Models and Methods for Planning and Scheduling in Iron and Steel Making: Review and Prospects

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Abstract: The analysis and optimization of material flows, planning and scheduling in a steel industry are of considerable interest to steelmakers and academic researchers. Numerous publications in this area indicate a great potential for significant benefits and innovations in modern steel production. This article provides a review and analysis of recent publications in English and Russian on modeling, planning and scheduling in metallurgy and steel production. The main attention is focused on inaccurate data and uncertain factors characterizing most planning and scheduling problems arising in iron and steel making. Current challenges to be overcome and promising areas of a future research on material flows in steel production are discussed including technological processes, models and methods used in steel production, planning and control of steel smelting-continuous casting, dynamic planning and energy consumption optimization.

Key-Words: steel making and continuous casting; production planning and scheduling; inaccurate data.

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1 Introduction

The importance of metallurgical production for modern industries and economy is difficult to overestimate. The central place is occupied by steel and iron production. Currently, the world industry produces about 1.9 billion tons of steel annually. It is more than in the entire 19th century. Global steel production has more than doubled since 2000 alone.

The world's largest steel producer is China accounting for 54% of the world's crude steel production in 2023. Eight out of top ten steel producers are Chinese companies, namely: China Baowu Group, HBIS Group, Shagang Group, Ansteel Group, Jianlong Group, Shougang Group and Shandong Steel Group.

China is followed by India, Japan and the USA in steel production. Russia ranked fifth place in the world in steel production, having smelted a record volume of steel 75.8 million tons in 2023.

The features of planning and scheduling in modern steel making are characterized by the multi-stage, multi-operational and multi-energy nature of the most technological processes, which are accompanied by the presence of uncertain factors and input data.

The bottleneck in most steel production is the process of steel smelting and continuous casting. In metallurgical technological processes, there are sudden events; some data are inaccurate and uncertain as follows:

(a) Possible changes in the duration of technological operations in different technological stages;

(b) Accidental technological disturbances and other unforeseen events;

(c) Sudden breakdowns of machines or equipment with needed maintenances;

(d) Needed corrections of the technological routes in steel making (e.g., repetition of some technological operations that may be time-consuming and expensive).

The possible uncertainties of the durations of most technological operations directly affect the production rhythm and may disrupt the dynamic balance between logistics and production time. This may reduce the flexible suitability between the structure of intermediate and final products, production capacity and customer orders.

Optimization of planning and scheduling steel smelting-continuous casting with possible technological changes and uncertain technological operation durations is a key for improving the efficiency of the equipment available in the steel mill. Better schedules allow reducing waiting times in the production process and reducing material and energy consumption.

2 Technological Processes Used in Modern Iron and Steel Making

It is no exaggeration to say that without developed metallurgy, technical progress may be impossible in many modern industries.

Ferrous metallurgy is an interconnected complex of enterprises for the production of cast iron, steel and rolled products. Cast iron is obtained from ore by reducing iron from oxides.

Steel is smelted from liquid pig iron and steel scrap (no more than 30%) by oxidizing excess impurities in steel making units (open-hearth furnaces, oxygen converters or electric furnaces). Smelted steel is secondary processing in ladle furnaces, vacuum treatment and casting [1].

Steel is released from the smelting furnaces into a ladle (out-of-furnace processing unit). If necessary, steel is refined to the required parameters, like alloying steel. The ladle is transported by an overhead crane to the place of steel casting.

The technology and organization of steel casting determine the quality of the finished metal and amount of waste during further processing of steel ingots. Most economical methods are continuous casting, which are performed on Continuous Casting Machines (CCM for short); see [1] and [2].

Metal from the ladle is discharged into an intermediate pouring ladle. Then metal enters the copper crystallizer. The intermediate ladle allows casting in several crystallizers (strands). Due to the continuity of casting and crystallization, rather complete uniformity of the ingot structure along its entire length is achieved.

CCMs are used to cast square blanks (blooms), rectangular blanks (slabs), round solid blanks and round hollow blanks [1], [3], [4]. At present, about 60% of continuous cast ingots are slabs.

The next stage of steel production is rolling production, cold or hot rolling. Ingots are processed into I-beams, channels, angles or sheets. Ingots are also used to make pipes, wheels, etc.

In recent years, due to the high productivity and low metal consumption, the oxygen-converter method has become the main method of steel production and has mostly replaced less efficient open-hearth production worldwide.

Currently in China, 90.5% of steel is produced in converters. A characteristic feature of China is a large number of converters with their relatively small unit capacity. In recent decades, a number of converter shops with units of 200–300 tons have been built in China.

For a comparison, in the EU in 2005 there were 91 converters, while in China in 2004 there were 292 converters. It is planned that only converters with a melting mass of more than 120 tons will be built in China in near future [2].

Baowu is the largest steel company not only in China but in the world, producing for 7% of world steel. Its main steel plant in Baoshan contains six oxygen converters (three 250-ton converters and three 300-ton converters) and two electric arc furnaces (150 tons).

Shandong Taishan Iron & Steel Corporation's main site contains three 70-ton converters, two ladle furnace units and a continuous slab caster [3]. Tangshan Iron and Steel Corporation's main site contains four converters, two refining furnaces, five casters and five rolling lines [4].

Russian metallurgical production is represented by 60 metallurgical plants and factories. The largest metallurgical plants are located in the following regions: Novolipetsk, Magnitogorsk, West Siberian, Chelyabinsk, Cherepovets, Nizhny Tagil and Tula.

The largest metallurgical plants in Russia are equipped with oxygen converters; see Table 1. The rest of Russian metallurgical plants are equipped with electric furnaces.

The table shows that almost all the largest metallurgical plants in Russia have one converter shop (except for Novolipetsk and West Siberian, which have two). The shops are equipped with large converters (from 160 to 370 tons).

Each of the seven largest metallurgical plants in Russia has several continuous casting machines, most of which are multi-strand. Almost all of them have two or three ladle furnaces (four at the Nizhny Tagil Metallurgical Plant alone).

In addition, there are vacuum degassers and metal finishing units. Each melt must go through 4 to 6 stages, each of which has one unit and no more than five units. It should be noted that the total number of units is not very large, most often these numbers are from 14 to 17 units.

Thus, one can conclude from the table that scheduling problems arising in Russian metallurgical production have moderate sizes.

Table. Converter shops at the large Russian metallurgical plants.

Steel Plant	Equipment
Lipetsk production site	Three 160-ton converters; two ladle-furnace units; four metal finishing units; one vacuum degasser; four 2-strand slab CCM (two vertical and two curved)
	Three 330-ton converters; five metal finishing units; two chemical metal preheating units; two ladle-furnace units; one vacuum degasser; four curved; one radial slab CCM
Magnitogorsk Iron and Steel Works	Three 370-ton oxygen converters; three ladle-furnace units; two vacuum degassers; two steel finishing units; five slab CCM (four curved 2-strand and one vertical 1-strand)
West Siberian Metallurgical Plant	Three 160-ton converters
	Two 300-ton converters; a 2-position ladle-furnace unit; a 2-strand slab CCM and an 8-strand billet CCM
Cherepovets Metallurgical Plant	Three 350-ton converters; four metal finishing units and one ladle-furnace unit; one vacuum degasser; and five slab 2-strand CCM
Nizhny Tagil Metallurgical Plant	Four 160-ton converters; four ladle-furnace units; two circulating vacuum degassers; four CCM (produce blooms and slabs)
Chelyabinsk Metallurgical Plant	Three 160-ton converters; three ladle-furnace units; one chamber vacuum degasser; two 6-strand billet and one 5-strand bloom CCM
Tula-Steel enterprise	One 160-ton converter; one 2-position ladle-furnace unit; one chamber vacuum degasser; one 6-strand billet CCM

The last large open-hearth furnace in Russia was closed in 2018 (while the last open-hearth furnace in Western Europe was closed in 1995). As of 2021, about 2% of steel in Russia was smelted using the open-hearth furnaces remained at the Petrostal plant, at the Magnitogorsk Metallurgical Plant and at the Guryevsky Metallurgical Plant.

In Russia, converter steel production has been significantly expanded and electric steelmaking has been launched. Powerful hot and cold sheet rolling mills have been built and put into operation. Electrical equipment control systems have reached a new level.

Different requirements are imposed on the processes and equipment in steel making. For example, when smelting steel in a converter, the main problem is to obtain a predetermined composition of steel by carbon, which basically comes down to determining the time when purging stops. This problem is very difficult since there is usually no direct and accurate information about the carbon content in metal.

Furthermore, the rate of carbon burnout is so high that one minute of purging leads to the production of a different grade of steel. It should be noted that the rate of carbon burnout changes significantly in the course of the purging.

To ensure the same casting qualities, it is required to maintain a constant metal level in the intermediate ladle and mold. The increase in this level can lead to an overflow of metal from the mold. Therefore, the decrease in this level below the permissible limit may lead to a thin ingot crust, rupture and the breakthrough of liquid metal.

During ingot cooling, it is required to prevent excessive cooling of the ingot shell and to ensure that the ingot solidifies evenly throughout its thickness.

An important problem is also to maximize the yield of dimensional blanks from the available mass of liquid metal.

The optimal control is to select a number of strands depending on the remaining metal balance in the intermediate ladle and a metal consumption to obtain an ingot of measured length. In such a case, it may turn out that the casting is completed (in the case of four-strand CCM) in four, three, two or even one mold.

The steel smelting-continuous casting process is a multi-stage process, where each stage is associated with specific equipment, their parameters and processing time. As a result, steels of different grades are often subject to different secondary treatment procedures (by type and number of units in the technological route, by the duration of steel processing on a unit of a certain type, etc.).

For different grades of steel, different standards have been determined for the minimum and for the maximum permissible exposures of metal in steel ladles. Different numbers of available aggregates, such as steel ladles, must be used for each stage. These numbers can vary considerably over time, such as the number of used strands in the mold.

When assigning multiple jobs to the same equipment and machine sequentially, some additional time is required to set it up and check its performance. This is because of different jobs may have different process characteristics and processing requirements.

3 Models of Technological Processes Used in Iron and Steel Making

The importance of metallurgical production for the economy determines the interest of engineers and scientists in the problems of increasing efficiency of these productions. Numerous publications in the world are annually devoted to the research on energy saving and materials saving.

Many publications address to increasing the efficiency of the use of technological equipment, their productivity and to the quality of metallurgical products. See the following review papers: [5], [6], [7], [8], [9], [10] and [11].

In the article [12], it is noted that the problems of planning, management and logistics of complex metallurgical processes were not sufficiently well investigated. The method of organizing production data was proposed. This method is based on the concept of "object" (like a plant, workshop, department, production operation, unit of production, etc.). The relationship between the considered objects was investigated.

In particular, the object "hot-rolled coil" is associated with the slab from which it is rolled. The slab is associated with the smelting, from which it is cast on a continuous casting machine. The "melting" object is associated with a converter, secondary processing units and a continuous casting machine. In [12], the representation of production as an interconnection of objects was used as a universal structure for modeling and optimizing technological processes.

The complexity of the multistage problem of integrated planning of steel production with different routes of the technological process leads to heuristic algorithms and artificial intelligence methods presented in the literature.

The article [5] presents an extensive review of the state of research on production planning for the process of smelting and continuous casting and hot rolling of steel, which includes developments related to artificial intelligence in the world published up to 2001.

The review article [6] addresses the research on planning in the metallurgical production to optimize the use of raw materials and energy for improving the product quality and reducing a price of the final products.

This review includes articles published up to 2019 on production planning and scheduling, real-time optimization and management decision-making in different steel manufacturing processes.

3.1 Mixed Integer Mathematical Programming

With diversified requirements and varying manufacturing environments, the optimal production planning for a steel mill becomes more flexible and complicated. This flexibility provides operators with auxiliary requirements through the implemented integrated production planning.

In the article [13], a mixed integer nonlinear programming (MINLP for short) model was proposed for the optimal planning that incorporates manufacturing constraints and flexibility in a steel plate mill.

Two strategies were developed to overcome the computational weakness in solving the MINLP. The first strategy was to transform the MINLP into an approximate mixed integer linear programming using a linearization. The second strategy was based on decomposing the model via the branch-and-bound method. Computational experiments are presented in terms of effectiveness and applicability. The computational results showed that the second method performs better in running time of the computations and solution accuracy.

The problem of planning the main equipment for the processes of steel smelting and continuous casting was investigated in [14]. A scheduling method was developed consisting of an algorithm for assigning equipment based on the dynamic programming and a conflict resolution algorithm based on linear programming.

Models, algorithms and computational results of the implementation of the developed computerized scheduling system were also presented. The developed system meets the following requirements. Each casting plan is implemented on time. Loads of the similar heats are processed continuously on the same continuous casting machine. The waiting time for loads in front of each available machine does not exceed the preliminary specified value.

The article [15] is devoted to the problem of assigning melts to processing units of a converter steel foundry. For a given set of machines (secondary processing units), it is required to construct a schedule of the jobs (steel ladles with hot metal melts) with determined constraints on the sequence of machines in the process chain.

The initial position of jobs is a converter. The final position of jobs is a continuous steel casting machine. The closed intervals including processing durations for each job on each machine and the minimum time of a transportation of products between machines are also specified before scheduling. For each job, the time of receipt of the blank and the time of exit of the product from the converter section are specified.

The considered scheduling problem can be classified as a Resource Constraint Project Scheduling Problem with non-fixed durations of the jobs and the given constraints on the start time and the completion time of processing of each final product.

In the article [14], a mixed integer linear programming problem with the criterion of minimizing the total duration of the product transportation was proposed. This problem corresponds to the maximum total flow time of all jobs and is a good indicator of the energy efficiency of the technological process.

In the computational experiment conducted on statistical data on the implemented production scenarios, an improvement in functionality of 19.16% of the multi-stage processing system was demonstrated.

For a global optimization, the algorithm of group evolution and the algorithm of mathematical programming are often used in production planning in flexible flow shops. When planning and optimizing new production jobs, time-consuming iterative calculations have to be performed anew every time. To reduce such hard calculations, a deep learning model was proposed in [16].

The proposed model is "trained" on historical production data and is used to obtain a correlation between information about production jobs and information about input data and planning results.

The energy-saving problem in the steel process planning was considered in articles [17] and [18]. All problems with energy saving in steel and iron productions are characterized by uncertain parameters. In fact, for a technological operation in steel production, only the minimum (or possibly maximum) required processing time can be specified before scheduling. The actual operation processing time may be exactly determined in the moment of implementing this operation in the compiled schedule.

In [19], the problem of scheduling of steel smelting and continuous casting with controlled processing times was considered. The optimality of a schedule was determined based on three criteria as follows: minimization of the total waiting time, minimization of a premature time (or, conversely, a delay time) of the total job executions and reducing the schedule cost.

The original complex problem was divided into two simpler sub-problems, namely, the parallel machine scheduling problem at the last stage and the hybrid flow shop scheduling problem at all previous stages. A hybrid evolutionary algorithm was proposed to solve the first problem and an iterative reverse list scheduling algorithm for the second problem.

The variability of job processing times affects the value of the completion time of the entire process. In [20], a multi-criteria mixed integer linear programming was developed for optimizing steel production. The minimum value and the maximum value of the criterion (the production completion time) were calculated.

Given the variability of the operations durations, the solution to the problem is transformed into a choice from a set of Pareto-optimal solutions with different production completion due dates. It was proposed to use a strategy based on what-if analysis in combination with the normal boundary crossing method for creating a series of uniformly distributed Pareto solutions adapted to possible production scenarios.

The influence of different criterion values within the boundaries of the values of other criteria along the Pareto frontier was analyzed in order to accurately determine the minimum cost strategy for each possible scenario.

The article [21] contains the investigation of the scheduling problem derived from the production of steel sheets in Shanghai Baoshan Iron and Steel Complex (Baosteel). A mixed integer programming (MIP for short) model for scheduling production on critical (bottleneck) operations in Baosteel was presented, where technological constraints were also considered. The main objective was to determine the starting and ending times of production routes of operations under capacity constraints for minimizing the sum of the weighted completion times of all given jobs.

In steelmaking and continuous casting processes, a sudden converter fault can lead to unexpected changes to the pre-specified converter-continuous caster production mode in such a way that the used schedule becomes unrealizable. In the article [22], the dynamic scheduling problem as a response to the converter fault is analyzed.

A multi-objective nonlinear programming model was established by introducing the production mode parameter and the production schedule parameters. The developed method considers changes in the production mode, production schedule of charge and the interval uncertainties of the job durations.

The combined problem of energy production and management in the steel industry is considered in the articles [23], [24] and [25]. In [23], an optimization model based on mixed integer linear programming with the criteria of profit maximization and greenhouse gas emission reduction was developed. The model aims to obtain a robust solution taking into account various constraints and different input data scenarios.

Three scenarios for running the robust model were considered, namely: demand changes, energy resource capacity changes, and electricity generation capacity changes. The proposed model was applied to a steel company with its own energy production system in Iran. The obtained results showed that among the production planning factors, the cost of electricity was more significant than resource reserves and labor costs.

3.2 Lagrangian Relaxation

Due to the large numbers of variables in the MIP and other optimization models, a decomposition methodology based on a combination of Lagrangian relaxation, linear programming and heuristics was developed. This method relaxes constraints by coupling integer variables with continuous variables, which were introduced to the objective function via Lagrangian multipliers.

In the article [26], the relaxed problem is decomposed into two sub-problems by separating continuous variables from integer ones. The first sub-problem with continuous variables is a linear programming, which is solved using a software package. The second sub-problem is an integer programming problem being solved via decomposition. The sub-gradient optimization is used to update Lagrangian multipliers. A production scheduling simulation system for Baosteel company is developed by embedding several heuristics. Computational results for problems with up to 100 orders showed that the proposed Lagrangian relaxation method is stable.

In the article [15], the heuristic algorithm based on Lagrangian relaxations was developed for the problem of rescheduling a hybrid flow shop for smelting and continuous casting of steel with uncertain data and possible disturbances. Different features of the problem were considered, such as restrictions on product batches and variable processing time at the last processing stage. Three approaches to Lagrangian relaxations were proposed in the form of decomposition of the problem into sub-problems with smaller numbers of jobs, batches and machines.

A heuristic based on the solution of the relaxation problem was presented for obtaining admissible rescheduling. The optimality criteria were not only the production efficiency, which includes the total weighted time of the job completion and the total waiting time, but also the stability of the schedule (i.e., the difference in the number of operations processed on different machines for different processing stages in the original schedule and in the modified schedule).

Similar two-stage scheduling methods are used in the literature, for example, in the form of rescheduling previously constructed schedules.

The feasibility of using Markov chains to model the probabilities of the uncertain job durations for the processes of steel smelting and continuous casting was investigated in [27] and [28].

A Markov chain transition matrix for accurate modeling of the probability of uncertain processing times was established in [27] for increasing the efficiency of the Lagrangian relaxation algorithm. It is based on the iterative optimization strategy with controlled gradient direction without predicting the optimal value. An optimization model of the refining system of steelmaking production is created based on the discrete Markov chain.

A two-stage dynamic scheduling method was proposed in [28]. This method includes the production planning of charges and the production time. Dynamic scheduling software was developed and applied to scheduling the iron and steel company BaoSteel in China. The real-time application showed that the developed method can reduce scheduling time, increase the outputs of converters and shorten the redundant waiting time for molten steel.

Order management is a complicated problem in the production process of the iron and steel industry. Orders are the bridge between customers and final products in different units. Therefore, the scheduling of orders is arranged by skilled production planners. The initial scheduling may be infeasible during the production process due to the frequently possible variations of the production environment. The practical order rescheduling problem was investigated in [29] to adapt various changes that affect the normal production. The problem is presented based on the mixed integer programming model with considering the original objective, the deviation from the initial schedules, and the equilibrium of production capacity. The genetic algorithm with proposed mutation and crossover operators is developed for finding a near-optimal schedule for this complicated problem.

Computational experiments were conducted on randomly generated instances and the instances from the practical production. Computational results showed that the proposed scheduling algorithm could obtain better solutions compared with four standard differential evolution algorithms. A practical decision support system based on production data was developed. The system includes the algorithm to monitor the production process, diagnose whether there are changes of the orders and units, and make rescheduling decisions if necessary.

4 Uncertain Factors and Parameters

Different studies in the steel industry have to consider the data inaccuracy and uncertainty of dynamically changing process indicators when developing mathematical models and methods for solving practical planning and scheduling problems.

As indicated in [30], different uncertain factors affecting the processes in the production system of steel smelting and continuous casting can be divided into the following eight main classes.

(1) Resource-related factors such as equipment breakdown, lack of raw materials or lack of employees.

(2) Different logistical problems and congestion;

(3) Possible unavailability of equipment, machines, or tools.

The technological operation-related factors include the following possible uncertainties:

(4) Unexpected insertion (or, conversely, cancellation) of a technological operation;

(5) Corrections of the predetermined due dates;

(6) Change of the processing times needed for technological operations;

(7) Change of the time of receipt of the scrap;

(8) Change of transportation time of the part.

There are also common uncertain factors connected with quality of final products, lack of available energy, etc.

Various methods are used for taking into account uncertainty factors as follows: undefined parameters are included as constraints in mixed mathematical programming models; indeterminate parameters are considered as stochastic or fuzzy; experts are involved in the selection of forecast values and a suitable schedule; a "rough" schedule is built that is valid for feasible implementations of the undefined values.

Common methods are stochastic modeling, fuzzy planning and management, adaptive and robust control, and using schedule stability respecting to possible variations of the numerical parameters.

4.1 Statistical and Stochastic Methods

In stochastic planning and scheduling of the steel smelting and continuous casting process, it is assumed that the duration of smelt processing, the heats transport time between machines and the moments of heat entry are stochastic variables with a known mathematical expectation and variance; see [30], [31] and [32].

Uncertain scheduling problems are formulated as stochastic and the mathematical expectation is optimized for the used optimality criterion.

In the article [30], a preference-based algorithm and a procedure for selecting the best solutions for the final output are proposed to minimize the average waiting time, the probability of casting breakage, and the excess of the waiting time.

For a similar scheduling problem, a two-layered method is developed in [31] using a particle swarm optimization algorithm and a dispatching heuristic for real-time decision-making.

A similar two-layered method is proposed in [32] for a two-criterion problem of minimizing the total weighted waiting time of the jobs and the weighted time to complete processing of the product batches. The decision-maker creates an initial robust schedule that maintains optimality and ensures the feasibility of the schedule over possible running time. To obtain the final schedule online, a dynamic optimization method triggered by already realized events was proposed in [32].

Statistical and stochastic methods have certain limitations since it is necessary to have a significant amount of information about the relationship between controlled indicators and control actions. Large arrays of statistical samples are required to determine the distribution law of each stochastic parameter that is difficult to obtain and use in practice.

The same limitations remain valid for neural networks [33], which training needs a large amount of statistical information that must be collected in real time based on the existing technological process.

4.2 Fuzzy Logic Methods

A multi-criteria model for optimizing energy consumption and total cost in a steel plant is presented in [34] and [35].

In [34], the data that cannot be measured directly are replaced by empirical values. A fuzzy set approach is used for selecting the optimal schedule corresponding to each objective function and for creating trade-off strategies for selecting the final schedule. The influence of variables such as pellet ratio, scrap to ore ratio, hot slab charging temperature, and excess gas utilization are quantitatively analyzed in [34].

In the article [35], a methodology combining machine learning algorithms and mathematical programming with probabilistic constraints was proposed. A fuzzy clustering algorithm using artificial neural networks and edge detection method was developed. Due to this algorithm the need for well-defined distributions to approximate uncertain data was eliminated. When implementing the proposed approach, an improvement in the energy consumption criterion values was 20–70% and an improvement in the Pareto optimal solutions was 50–100%.

For the process of steelmaking in an electric arc furnace, a fuzzy multi-agent system was proposed in [36], where each process was assigned to one of the considered agents. The ingot casting agent determines the processing time at the stage, which differs for ingots of different weights. The vacuum degassing agent is responsible for controlling the degassing process, which includes determining the degassing parameters and calculating the duration of the treatment. An adaptive neuro-fuzzy inference system was used to create the agents' knowledge. When fuzzy systems were used, multi-agent systems more accurately reflected the real process.

An intelligent model for predicting the output characteristics of metal products based on fuzzy modeling and fuzzy Takagi-Sugeno-Kang inference was proposed in [37] for taking into account the influence of the uncertain factors. To ensure adequate functioning of the system and to minimize the impact of uncertainties on the decision-making process, an iterative reduction of the closed intervals of possible durations of the operations was proposed.

Despite the fact that the mathematical apparatus for fuzzy logic control is widely used in metal production, it does not allow to manage quickly the quality of technological operations, since the contribution of individual influencing parameters on the product quality is not clearly determined.

The fuzzy logic method also does not make it possible to predict the quality of innovative or non-standard products. Furthermore, in order to construct a fuzzy set, it is necessary to know the probability distribution of stochastic parameter within this set. However, information about this is often lacking, and variables can take any values from the given closed interval.

4.3 Robust Methods

The robust approach is used to solve heuristically optimization problems in technological systems that are subject to various types of perturbations. Undefined input data are often represented by the bounds of possible closed intervals.

A two-stage robust approach was proposed in the article [38] for the problem of planning the steel smelting and continuous casting process and in the article [39] for the problem of determining the production volume and distribution of finished products by order with the uncertain demand, the possibility of product substitution and different modes of steel production.

In [38], the considered problem is reduced to the hybrid flow shop scheduling problem with taking into account the uncertainty of steel production. In the offline stage, forecasting time series of steel output using historical data is created. The forecasting accuracy in the model is also calculated.

In the online stage, a robust schedule is created using Monte Carlo simulation and a genetic algorithm for the probabilistic constraint programming model of the previous stage.

The imbalance between fluctuating oxygen demand and a stable oxygen supply in the steel industry usually leads to excessive oxygen emissions and energy overconsumption. The article [40] addresses the problem of robust proactive process planning in a steel mill with parallel machines and uncertain maintenance times. The objective was to improve the machine assignment and processing routes rationality, to reduce fluctuations in oxygen consumption, and to ensure energy savings. It was formulated a scheduling problem with criteria of minimization of the average process time, the degree of schedule change, and fluctuations in oxygen consumption.

The problem of optimal oxygen distribution in the steel industry was considered in the articles [41] and [42].

The two-stage models of robust optimization with uncertain oxygen demand are proposed. An integrated planning model based on mixed integer linear programming was proposed with taking into account the probability distribution of oxygen demand. This model can become a reference model for other complex scheduling problems of a hybrid shop with different routes used for processing jobs on machines.

An adaptive robust optimization system is proposed in the paper [24] to evaluate the dispatch ability under uncertainty of steam and electric loads of the power supply system used in the steel plant.

In [25], a distributed adaptive robust optimization framework is proposed for an integrated energy system in the steel industry. Gas, steam and electricity networks are considered as distributed participants. The objective function is defined as the minimum cost of the system over several periods of operation. The model includes the cost of carbon emissions and economic operating costs (i.e., the cost of gas emissions, steam emissions, energy purchase, start-up and shutdown costs of energy equipment).

A robust optimization model was developed to remove the forecast uncertainties. In the optimization stage, the predicted result for each energy source is considered as uncertain interval of possible values of parameters.

A combination of robust and stochastic approaches is used in [43] to solve the scheduling problem of steel smelting and continuous casting planning, taking into account the uncertainty of the processing parts durations. A model of stochastic planning is proposed with the search for the robust schedule that is acceptable for uncertain input data.

An adaptive optimization for the worst case is carried out (i.e., an adaptive robust model is built). A schedule is then built based on adaptive stochastic programming without the need to solve a new optimization problem again. The value of the optimality criterion is the expected lead time, which is determined as the casting completion time for uncertain implementations.

5 Optimizing the energy consumption in multi-energy iron and steel production

A review of energy efficiency improvements is presented in [11]. Improving energy efficiency is based on cost-effective methods to advancing energy conservation via reducing emissions and achieving sustainable development of the multi-energy iron and steel production.

The review [11] presents main component units and energy networks within the steel production system. A comprehensive analysis is conducted from multiple perspectives: energy evaluation, diagnosis, benchmarking analysis, optimization and energy-saving measures. The review examines the diverse challenges encountered by steel manufacturing in realizing energy efficiency improvements and exploring energy-saving potential.

It is noted in [11] that a future research directions must include multi-level evaluation systems, implementation of energy efficiency improvement technologies, and adoption of intelligent energy systems that are crucial for modern manufacturing in the steel sector.

In [17], the production process is considered in the form of a problem of planning a flexible flow shop with parallel machine and batch processing at the middle stage. It is proposed an algorithm of an artificial bee colony with feedback with criteria for increasing the speed of response and minimizing the total delay and total energy consumption.

A two-tier strategy to simultaneously improve production efficiency (in terms of speed, average energy consumption per load, and fluctuations in energy consumption) and reduce energy costs was used in [18]. To solve this problem, a multi-criteria evolutionary algorithm was proposed.

In [44], a molten iron allocation problem is studied. The problem includes an allocation of molten iron from blast furnaces to steel-making furnaces. The input data include the release dates of the molten iron determined by the draining plan of the blast furnaces and the transport time between the iron-making and steel-making stages. Time constraints for processing the molten iron must be satisfied to avoid freezing. The objective function is to find a schedule with minimal total weighted completion time. This function reflects the practical aim of improving energy efficiency and reducing cost caused by the need for reheating. This considered problem is an NP-hard parallel machine scheduling problem with restricted time windows.

The problem is formulated as an integer mathematical programming and solved by applying the Dantzig-Wolfe decomposition and branch-and-price algorithm. It is also proposed a state-space relaxation-based dynamic programming algorithm for each sub-problem. The obtained computational results showed that the proposed algorithms are capable of solving problems with up to 100 jobs to optimality within a reasonable running time.

In steel-making plants, complex chains of processes need to be scheduled efficiently to minimize time, energy cost, and maximizes productivity. The scheduling problem deals with optimizing the transport of hot metal in a steel production plant. In the article [45], an algorithm utilizing a multi-stage simulated annealing process adapted for the provided lexicographic evaluation is developed. It is based on two rounds of simulated annealing, each using a specific objective tailored for the corresponding part of the goals, with an emphasis on efficient moves.

The article [46] is devoted to the practical scheduling steelmaking problem with batching decisions and energy constraints. Batching is used to decide how to group and order a number of jobs to form groups to meet a batch production mode. Incorporating energy cost consideration into the scheduling process is motivated by practical needs and the potential of reducing the energy bill through optimal scheduling.

Based on the proposed energy expressions, a mathematical integer linear programming model is formulated and solved using the branch and bound algorithm, which is enhanced by the decomposition strategy working as a heuristic. The benefits of the proposed scheduling and stability analysis results are demonstrated on illustrative examples.

Computational experiments on randomly generated instances showed that the energy expression outperforms the state-of-the-art commercial solvers. The decomposition performs well running as an independent heuristic.

In the article [47], a method is proposed to solve the molten iron scheduling problem; see also [44]. A Boolean mixed-integer nonlinear programming model is developed for a special case of the considered scheduling problem. A method to solve this special case is described. This method includes assigning machine strategy and determining a mathematical model of eliminating machine conflicts.

Based on the developed algorithms, a heuristic algorithm based on the dispatching rule of First Come, First Served principle is developed to solve a general molten iron scheduling problem. The molten iron scheduling system with the two-stage method was applied to Shanghai Baosteel Company to realize molten iron scheduling. The results obtained in practice showed that this method is effective.

The article [48] is devoted to Steelmaking–Refining–Continuous Casting (SRCC for short). It is shown that SRCC is a key electricity-intensive and flexible process in the iron and steel production, having great potential in providing demand-side flexibility.

Optimal scheduling of resources for SRCC is very important for ensuring maximal flexibility. On the other hand, optimal scheduling for SRCC is seldom studied, which leads to a low ability for providing flexibility at iron and steel production and an inability to support the participation in flexibility transactions and reducing energy costs.

In [48], a basic resource–task network (RTN for short) and a corresponding model for SRCC scheduling are developed. For minimizing the power cost of the process (it is the objective function), the RTN model takes various load modes of refining ladle furnaces into account and provides an approach for scheduling flexible resources in the iron and steel plants. The energy cost of iron and steel under the RTN model for flexible resources was compared with that of the same model without flexible resources. The results of a 2-minute time-step case study showed 12000 Yuan can be saved by implementing such a flexibility-based schedule.

The one-ladle technology requires an efficient iron-making and steel-making interface. The scheduling of the hot metal ladle in the steel plant determines the overall operational efficiency of the production. Taking into account the uncertainties of real-world steel productions, the article [49] studies the dynamic scheduling problem of hot metal ladles and develops an uncertain data-driven three-layer method for solving this scheduling problem.

A dynamic optimization of the hot metal ladle operation with a minimum average turnover time (it is the objective function) was constructed in [49]. The smart perception of industrial scenes and autonomous identification of possible disturbances, adaptive configuration of dynamic procedures, and real-time adjustment of constructed schedules were realized.

The upper layer of scheduling generates a demand-oriented predictive schedule for hot metal ladles. The middle layer of scheduling adaptively adjusts this predictive schedule to obtain a feasible schedule according to the factual supply-demand relationship. In the lower layer of scheduling, three dynamic scheduling strategies were designed based on the characteristics of the dynamic disturbance, namely: real-time flexible fine-tuning, local machine adjustment, and global rescheduling.

Tested cases with 24h production data on a day during the system operation of a steel plant showed that the method effectively reduced the variation and operation time of the hot metal ladle and improved the stability of the iron making and steelmaking interface production rhythm. The data-driven dynamic scheduling strategy was effective, and the proposed method improved the operation efficiency of hot metal ladles.

6 Discussion and Prospects

In the article [50], comparing production planning and scheduling in metallurgy with other industries was realized. It was shown that metallurgical production is one of the most complexes in terms of production planning and scheduling.

The following six features of scheduling in metallurgy have been analyzed in [50].

1. A large assortment (tens of thousands of items), the need to take into account the specific requirements of customers that are fixed in the order (dimensions, chemistry, permissible deviations) when planning and scheduling.

2. A large number of redistributions (dozens of redistributions), numerous of alternative technological routes and units (alternative furnaces, converters, continuous casting machines, slitting and cross-cutting machines), equipment operating modes.

3. The need to take into account changeovers, which take a lot of time and, accordingly, significantly affect production volumes.

4. The need to take into account specific rules and requirements related to the peculiarities of equipment operation (serial heats, rules for rolling).

5. The need to take into account the possibility of picking a blank from the warehouse.

6. The need to transfer production plans to the workshop and obtains information on the actual production in real time (or sufficiently close to real time).

The review presented in Sections 1–5 shows that only a part of published research for scheduling in steel making takes into account the uncertainty of input data, possible changes of the technological routes, and disruptions of machines or equipment.

The most commonly used approaches used for production planning and scheduling in the steelmaking industry are based on mixed integer linear programming and different heuristic algorithms. Therefore, if unexpected events arise and the initial plan and schedule must be changed, then the changed plan and schedule for continuing the modified technological processes must be newly calculated.

Most published models, methods and algorithms for scheduling in the steel industry cannot provide stability and close correlation with already partially realized schedules and newly constructed ones due to unexpectedly changed technological routes or changed data. To overcome these drawbacks, we propose to use a stability method for scheduling jobs in some appropriate steel-making processes.

Several versions of the stability method were described in the articles [51], [52], [53], [54] for scheduling problems and in the articles [55] and [56] for other discrete optimization problems with multiple criteria.

6.1 Optimization Methods Based on Solution Stability

Some classical scheduling problems arising in steel smelting and continuous casting can be considered as a problem of scheduling jobs in a flow-shop or job-shop with minimizing schedule lengths (or production cost or penalty for due-date delay).

Most production processes in steel making include several stages for using dedicated (different) machines for a single direction of product (job) movement. Such a problem of creating an optimal schedule is called a flow-shop scheduling problem with uncertain duration of technological operations and the criterion of minimizing the schedule length (or other schedule parameters).

Only closed intervals of possible operation durations are assumed to be known before scheduling. Using the three-field notation specified in [57], this uncertain scheduling problem may be denoted as $F \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ provided that all machines are different and as $HF \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$, if there are identical machines in the flow-shop.

The stability method used for solving scheduling problems with uncertain parameters include the following sequence of steps completely or partially (see articles [51] and [52]):

- The analysis of stability of the optimal schedule to possible variations of input data (the stability region or radius of the optimal schedule has to be calculated);
- At the off-line stage, the construction of the Minimum Dominant Set of semi-active schedules (MDS for short) before the start of technological operations;
- A multi-stage decision-making process before the start of technological operations and at the on-line stage when some of the technological operations have been completed;
- To select an optimal schedule from the remaining MDS when it is possible; otherwise, to use an appropriate heuristic for choosing a schedule for processing the remaining technological operations.

In the stability method, it is usually assumed that the durations of the technological operation (and other numerical parameters) can take any real values from the specified closed intervals (segments).

The MDS must contain at least one optimal schedule for each possible scenario of input data; see [51] and [52]. Since there is usually no schedule that remains optimal for all scenarios of possible operation durations, it is advisable to minimize the cardinality of the constructed MDS.

A two-stage flow-shop scheduling system with uncertain operation durations and makespan criterion $F2 \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ was considered in the article [50], where the MDS was represented as a circuit-free digraph of schedule dominance. The properties of the schedule dominance digraph have been investigated in [51].

In the article [52], precise and heuristic algorithms for solving the job-shop problem $J \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ (in the job-shop, machine routes are given different for different jobs) are developed for selecting an effective schedule from the constructed MDS. The developed algorithms are polynomial in the number of jobs n, the asymptotic complexity being $O(n^2)$.

The constructed MDS allows the scheduler to quickly make good decisions about optimal scheduling the remaining jobs in real time when additional information about the jobs already performed becomes available.

The article [51] discusses on-line decision-making, when local information about the performance of some technological operations becomes known. On-line planning algorithms with asymptotic complexity $O(n^2)$ have been developed, and a computational experiment has been carried out to show the effectiveness of the proposed stability method for solving optimization problems under uncertainty of the numerical parameters.

6.2 Possible Generalization of the Stability Methods for Using in Steel and Iron Production

The published articles on the stability method deal with classical scheduling problems. The published stability methods allow considering possible changes in the duration of technological operations in one or more technological stages; see point (a) in Introduction and point (6) in Section 4.

We next outline possible extensions of the stability method such that the stability method will be available for constructing schedules for steel and iron making if input data will be uncertain. Thus, our aim is to describe how to present uncertainties (b), (c), (d), (4), (5), (7) and (8) in the weighted mixed graph models used in the stability method.

It should be noted that input data and scheduling methods were based on the networks in the form of the weighted mixed graphs in articles [51], [52] and other publications on the stability approach used for classical scheduling problems.

Let input data of the general scheduling problem $G \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ are represented as a weighted mixed graph $G=(Q,A,E)$. In the mixed graph $G=(Q,A,E)$, all technological operations are represented by the vertices of the set Q, the precedence relations determined on the set of operations are represented by the arcs of the set A, and the prohibitions of performing operations on the same equipment (machines or tools) are represented by the edges of the set E.

If the execution times of all technological operations Q are precisely known before the start of planning the production process, then we associate a non-negative real weight p_i (this is the duration of operation $i \in Q$) with each vertex $i \in Q$ in the mixed graph $G=(Q,A,E)$. As a result, we obtain a weighted mixed graph $G(p)=(Q(p),A,E)$.

The set of circuit-free digraphs $\Lambda(G)=\{G_1,G_2,...,G_\lambda\}$ generated by the mixed graph $G=(Q,A,E)$ by replacing its edges with arcs determines the set of all semi-active schedules of the problem $G \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$; see [58].

If vector $p=(p_1,p_2,...,p_{|Q|})$ of the durations of all technological operations Q is known, the weighted digraph of the set $\Lambda(G)=\{G_1,G_2,...,G_\lambda\}$ uniquely determines the earliest completion time of each technological operation. The feasibility of the schedule does not depend on the vector $p=(p_1,p_2,...,p_{|Q|})$ of operations durations. In other words, the set $\Lambda(G)=\{G_1,G_2,...,G_\lambda\}$ of circuit-free digraphs defining the set of semi-active schedules of the problem $G \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ is completely determined by the mixed graph $G=(Q,A,E)$ without any weights. Information about the vector $p=(p_1,p_2,...,p_{|Q|})$ of processing times is needed to determine whether the corresponding schedule is optimal or not, i.e., the optimality of the schedule is determined by the weighted mixed graph $G(p)=(Q(p),A,E)$.

Due to the high uncertainty of input data in steel-making processes, it is impossible to construct an optimal schedule in most cases. Thus, the stability method may be based on constructing the following Minimal Dominant Set of Approximate Schedules (MDAS for short) with a guaranteed error.

Definition: *A subset* $\Lambda^{\min}(G)$ *of the set of circuit-free digraphs* $\Lambda(G)=\{G_1,G_2,...,G_\lambda\}$ *generated by a mixed graph* $G=(Q,A,E)$ *by replacing its edges with arcs is MDAS if for each fixed vector* $p=(p_1,p_2,...,p_{|Q|})$ *of the operation durations, the subset* $\Lambda^{\min}(G)$ *of the set*

$\Lambda(G) = \{G_1, G_2, ..., G_\lambda\}$ *contains at least one approximate schedule with a given error.*

In contrast to the shop scheduling problems $F \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$, $G \mid p_i^L \le p_i \le p_i^U \mid C_{\max}$ and other classical scheduling problems, where a single machine performs each operation (task), in the multi-stage system, with multiprocessor tasks, the task may require several dedicated processors during the entire time of processing this task; see [58] and [59].

In the article [59], the mixed graph model was developed for scheduling problems $GMPT \,||\, C_{\max}$ with multiprocessor tasks. Two types of the precedence relations can be given in input data, namely: the operation must be completed before starting another operation; and the start-start precedence constraint when one operation must start before starting another operation.

It may be required that a subset of multiprocessor tasks has to be performed simultaneously in each feasible schedule.

Moreover, realize dates of the jobs and due dates for job completions are presented in the mixed graph models used in the article [59].

The above generalizations of the mixed graph models allow presenting the uncertain factors (4), (5), (7), and (8) described in Section 4.

The scheduling problems for steel making are usually multi-criteria in nature. The stability approach to multi-criteria optimizations was developed in articles [55] and [56].

7 Conclusion

This article provides a condensed review of the resent publications in English and Russian on modeling and scheduling in metallurgy and steel production. The attention is focused on the inaccurate data and uncertain factors characterizing most planning and scheduling problems arising in modern iron and steel making.

Complex metallurgical production systems operate with incomplete and imprecise initial data. This uncertainty must be taken into account when developing models of such systems for designing blank processing schemes and predicting the properties of processed materials.

Planning of technological operations in the steel smelting-continuous casting process is a "bottleneck" in steelmaking. Therefore, production planning and scheduling for steel smelting-continuous casting is an uncertain scheduling problem with multiple criteria. This problem can be formulated as an optimization scheduling problem on the weighted mixed graph presenting input data.

To solve this problem, methods and models of scheduling theory are used. The article proposes to use a method based on the stability of approximate schedules. This method uses the concept of a minimum dominant set of schedules. For steel production planning, the concept of MDS should be generalized to a dominant set of approximate schedules with a given error.

In forthcoming research, we plan to develop appropriate network models based on weighted mixed graph and multi-graph for presenting uncertain input data for some steel making technological processes. Stable scheduling algorithms will use these models to construct schedules that are fairy close to factually optimal schedules.

References:

[1] Dyudkin, D.A., Kisilenko, V.V. Steel production. Vol. 1. Processes of smelting, extra-furnace treatment and continuous casting, Moscow: "Teplotekhnik", 2008. 528 p. (in Russian). https://rusneb.ru/catalog/000199_000009_003368371/

[2] Converter progress. Available from https://readmetal.com/?p=12809 (21/09/2024) (in Russian).

[3] Chen, P.D., Hao, J., Zhai, Y., Bruckner, C. Modernization of converters for smelting corrosion-resistant steels at the plant of Taishan Iron & Steel Co. *China Metallurgical production and technology*, Vol. 2, 2014, pp. 22–31 (in Russian). https://rudmet.net/media/articles/Article_MPT_02_14_pp.22-31.pdf

[4] Gu, Z., Xu, A., Chang, J., Li, S., Xiang, Y. Optimization of the production organization pattern in Tangshan Iron and Steel Co., Ltd. *Journal of Iron and Steel Research, International,* Vol. 21 (S1), 2014, pp. 17–22. DOI: 10.1016/S1006-706X(14)60114-X

[5] Tang, L., Liu, J., Rong, A., Yang, Z. A review of planning and scheduling systems and methods for integrated steel production. *European Journal of Operational Research*, Vol. 133, 2001, pp. 1–20. DOI: 10.1016/S0377-2217(00)00240–X

[6] Backman, J., Kyllönen, V., Helaakoski, H. Methods and tools of improving steel manufacturing processes: current state and future methods. *IFAC PapersOnLine*, Vol. 52, No. 13, 2019, pp. 1174–1179. DOI: 10.1016/j.ifacol.2019.11.355

[7] Malakizadi, A., Mallipeddi, D., Dadbakhsh, S., M'Saoubi, R., Krajnik, P. Post-processing of additively manufactured metallic alloys – a review. *International Journal of Machine Tools & Manufacture*, Vol. 179, 2022, pp. 1–39. DOI:10.1016/j.ijmachtools.2022.103908

[8] Abadi, M.M., Tang, H., Rashidi, M.M. A review of simulation and numerical modeling of electric arc furnace (EAF) and its processes. *Heliyon,* Vol. 10(11), 2024, No. e32157, pp. 1–21. DOI: 10.1016/j.heliyon.2024.e32157

[9] Li, X., Liu, H., Zhang, Y., Mahlknecht, J., Wang, C. A review of metallurgical slags as catalysts in advanced oxidation processes for removal of refractory organic pollutants in wastewater. *Journal of Environmental Management.* Vol. 352, 2024, No. 120051, pp. 1–16. DOI: 10.1016/j.jenvman.2024.120051

[10] Wang, Y., Liu, Z., Zhang, J., Mao, R., Zhang, Y. Advanced converter sludge utilization technologies for the recovery of valuable elements: a review. *Journal of Hazardous Materials,* Vol. 381, 2020, No. 120902, pp. 1–3. DOI: 10.1016/j.jhazmat.2019.120902

[11] Yuan, Y., Na, H., Chen, C., Qiu, Z., Sun, J., Zhang, L., Du, T., Yang, Y. Status, challenges, and prospects of energy efficiency improvement methods in steel production: a multi-perspective review. *Energy,* Vol. 304, 2024, No. 132047, pp. 1–24. DOI:10.1016/j.energy.2024.132047

[12] Peng, G., Wu, Y., Zhang, C., Shen, W. Integrated optimization of storage location assignment and crane scheduling in an unmanned slab yard. *Computers & Industrial Engineering*, Vol. 161, 2021, No. 107623, pp. 1–12. DOI: 10.1016/j.cie.2021.107623

[13] Lu, S., Su, H., Johnson, C., Wang, Y., Xie, L. Modeling and optimization methods of integrated production planning for steel plate mill with flexible customization. *Chinese Journal of Chemical Engineering*, Vol. 23, 2015, pp. 2037–2047. DOI: 10.1016/j.cjche.2015.10.003

[14] Kibzun, A.I., Rasskazova, V.A. Model of integer linear programming as mathematical support for the system of optimal planning of flow production at the stage of operational scheduling. *Automation and telemetry*, Vol. 5, 2023, pp. 113–132 (in Russian). DOI: 10.31857/S0005231023050069

[15] Sun, L., Luan, F., Ying, Y., Mao, K. Rescheduling optimization of steelmaking-continuous casting process based on the Lagrangian heuristic algorithm. *Journal of Industrial and Management Optimization*, Vol. 13 (3), 2017, pp. 1431-1448. DOI: 10.3934/jimo.2016081

[16] Wang, J., Tang, H., Lei, D. A robust dynamic scheduling approach based on release time series forecasting for the steelmaking-continuous casting production. *Applied Soft Computing Journal*, Vol. 153, 2024, No. 111254, pp. 1–12. DOI: 10.1016/j.asoc.2024.111254

[17] Lian, X., Zheng, Z., Wang, C., Gao, X. An energy-efficient hybrid flow shop scheduling problem in steelmaking plants. *Computers & Industrial Engineering*, Vol. 162, 2021, No. 107683, pp. 1–15. DOI: 10.1016/j.cie.2021.107683

[18] Jiang, S. Liu, M., Hao, J., Qian, W. A bi-layer optimization approach for a hybrid flow shop scheduling problem involving controllable processing times in the steelmaking industry. *Computers & Industrial Engineering*, Vol. 87, 2015, pp. 518–531. DOI: 10.1016/j.cie.2015.06.002

[19] Su, P., Zhou, Y., Wu, J. Multi-objective scheduling of a steelmaking plant integrated with renewable energy sources and energy storage systems: balancing costs, emissions and makespan. *Journal of Cleaner Production*, Vol. 428, 2023, No. 139350, pp. 1–12. DOI: 10.1016/j.jclepro.2023.139350

[20] Pan, Q.-K., Gao, L., Wang, L. A multi-objective hot-rolling scheduling problem in the compact strip production. *Applied Mathematical Modelling*, Vol. 73, 2019, pp. 327–348. DOI: 10.1016/j.apm.2019.04.006

[21] Tang, L., Lin, G. A mathematical programming model and solution for scheduling production orders in Shanghai Baoshan Iron and Steel Complex. *European Journal of Operational Research*, Vol. 182, 2007, pp. 1453–1468. DOI:10.1016/j.ejor.2006.09.090

[22] Yu, S., Chai, T., Wang, H., Pang, X., Zheng, B. Dynamic Optimal Scheduling Method and Its Application for Converter Fault in Steelmaking and Continuous Casting Production Process. *Proceedings of the 18th World Congress The International Federation of Automatic Control Milano (Italy) August 28 - September* 2, 2011, pp. 9506–9511. DOI: 10.3182/20110828-6-IT-1002.0179

[23] Karimi-Zare, A., Shakouri, G.H., Kazemi, A., Kim, E.-H. Aggregate production planning and energy supply management in steel

industry with an onsite energy generation system: A multi-objective robust optimization model. *International Journal of Production Economics*, Vol. 269, 2024, No. 109149, pp. 1–13. DOI: 10.1016/j.ijpe.2024.109149

[24] Wang, Q., Xu, X., Jia, H. Dispatchable onsite electricity generation evaluation of steel plants considering load uncertainty. *Energy Reports*, Vol. 9, 2023, pp. 939–946. DOI:10.1016/j.egyr.2023.05.017

[25] Wang, Z., Han, Z., Zhao, J., Wang, W. Distributed robust scheduling optimization for energy system of steel industry considering prediction uncertainties. *Information Sciences*, Vol. 666, 2024, No. 120431, pp. 1–19.

[26] Tang, L., Liu, G. A mathematical programming model and solution for scheduling production orders in Shanghai Baoshan Iron and Steel Complex. *European Journal of Operational Research*, Vol. 182, 2007, pp. 1453–1468. DOI:10.1016/j.ejor.2006.09.090

[27] Shiryaeva, E.N., Polyakova, M.A., Golubchik, E.M. Accounting information uncertainty in constructing models of multi-stage multi-operation technological processes for the production of metal products. *Technologies of metallurgy, mechanical engineering and materials processing*, Vol. 22, 2023, pp. 67–72, (in Russian) https://elibrary.ru/item.asp?id=59757569

[28] Sun, L., Lu, T., Li, Z., Li, Y., Yu, Y., Liu, J. Research on steelmaking-continuous casting production scheduling problem with uncertain processing time based on Lagrangian relaxation framework. *International Journal of Automation and Control*, Vol. 16 (1), 2022, pp. 87–107. DOI:10.1504/IJAAC.2022.119425

[29] Guoa, Q., Tang, L. Modelling and discrete differential evolution algorithm for order rescheduling problem in steel industry. *Computers & Industrial Engineering*, Vol. 130, 2019, pp. 586–596. DOI: 10.1016/j.cie.2019.03.011

[30] Jiang, S.-L., Zheng, Z., Liu, M. A preference-inspired multi-objective soft scheduling algorithm for the practical steelmaking-continuous casting production, *Computers & Industrial Engineering*, Vol. 115, 2018, pp. 582–594. DOI: 10.1016/j.cie.2017.10.028

[31] Hao, J., Liu, M., Jiang, S., Wu, C. A soft-decision based two-layered scheduling approach for uncertain steelmaking-continuous casting process. *European Journal of Operational Research*, Vol. 244, 2015, pp. 966–979. DOI: 10.1016/j.ejor.2015.02.026

[32] Jiang, S.-L., Liu, M., Lin, J., Zhong, H. A prediction-based online soft scheduling algorithm for the real-world steelmaking-continuous casting production. *Knowledge-Based Systems*, Vol. 111, 2016, pp. 159–172. DOI: 10.1016/j.knosys.2016.08.010

[33] Pivovarova, K. G., Tagirova V. M. Analysis of modern methods of metal products quality management. *Technologies of metallurgy, mechanical engineering and materials processing*, Vol. 22, 2023, pp. 125–134 (in Russian). https://elibrary.ru/item.asp?id=59757582

[34] Yuan, Y., Na, H., Du, T., Qiu, Z., Sun, J., Yan, T., Che, Z. Multi-objective optimization and analysis of material and energy flows in a typical steel plant. *Energy,* Vol. 263, 2023, No. 125874, pp. 1–14. DOI: 10.1016/j.energy.2022.125874

[35] Pantula, P.D., Mi, K. A data-driven approach towards finding closer estimates of optimal solutions under uncertainty for an energy efficient steel casting process. *Energy*, Vol. 189, 2019, No. 116253, pp. 1–14. DOI: 10.1016/j.energy.2019.116253

[36] Fazel Zarandi, M.H., Ahmadpour, P. Fuzzy agent-based expert system for steel making process. *Expert Systems with Applications*, Vol. 36, 2009, pp. 9539–9547. DOI:10.1016/j.eswa.2008.10.084

[37] Kabulova, E.G. Intelligent control of multi-stage systems of metallurgical production. *Modeling, optimization and information technologies.* Vol. 7 (1), 2019, pp. 341–351, (in Russian). DOI: 10.26102/2310-6018/2019.24.1.022.

[38] Long, J., Sun, Z., Pardalos, P. M., Bai, Y., Zhang, S., Li, C. A robust dynamic scheduling approach based on release time series forecasting for the steelmaking-continuous casting production. *Applied Soft Computing Journal*, Vol. 92, 2020, No. 106271, pp. 1–17. DOI: 10.1016/j.asoc.2020.106271

[39] Wang, G., Wu, J., Yang, Y., Su, L. Robust optimization for a steel production planning problem with uncertain demand and product substitution. *Computers & Operations Research*, Vol. 165, 2024, No. 106569, pp. 1–12. DOI: 10.1016/j.cor.2024.106569

[40] Lian, X., Zheng, Z., Zhu, M., Ga, X. Proactive scheduling for steel plants with unrelated parallel machines and time uncertainty. *Computers & Industrial*

Engineering, Vol. 188, 2024, No. 109890, pp. 1–17.
DOI:10.1016/j.cie.2024.109890

[41] Jiang, S.-L., Peng, G., Bogle, I. D. L., Zheng, Z. Two-stage robust optimization approach for flexible oxygen distribution under uncertainty in integrated iron and steel plants. *Applied Energy*, Vol. 306, 2022, No. 118022, pp. 1–14.
DOI: 10.1016/j.apenergy.2021.118022

[42] Zhang, L., Zhang, K., Zheng, Z., Chai, Y., Lian, X., Zhang, K., Xu, Z., Chen, S. Two-stage distributionally robust integrated scheduling of oxygen distribution and steelmaking-continuous casting in steel enterprises. *Applied Energy*, Vol. 351, 2023, No. 121788, pp. 1–17.
DOI: 10.1016/j.apenergy.2023.121788

[43] Kammammettu, S., Li, Z. Multistage adaptive optimization for steelmaking and continuous casting scheduling under processing time uncertainty. *IFAC PapersOnLine*, Vol. 51, No. 21, 2018, pp. 262–267.
DOI: 10.1016/j.ifacol.2018.09.429

[44] Tang, L., Wang, G., Liu, J. A branch-and-price algorithm to solve the molten iron allocation problem in iron and steel industry. *Computers & Operations Research,* Vol. 34, 2007, pp. 3001–3015.
DOI: 10.1016/j.cor. 2005.11.010.

[45] Kletzander, L., Musliu, N. A multi-stage simulated annealing algorithm for the Torpedo scheduling problem. In: Salvagnin D., Lombardi M. (eds). *Integration of AI and OR techniques in constraint programming*. CPAIOR 2017. Lecture Notes in Computer Science. Vol. 10335. *Springer, Cham;* 2017. pp. 344–358.
DOI: 10.1007/978-3-319-59776-8_28

[46] Xu, W., Tang, L., Pistikopoulos, E.N. Modeling and solution for steelmaking scheduling with batching decisions and energy constraints. *Computers and Chemical Engineering*, Vol. 116, 2018, pp. 368–84.
DOI: 10.1016/j.compchemeng.2018.03.010.

[47] Huang, H., Chai, T., Luo, X., Zheng, B., Wang, H. Two-Stage method and application for molten iron scheduling problem between iron-making plants and steel-making plants. *IFAC Proceedings.* Vol. 44, 2011, pp. 9476–9481.
DOI: 10.3182/20110828-6-IT-1002.01373.

[48] Wang, J., Wang, Q., Sun, W. Optimal power system flexibility-based scheduling in iron and steel production: a case of steelmaking–refining–continuous casting process. *Journal of Cleaner Production*, Vol. 414, 2023, No. 137619. DOI: 10.1016/j.jclepro.2023.137619.

[49] Zeng, L., Zheng, Z., Lian, X., Zhang, K., Zhu, M., Zhang, K., Xu, C., Wang, F. Intelligent optimization method for the dynamic scheduling of hot metal ladles of one-ladle technology on ironmaking and steelmaking interface in steel plants. *International Journal of Minerals, Metallurgy and Materials,* Vol. 30, 2023, pp. 1729–1739.
DOI: 10.1007/s12613-023-2625-6.

[50] Knyazhev, A. Planning and optimization of production of a metallurgical enterprise. *Metal Supply and Sales*, Vol. 10, 2009, pp. 68–71 (in Russian). https://www.metalinfo.ru/ru/magazine/2009/10/920

[51] Matsveichuk, N.M., Sotskov, Y.N., Werner F. The dominance digraph as a solution to the two-machine flow-shop problem with interval processing times. *Optimization,* Vol. 60, 2011, pp. 1493–1517.
DOI: 10.1080/02331931003657691

[52] Sotskov, Y.N., Matsveichuk, N.M., Hatsura, V.D. Schedule execution for two-machine job-shop to minimize makespan with uncertain processing times. *Mathematics,* Vol. 8, No. 1314, 2020, pp. 1–51.
DOI: 10.3390/math8081314.

[53] Allahverdi, A. Two-machine flowshop scheduling problem to minimize total completion time with bounded setup and processing times. *International Journal of Production Economics,* Vol. 103, No. 1, 2006, pp. 386–400. DOI: 10.1016/j.ijpe.2005.10.002

[54] Aydilek, A., Aydilek, H., Allahverdi, A. Production in a two-machine flowshop scheduling environment with uncertain processing and setup times to minimize makespan. *International Journal of Production Research*, Vol. 53, No. 9, 2015. pp. 2803–2819.
DOI: 10.1080/00207543.2014.997403

[55] Emelichev, V., Podkopaev, D. Quantitative stability analysis for vector problems of 0-1 programming. *Discrete Optimization,* Vol. 7, No. 1–2, 2010, pp. 48–63.
DOI: 10.1016/j.disopt.2010.02.001

[56] Emelichev, V., Kuzmin, K.A. General approach to studding the stability of a Pareto optimal solution of a vector integer linear programming problem. *Discrete Mathematics and Applications*, Vol. 17, No. 4, 2007, pp. 340–354. DOI: 10.1515/dma.2007.029

[57] Graham, R.E., Lawler, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G. Optimization and approximation in deterministic sequencing and scheduling: a survey, *Annals of Discrete Mathematics,* Vol. 5, 1979, pp. 287–326. DOI: 10.1016/S0167-5060(08)70356-X

[58] Brucker, P. Scheduling algorithms. Berlin: "Springer", 1995, 326 p. link.springer.com› book/10.1007/978-3-540-69516-5

[59] Mihova, E.I., Sotskov, Y.N. Mixed graph colouring as scheduling a partially ordered set of interruptible multi-processor tasks with integer due dates. *Algorithms*, Vol. 17, No. 299, 2024, pp. 1–32. DOI: 10.3390/a17070299

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The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.