The Permanence of a Nonautonomous Single-species Model with Stage-Structure and Feedback Control

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Abstract: - A nonautonomous single-species model with stage structure and feedback control is revisited in this paper. By applying the differential inequality theory, a set of delay-dependent conditions ensures the permanence of the system is obtained; Next, by further developing the analytical technique of Chen et al, we prove that the system is always permanent. Numeric simulation supports our findings. Also, the numeric simulation shows that the feedback control variable harms the final density of the species, and this may increase the chance of the extinction of the species. Our results supplement and complement some known results.

Key-Words: Systems Theory, Dynamical Systems, stage structure, feedback control, permanence.

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1 Introduction

During the last decade, many scholars investigated the dynamic behaviors of the ecosystem, see [1]-[53] and the references cited therein. Such topics as extinction, persistence, and stability are extensively investigated.

It is well known that many species take several stages throughout their life, and to model such kind of phenomenon, many scholars ([1]-[17],[30], [52]-[53]) proposed the stage-structured population system. Aiello Freedman [30] proposed the following single-species stage-structured model

$$\frac{dx_1}{dt} = \alpha x_2 - \gamma x_1 - \alpha e^{-\gamma\tau} x_2(t-\tau),$$
$$\frac{dx_2}{dt} = \alpha e^{-\gamma\tau} x_2(t-\tau) - \beta x_2^2. \tag{1.1}$$

The authors of [30] showed that the system (1.1) admits a unique positive equilibrium that is globally asymptotically stable. Based on the work of [30], many scholars proposed the delayed stage-structured model, for example, Lin et al[10] studied the persistent property of the following stage-structured predator-prey model

$$x_1'(t) = r_1 x_2(t) - d_{11}x_1(t) - r_1 e^{-d_{11}\tau_1} x_2\left(t-\tau_1\right),$$
$$x_2'(t) = r_1 e^{-d_{11}\tau_1} x_2\left(t-\tau_1\right) - d_{12}x_2(t) - bx_2^2(t) - \frac{a_1 y_2(t) x_2(t)}{x_2(t)+k_1},$$
$$y_1'(t) = r_2 y_2(t) - d_{22}y_1(t) - r_2 e^{-d_{22}\tau_2} y_2\left(t-\tau_2\right),$$
$$y_2'(t) = r_2 e^{-d_{22}\tau_2} y_2\left(t-\tau_2\right) - d_{21}y_2(t) - \frac{a_2 y_2^2(t)}{x_2(t)+k_2} \tag{1.2}$$

Their study indicates that for a stage-structured predator-prey community, both stage structure and the death rate of the mature species are the important factors that lead to the permanence or extinction of the system. For more work on the stage-structured model incorporating time delay, one could refer to [1]-[12] and the references cited therein.

On the other hand, ecosystems in the real world are continuously disturbed by unpredictable forces which can result in changes in biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can stand those unpredictable disturbances that persist for a finite period of time. In the language of control variables, we call the disturbance functions control variables. Gopalsamy and Weng[26] proposed the following single-species feedback control ecosystem

$$\dot{n} = rn\left[1 - \frac{a_1 n(t) + a_2 n(t-\tau)}{K} - cu(t)\right],$$
$$\dot{u} = -au(t) + bn(t). \tag{1.3}$$

They showed that the inequality a1 > a2 is enough to ensure the existence of a unique globally asymptotically stable positive equilibrium. Chen, Yang, and Chen [29] studied the following single-species feedback control ecosystem

$$\dot{N} = r(t)N(t)\left[1 - \frac{N^2\left(t-\tau_1(t)\right)}{K^2(t)}\right.$$

$$-c(t)u\left(t-\tau_2(t)\right)\Big], \qquad (1.4)$$
$$\dot{u}=-a(t)u(t)+b(t)N(t)$$

By establishing a new integral inequality, they can show that the system (1.4) is always permanent.
That is to say, the feedback control variable does not influence the persistent property of the system (1.4).

Though there are many works on feedback control ecosystems ([26]-[49]), there is still little work on stage-structured ecosystems with feedback controls. Ding and Cheng[27] proposed the following single-species stage-structured model with feedback control:

$$\begin{aligned}\frac{dx_1(t)}{dt} &= bx_2(t)-d_1x_1(t)\\ &\quad -be^{-d_1\tau}x_2(t-\tau),\\ \frac{dx_2(t)}{dt} &= be^{-d_1\tau}x_2(t-\tau)-cx_2^2(t) \qquad (1.5)\\ &\quad -cx_2(t)u(t),\\ \frac{du(t)}{dt} &= -fu(t)+ex_2(t).\end{aligned}$$

In [27], it shows that if $fa > ce$ then system (1.5) admits a unique positive equilibrium that is globally attractive. Recently, Yang[28] argued that the nonautonomous case is more suitable since the circumstance is varying with time. They proposed the following non-autonomous feedback control ecosystem

$$\begin{aligned}\frac{dx_1(t)}{dt} &= b(t)x_2(t)-d_1(t)x_1(t)\\ &\quad -b(t-\tau)e^{-\int_{t-\tau}^{t}d_1(s)ds}x_2(t-\tau),\\ \frac{dx_2(t)}{dt} &= b(t-\tau)e^{-\int_{t-\tau}^{t}d_1(s)ds}x_2(t-\tau)\\ &\quad -a(t)x_2^2(t)-c(t)x_2(t)u(t),\\ \frac{du(t)}{dt} &= -f(t)u(t)+e(t)x_2(t).\end{aligned} \qquad (1.6)$$

Under the assumption $b(t), d_1(t), a(t), f(t)$ and e(t) are all continuous positive T -periodic functions, by using the coincidence degree theory, the author showed that system (1.6) admits at least one positive T -periodic solution.

Since the environment is varied with season, it is natural to consider the general non-autonomous case of the system (1.6), i.e., it is natural to consider the system (1.6) under the following assumption:

(H_1) $b(t), d_1(t), a(t), c(t), f(t),$ and $e(t)$ are all continuous functions bounded above and below by positive constants.

For general non-autonomous cases, the persistent property is one of the most important topics in the study of population dynamics, however, Yang[28] did not investigate the persistent property of the system (1.6). The aim of this paper is, by applying the comparison theorem of the differential equation and developing the analytical technique of Chen, Yang, and Chen[29], to obtain two sets of sufficient conditions that guarantee the permanence of the system (1.6).

The rest of the paper is arranged as follows: We will state several lemmas in the next section, and give the first set of sufficient conditions in Section 3. Then we will use the idea of Chen, Yang, and Chen[29] to establish another set of sufficient conditions in Section 4. An example together with its numeric simulation is presented in Section 5 to show the feasibility of the main results. We end this paper with a brief discussion.

2 Lemmas

Now let us state several lemmas which will be useful in proving the main results.

Lemma 2.1. [10]Consider the following equation:

$$x'(t)=bx(t-\delta)-a_1x(t)-a_2x^2(t),$$
$$x(t)=\phi(t)>0, \quad -\delta\le t\le 0,$$

and assume that $b, a_2 > 0, a_1 \geq 0$ and $\delta \geq 0$ is a constant. Then

(i) *If* $b\ge a_1$, *then* $\lim\limits_{t\to+\infty}x(t)=\dfrac{b-a_1}{a_2}$;

(ii) *If* $b\le a_1$, *then* $\lim\limits_{t\to+\infty}x(t)=0.$

Lemma 2.2. [29] *Assume that* $a > 0, b(t) > 0$ *is a bounded continuous function and* $x(0) > 0$ *Further suppose that*

(i)

$$\frac{dx(t)}{dt}\le -ax(t)+b(t),$$

Then for all $t \geq s$,

$$x(t)\le x(t-s)\exp\{-as\}+\int_{t-s}^{s}b(\tau)\exp\{a(t-\tau)\}d\tau.$$

Especially, if b(t) is bounded above with respect to M, then

$$\limsup_{t\to+\infty}x(t)\le\frac{M}{a}.$$

(ii)

$$\frac{dx(t)}{dt} \geq -ax(t) + b(t),$$

Then for all $t \geq s$,

$$x(t) \geq x(t-s)\exp\{-as\} + \int_{t-s}^{s} b(\tau)\exp\{a(t-\tau)\}d\tau.$$

Especially, if b(t) is bounded below with respect to m, then

$$\liminf_{t\to+\infty} x(t) \geq \frac{m}{a}.$$

3 Permanence of system (1.6) (I)

The aim of this section is, by developing the analysis technique of Chen et al([1]-[4]), more precisely, by applying the differential inequality theory, to investigate the persistent property of the system (1.6).

We adopt the following notations throughout this paper:

$$g^M = \sup_{t\in[0,+\infty)} g(t),$$
$$g^L = \inf_{t\in[0,+\infty)} g(t), \tag{3.1}$$

where *g(t)* is a continuous function on [0,+∞).

Lemma 3.1. The first equation of system (1.6) is equivalent to

$$x_1(t) = \int_{t-s}^{t} b(s)e^{\int_s^t d_1(u)du} x_2(s)ds.$$

Proof. From (3.2), one has

$$\begin{aligned}
&\dot{x}_1(t)\\
=\ & b(t)e^{-\int_t^t d_1(u)du} x_2(t)\\
&-b(t-\tau)e^{-\int_{t-\tau}^t d_1(u)du} x_2(t-\tau)\\
&+\int_{t-\tau}^t b(s)e^{-\int_s^t d_1(u)du} x_2(s)ds\left(-\int_s^t d_1(u)du\right)'_t\\
=\ & b(t)x_2(t) - b(t-\tau)e^{-\int_{t-\tau}^t d_1(u)du} x_2(t-\tau)\\
&+\int_{t-\tau}^t b(s)e^{-\int_s^t d_1(u)du} x_2(s)ds\left(-d_1(t)\right)\\
=\ & b(t)x_2 - d_1(t)x_1\\
&-b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds} x_2(t-\tau).
\end{aligned}$$

The above analysis shows that the conclusion of Lemma 3.1 holds. This ends the proof of Lemma 3.1.

Theorem 3.1. *In addition to* (H_1) *, assume further that*

$$b^L e^{-d_1^U\tau} f^L a^L > c^U e^U b^U e^{-d_1^L\tau} \tag{3.2}$$

holds, then system (1.6) is permanent.

The proof of Theorem 3.1 immediately follows from the proof of Theorem 3.2 and 3.3.

Remark 3.1. If we assume that the coefficients of the system (1.6) are all positive constants, then condition (3.2) degenerates to

$$fa > ce, \tag{3.3}$$

from the introduction section, we know that condition (3.3) is enough to ensure that system (1.5) admits a unique globally asymptotically stable positive equilibrium.

Theorem 3.2. Let $(x_1(t), x_2(t), u(t))$ be any positive solution of the system (1.6), then

$$\limsup_{t\to+\infty} x_i(t) \leq M_i, i = 1, 2,$$
$$\limsup_{t\to+\infty} y(t) \leq M_3. \tag{3.4}$$

where

$$M_1 = b^U e^{-d_1^L\tau}\tau M_2,$$
$$M_2 = \frac{b^U e^{-d_1^L\tau}}{a^L},$$
$$M_3 = \frac{e^U M_2}{f^L}.$$

Proof. Let $(x_1(t), x_2(t), u(t))$ be any positive solution of system (1.6), then from the second equation of (1.6), we have

$$\begin{aligned}
&\frac{dx_2}{dt}\\
=\ & b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds} x_2(t-\tau)\\
&-a(t)x_2^2(t) - c(t)x_2(t)y(t) \qquad (3.5)\\
\leq\ & b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds} x_2(t-\tau)\\
&-a(t)x_2^2(t)\\
\leq\ & b^U e^{-d^L\tau} x_2(t-\tau) - a^L x_2^2(t).
\end{aligned}$$

By applying Lemma 2.1 to (3.5), it immediately follows that

$$\limsup_{t\to+\infty} x_2(t) \leq \frac{b^U e^{-d^L\tau}}{a^L} \overset{\text{def}}{=} M_2. \tag{3.6}$$

For any enough small positive constant $\varepsilon_1 > 0$, there exists a $T_1 > 0$ such that

$$x_2(t) < M_2 + \varepsilon_1 \text{ for all } t \geq T_1. \tag{3.7}$$

(3.7) together with the third equation of system (1.6) leads to

$$\begin{aligned}
\frac{dy}{dt} &= -f(t)y(t) + e(t)x_2(t)\\
&\leq -f^L y(t) + e^U\left(M_2 + \varepsilon\right). \qquad (3.8)
\end{aligned}$$

Applying Lemma 2.2 to (3.8) leads to

$$\limsup_{t\to+\infty} y(t) \le \frac{e^U\left(M_2+\varepsilon_1\right)}{f^L}. \quad (3.9)$$

Setting $\varepsilon_1 \to 0$ in (3.9) leads to

$$\limsup_{t\to+\infty} y(t) \le \frac{e^U M_2}{f^L} \stackrel{\text{def}}{=} M_3. \quad (3.10)$$

From Lemma 3.1 we have

$$x_1(t) = \int_{t-\tau}^{t} b(s) e^{-\int_s^t d_1(u)du} x_2(s)ds. \quad (3.11)$$

This, together with (3.7) leads to

$$x_1(t) \le b^U e^{-d_1^L\tau}\tau\left(M_2+\varepsilon_1\right). \quad (3.12)$$

Setting $\varepsilon_1 \to 0$ in (3.12) leads to

$$\limsup_{t\to+\infty} x_1(t) \le b^U e^{-d_1^L\tau}\tau M_2 \stackrel{\text{def}}{=} M_1. \quad (3.13)$$

(3.6), (3.10), and (3.13) show that the conclusion of Theorem 3.1 holds. This ends the proof of the Theorem 3.1.

Theorem 3.3. In addition to (H1), assume further that (3.2) holds, then

$$\liminf_{t\to+\infty} x_i(t) \ge m_i, i=1,2,$$
$$\liminf_{t\to+\infty} y(t) \ge m_3, \quad (3.14)$$

where

$$m_1 = b^L e^{-d_1^U\tau}\tau m_2,$$
$$m_2 = \frac{b^L e^{-d_1^U\tau} - c^U M_3}{a^L}, \quad (3.15)$$
$$m_3 = \frac{e_1^L m_2}{f^U}.$$

Proof. Let $(x_1(t), x_2(t), u(t))$ be any positive solution of system (1.6). One could easily check that inequality (3.2) is equivalent to

$$b^L e^{-d_1^U\tau} > c^U M_3.$$

Hence, for enough small positive constant $\varepsilon_2 > 0$, the inequality

$$b^L e^{-d_1^U\tau} > c^U\left(M_3+\varepsilon_2\right) \quad (3.16)$$

holds. It follows from (3.6) that for the above $\varepsilon_2 > 0$, there exists a $T_2 > T_1$, for $t > T_2$,

$$y(t) < M_3 + \varepsilon_2 \quad (3.17)$$

holds. Hence, for $t \ge T_2$, from the second equation of (1.6), one has

$$\begin{aligned}\frac{dx_2}{dt} &= b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds}x_2(t-\tau) \\ &\quad -a(t)x_2^2(t) - c(t)x_2(t)y(t) \\ &\ge b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds}x_2(t-\tau) \quad (3.18) \\ &\quad -a(t)x_2^2(t) - c(t)\left(M_3+\varepsilon_2\right)x_2(t) \\ &\ge b^U e^{-d^L\tau}x_2(t-\tau) - a^L x_2^2(t) \\ &\quad -c^U\left(M_3+\varepsilon_2\right)x_2(t).\end{aligned}$$

Applying Lemma 2.2 to (3.18) leads to

$$\liminf_{t\to+\infty} x_2(t) \ge \frac{b^U e^{-d^L\tau} - c^U\left(M_3+\varepsilon_2\right)}{a^L}. \quad (3.19)$$

Setting $\varepsilon_2 \to 0$ in(3.19) leads to

$$\liminf_{t\to+\infty} x_2(t) \ge \frac{b^U e^{-d^L\tau} - c^U M_3}{a^L} \stackrel{\text{def}}{=} m_2. \quad (3.20)$$

For any enough small positive constant $\varepsilon_3 > 0$ (without loss of generality, we may assume that $\varepsilon_3 > \frac{1}{2}m_2$), it follows from (3.20) that there exist a $T_3 > T_2$,such that

$$x_2(t) > m_2 - \varepsilon_3 \ for\ all\ t \ge T_3. \quad (3.21)$$

For $t > T_3$, it follows from the third equation of system (1.6) that

$$\begin{aligned}\frac{dy}{dt} &= -f(t)y(t) + e(t)x_2(t) \\ &\ge -f^U y(t) + e^L\left(m_2-\varepsilon\right). \quad (3.22)\end{aligned}$$

Applying Lemma 2.1 to (3.22) leads to

$$\liminf_{t\to+\infty} y(t) \ge \frac{e^L\left(m_2-\varepsilon_3\right)}{f^U}. \quad (3.23)$$

Since ε_3 is enough small positive constant, setting $\varepsilon_3 \to 0$ in (3.23) leads to

$$\liminf_{t\to+\infty} y(t) \ge \frac{e^L m_2}{f^U} \stackrel{\text{def}}{=} m_3. \quad (3.24)$$

It follows from Lemma 3.1 that

$$x_1(t) = \int_{t-\tau}^{t} b(s) e^{-\int_s^t d_1(u)du} x_2(s)ds. \quad (3.25)$$

(3.25) combine with (3.21) leads to

$$x_1(t) \ge b^L e^{-d_1^U\tau}\tau\left(m_2-\varepsilon_3\right) \ for\ all\ t \ge T_3.$$

Setting $\varepsilon_3 \to 0$ in the above inequality, we have

$$\liminf_{t\to+\infty} x_1(t) \ge b^L e^{-d_1^U\tau}\tau m_2. \quad (3.26)$$

(3.20), (3.24), and (3.26) show that the conclusion of Theorem 3.3 holds. This ends the proof of Theorem 3.3.

4 Permanence of system (1.6)(II)

In the previous section, we showed that under the assumption (3.2) holds. System (1.6) is permanent. One interesting issue is to investigate the dynamic behaviors of the system (1.6) if inequality (3.2) does not hold.

To give some hints on this direction, already, Yang[28] had showed that system (1.6) admits at least one positive T -periodic solution if the coefficients of the system are all positive T-periodic functions. Since the periodic solution means the species could be survived in a fluctuating form. This motivated us to propose the conjecture:

Conjecture. System (1.6) is permanent if the condition (H_2) holds.

The aim of this section is to give the affirmative answer to this conjecture. More precisely, we will obtain the following result.

Theorem 4.1. System (1.6) is permanent if condition (H_2) holds.

Proof. We will prove Theorem 4.1 by developing the idea of Chen et al[29].
Let $(x_1(t), x_2(t), u(t))$ be any positive solution of system (1.6). Theorem 3.2 had shown that

$$\limsup_{t\to+\infty} x_i(t) \le M_i, i=1,2, \qquad \limsup_{t\to+\infty} y(t) \le M_3, \tag{4.1}$$

Hence, for any enough small positive constant $\varepsilon > 0$,
there exists a $T > 0$, such that

$$\begin{aligned} x_1(t) &< M_1 + \varepsilon \overset{def}{=} M_1^\varepsilon, \\ x_2(t) &< M_2 + \varepsilon \overset{def}{=} M_2^\varepsilon, \\ y(t) &< M_3 + \varepsilon \overset{def}{=} M_3^\varepsilon. \end{aligned} \tag{4.2}$$

for all $t \ge T$.

From the second equation of system (1.6), we have

$$\begin{aligned} \frac{dx_2}{dt} &= b(t-\tau)e^{-\int_{t-\tau}^t d_1(s)ds} x_2(t-\tau) \\ &\quad - a(t)x_2^2(t) - c(t)x_2(t)y(t) \\ &\ge -a(t)x_2^2(t) - c(t)x_2(t)y(t) \\ &\ge x_2(t)\left[-a^U M_2^\varepsilon - c^U M_3^\varepsilon\right] \\ &\overset{\Delta}{=} \Gamma x_2(t), \end{aligned} \tag{4.3}$$

Obviously,

$$\Gamma < 0. \tag{4.4}$$

Integrate (4.3) from τ_1 to t lead to

$$\frac{x_2(t)}{x_2(\tau_1)} \ge \exp\left\{\int_{\tau_1}^t \Gamma ds\right\} \tag{4.5}$$

Thus,

$$x_2(\tau_1) \le x_2(t)\exp\{-\Gamma(t-\tau_1)\}. \tag{4.6}$$

From the third equation of system (1.6), we have

$$\frac{dy}{dt} \le -f^L y(t) + e^U x_2(t). \tag{4.7}$$

Applying Lemma 2.2 to the above inequality, we have

$$\begin{aligned} & y(t) \\ &\le y(t-s)\exp\{-f^L s\} \\ &\quad + \int_{t-s}^t e^U x_2(\tau_1)\exp\{f^L(\tau_1 - t)\}d\tau_1 \\ &\le y(t-s)\exp\{-\Gamma(t-\tau_1)\} \\ &\quad + \int_{t-s}^t e^U x_2(\tau_1)\exp\{-\Gamma(t-\tau_1)\}\times \\ &\quad \exp\{f^L(\tau_1 - t)\}d\tau_1 \\ &\le y(t-s)\exp\{-f^L s\} \\ &\quad + e^U x_2(\tau_1)\int_{t-s}^t \exp\{-\Gamma(t-\tau_1)\}d\tau_1 \\ &\le y(t-s)\exp\{-f^L s\} \\ &\quad + \frac{e^U}{\Gamma}(1-\exp\{-\Gamma s\})x_2(t), \end{aligned} \tag{4.8}$$

here we have to use the fact

$$\max_{\tau_1\in[t-s,t]} \exp\{f^L(\tau_1 - t)\} = \exp\{0\} = 1.$$

Choose K enough large, such that

$$K > \max\left\{\frac{1}{f^L}\ln\frac{2c^U M_3^\varepsilon}{b_1^L \exp\{-d_1^U\tau\}}, 0\right\},$$

then

$$c^U M_3^\varepsilon \exp\{-f^L K\} < \frac{1}{2} b^L \exp\{-d_1^U \tau\}. \tag{4.9}$$

For above K, there exists a $T_1 > T + K$, for $t \ge T_1$,
Take $s = K$ in (4.8) leads to

$$\begin{aligned} & y(t) \\ &\le y(t-K)\exp\{-f^L K\} \\ &\quad + \frac{e_1^U}{\Gamma}(1-\exp\{-\Gamma K\})x_2(t) \\ &\le M_3^\varepsilon \exp\{-f^L K\} \\ &\quad + \frac{e^U}{\Gamma}(1-\exp\{-\Gamma K\})x_2(t) \\ &\le M_3^\varepsilon \exp\{-f^L K\} + Hx_2(t), \end{aligned} \tag{4.10}$$

where $H = \frac{e^U}{\Gamma}(1-\exp\{-\Gamma K\}) > 0.$

Substituting (4.9) and (4.10) to the second equation of the system (1.6), for $t \geq T_1$, one has

$$\begin{aligned}\frac{dx_2}{dt} &= b(t-\tau)e^{\int_{t-\tau}^{t} d_1(s)ds} x_2(t-\tau) \\ &\quad - a(t)x_2^2(t) - c(t)x_2(t)y(t) \\ &\geq b^L e^{-d_1^U \tau} x_2(t-\tau) - a^U x_2^2(t) \\ &\quad - c^U\left(M_3^\varepsilon \exp\{-f^L K\} + Hx_2(t)\right)x_2(t) \\ &= b^L e^{-d_1^U \tau} x_2(t-\tau) - \left(a^U + c^U H\right)x_2^2(t) \\ &\quad - c^U M_3^\varepsilon \exp\{-f^L K\}x_2(t),\end{aligned} \tag{4.11}$$

Applying Lemma 2.1 to (4.11) leads to

$$\liminf_{t\to+\infty} x_2(t) \geq \frac{\Delta}{a^U + c^U H} \overset{def}{=} m_2. \tag{4.12}$$

where $\Delta = b^L e^{-d_1^U \tau} - c^U M_3^\varepsilon \exp\{-f^L K\}$. For any enough small positive constant $\varepsilon_1 > 0$, without loss of generality, we may assume that $\varepsilon_1 < \frac{1}{2}m_2$, it follows from (4.12) that there exists a $T_2 > T_1$ such that

$$x_2(t) > m_2 - \varepsilon_1 \text{ for all } t > T_2. \tag{4.13}$$

From (4.13) and the third equation of system (1.6), for $t > T_2$, we have

$$\frac{dy}{dt} \geq -f^U y(t) + e^L(m_2 - \varepsilon_1). \tag{4.14}$$

Applying Lemma 2.2 to (4.14) leads to

$$\liminf_{t\to+\infty} y(t) \geq \frac{e^U(m_2 - \varepsilon_1)}{f^U}. \tag{4.15}$$

Since ε_1 is an arbitrarily small positive constant, setting $\varepsilon_1 \to 0$ in (4.15) leads to

$$\liminf_{t\to+\infty} y(t) \geq \frac{e^L m_2}{f^U}. \tag{4.16}$$

Lemma 3.1 had shown that

$$x_1(t) = \int_{t-\tau}^{t} b(s)e^{-\int_s^t d_1(u)du} x_2(s)ds. \tag{4.17}$$

(4.17) together with (4.13) leads to

$$x_1(t) \geq b^L e^{-d_1^U \tau}\tau(m_2 - \varepsilon_1) \text{ for all } t > T_2 + \tau. \tag{4.18}$$

Setting $\varepsilon_1 \to 0$ leads to

$$\liminf_{t\to+\infty} x_1(t) \geq b^L e^{-d_1^U \tau}\tau m_2. \tag{4.19}$$

Theorem 3.2, (4.12), (4.16), and (4.19) show that system (1.6) is permanent. This ends the proof of Theorem 4.1.

5 Numeric simulations

The aim of this section is to give some numeric simulations to show the feasibility of the main results.

Example 5.1.

$$\begin{aligned}\frac{dx_1(t)}{dt} &= \left(\frac{5}{2} + \frac{1}{2}\sin(t)\right)x_2(t) - x_1(t) \\ &\quad - \left(\frac{5}{2} + \frac{1}{2}\sin(t-1)\right)e^{-1}x_2(t-1), \\ \frac{dx_2(t)}{dt} &= \left(\frac{5}{2} + \frac{1}{2}\sin(t-1)\right)e^{-1}x_2(t-1) \\ &\quad - (3+\cos(t))x_2^2(t-1) \\ &\quad - \left(1 - \frac{1}{2}\sin(t)\right)x_2(t)u(t), \\ \frac{du(t)}{dt} &= -(4+\sin(t))u(t) \\ &\quad + \left(1 - \frac{\cos(t)}{2}\right)x_2(t).\end{aligned} \tag{5.1}$$

Here, corresponding to the system (1.6), we take

$$b(t) = \frac{5}{2} + \frac{1}{2}\sin(t), d_1(t) = 1, \tau(t) = 1.$$
$$f(t) = 4 + \sin(t), a(t) = 3 + \cos(t).$$
$$e(t) = 1 - \frac{\cos(t)}{2}, c(t) = 1 - \frac{\sin(t)}{2}.$$

It follows from Theorem 4.1 that system (5.1) is permanent, also, from the main result of Yang[27], system (1.6) admits at least one positive T -periodic solution. Fig.1-3 support those assertions.

Example 5.2.

$$\begin{aligned}\frac{dx_1(t)}{dt} &= \left(\frac{5}{2} + \frac{1}{2}\sin(t)\right)x_2(t) - x_1(t) \\ &\quad - \left(\frac{5}{2} + \frac{1}{2}\sin(t-1)\right)e^{-1}x_2(t-1), \\ \frac{dx_2(t)}{dt} &= \left(\frac{5}{2} + \frac{1}{2}\sin(t-1)\right)e^{-1}x_2(t-1) \\ &\quad - (3+\cos(t))x_2^2(t) \\ &\quad - c(t)x_2(t)u(t),\end{aligned} \tag{5.2}$$

$$\frac{du(t)}{dt} = -(4+\sin(t))u(t) + \left(1-\frac{\cos(t)}{2}\right)x_2(t).$$

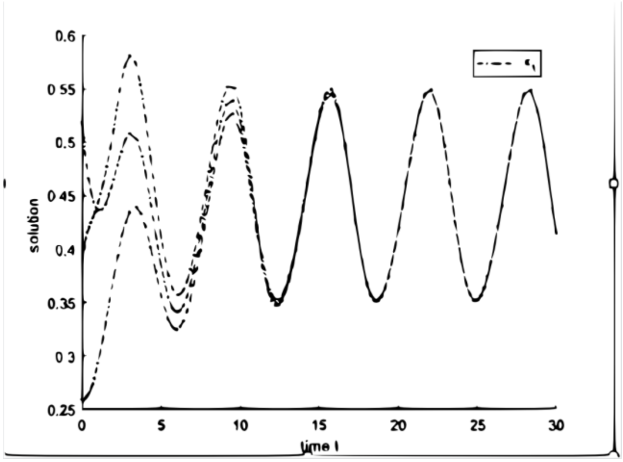

Figure 1: Dynamic behaviors of the first component of the system (5.1).

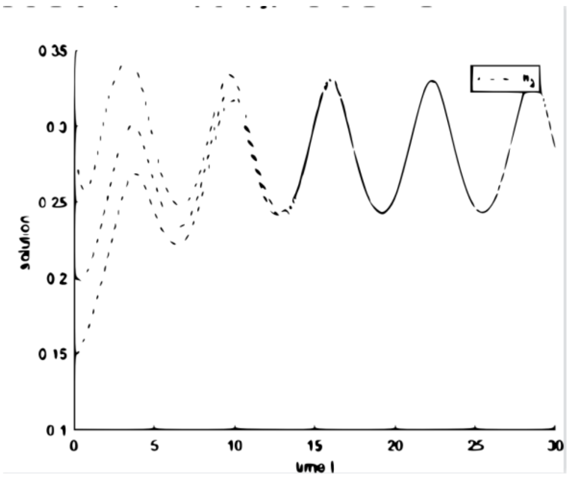

Figure 2: Dynamic behaviors of the second component of the system (5.1).

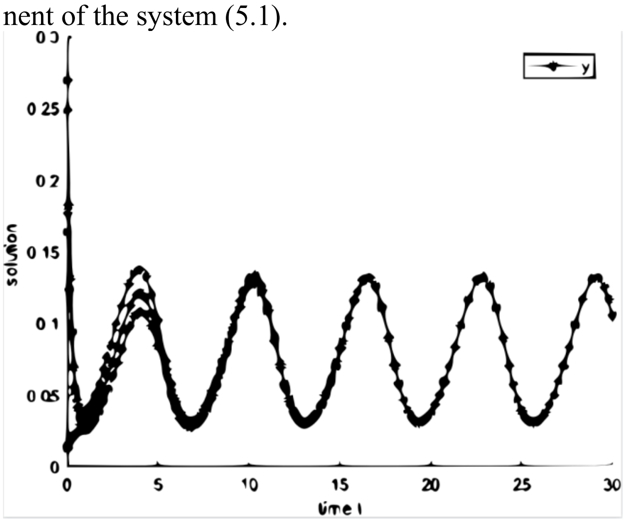

Figure 3: Dynamic behaviors of the third component of the system (5.1).

Here, all the other coefficients are the same as that of the system (5.1), only with $c(t)$ be determined late. Now let's choose $c(t)=3+\sin(t), 2+\sin(t)$ and $1+\sin(t)$, respectively, Fig.4 shows that in this case, for the same initial value, with the increasing of the $c(t)$, the density of mature species $x_2(t)$ is decreasing.

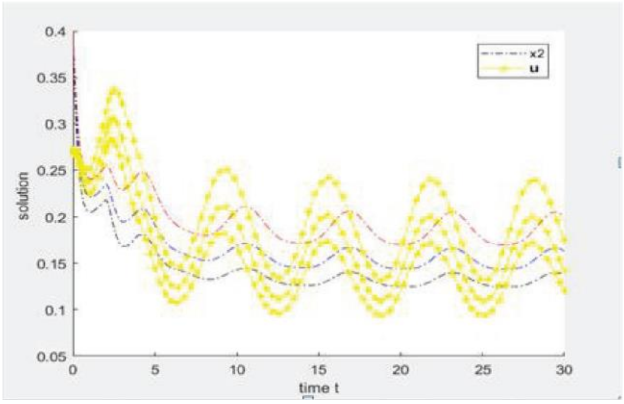

Figure 4: Numeric simulation of the system (5.2). Here we choose $c(t)=3+\sin(t), 2+\sin(t)$ and $1+\sin(t)$, respectively.

6 Discussion

Yang[28] proposed system (1.6), under the assumption all the coefficients are positive T -period function, they showed that system (1.6) admits at least one positive periodic solution.However, they did not investigate the persistent property of the system. In this paper, by using the differential inequality theory,we first obtain a set of sufficient conditions (Theorem3.1) which ensure the permanent of the system. After that, by comparing the results of Yang [28] and Chen et al[29], we propose a conjecture: the feedback control has no influence on the system. We give a strict proof of this conjecture in section 4. Numeric simulation (Fig. 1-3) also supports our findings.

Though feedback control has no influence to the persistent property of the system, example 5.2 shows that with the increasing of the coefficient $c(t)$, the final density of the species is decreasing, it is well known that with the decreasing of the density, the less chance for the species to meet suitable partner, and this increasing the extinct chance of the species. It is in this sense that the feedback control variable has the in stable effect.

We mention here that this is the first time that we find the feedback control variables has no influence to the persistent property of the stage structured ecological modelling. However, whether this conclusion still hold or not for the complicate system, for example, stage structured predator prey system is still unknown. We will try to investigate some more complicated ecological modelling in the future.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.