Roberson, E. Villebrun, and P. Hoher, A comparison of optimal and sub-optimal MAP decoding algorithms operating in the Log domain, *in Proc. IEEE International Conference on Communications*, 1995, pp. 1009-1013.

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