Modeling and Research of Electromechanical Processes in Power Systems

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Abstract: - More and more heavy demands are placed on the research accuracy of the transient processes and the stability of the electrical power system (EPS). A method for automatically compiling a mathematical model of EPS is proposed, with the help of which transient electromechanical processes in EPS with an arbitrary distribution network and an arbitrary composition (number of elements) of the system (generators, motors, static loads) can be studied. As a result of the proposed method, the common model of the system consists of an algebraic system of equations whose node voltages are calculated non-iteratively, and separate systems of differential equations for each element of the EPS. An example of the study of transient processes in a specific system is given.

Key-Words: - power system, mathematical model, generator, motor, static load, converter, electrical network, disturbances, electromechanical transient processes, stability, automated model creation, numerical calculation methods.

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1 Introduction

Modern society and economy depend to a very large extent on electrical energy, therefore ensuring a reliable electricity supply is every country's main task. This is done not only by increasing the production of electricity, but also by the necessary transmission and distribution infrastructure.

Power systems are complex multi-connected systems consisting of different types of generators, static and dynamic loads, an increasing number of static converters, regulators and control systems. All of the aforementioned elements of power systems are connected by an electrical network with a complex topology. Individual countries connect their energy systems into a united power system (for example, the European one).

The main goal is to ensure the sustainable operation of the electricity system. For the purposes of designing and developing the electric power system (EPS), the regulators used and control systems and in order to ensure optimal, reliable and sustainable operation, modeling of the transient modes and processes under various operating and disturbing effects is used.

The article examines models of transient electromechanical processes in power systems in the time frame from 10^{-4} to 10^{2} sec. In this way, the following processes can be studied: switching surges, stator transient, sub-synchronous resonance, transient stability, protections, frequency control, term dynamics, [1].

The transient processes in EPS have a significant impact on the operating modes of generators and consumers, on their sustainable operation and on the quality of electrical energy. The state of the system is characterized by the operating modes, which are distinguished by mode parameters - powers, voltages, currents, and frequency, and by system parameters - resistances, conductivities, time constants, transformation coefficients, gain coefficients, etc. A reason for the occurrence of transients in EPS is the change of the system parameters, which leads to the change of the mode parameters.

The study of transient processes even in a separate system element, for example, a generator, is associated with significant difficulties due to the complex interrelated processes and dependencies in it, hence their complex mathematical description. These difficulties are greatly increased when the individual elements are connected into a system, when the elements acquire new properties and the degrees of freedom increase enormously.

The mathematical description of processes in EPS elements (generators, motors, static loads, regulators) has one main drawback - they are non-linear and contain variable coefficients. The main mathematical device used to describe the processes in them is their conversion to a rotating Cartesian system of coordinate axes d,q.

In many research cases it is not necessary to study the entire transient process at all EPS points, and a very high accuracy of the obtained results and characteristics is not required. In these cases, approximate methods for studying the transient processes are used: by simplifying the mathematical description based on physical considerations and mainly achieved by:

- neglecting the small and unimportant parameters and variables for the researched process, i.e., obtaining the equivalent replacement scheme of the elements;
- equating a group of generators or motors or whole parts of the EPS;
- dividing the systems of equations into EPS that are weakly connected - while the occurrences that are specific and important for one of the groups and secondary for the other, are taken into account approximately in the second;
- separate consideration of transient processes depending on their speed - electromagnetic, mechanical, thermal;
- linearization of the equations describing the processes in EPS.

As a result of the simulations, using the help of the mathematical models, information about the transient processes is obtained in four main forms, and each form corresponds to specific solution methods:

1. The variation of one or several mode parameters is calculated by integrating the equations.
2. Approximate solution or partial integration of the equations, that is, only some maximum values of essential parameters of the regime are studied during a small period after the disturbance.
3. Methods for qualitative research of the equations are used, while only characteristic ratios between the parameters and their variables are found;
4. Algebraic and frequency methods are used to study the stability of EPS.

The main tasks that need to be solved in the study of transient electromechanical processes in EPS are - the selection of a mathematical model of EPS elements (generators, motors, static loads, power converters); the choice of a model of the electrical network; the selection of numerical methods for process research. In addition, one of the main requirements is a minimum computing capacity for research.

The EPS is naturally decomposed (split into parts), with the elements included in the system with only the voltage vectors at the connection node and the element current. We will investigate the EPS with the RL scheme of the distribution network.

The article deals with the method of automatically compiling the model of the studied EPS, not imposing restrictions on the number of elements and the topology of the distribution network of the electrical system and the possible regulating and disturbing effects. The article is a continuation of the author's previous research, [2], [3], [4].

2 Models of Power System Elements

Different forms of presenting the equations of EPS elements are used, which depend on the neglected characteristics, on the chosen directions of the flux linkage vectors, currents, voltages, and phase sequence, on the per units, [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16].

The EPS model consists of an algebraic system for calculating nodal voltages (in which transients in the network are not considered) and a differential system of equations for element currents. Two methods are used for their numerical solution: 1. A separate sequential solution; 2. A simultaneous solution. In the first method, there is a "delay", that is, the algebraic variables are frozen at their old values while the value variables, which are calculated using differential equations, are calculated. Thus, iterations are required, which can cause numerical instability. In the second method, the algebraic and differential systems of equations are solved simultaneously, that is, there is no "delay". However, the overall system of equations has a very high order and the Newton method is usually used, that is, there is again an iterative calculation and factorization of the Jacobi matrix.

Electrical networks consist of transmission lines and transformers, each of which can be modeled by the π-equivalent circuits. The individual models are combined with network models by forming the nodal network equations.

The following generator models are used [12]:

- 5-th order model $(\dot{\delta}, \dot{\omega}, \dot{E}_d'', \dot{E}_q'', \dot{E}_q')$, $X_q' = X_q$ and $E_d' = 0$;
- 4-th order model $(\dot{\delta}, \dot{\omega}, \dot{E}_d', \dot{E}_q')$, damper windings are neglected and SG is represented by the transient emfs E_d' and E_q' and transient reactances X_d' and X_q';
- 3-rd order model $(\dot{\delta}, \dot{\omega}, \dot{E}_q')$, $X_q' = X_q$ and $E_d' = 0$;
- 2-nd order model (classical model) $(\dot{\delta}, \dot{\omega})$, I_d=const, E_f=const, that is generator is represented by E'=const behind the transient reactance X_d' and the swing equations.

Unlike the mentioned methods, this paper does not neglect the transients in the network and uses the full differential equations of the generators, motors, static loads and power electronic converters.

Equations of the electrical machines are writhen a rotating Cartesian coordinate system d,q. We will write the equations of the machine in a „per unit“ system of equal mutual inductances and magnetizing forces MF, [17].

2.1 Synchronous Generator

The equations are presented in Cauchy form in terms of currents:

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \\ i_f \\ i_g \\ i_h \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} . \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_g \\ i_h \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & 0 \\ b_{31} & 0 & b_{33} \\ b_{41} & 0 & b_{43} \\ 0 & b_{52} & 0 \end{bmatrix} . \begin{bmatrix} u_d \\ u_q \\ u_f \end{bmatrix} = \quad (1)$$

$$= \frac{d}{dt}\begin{bmatrix} \boldsymbol{I}_s \\ \boldsymbol{I}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{ss} & \boldsymbol{A}_{sr} \\ \boldsymbol{A}_{rs} & \boldsymbol{A}_{rr} \end{bmatrix} . \begin{bmatrix} \boldsymbol{I}_s \\ \boldsymbol{I}_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_{ss} & \boldsymbol{B}_{sr} \\ \boldsymbol{B}_{rs} & \boldsymbol{B}_{rr} \end{bmatrix} . \begin{bmatrix} \boldsymbol{U}_s \\ \boldsymbol{U}_r \end{bmatrix} = \boldsymbol{H} + \boldsymbol{B}.\boldsymbol{U}$$

$$\frac{d}{dt}\omega_k = \frac{1}{\tau_m}.(T_m - T_e) \quad (2)$$

where: by f is marked field, g, h – direct and quadrature damper windings; the model parameters a_{ij}, b_{ij} are a function of its parameters and the angular velocity of rotation ω_k; T_m, T_e - mechanical and electrical torques; τ_m - mechanical time constant; $T_e = x_q.i_q.i_d - (x_d.i_d + x_{ad}.i_f).i_q$.

2.2 Induction Motor

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \\ i_{rd} \\ i_{rq} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} . \begin{bmatrix} i_d \\ i_q \\ i_{rd} \\ i_{rq} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ b_{31} & 0 \\ 0 & b_{42} \end{bmatrix} . \begin{bmatrix} u_d \\ u_q \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{A}_{ss} & \boldsymbol{A}_{sr} \\ \boldsymbol{A}_{rs} & \boldsymbol{A}_{rr} \end{bmatrix} . \begin{bmatrix} \boldsymbol{I}_s \\ \boldsymbol{I}_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_s \\ \boldsymbol{B}_r \end{bmatrix} . [\boldsymbol{U}_s] = \boldsymbol{H} + \boldsymbol{B}.\boldsymbol{U} \quad (3)$$

$$\frac{d}{dt}\omega_r = \frac{1}{\tau_m}.(T_e - T_b) \quad (4)$$

where: s,r marks stator and rotor parameters and variables; the model parameters a_{ij}, b_{ij} are a function of its parameters and the angular speed of rotation coordinate axes ω_k and rotor ω_r; T_e, T_b - electrical and braking torques; τ_m - mechanical time constant; $T_e = x_{ad}.(x_{rd}.i_q - i_{rq}.i_d)$.

2.3 Static Active Inductive Load

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} . \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} . \begin{bmatrix} u_d \\ u_q \end{bmatrix} =$$

$$= \frac{d}{dt}\boldsymbol{I} = \boldsymbol{A}.\boldsymbol{I} + \boldsymbol{B}.\boldsymbol{U} = \boldsymbol{H} + \boldsymbol{B}.\boldsymbol{U} \quad (5)$$

where: $a_{11} = a_{22} = -r_l/l_l$; $a_{12} = -a_{21} = \omega_k$; r_l, l_l – respectively the active and inductive resistance of the load; ω_k – angular speed of rotation coordinate axes.

The mathematical models of the other types of motors (synchronous, permanent magnet synchronous, reluctance, direct current) are also written in the same Cauchy form of currents.

2.4 Power Electronic Converter

The power converter model is a changing structure system, modeled with switching functions [18] and the output voltage of the rectifier:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{L}.\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} - \frac{R}{L}.\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \quad (6)$$

$$-\frac{U_{dc}}{L}.\begin{bmatrix} (2S_a - S_b - S_c)/3 \\ (2S_b - S_c - S_a)/3 \\ (2S_c - S_a - S_b)/3 \end{bmatrix} = \frac{d}{dt}I_{abc} = \frac{1}{L}.\boldsymbol{U}_{abc} - \frac{R}{L}.\boldsymbol{I}_{abc} - \frac{\boldsymbol{U}_{in}}{L}.$$

where: $\boldsymbol{U_{abc}}$, $\boldsymbol{I_{abc}}$ - voltage and current vectors; $\boldsymbol{S}$ - the vector of the switching functions, whose elements can take a value of 1 or 0, according to the state of the switches; $\boldsymbol{U_{in}}$ - the input voltage of the converter; R, L - active and reactive resistance of the converter.

After the transformation to *d,q* axes, the system of equations will take the following expression:

$$\mathrm{d}/\mathrm{d}\boldsymbol{I}_{\mathrm{dq}} = \frac{1}{L}.\boldsymbol{U}_{\mathrm{dq}} - \frac{R}{L}.\boldsymbol{I}_{\mathrm{dq}} - \frac{U_R}{L}.\boldsymbol{P}.\boldsymbol{S} - \\ -\boldsymbol{W}.\boldsymbol{I}_{\mathrm{dq}} = \mathrm{d}/\mathrm{dt}\boldsymbol{I} = \boldsymbol{H} + \boldsymbol{B}.\boldsymbol{U} \tag{7}$$

where: $\boldsymbol{P}$ - matrix of the direct transformation of Park.

$$\boldsymbol{W} = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix}. \tag{8}$$

3 Models of Transmission Network

For EPS with multiple elements and with a developed electrical network, time-domain modeling is the most appropriate. In most cases, the general system of differential equations represents a large system of equations that requires computers with high computing power, [4]. Transients in the network, which are represented by impedances, are usually neglected.

Moreover, the resulting system of differential equations is a "stif" system with time constants differing by 10^3-10^4 times. The differential equations in the split form with E=const and τ=const, the stiff and the non-stiff parts are linear in their respective states.

In [19], the electrical machines are presented as current sources with commutations in the network, in order to exclude false voltage peaks, a small capacitance or a high resistance is placed from the terminals of the machine to the ground.

3.1 Algorithm for Automatic Formation of a Power System Model

We will consider an electrical network in which the network lines are represented by active inductive resistance (Figure 1).

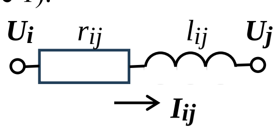

Fig. 1: RL_{ij} line of electrical network

The equations of the line, assuming that equations of the elements (generators, motors, loads, converters) of power system connected, at nodes i and j are written in different coordinate axes dq_i and dq_j:

$$\begin{bmatrix} u_{di} \\ u_{qi} \end{bmatrix} - \begin{bmatrix} cos\delta_{ij} & sin\delta_{ij} \\ -sin\delta_{ij} & cos\delta_{ij} \end{bmatrix} . \begin{bmatrix} u_{dj} \\ u_{qj} \end{bmatrix} = r_{ij} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + l_{ij} . \frac{d}{dt} \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + \begin{bmatrix} 0 & -l_{ij}.\omega_{ki} \\ l_{ij}.\omega_{ki} & 0 \end{bmatrix} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} =$$

$$= \begin{bmatrix} r_{ij} & -l_{ij}.\omega_{ki} \\ l_{ij}.\omega_{ki} & r_{ij} \end{bmatrix} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + l_{ij} . \frac{d}{dt} \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} = \tag{9}$$

$$= \boldsymbol{U}_{ij} = \boldsymbol{U}_i - \boldsymbol{T}_{ij} . \boldsymbol{U}_j = \boldsymbol{Z}_{ij} . \boldsymbol{I}_{ij} + \boldsymbol{L}_{ij} \frac{d}{dt} \boldsymbol{I}_{ij}$$

If the equations of element currents and their derivatives are converted to coordinate axes rotating synchronously ω_{syn}=1.0 p.u. the line equation will look like this:

$$\begin{bmatrix} u_{di} \\ u_{qi} \end{bmatrix} - \begin{bmatrix} u_{dj} \\ u_{qj} \end{bmatrix} = r_{ij} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + l_{ij} . \frac{d}{dt} \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + \begin{bmatrix} 0 & -l_{ij}.\omega_{ki} \\ l_{ij}.\omega_{ki} & 0 \end{bmatrix} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} =$$

$$= \begin{bmatrix} r_{ij} & -l_{ij}.\omega_{ki} \\ l_{ij}.\omega_{ki} & r_{ij} \end{bmatrix} . \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} + l_{ij} . \frac{d}{dt} \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} =$$

$$= \boldsymbol{U}_{\mathrm{ij}} = \boldsymbol{U}_{\mathrm{i}} - \boldsymbol{U}_{\mathrm{j}} = \boldsymbol{Z}_{\mathrm{ij}} . \boldsymbol{I}_{\mathrm{ij}} + \boldsymbol{L}_{\mathrm{ij}} \frac{d}{dt} \boldsymbol{I}_{\mathrm{ij}} \tag{10}$$

A characteristic feature of the EPS is that all its elements connected to the distribution network by two vectors: the node voltage vector to which the element is connected, and the current vector of the element itself. Therefore, naturally, the systems of equations describing processes in individual elements are compiled separately, especially since they are presented in Cauchy form of currents.

Combining the equations of all elements into a common system must be done via network equations. By using them, the vectors of nodal voltages, which are term of the right side of the systems of differential equations of elements (1), (3), (5), (7), should be calculated. Thus, the key issue when calculating transient processes in EPS is the choice of a method and an algorithm for calculating nodal voltages.

In contrast to the aforementioned methods for constructing a general model of EPS, a method for constructing a model is proposed, with the help of which separate systems of differential equations are obtained for each element of the system (generators, motors, loads, converters) in the form of Koshy for their currents, and through an algebraic system of equations the nodal voltages are calculated. Transient processes in the stators of electrical machines (the so-called „transformer electromotive voltages“) and in the distribution network are not neglected. In this way, the order of the solved systems of equations is reduced and iterations in the numerical calculation process are avoided. The method does not limit the number of EPS elements and the topology of the distribution network. The creation method is formalized and can be automated.

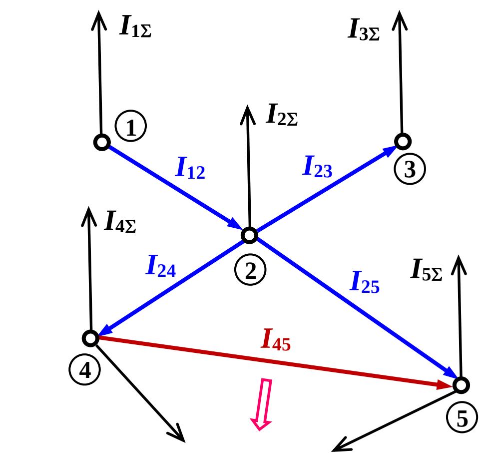

Fig. 2: Example of electrical network

The structure (topology) of an electrical network is divided into tree branches and links (as shown on Figure 2). Link equations are treated as RL loads at the respective nodes they connect. A model for the thus obtained structure of a network representing a graph-tree, that is, a radial network, is created.

Sum vectors of the currents of the respective nodes $\boldsymbol{I}_{j\Sigma}$, which include the currents of the elements, which are connected to this node:

$$\boldsymbol{I}_{\mathbf{j\Sigma}} = \sum\nolimits_{\mathrm{k}} m_k . \boldsymbol{I}_{\mathbf{k}} \tag{11}$$

where: $m_k = S_k/S_\Sigma$ – scale factor; S_k – apparent power of k^{th} element; S_Σ – apparent total power of EPS.

Using Kirchhoff's first law, line currents are replaced by sum vectors of node currents (example on Figure 2):

Node 1: $\boldsymbol{I}_{1\Sigma} = -\boldsymbol{I}_{12}$.

Node 2: $\boldsymbol{I}_{2\Sigma} = \boldsymbol{I}_{12} - \boldsymbol{I}_{23} - \boldsymbol{I}_{24} - \boldsymbol{I}_{25}$.

Node 3: $\boldsymbol{I}_{3\Sigma} = \boldsymbol{I}_{23}$.

Node 4: $\boldsymbol{I}_{4\Sigma} = \boldsymbol{I}_{24} - \boldsymbol{I}_{45}$.

Node 5: $\boldsymbol{I}_{5\Sigma} = \boldsymbol{I}_{25} + \boldsymbol{I}_{45}$.

(12)

After solving system (12), that is, substitution of the line currents, we get:

$$\boldsymbol{I}_{1\Sigma} + \boldsymbol{I}_{2\Sigma} + \boldsymbol{I}_{3\Sigma} + \boldsymbol{I}_{4\Sigma} + \boldsymbol{I}_{5\Sigma} = \mathbf{0} \tag{13}$$

For the general case of an n-node EPS:

$$\sum\nolimits_{1}^{n} \boldsymbol{I}_{j\Sigma} = \mathbf{0} \tag{14}$$

For one of the nodes (for example, the first one), Kirchhoff's first law is written in differential form:

$$\frac{d}{dt}\boldsymbol{I}_{1\Sigma} + \frac{d}{dt}\boldsymbol{I}_{2\Sigma} + \ldots + \frac{d}{dt}\boldsymbol{I}_{n\Sigma} = \mathbf{0} \tag{15}$$

This equation is true because these currents are continuous functions because they flow through active-inductive circuits.

For each of the remaining nodes of the EPS, the line equation (10) is used for its voltage through the voltage of the neighboring node:

$$\boldsymbol{U}_{\mathrm{i}} - \boldsymbol{U}_{\mathrm{j}} = \boldsymbol{Z}_{\mathrm{ij}}.\boldsymbol{I}_{\mathrm{ij}} + \boldsymbol{L}_{\mathrm{ij}}\frac{d}{dt}\boldsymbol{I}_{\mathrm{ij}} \tag{16}$$

We replace the derivatives of the sum currents of the nodes in the systems of equations (15) and (16) by the right-hand parts of the systems of differential equations (1), (3), (5), (7) and group the individual terms:

$$(\boldsymbol{K}_1 + \boldsymbol{L}.\boldsymbol{K}_2.\boldsymbol{B}).\boldsymbol{U} = -[\boldsymbol{L}.\boldsymbol{K}_2.\boldsymbol{H} + (\boldsymbol{L}.\boldsymbol{K}_2. + \boldsymbol{Z}.\boldsymbol{K}_2).\boldsymbol{I}] \tag{17}$$

where:

$\boldsymbol{U}=[\boldsymbol{U}_1, \ldots, \boldsymbol{U}_n]^T$; $\boldsymbol{I}=[\boldsymbol{I}_{1\Sigma}, \ldots, \boldsymbol{I}_{n\Sigma}]^T$; $\boldsymbol{L}=\mathrm{diag}[\boldsymbol{L}_{ij}]$; $\boldsymbol{Z}=\mathrm{diag}[\boldsymbol{Z}_{ij}]$; $\boldsymbol{B}=\mathrm{diag}[\boldsymbol{B}_{i\Sigma}]$;

$\boldsymbol{L}_{\mathrm{ij}} = \begin{bmatrix} l_{\mathrm{ij}} & 0 \\ 0 & l_{\mathrm{ij}} \end{bmatrix}$; $\boldsymbol{B}_{\mathbf{i\Sigma}} = \begin{bmatrix} b_{11\mathrm{ij}} & 0 \\ 0 & b_{22\mathrm{ij}} \end{bmatrix}$;

$\boldsymbol{Z}_{\mathbf{ij}} = \begin{bmatrix} r_{\mathrm{ij}} & -l_{\mathrm{ij}}.\omega_{\mathbf{kj}} \\ l_{\mathrm{ij}}.\omega_{\mathbf{kj}} & r_{\mathrm{ij}} \end{bmatrix}$; n - number of nodes.

The system of equations (17) is an algebraic system where the state variables, currents and angular speeds are known. It can be used to calculate the nodal voltages of the EPS. Most matrices are diagonal or quasi-diagonal, that is, sparsely filled.

The algorithm for calculating the transient electromechanical processes in EPS consists in the sequential solution of differential equations (1), (3), (5), (7) and the algebraic system of equations (17), that is, we obtained a non-iterative algorithm for numerical study of the processes.

3.2 Automation of the Research Process

Let's consider the process of automatic creation of the algebraic system of equations (17) based on the example from Figure 2.

The topological matrices $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ and $\boldsymbol{K}_3$ have the following form:

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$$\boldsymbol{K}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \qquad (18)$$

$$\boldsymbol{K}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (19)$$

where the rows correspond to the nodes and the columns to the edges of the graph.

Creating these two topology matrices is easy using a table that shows the relationship between nodes and edges. For the example shown in Figure 2 (for the directed graph):

Node **1**: −12
Node **2**: 12, −23, −24, −25
Node **3**: 23
(20)
Node **4**: 24, −45
Node **5**: 25, 45
Edge **1**: 12
Edge **2**: 23
Edge **3**: 24
(21)
Edge **4**: 25
Edge **5**: −45
Edge **6**: 45

The incidence matrix undirected graph:

$$\boldsymbol{D}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = -\boldsymbol{K}_1^{\mathrm{T}} \qquad (22)$$

That is, the topological matrix $\boldsymbol{K}_1$ is equal to the minus transposed incidence matrix of the graph:

$$\boldsymbol{K} = -\boldsymbol{D}^{\mathrm{T}} \qquad (23)$$

The creation of the topological matrix $\boldsymbol{K}_2$ is more complicated. To form this matrix, the undirected graph matrix of EPS $\boldsymbol{D}_2$ is used. Matrix $\boldsymbol{K}_2$ is formed as follows. It first equates $\boldsymbol{K}_2 = \boldsymbol{D}_2$, then performs sequential logical bitwise multiplication on each row (starting from the last of the previous ones):

$$\begin{gathered} \mathbf{D}_{2,i} \cap \mathbf{D}_{2,i-1} = \boldsymbol{K}_{i-1} \\ \mathbf{D}_{2,i} \cap \mathbf{D}_{2,i-2} = \boldsymbol{K}_{i-2} \\ \cdots\cdots\cdots\cdots \\ \mathbf{D}_{2,i} \cap \mathbf{D}_{2,i-k} = \boldsymbol{K}_{i-k} \\ \cdots\cdots\cdots\cdots \\ \mathbf{D}_{2,i} \cap \mathbf{D}_{2,1} = \boldsymbol{K}_1 \end{gathered} \qquad (24)$$

where i - the sequence number of the node.

At the same time, the $\boldsymbol{K}_{i-k}$ score obtained is analyzed for difference from zero. In the case of a non-zero result, the number of the row i-k is fixed and a logical bitwise summation of this row with the row of the product is performed:

If

$$\begin{gathered} \boldsymbol{K}_{i-1} \neq 0, \text{ then } \boldsymbol{D}_{2,i-1} = \boldsymbol{D}_{2,i} \cup \boldsymbol{D}_{2,i-1} \\ \boldsymbol{K}_{i-2} \neq 0, \text{ then } \boldsymbol{D}_{2,i-2} = \boldsymbol{D}_{2,i} \cup \boldsymbol{D}_{2,i-2} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \boldsymbol{K}_{i-k} \neq 0, \text{ then } \boldsymbol{D}_{2,i-k} = \boldsymbol{D}_{2,i} \cup \boldsymbol{D}_{2,i-k} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \boldsymbol{K}_1 \neq 0, \text{ then } \boldsymbol{D}_2 = \boldsymbol{D}_{2,i} \cup \boldsymbol{D}_2 \end{gathered} \qquad (25)$$

In case the logical multiplication results in a zero ($\boldsymbol{K}_{i-k}=0$), the corresponding row $\boldsymbol{D}_{2,i-k}$ of matrix $\boldsymbol{D}_2$ does not change.

In expressions (24) and (25) the following notations are adopted: i - matrix row number $\boldsymbol{K}_2$ and $\boldsymbol{D}_2$; $i = \overline{1,n}$; $k = \overline{1,i-1}$

∩ and U – logical multiplication and addition signs.

In the considered example the graph of the EPS network will be formed as follows.

$$\boldsymbol{K}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (26)$$

The uncircled units in matrix $\boldsymbol{K}_2$ correspond to the initial matrix $\boldsymbol{D}_2$, and the circled units appear as a result of the formation according to the algorithm shown above. The blue arrows connect the rows where ones appear in the non-zero result after logical multiplication and logical addition.

This transformation of the topological matrix $\boldsymbol{D}$ physically corresponds to replacing the currents of the lines of the EPS network $\boldsymbol{I}_{ij}$ with the sum currents of the nodes $\boldsymbol{I}_{j\Sigma}$. In this way, the topological matrix $\boldsymbol{K}_2$ is represented by its two submatrices – of the tree matrix of the graph $\boldsymbol{K}_2^{\mathrm{T}}$ and the link submatrix $\boldsymbol{K}_2^{\mathrm{L}}$:

$$\boldsymbol{K}_2 = \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\boldsymbol{K}_2^{\mathrm{T}} \,|\, \boldsymbol{K}_2^{\mathrm{L}}\right] \tag{27}$$

From the example given of an EPS that only has 5 nodes, it can be seen that the matrices of the algebraic system of equations (17) are sparsely filled. Moreover, numerical multiplications and additions in this system will lead to the accumulation of errors. Therefore, an algorithm is proposed for forming this system of equations directly:

$$\boldsymbol{A.U} = \boldsymbol{Y} \tag{28}$$

where:

$$\begin{aligned} \boldsymbol{A} &= (\boldsymbol{K}_1 + \boldsymbol{L}.\boldsymbol{K}_2.\boldsymbol{B}) \\ \boldsymbol{Y} &= -[\boldsymbol{L}.\boldsymbol{K}_2.\boldsymbol{H} + (\boldsymbol{L}.\boldsymbol{K}_2.+\boldsymbol{Z}.\boldsymbol{K}_2).\boldsymbol{I}] \end{aligned} \tag{29}$$

The structure of matrix $\boldsymbol{A}$ is determined by the structure of the topological matrices $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$. The matrix $\boldsymbol{A}$ has elements symmetric on the main diagonal and coincident in places with the non-zero elements of the incidence matrix $\boldsymbol{D}$, since $\boldsymbol{K}_1$ is lower triangular and $\boldsymbol{K}_2$ is upper triangular and both are functions of the incidence matrix of the distribution network graph. Therefore, the following matrix can be used as a prototype matrix:

$$\boldsymbol{A}_{\mathrm{p}} = \boldsymbol{D} + \boldsymbol{K}_2 = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \tag{30}$$

The presence of 2 in this prototype matrix means that matrix $\boldsymbol{A}$ has non-zero elements that are symmetric about the main diagonal, that is, it has a lower triangular part.

By scanning $\boldsymbol{A}_{\mathrm{p}}$, matrix $\boldsymbol{A}$ is directly formed. The same procedure is followed with the vector $\boldsymbol{Y}$, that is, $\boldsymbol{A}_{\mathrm{p}}$ is scanned again and the lower triangular matrix $\boldsymbol{K}_1$ is formed. The remaining matrices in a vector $\boldsymbol{Y}$ are diagonal and the multiplication and addition algorithm is very simple.

3.3 Simulation of Disturbances in the Power System

Disturbances can be simulated in three ways: by changing the parameters of elements and regulators; by changing the number of elements; and by changing the structure (topology) of the network. By changing the parameters of the elements and regulators we can simulate different normal and emergency modes. For example, the short circuit (s.c.) is simulated by changing the parameters of a symmetrical RL load connected to the s.c. point: $\boldsymbol{Z}_l \Rightarrow \boldsymbol{Z}_{sc}$. The exclusion of short circuit is simulated by restoring the previous one Z_l. By changing the braking torque T_b of the motor, we simulate the change in the load of the electric drive, etc.

The most typical modes in an EPS, such as start/stop of electrical drive, switch on/off of static load, and synchronization of synchronous generators, are simulated by changing the number model of the elements. In these cases, the model of the corresponding element (generator, motor, static load) is added or removed. Accordingly, the sum parameters and variables ($\boldsymbol{I}_{j\Sigma}$, $\boldsymbol{H}_{j\Sigma}$, $\boldsymbol{B}_{j\Sigma}$) of the corresponding EPS node [the system of algebraic equations (17)] to which the element is included or excluded are recalculated.

Transient modes caused by changes in the topology of the network are the most complicated to simulate. In this case, not only the number of the elements is changed (adding or removing their respective models), but the algebraic system for calculating the nodal stresses (17) is formed again. Most often, the change in the structure of the distribution network occurs when one of the protections is triggered, dividing the EPS into two autonomous parts. But there is an alternative parametric way of simulating a change in the network topology – by changing the parameters of the EPS line being turned off/on. When the line is turned off, its parameters increase hundreds of times, and when turned on, its normal parameters are restored. In this way, no changes are required in the system of equations (17).

4 Transient Processes Simulation

As can be seen from the above, the models of transients in EPS were derived and presented in a form that allows formalization of the composition process and allows this process to be automated. The models of all EPS elements are represented in Cauchy form and they are the same for individual element types (1), (2), (3), (4), (5), and (7), therefore the individual element models will differ only in their parameters and their initial values. Nodal voltages are calculated using the algebraic system of equations (17), the formation of which was shown above.

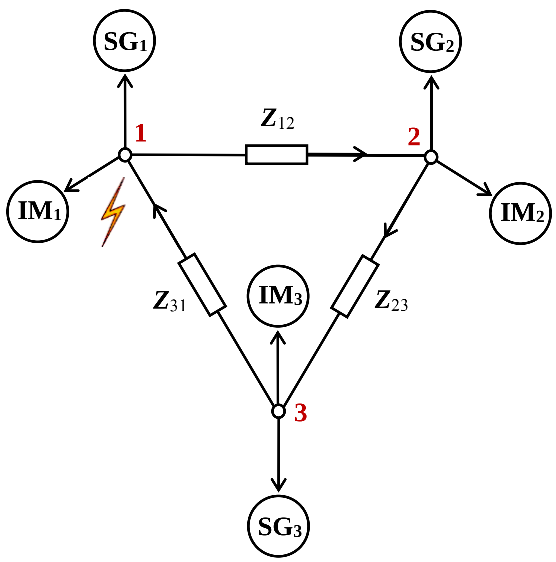

Fig. 3: Scheme of investigation system

Figure 3 shows the scheme of the investigation system, which has 3 nodes, 3 synchronous generators SG and 3 induction motors IM. Network parameters: x_{12}=0,0086, r_{12}=0.0005, x_{23}=0,015, r_{23}=0.01364, x_{31}=0,015, r_{31}=0.01364. Synchronous generators parameters: SG_1 x_{ad}=1,8335, x_l=0,1065, x_{fl}=0,103, x_{gl}=0,0326, x_{aq}=1,8335, x_h=1,856, r_s=0,0105, r_f=0,001936, r_g=0,0067, r_h=0,01, τ_m=1570, m_1=0,5; SG_2 x_{ad}=1,92, x_l=0,056, x_{fl}=0,128, x_{gl}=0,0975, x_{aq}=0,89, x_h=0,964, r_s=0,008, r_f=0,00183, r_g=0,0295, r_h=0,0052, τ_m=5133,8, m_1=0,25; SG_3 x_{ad}=1,92, x_l=0,056, x_{fl}=0,128, x_{gl}=0,0975, x_{aq}=0,89, x_h=0,964, r_s=0,008, r_f=0,00183, r_g=0,0295, r_h=0,0052, τ_m=5133,8, m_1=0,25; Induction motors: IM_1: x_{ad}=2,01, x_l=0,12, x_{rl}=0,096, r_s=0,0192, r_r=0,262, τ_m=86,92, m_{m1}=0,25; IM_2: x_{ad}=1,925, x_l=0,105, x_{rl}=0,092, r_s=0,0267, r_r=0,214, τ_m=78,5, m_{m1}=0,2667; IM_3: x_{ad}=1,925, x_l=0,105, x_{rl}=0,092, r_s=0,0267, r_r=0,214, τ_m=78,5, m_{m1}=0,2667; Standard mechanical speed governing system model [20] and excitation system model [21] were used in the research process.

The Figure 4, Figure 5 and Figure 6 show part of the results obtained during the simulations. At t=0÷62.8 turn on IM_1, at t=62.8÷94.2 short circuit in node 1, t=94.2÷120 turn off short circuit.

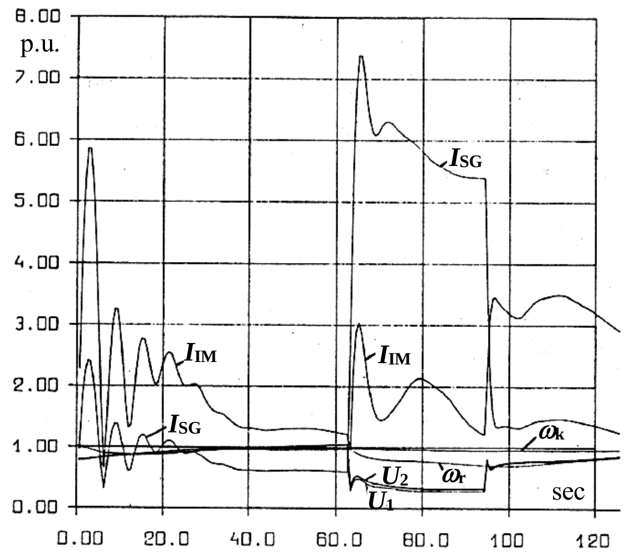

Fig. 4: Experimental results of the transition

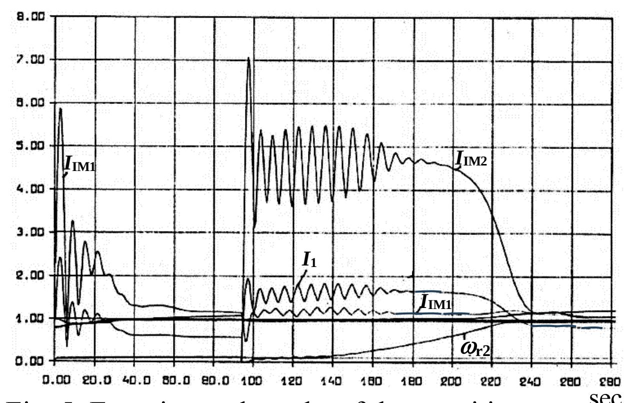

Fig. 5: Experimental results of the transition

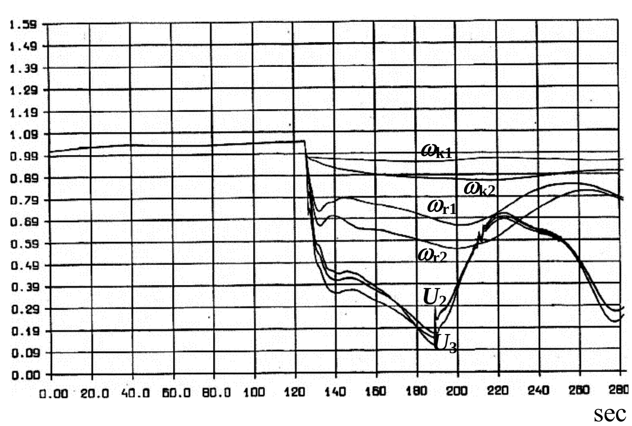

Fig. 6: Experimental results of the transition

Various programming languages are used to study the processes in EPS, [1]: legacy system languages (e.g., FORTRAN, C, C++); modern system languages (Delphi, Java, C++, Scala); scripting languages (Python, Perl); scientific-oriented languages (Matlab, Octave).

The presented method and algorithm for the study of transient electromechanical processes in EPS is implemented in the Matlab environment, and arbitrary numerical calculation methods can be used, [22], [23].

5 Conclusions

The requirements for increasing the accuracy and reliability of the study of the transient electromechanical processes in the EPS require consideration of the transient processes in the network. The presented EPS element models are complete without ignoring parameters and variables. A method is presented, an algorithm for the automatic generation of a mathematical model of an EPS with an arbitrary composition of generators, motors, and power converters, taking into account the transient processes in all parts of the system, including the network. The method automates simulations of random perturbations in random parts of the elements and the network.

The method presented requires minimum computational capabilities of the computer, since the general model consists of the n-number of systems of differential equations of the elements (n – number of elements) with minimum order and an algebraic system for calculating the nodal voltages of the network with order $2m$ (m – number of system nodes).

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare