WSEAS TRANSACTIONS on CIRCUITS and SYSTEMS
DOI: 10.37394/23201.2022.21.34

Global Attractivity of a Single Species Model with both Infinite Delay Merdan Type Allee effect

XIAORAN LI[a, b], QIN YUE[b], FENGDE CHEN[c]
[a] Faculty of Economics and Business, Universiti Malaysia Sarawak Kota Samarahan, 94300, Sarawak, MALAYSIA
[b] College of Finance and Mathematics, West Anhui University Yueliang Dao, West of Yunlu Bridge, Lu An, An Hui, CHINA
[c] College of Mathematics and Statistics, Fuzhou University No. 2, wulongjiang Avenue, Minhou County, Fuzhou, CHINA

Abstract: - This paper proposes and investigates a single-species model with infinite delay and Merdan- type Allee effect. The model takes the form

$$\frac{dx}{dt}=x(a-bx-c\int_{-\infty}^{t}K(t-s)x(s)ds)\frac{x}{\beta+x},$$

where a, b, c, and β are all positive constants. The differential inequality theory and iterative method are used to obtain sufficient conditions that ensure the global attractivity of the system's positive equilibrium. Our research shows that the Allee effect does not affect the final density of the species; however, the numerical simulations show that with the increase of the Allee effect, the system takes much time to the stale state.

Key-Words: Dynamical Systems, Systems Theory, Single species; Infinite delay; Global attractivity; Allee effect
Received: April 17, 2022. Revised: November 3, 2022. Accepted: November 22, 2022. Published: December 31, 2022.

1 Introduction

The paper aims to investigate the dynamic behaviors of the following single-species model with both infinite delay and the Allee effect

$$\frac{dx}{dt}=x\times\frac{x}{\beta+x}\times(a-bx \qquad (1.1)$$

$$-c\int_{-\infty}^{t}K(t-s)x(s)ds),$$

where a,b,c and β are all positive constants. $x(t)$ is the density of the d species at the time t. We shall consider (1.1) together with the initial conditions

$$x(t)=\phi(t),t\in(-\infty,0],i=1,2; \qquad (1.2)$$

Where $\phi_i=BC^+$ and

$BC^+=\{\phi\in C((-\infty,0],[0,+\infty)):\phi(0)>0$, and

ϕ bounded$\}$.

The delay kernel $K:[0,+\infty)\to(0,+\infty)$ is a continuous function such that

$$\int_0^{+\infty}K(s)ds=1. \qquad (1.3)$$

According to the fundamental theory of functional differential equations [39], system (1.1) has a unique solution $x(t)$ satisfying the initial condition (1.2). We can easily demonstrate $x(t)>0$ for all $t\geq 0$.

Many scholars have studied the dynamic behaviors of population models over the last few decades ([1]-[37]). Among those works, the model with infinite delays has been studied by many scholars ([1]-[18]). For example, Chen, Xie, and Wang [7] proposed a competition model of plankton allelopathy with infinite delay,

$$\dot{x}_1(t)=x_1(t)[K_1-\alpha_1x_1(t)$$
$$-\beta_{12}\int_{-\infty}^{t}K_{12}(s)x_2(t-s)ds$$
$$-\gamma_1x_1(t)\int_{-\infty}^{t}f_{12}(s)x_2(t-s)ds],$$
$$\dot{x}_2(t)=x_2(t)[K_2-\alpha_2x_2(t)$$

$$-\beta_{21}\int_{-\infty}^{t} K_{21}(s)x_1(t-s)ds$$
$$-\gamma_2 x_2(t)\int_{-\infty}^{t} f_{21}(s)x_1(t-s)ds], \quad (1.4)$$

They showed that for this system, delay and toxic substances are harmless for the stability of the interior equilibrium point.

On the other hand, Merdan [19] proposed the following predator-prey system with the Allee effect on prey species:

$$\begin{aligned} \frac{dx}{dt} &= rs(1-x)\frac{x}{\beta+x} - axy, \\ \frac{dy}{dt} &= ay(x-y), \end{aligned} \quad (1.5)$$

where β is a positive constant, which describes the intensity of the Allee effect. Merdan showed that the Allee effect harms the species. Since then, many scholars have proposed an ecosystem incorporating a Merdan-type Allee effect ([19]-[34]). The Merdan-type Allee effect appears to have a different impact on different population systems; for example, Lin[21] proposed a Lotka-Volterra commensal symbiosis model with the first species subject to the Allee effect

$$\begin{aligned} \frac{dx}{dt} &= x(b_1 - a_{11}x)\frac{x}{\beta+x} + a_{12}xy, \\ \frac{dy}{dt} &= y(b_2 - a_{22}y), \end{aligned} \quad (1.6)$$

he showed that the final density of the species subject to the Allee effect is also increased.

It draws our attention to the fact that all of those works ([19]-[34]) did not consider the influence of the delay, which is one of the most important factors to determine the dynamic behaviors of the species. This motivated us to propose the system (1.1), which is the most simple single species model with infinite delay and the Allee effect.

As far as the system (1.1) is concerned, one interesting issue is finding out the influence of the Allee effect. One may think it is possible to investigate the dynamic behaviors of the system (1.1) by constructing some suitable Lyapunov function similar to [5, 6, 8]. i.e.,

$$V(t) = \left|\ln x(t) - \ln x^*\right| + c\int_0^{+\infty} k(s)\int_{t-s}^{t} \left|x(\theta) - x^*\right| ds dt.$$

However, by simple computation, with the Allee effect term, it seems impossible to obtain conditions to ensure the negative of the D+V (t), hence, one could not apply the method of [5, 6, 8] to investigate the dynamic behaviors of the system (1.1).

The paper aims to find out the influence of the Allee effect on the system (1.1), the rest of the paper is organized as follows. In Section 2, by using the iterative method, we obtain a set of sufficient conditions that ensure the system (1.1) admits a unique globally attractive positive equilibrium. In Section 3, we give a numeric simulation to show the feasibility of the main result. We end this paper with a brief discussion.

2 Main result

Concerned with the stability of the system (1.1), we have the following result.

Theorem 2.1.
Assume that

$$b > c \quad (2.1)$$

then the system (1.1) admits a unique positive equilibrium $x^* = \frac{a}{b+c}$, which is globally attractive.

Before we begin to prove the main result, we need several Lemmas.

Lemma 2.1. [27] Consider the following equation:

$$\frac{dy}{dt} = y(a_2 - b_2 y)\frac{y}{u+y}, \quad (2.2)$$

the unique positive equilibrium $y^* = \frac{a_2}{b_2}$ is global stable.

Following Lemma 2.2 is Lemma 3 of Francisco Montes de Oca and Miguel Vivas[9].

Lemma 2.2. Let $x: R \to R$ be a bounded nonnegative continuous function, and let $k: [0,+\infty) \to (0,+\infty)$ be a continuous kernel such that $\int_0^{\infty} k(s)ds = 1$. Then

$$\begin{aligned} &\liminf_{t\to+\infty} x(t) \\ &\le \liminf_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)ds \\ &\le \limsup_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)ds \\ &\le \limsup_{t\to+\infty} x(t). \end{aligned}$$

Lemma2.3[27] System (1.1) has a unique positive equilibrium $x^* = \frac{a}{b+c}$.

Proof The equilibria of system (1.1) satisfy the equation

$$x(a - bx - c\int_{-\infty}^{t} K(t-s)xds)\frac{x}{\beta+x} = 0. \quad (2.3)$$

Noting that

$$\int_{-\infty}^{t} K(t-s)ds = 1,$$

the positive solution of the system (1.1) satisfies the equation

$$a - bx - cx = 0.$$

That is, system (1.1) allows for a unique positive equilibrium $x^* = \frac{a}{b+c}$.

This ends the proof of Lemma 2.3.

Lemma 2.4. Let $x(t)$ be any solution of $(1.1) \rightarrow (1.2)$, then $x(t) > 0$ for all $t \geq 0$.

Proof. We note that in (1.1), one has

$$\frac{dx(t)}{dt} = P(t)x(t), \qquad (2.4)$$

where

$$P(t) = \frac{x}{\beta + x} \times (a - bx - c\int_{-\infty}^{t} K(t-s)x(s)ds). \qquad (2.5)$$

Hence,

$$x(t) = x(0)\exp\{\int_0^t P(s)ds\} > 0. \qquad (2.6)$$

This ends the proof of Lemma 2.4.

Following, we will develop the method of Chen, Xie, and Wang [6] and Wu, Gao, and Chen [16] to prove the main result.

Proof of Theorem 2.1. From (1.7), it follows that there exists a $\varepsilon > 0$ sufficiently small such that

$$r_1 > \frac{b_1(\frac{r_2}{a_2} + \varepsilon)}{1 + (\frac{r_2}{a_2} + \varepsilon)} + a_1\varepsilon,$$
$$r_2 > \frac{b_2(\frac{r_1}{a_1} + \varepsilon)}{1 + (\frac{r_1}{a_1} + \varepsilon)} + a_2\varepsilon. \qquad (2.7)$$

Let $x(t)$ be any positive solution of system (1.1) with initial condition (1.1). From system (1.1) it follows that

$$\frac{dx}{dt} \leq x(a - bx)\frac{x}{\beta + x}. \qquad (2.8)$$

Now let's consider the auxiliary equation

$$\frac{du}{dt} = u(a - bu)\frac{u}{\beta + u}. \qquad (2.9)$$

It follows from Lemma 2.1 that the unique positive equilibrium $u^* = \frac{a}{b}$ is globally stable. That is,

$$\lim_{t\to+\infty} u(t) = \frac{a}{b},$$

As a result of employing the differential inequality theory, one has

$$\limsup_{t\to+\infty} x(t) \leq \frac{a}{b}, \qquad (2.10)$$

It follows from Lemma 2.2 that

$$\limsup_{t\to+\infty}\int_{-\infty}^{t} K(t-s)x(s)ds \leq \limsup_{t\to+\infty} x(t) \leq \frac{a}{b}, \qquad (2.11)$$

Hence, for sufficiently small $\varepsilon > 0$, which satisfies (2.7), it follows from (2.10) and (2.11) that there exists a $T_1 > 0$ such that for all $t \geq T_1$

$$x(t) < \frac{a}{b} + \varepsilon \stackrel{def}{=} M^{(1)}. \qquad (2.12)$$

$$\int_{-\infty}^{t} K(t-s)x(s)ds < \frac{a}{b} + \varepsilon \stackrel{def}{=} M^{(1)}. \qquad (2.13)$$

For $t > T_1$, it follows from the system (1.1) and (2.13) That

$$\frac{dx(t)}{dt} \geq x[a - bu - cM^{(1)}]\frac{x}{\beta + x}. \qquad (2.14)$$

Now let's consider the auxiliary equation

$$\frac{du}{dt} = u(a - bu - cM^{(1)})\frac{u}{\beta + u}. \qquad (2.15)$$

It follows from Lemma 2.1 that the unique positive equilibrium $u^* = \frac{a - cM^{(1)}}{b}$ is globally stable. That is,

$$\lim_{t\to+\infty} u(t) = \frac{a - cM^{(1)}}{b}.$$

As a result of employing the differential inequality theory, one has

$$\liminf_{t\to+\infty} x(t)\frac{a - cM^{(1)}}{b}, \qquad (2.16)$$

It follows from Lemma 2.2 and (2.16) that

$$\liminf_{t\to+\infty}\int_{-\infty}^{t} K(t-s)x(s)ds$$

$$\geq \liminf_{t\to+\infty} x(t) \geq \frac{a-cM^{(1)}}{b}, \tag{2.17}$$

Hence, for enough small $\varepsilon > 0$, which satisfies (2.7), it follows from (2.7) and (2.17) that there exists a $T_2 > 0$ such that for all $t \geq T_2$,

$$x(t) > \frac{a-cM^{(1)}}{b} - \varepsilon \overset{def}{=} m^{(1)}. \tag{2.18}$$

$$\int_{-\infty}^{t} K(t-s)x(s)ds > \frac{a-cM^{(1)}}{b} - \varepsilon \overset{def}{=} m^{(1)}. \tag{2.19}$$

For $t \geq T_2$, it follows from the system (1.1) and (2.19) that

$$\frac{dx(t)}{dt} \leq x[a-bx-cm^{(1)}]\frac{x}{\beta+x}. \tag{2.20}$$

Now let's consider the auxiliary equation

$$\frac{du}{dt} \leq u[a-bu-cm^{(1)}]\frac{u}{\beta+u}. \tag{2.21}$$

It follows from Lemma 2.1 that the unique positive equilibrium $u^* = \frac{a-cm^{(1)}}{b}$ is globally stable. That is,

$$\lim_{t\to+\infty} u(t) = \frac{a-cm^{(1)}}{b}.$$

Thus, by using the differential inequality theory, one has

$$\limsup_{t\to+\infty} x(t) \leq \frac{a-cm^{(1)}}{b}, \tag{2.22}$$

It follows from Lemma 2.2 that

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K(t-s)x(s)ds \leq \limsup_{t\to\infty} x(t) \leq \frac{a-cm^{(1)}}{b}. \tag{2.23}$$

Hence, for $\varepsilon > 0$ which satisfies (3.1), it follows from (3.15)-(3.16) that there exists a $T_3 > 0$ such that for all $t \geq T_3$,

$$y_1(t) < \frac{a-cm^{(1)}}{b} + \frac{\varepsilon}{2} \overset{def}{=} M^{(2)}. \tag{2.24}$$

$$\int_{-\infty}^{t} K(t-s)x(s)ds < \frac{a-cm^{(1)}}{b} + \frac{\varepsilon}{2} \overset{def}{=} M^{(2)}. \tag{2.25}$$

For $t \geq T_3$, it follows from the system (1.1) and (2.25) That

$$\frac{dx(t)}{dt} \geq x(t)[a-bx-cM^{(2)}]\frac{x}{\beta+x}. \tag{2.26}$$

Now let's consider the auxiliary equation

$$\frac{du}{dt} = u[a-bu-cM^{(2)}]\frac{u}{\beta+u}. \tag{2.27}$$

It follows from Lemma 2.1 that the unique positive equilibrium $u^* = \frac{a-cM^{(2)}}{b}$ b is globally stable. That is,

$$\lim_{t\to+\infty} u(t) = \frac{a-cM^{(2)}}{b}.$$

Thus, by using the differential inequality theory, one has

$$\liminf_{t\to+\infty} x(t) \geq \frac{a-cM^{(2)}}{b}, \tag{2.28}$$

It follows from Lemma 2.3 that

$$\liminf_{t\to+\infty} \int_{-\infty}^{t} K(t-s)x(s)ds \geq \liminf_{t\to+\infty} x(t) \geq \frac{a-cM^{(2)}}{b}. \tag{2.29}$$

Hence, for $\varepsilon > 0$ which satisfies (3.1), it follows from (2.28) and (2.29) that there exists a $T_4 > 0$ such that for all $t \geq T_4$,

$$x(t) > \frac{a-cM^{(2)}}{b} - \varepsilon \overset{def}{=} m^{(2)}. \tag{2.30}$$

$$\int_{-\infty}^{t} K(t-s)x(s)ds > \frac{a-cM^{(1)}}{b} - \varepsilon \overset{def}{=} m^{(2)}. \tag{2.31}$$

One could easily see that

$$M^{(2)} = \frac{a-cm^{(1)}}{b} + \frac{\varepsilon}{2} < \frac{a}{b} + \varepsilon = M^{(1)};$$

$$m^{(2)} = \frac{a-cM^{(2)}}{b} - \frac{\varepsilon}{2} > \frac{a-cM^{(1)}}{b} - \varepsilon = m^{(1)}. \tag{2.32}$$

Repeating the above procedure, we get four sequences $M^{(n)}, m^{(n)}, n = 1,2,\cdots$, such that for $n \geq 2$

$$M^{(n)} = \frac{a - cm^{(n-1)}}{b} + \frac{\varepsilon}{n};$$
$$m^{(n)} = \frac{a - cM^{(n)}}{b} - \frac{\varepsilon}{n}. \quad (2.33)$$

Obviously,

$$m^{(n)} < x(t) < M^{(n)}, \text{ for } t > T_{2n}. \quad (2.34)$$

By induction, similar to the analysis of Chen, Xie, and Wang[7], we could prove that sequences $M^{(n)}$ are non-increasing, and sequences $m^{(n)}$ are non-decreasing. Therefore,

$$\lim_{t\to+\infty} M^{(n)} = \bar{x}, \lim_{t\to+\infty} m^{(n)} = \underline{x}. \quad (2.35)$$

Letting $n \to +\infty$ in (2.33), we obtain

$$b\bar{x} = a - c\underline{x},$$
$$b\underline{x} = a - c\bar{x}. \quad (2.36)$$

It follows from (2.36) that

$$(b+c)(\bar{x} - \underline{x}) = 0.$$

Hence,

$$\bar{x} = \underline{x}. \quad (2.37)$$

Again, substituting (2.37) with (2.36) leads to

$$\bar{x} = \underline{x} = \frac{a}{b+c} = x^*.$$

that is

$$\lim_{t\to+\infty} x(t) = x^*.$$

Thus, the unique interior equilibrium x^* is globally attractive. This completes the proof of Theorem 2.1.

3 Numeric simulations

Now let us consider the following examples.

Example 3.1

$$\frac{dx}{dt} = x(6 - 2x - \int_{-\infty}^{t} e^{-(t-s)} x(s)ds) \frac{x}{\beta + x}, \quad (3.1)$$

where β is a positive constant.

Let $y = \int_{-\infty}^{t} e^{-(t-s)} x(s)ds$, then system (3.1) is equivalent to the system

$$\frac{dx}{dt} = x(6 - 2x - y)\frac{x}{\beta + x},$$
$$\frac{dy}{dt} = x - y. \quad (3.2)$$

It follows from Theorem 2.1 that system (3.1) admits a unique globally attractive positive equilibrium $x^* = 2$. Consequently, system (3.2) admits a unique globally attractive positive equilibrium $(x^*, y^*) = (2,2)$. Fig.1 supports this assertion.

Now let's choose $\beta = 2$, 5, and 20, respectively. Fig. 2 shows that as β (i.e., Allee effect) increases, the solution takes longer to reach its steady state.

Dynamic behaviors of the system (3.2)

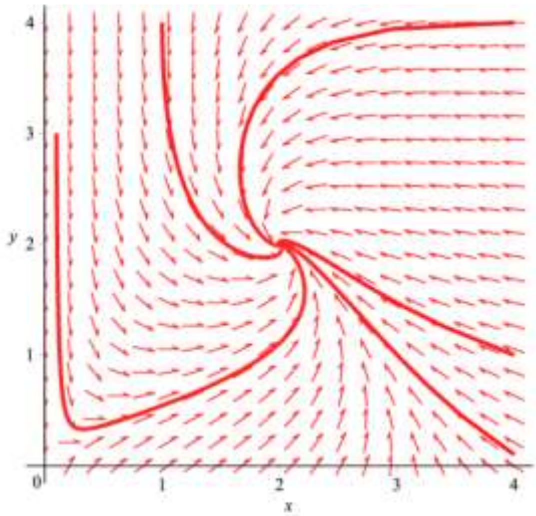

Figure 1: Dynamic behaviors of the system (3.2), the initial condition $(x(0), y(0)) = (4,0.1), (0.1,3), (1,4)$, $(4,1)$ and $(4,4)$, respectively.

4 Discussion

Merdan [19] proposed a predator-prey system with the Allee effect on prey species, see system (1.5), his study showed that this harms the species. The final density of the species will decrease as the Allee effect increases. Since then, many scholars ([19]-[34]) have investigated the dynamic behaviors of the population system with the Merdan-type Allee effect, and for different systems, the Allee effect has different influences.

In this paper, we further incorporate the Merdan-type Allee effect into the single-species delayed system. As demonstrated in Theorem 2.1, system (1.1) admits a unique globally attractive positive equilibrium x^* under the assumption $b > c$. Noting that this condition is also necessary to ensure the system

$$\frac{dx}{dt} = x(a - bx - c\int_{-\infty}^{t} K(t-s)x(s)ds), \quad (4.1)$$

admits the unique globally attractive positive equilibrium. Also, noting that x^* is independent of β. Hence, we can conclude: the Allee effect does not influence the existence and stability of the positive equilibrium. However, Fig. 2 shows that with the increasing Allee effect, the system should take more time to approach its steady state. It is in this sense that the Allee effect harms the stability property of the system.

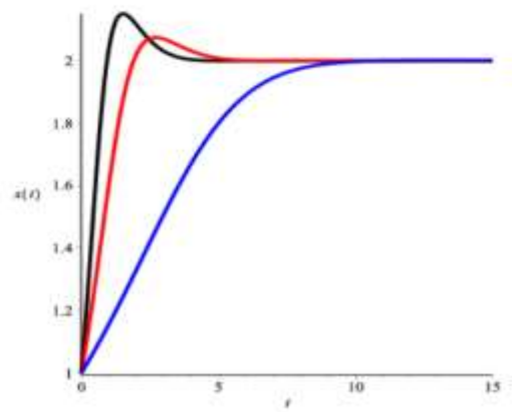

Figure 2: Numeric simulations of $x(t)$, with β = 2, 5, 20, and $(x(0), y(0)) = (1,1)$, respectively, where the black curve is the solution of β = 2, the red curve is the solution of β = 5, and the blue curve is the solution of $\beta = 20$.

We would like to mention at the end of the paper that in [40], one of our recent works, we discovered that for the discrete commensalism model with Merdan type Allee effect, the Allee effect increased the stability property of the system in the sense that, without Allee effect, the system may be chaos, however, only with the increasing of Allee effect, if the Allee effect is enough large, the system may become stable. The influence of the Merdan-type Allee effect appears to be quite different for the continuous and discrete systems. To this day, no scholar has proposed a discrete model with infinite delay and a Merdan-type Allee effect, we will try to do work in this direction in the future.

References:

[1] Yang K., Miao Z. S., Chen F. D., et al, Influence of single feedback control variable on an autonomous Holling-II type cooperative system, *Journal of Mathematical Analysis and Applications*, Vol.435, No.1, 2016, pp.874-888.

[2] Chen F. D., Xie X. D., Chen X. F., Dynamic behaviors of a stage-structured cooperation model, *Communications in Mathematical Biology and Neuroscience*, Vol 2015, 2015, Article ID 4.

[3] Chen F., Li Z., Huang Y., Note on the and feedback controls, *Nonlinear Analysis: Real World Applications*, Vol.8, No.2, 2007, pp. 680- 687.

[4] Li T. T., Chen F. D., Chen J. H., et al, Stability of a stage-structured plant-pollinator mutualism model with the Beddington-DeAngelis functional response, *Journal of Nonlinear Functional Analysis,* Vol. 2017 2017, Article ID 50, pp. 1-18.

[5] Shi C., Li Z., Chen F., Extinction in a nonautonomous Lotka-Volterra competitive system with infinite delay and feedback controls, *Nonlinear Analysis: Real World Applications,* Vol.13, No.5, 2012, pp. 2214-2226.

[6] Li Z., Han M., Chen F., Influence of feedback controls on an autonomous Lotka-Volterra competitive system with infinite delays, *Nonlinear Analysis: Real World Applications*, Vol.14, No.1, 2013, pp. 402-413.

[7] Chen F., Xie X., Wang H., Global stability in a competition model of plankton allelopathy with infinite delay, *Journal of Systems Science and Complexity,* Vol.28, No.5, 2015, pp. 1070-1079.

[8] Chen F., Global asymptotic stability in n-species nonautonomous Lotka-Volterra competitive systems with infinite delays and feedback control, *Applied Mathematics and Computation,* Vol.170, No.2, 2005, pp. 1452-1468.

[9] De Oca F. M., Vivas M., Extinction in a two-dimensional Lotka-Volterra system with infinite delay, *Nonlinear Analysis: Real World Applications,* Vol.7, No.5, 2006, pp. 1042-1047.

[10] Xie X. D., Chen F. D., Yang K. and Xue Y. L., Global attractivity of an integrodifferential model of mutualism, *Abstract and Applied Analysis*, Volume 2014, 2014, Article ID 928726.

[11] Shi C., Chen X., Wang Y., Feedback control effect on the Lotka-Volterra prey-predator system with discrete delays, *Advances in Difference Equations,* Vol.2017, 2017, pp. 1-13.

[12] Shi C., Wang Y., Chen X., et al. Note on the persistence of a nonautonomous Lotka-Volterra competitive system with infinite delay and feedback controls, *Discrete Dynamics in Nature and Society,* Volume 2014, 2014, Article ID 682769.

[13] Yang W. S., Li X. P., Permanence of a discrete nonlinear N-species cooperation system with time delays and feedback controls, *Appl. Math. Comput.,* Vol.218, No.7, 2011, pp.3581- 3586.

[14] Wu R. X., Li L., Zhou X. Y., A commensal symbiosis model with Holling type functional response, *Journal of Mathematics and Computer Science,* Vol.16, No.3, 2016, pp.364-371.

[15] Xie X., Xue Y., Wu R., Global attractivity of a discrete competition model of plankton allelopathy with infinite deviating arguments, *Advances in Difference Equations,* Vol. 2016, 2016, pp.1- 12.

[16] Wu R., Gao Z., Chen F., Dynamic behaviors of a two-species competitive system with nonlinear inter-inhibition terms and infinite delay, *Journal of Mathematics and Computer Science,* Vol.21, No.1, 2020, pp.45-56.

[17] Chen B., Permanence for the discrete competition model with infinite deviating arguments, *Discrete Dynamics in Nature and Society,* Volume 2016, 2016, Article ID 1686973.

[18] Chen B., Global attractivity of an integrodifferential model of competition, *Advances in Difference Equations,* Vol. 2017, 2017, pp. 1-13.

[19] Merdan H., Stability analysis of a Lotka-Volterra type predator-prey system involving Allee effects,

The ANZIAM Journal, Vol.52, No.2, 2010, pp. 139-145.

[20] Q. F. Lin, Stability analysis of a single species logistic model with Allee effect and feedback control, *Advances in Difference Equations,* 2018, 2018(1): 1-13.

[21] Lin Q. F., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, *Advances in Difference Equations,* Vol.2018, 2018, Article ID 196.

[22] Lei C., Dynamic behaviors of a Holling type commensal symbiosis model with the first species subject to Allee effect, *Commun. Math. Biol. Neurosci.,* Vol. 2019, 2019, Article ID 3.

[23] Wei Z., Xia X., Zhang T., Stability and bifurcation analysis of an amensalism model with weak Allee effect, *Qualitative Theory of Dynamical Systems,* Vol.19, No.1, 2020, pp. 1-15.

[24] X. Guan, Y. Liu, X. Xie, Stability analysis of a Lotka-Volterra type predator-prey system with Allee effect on the predator species, *Commun. Math. Biol. Neurosci.,* 2018, 2018: Article ID 9.

[25] Guan X., Chen F., Dynamical analysis of a two-species amensalism model with Beddington-DeAngelis functional response and Allee effect on the second species, *Nonlinear Analysis: Real World Applications,* Vol.48, No.1, 2019, pp. 71- 93.

[26] Lv Y., Chen L., Chen F., Stability and bifurcation in a single species logistic model with additive Allee effect and feedback control, *Advances in Difference Equations,* Vol. 2020, 2020, pp. 1- 15.

[27] Wu R., Li L., Lin Q., A Holling type commensal symbiosis model involving Allee effect, *Commun. Math. Biol. Neurosci.,* Vol. 2018, 2018, Article ID 6.

[28] Chen F., Guan X., Huang X., et al. Dynamic behaviors of a Lotka-Volterra type predator-prey system with Allee effect on the predator species and density-dependent birth rate on the prey species, *Open Mathematics,* Vol.17, No.1, 2019, pp. 1186-1202.

[29] Li T., Huang X., Xie X., Stability of a stage-structured predator-prey model with Allee effect and harvesting, Commun. *Math. Biol. Neurosci.,* Vol. 2019, 2019, Article ID 13.

[30] Xiao Z., Xie X., Xue Y., Stability and bifurcation in a Holling type II predator-prey model with Allee effect and time delay, *Advances in Difference Equations,* Vol. 2018, 2018, pp. 1-21.

[31] Xiao Z., Li Z., Stability and bifurcation in a stage-structured predator-prey model with Allee effect and time delay, *IAENG International Journal of Applied Mathematics,* Vol.49, No.1, 2019, pp.6-13.

[32] Zhu Z., He M., Li Z., et al, Stability and bifurcation in a Logistic model with Allee effect and feedback control, *International Journal of Bifurcation and Chaos,* Vol.30, No.15, 2020, Article ID 2050231.

[33] Lai L., Zhu Z., Chen F., Stability and bifurcation in a predator-prey model with the additive Allee effect and the fear effect, *Mathematics,* Vol.8, No.8, 2020, Article ID 1280.

[34] Huang Y., Zhu Z., Li Z., Modeling the Allee effect and fear effect in a predator-prey system incorporating a prey refuge, *Advances in Difference Equations,* Vol. 2020, 2020, pp. 1-13.

[35] Yue Q., Permanence of a delayed biological system with stage structure and density-dependent juvenile birth rate, *Engineering Letters,* Vol.27, No.2, 2019, pp.263-268.

[36] Yue Q., Wang Y., Dynamic behaviors a single species stage structure model with density-dependent birth rate and non-selective harvesting in a partial closure, *IAENG International Journal of Applied Mathematics,* Vol. 50, No.1, 2020, pp. 1- 6.

[37] Yue Q., The Influence of positive feedback control to a single species stage structure system, *Engineering Letters,* Vol.28, No.2, 2020, pp. 1-5.

[38] Chen L. S., Mathematical Models and Methods in Ecology, *Science Press,* Beijing, (1988), (in Chinese).

[39] Kuang Y., Delay Differential Equations with Applications in Population Dynamics, *Academic Press*, New York, 1993.

[40] Zhou Q. M., Chen F. D., Dynamical analysis of a discrete amensalism system with the Beddington-DeAngelis functional response and Allee effect for the unaffected species, *Qualitative Theory of Dynamical Systems,* Vol.22, No. 16, 2023, pp. 1- 30

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Xiaoran Li carried out the computation and wrote the draft.
Qin Yue carried out the simulation.
Fengde Chen was responsible for the proposal of the problem.

Sources of funding for research presented in a scientific article or scientific article itself

Social Science Project of Anhui Provincial Department of Education (2022AH051657).

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.