Reducing Environmental Hazards of Prismatic Storage Tanks under Vibrations

OLENA SIERIKOVA
Applied Mechanics and Environmental Protection Technologies Department
National University of Civil Defence of Ukraine
vul. Chernishevska, 94, Kharkiv, Ukraine, 61023
UKRAINE

ELENA STRELNIKOVA, DENYS KRIUTCHENKO, VASIL GNITKO
Department of Hydroaeromechanics of Power Machines
A.M. Pidhorny Institute for Mechanical Engineering Problems NAS of Ukraine
vul. Pozharskoho, 2/10, Kharkiv, Ukraine 61046
UKRAINE

Abstract: - Regular operation, pre-repair and repair work on tanks, as well as outflows of oil products and other flammable liquids under the influence of seismic loads, fires and explosions on tanks are the source of technogenic impact on the environment. Therefore, the influence treatment of the fluctuations and vibrations on the storage tanks for environmentally hazardous liquids and the assessment of reducing the load on nature is a very relevant scientific and practical issue to improve the environmental safety of areas adjacent to the tanks. This paper treats free and forced liquid vibrations in prismatic tanks filled with an incompressible ideal liquid. The developed method allows us to estimate the level of the free surface elevation in prismatic tanks under suddenly enclosed loadings. The proposed method makes it possible to determine a suitable place with a proper height for installation of the baffles in tanks by using numerical simulation and thus shortening the expensive field experiments. The proposed approach could be applied to various environmentally hazardous liquids. This will increase the environmental safety level of the territories adjacent to stationary tanks with environmentally hazardous liquid. It will also be possible to prevent emergencies.

Key-Words: - Dynamical Systems, Distributed Systems, Applied Systems Theory, oil products, flammable liquids, environmental safety, seismic loads, storage tanks

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1 Introduction

Prismatic tanks are widely used in various fields of modern technology and construction as tanks, containers, locks of hydraulic structures. They are also used to store the variety of liquids: drinking and firefighting water, oil, liquefied natural gas, wine, etc.

On large farms, liquid fertilizers, fuel for agricultural machinery, manure, silage, and so on are stored in such tanks. These substances often store centrally in large quantities, which increases the risk of environmental damage. Leakage of stored substances could be associated with insufficient tightness of storage tanks, accidents or operators unintentional actions or maintenance personnel, as well as seismic loads. The environmental hazard of substances used in agriculture is mainly caused by the high concentration of nutrients, as well as the possible presence of pathogens, veterinary drugs [1,2], or toxicity to aquatic organisms (for example gasoline, certain pesticides). Agricultural waste could also have a high biological oxygen demand, where the biological oxygen demand of bovine slurry could be up to 50 times higher than domestic wastewater. The result may be the death of aquatic organisms [3].

Reservoirs for storing oil products are environmentally dangerous sources of technogenic impact on the environment, acting as objects of uncontrolled emissions of steam-air or steam-gas-air mixtures and oil products spills, followed by fires and explosions. The environmental significance of storage largely depends on its potential to pollute the environment and on the physical and chemical properties of the stored substances. Oil tanks are used on the farm to store gasoline and diesel fuel. Properly designed oil storage facilities must prevent leakage and potential contamination of soil, surface or groundwater. Soil contamination includes the hazardous waste, oil spills, sludge from the treatment process, and coke dust. Soil contamination reduces the fertility of the soil and introduces foreign particles, which may affect the growth and quality of crops. One of the environmental impacts that may arise out of the implementation and operation of the storage tank is oil spill and evaporation of products that pollute the area. Each activity involved in the operation of a tank farm has a potential spill risk and evaporation of different types of gases causing pollution [4].

2 Problem Formulation

The analysis of the environmental impact sources during the tanks operation indicates that steel vertical tanks, even during normal operation, are environmentally hazardous. Fires and explosions on tanks from combustible substances and flammable liquids often occur during cleaning and preparation for repairs, as well as during the repair work itself. Regular operation, pre-repair and repair work on tanks with oil products, as well as outflows of oil products under the influence of seismic loads are the source of technogenic impact on the environment due to the occurrence of environmentally hazardous situations, accompanied by explosions and fires, and posing the real threat to the population life and health. Therefore, the influence treatment of the fluctuations and vibrations on the storage tanks for environmentally hazardous liquids and the assessment of reducing the load on nature is a very relevant scientific and practical issue to improve the environmental safety of areas adjacent to the tanks [5,6].

In previous studies [7], the authors have considered seismic loads on cylindrical reservoirs [8,9], the use of composite materials with nanoinclusions as reservoir materials, as well as the effect of flooding and flooded soils on technogenic objects and on the seismic activity increasing in [10,11]. This paper considers prismatic storage tanks for environmentally hazardous liquids. The environment inside the tank has significantly affected their dynamic characteristics (frequencies and vibration modes) and stress-strain state. If a tank structure resting on the ground experiences dynamic or kinematic effects as a result of earthquakes, the impulsive wave pressure component and other perturbations, then its response (displacements, forces, frequencies and vibration modes) depends significantly on the degree of vessel filling, the elastic properties of the foundations, and also stiffness inertial characteristics of the structure itself.

Failures of these tanks, following destructive earthquakes or explosives, may lead to environmental catastrophes, loss of valuable contents, and disruption of fire-fighting efforts. Liquid sloshing is also a reason for damage of roofs and upper walls of storage tanks. Inadequately designed tanks and liquid storages were damaged in past extensive earthquakes.

Comprehensive reviews of the phenomenon of sloshing [9], including analytical predictions and experimental observations were done in the different works [7,12]. It has been proved that exact solutions for the linear liquid sloshing are limited by tanks geometry with straight walls, as rectangular and cylindrical containers.

Furthermore, it is difficult or impossible to obtain analytical solutions for reservoirs and tanks with complex geometric shapes. Therefore, many numerical methods have been implemented to solve linear boundary value problems of liquid sloshing. Diverse simplified theoretical studies have been provided [12,13], and other numerical methods have been developed. Some of these researches have been used as the basis for current design standards. Raynovskyy and Tymokha [14] have developed the analytical linear multimodal method to analyze 2D liquid sloshing in horizontal cylindrical tanks. The liquid volume method has been developed at [12,13]. Dynamic analysis of fluid-filled shell structures using compatible finite element and boundary element methods has been performed at [15,16].

Many slosh suppression devices have been proposed to damp fluid motion, reduce structural loads caused by fluid spillage, and prevent instability [17]. These devices are rigid or elastic with different sizes and orientations, annular partitions, and different plates that partially cover the free surface [18]. The suppression systems design requires the quantitative knowledge of the slosh characteristics.

In practice, the baffles effect could be seen after the baffles installation. However, such experimental research is too expensive. Therefore, the computing technologies development for qualitative numerical modeling is a very relevant issue.

The paper treats the free and forced liquid vibrations in prismatic tanks filled with the incompressible ideal liquid. Reservoirs with both horizontal and vertical baffles have been considered. The changing dynamical reservoir behavior with the internal baffle has been established. The harmonic and impulse loads have been supposed to be applied to the considered structures.

3 Problem Solution

Issues concerned with liquid vibrations in rigid prismatic and cylindrical tanks have been considered in the paper. The considered reservoirs have been presented on Figure 1. The horizontal and vertical baffles have been installed into cylindrical tanks. At the first stage, reservoirs have their own modes and frequencies. Their own modes have been used as basic systems for the forced vibration problems under harmonic and impulse loads.

It has been supposed the liquid in the containers as the ideal and incompressible one, and the fluid motion caused by shell vibrations as irrotational. Then its relative velocity ***V*** has a potential $\Phi = \Phi(t, x, y, z)$, so that

$$V_x = \frac{\partial \Phi}{\partial x}; V_y = \frac{\partial \Phi}{\partial y}; V_z = \frac{\partial \Phi}{\partial z}. \quad (1)$$

The moistened shell surface has been designated by S_1, and the free surface by S_0. Suppose the Cartesian coordinate system $0xyz$ has been connected with considered containers, the liquid free surface S_0 coincides with the plane $z = 0$ at the rest state. If the liquid-filled shell is under the force acting in the horizontal plane, then the coordinate axes with orts may be chosen so that acceleration $\mathbf{a}_s$ has been considered as

$$\mathbf{a}_s = a_s(t)\,\mathbf{i}, \quad (2)$$

where the factor $a_s(t)$ depends only on time t, and $\mathbf{i}$ is the unit vector along Ox.

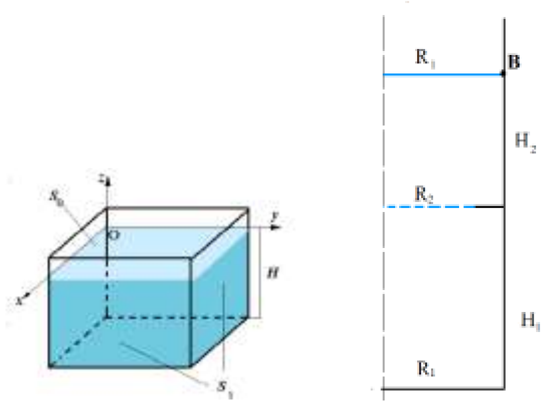
Firstly it has been obtained the relation between the velocity potential, accelerations due to driving forces, and the liquid pressure. Equation (2) has been acquired in the following form:

$$\mathbf{a}_s = \nabla[x \cdot a_s(t)] \quad (3)$$

Note that for the gravitational acceleration there have been obtained form

$$\mathbf{g} = -\nabla(\rho g z),$$

where g is the gravity acceleration, z is the vertical coordinate of a point in the liquid.



a) b)

Figure 1: Partially filled with the liquid reservoirs

Motion equations for the ideal liquid could be represented in the vector form as follows:

$$\rho \mathbf{w} + \nabla(\rho x a_s(t)) + \nabla(\rho g z) = -\nabla p,$$

where $\mathbf{w}$ is the fluid flow acceleration, ρ is the liquid density, and p is the fluid pressure. When $\rho = \text{const}$ the following relation is valid:

$$\mathbf{w} + \nabla(x a_s(t)) + \nabla(g z) = -\frac{\nabla p}{\rho}. \quad (4)$$

Therefore, the liquid particle's acceleration under gravitational forces and horizontal excitations always has the potential (an analog of the Prandtl's potential). The driving forces are usually considered starting from the rest state. Thus, according to Kelvin's theorem, if the liquid motion starts from rest, then its relative velocity ***V*** has the potential.

Formula has been obtained for the pressure (Bernoulli's equation) considering the gravity and horizontal accelerations. Using equation (4) and assuming that the flow is irrotational, Bernoulli equation has been received in the following form:

$$p - p_0 = -\rho\left[\frac{\partial \Phi}{\partial t} + a_s(t)x + gz + \frac{1}{2}|\nabla \Phi|^2\right], \quad (5)$$

where p_0 is for atmospheric pressure.

If small liquid oscillations have been considered (the linearized formulation has been studied), then $|\nabla \Phi|^2 << 1$, and there have been obtained formula

$$p - p_0 = -\rho\left[\frac{\partial \Phi}{\partial t} + a_s(t)x + gz\right]. \quad (6)$$

Pressure p on the shell walls in the absence of the horizontal volume force has been determined from the linearized Bernoulli equation by the formula

$$p = -\rho\left(\frac{\partial \Phi}{\partial t} + gz\right) + p_0, \quad (7)$$

Formulas (5)-(7) shows that to receive the liquid pressure it is necessary to obtain the liquid potential Φ. It has been assumed the flow to be inviscid and incompressible, the irrotational fluid motion in the 3D reservoir described by the Laplace equation for the velocity potential Φ

$$\nabla^2 \Phi = 0. \quad (8)$$

The mixed boundary value problem for the Laplace equation has been formulated to determine this potential. The non-penetration condition on the wetted tank surfaces S_1 has been applied [16]

$$\left.\frac{\partial \Phi}{\partial \mathbf{n}}\right|_{S_1} = 0.$$

On the free surface ($z = 0$), the following dynamic and kinematic boundary conditions must be satisfied:

$$\left.\frac{\partial \Phi}{\partial n}\right|_{S_0} = \frac{\partial \zeta}{\partial t}; \quad p - p_0\big|_{S_0} = 0,$$

where the unknown function $\zeta = \zeta(x, y, t)$ describes the free surface shape and position.

Therefore, for the velocity potential there have been obtained the following boundary-value problem

$$\nabla^2\Phi = 0\,;\ \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_1} = 0\,;\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t};\quad \left.p - p_0\right|_{S_0} = 0\,. \quad (9)$$

where $p - p_0$ has been received from equation (6) at $z = \zeta(t, x, y)$.

To calculate the liquid vibrations in the presence of baffles, consider at first the cylindrical shell with a ring baffle, Fig. 1b). Let wetted surface be $S_1 = S_{11} \cup S_{12} \cup S_{\text{bot}} \cup S_{\text{baf}}$,

The multi-domain (boundary super-elements) method has been used. Note, it has introduced the "artificial" interface surface Sint [13], and divided the region filled with the liquid on the two parts $\Sigma_1;\Sigma_2$, bounded by surfaces S_{11}, S_{baf}, S_{int} , $S_{\text{ bot}}$ and S_{12}, S_{baf}, S_{int}, S_0, respectively. On the interface surface S_{int}, the following boundary conditions have been set [19,20]:

$$\left.\Phi\right|_{S_{\text{int}}\cap\partial\Sigma_1} = \left.\Phi\right|_{S_{\text{int}}\cap\partial\Sigma_2};\quad \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_{\text{int}}\cap\partial\Sigma_1} = -\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_{\text{int}}\cap\partial\Sigma_2} \quad (10)$$

The zero eigenvalue obviously exists for problem (9), but it has been excluded with the help of the following orthogonality condition:

$$\iint\limits_{S_0}\frac{\partial\Phi}{\partial\mathbf{n}}dS_0 = 0\,. \quad (11)$$

So, equations (9)-(11) for calculating the unknown functions $\zeta = \zeta(x, y, t)$ and $\Phi = \Phi(t, x, y, z)$ have been obtained.

Consider the cylindrical quarter tank, Fig. 1d). Suppose R is tank radius, and H is for filling level. Using cylindrical coordinate system there have been obtained the following boundary value problem

$$\nabla^2\Phi = 0\,,\quad \left.\frac{\partial\varphi}{\partial r}\right|_{r=R} = 0,\quad \left.\frac{\partial\varphi}{\partial z}\right|_{z=-h} = 0,\quad \left.\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right|_{\theta=0,\theta=\frac{\pi}{2}} = 0,$$

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t};\quad \left.p - p_0\right|_{S_0} = 0 \quad (12)$$

with orthogonality condition (11) and $p - p_0$ obtained from (7) at $z = \zeta(t, x, y)$.

For the prismatic tank depicted in Fig. 1a) there could be solved the boundary value problem (9) with the performance of equation (11).

3.1 Mode decomposition method

For all presented boundary value problems let consider the auxiliary boundary value problems have been received

$$\nabla^2\psi = 0\,,\ \left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{S_1} = 0\,,\ \left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{S_0} = \frac{\partial\varsigma}{\partial t};\ \ \frac{\partial\psi}{\partial t} + g\zeta = 0\,. (13)$$

Differentiate the second correlation in (13) by respect to t and substitute received equality for $\frac{\partial\varsigma}{\partial t}$ into the first relation. Further the auxiliary function ψ as $\psi(t, x, y, z) = e^{i\chi t}\varphi(x, y, z)$ has been presented.

It has been come to the eigenvalue problem

$$\nabla^2\varphi = 0\,,\ \left.\frac{\partial\varphi}{\partial\mathbf{n}}\right|_{S_1} = 0\,,\ \left.\frac{\partial\varphi}{\partial\mathbf{n}} = \frac{\chi^2}{g}\varphi\right|_{S_0}\,,\ \iint\limits_{S_0}\frac{\partial\varphi}{\partial\mathbf{n}}dS_0 = 0\,. \quad (14)$$

Suppose the solutions of eigenvalue problem (14) are their own modes φ_k with corresponding own frequencies χ_k . Consider now the potential $\mathbf{\Phi}$ in the next form:

$$\Phi = \sum_{k=1}^{M}\dot{d}_k\varphi_k\,. \quad (15)$$

As the equation for the free surface there have been gained the following expression:

$$\zeta = \sum_{k=1}^{M} d_k\frac{\partial\varphi_k}{\partial n}\,. \quad (16)$$

3.2 Reducing to systems of boundary integral equations

For the prismatic reservoirs, the boundary value problem (9), (11) has been solved analytically using the Fourier method of variable separation.

Shell of revolution with and without baffles

Consider the boundary value problem (9),(11) for the cylindrical shell without baffles.

It has been supposed the structure under consideration is a shell of revolution, in cylindrical coordinates system (r, z, θ) there have been gained

$$\varphi_k(r, z, \theta) = \varphi_k(r, z)\cos\alpha\theta,\quad \alpha = 0,1,\ldots,\quad k = 1,2,\ldots\,. (17)$$

Here α is a harmonica number, indexes k is for mode numbers, corresponding to α. Thus, frequencies and modes of free vibrations are considered separately for different values of α.

Dropping indices k, the main relation for determining functions φ_k could be written in the form [19,20]

$$2\pi\varphi(P_0) = \iint\limits_S\frac{\partial\varphi}{\partial\mathbf{n}}\frac{1}{|P - P_0|}dS - \iint\limits_S\varphi\frac{\partial}{\partial\mathbf{n}}\frac{1}{|P - P_0|}dS\,. (18)$$

Here $S = S_1 \cup S_0$; both points P and P_0 belong to the surface S. By $|P - P_0|$ there have been denoted the Cartesian distance between points P and P_0.

With boundary conditions (9), there have been gained the system of integral equations in the form [6,7]

$$\begin{cases} 2\pi\psi_1 + \iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS_1 - \frac{\kappa^2}{g} \iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0 + \iint\limits_{S_0} \psi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dS_0 = 0; \\ -\iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS_1 - 2\pi\psi_0 + \frac{\kappa^2}{g} \iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0 = 0. \end{cases} \quad (19)$$

Here for convenience it has been denoted by ψ_0 values of the potential on the free surface and by ψ_1 its values on the shell walls.

There have been searched solutions in form (17) for the system (19).

The following integral operators have been presented below:

$$A\psi_1 = 2\pi\psi_1 + \iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n} \frac{1}{r(P,P_0)} dS_1;$$

$$B\psi_0 = \iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0;\ C\psi_0 = \iint\limits_{S_0} \psi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dS_0;$$

$$D\psi_1 = -\iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n} \frac{1}{|P-P_0|} dS_1;\ F\psi_0 = \iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0 . \quad (20)$$

Then the boundary value problem (9) takes the form

$$A\psi_1 = \frac{\kappa^2}{g} B\psi_0 - C\psi_0;\ P_0 \in S_1;$$

$$D\psi_1 = 2\pi E\psi_0 - \frac{\kappa^2}{g} F\psi_0;\ P_0 \in S_0 .$$

After excluding function ψ_1 from these relations the following eigenvalue problem relative to ψ_0 have been obtained

$$(DA^{-1}C + E)\psi_0 - \lambda(DA^{-1}B + F)\psi_0 = 0;\quad \lambda = \frac{\chi^2}{g}.$$

Its solution gives natural modes and frequencies of liquid sloshing in rigid tank. Evaluation of integral operators in (21) has been carried out by the method proposed in [9,19,20].

Having defined the basic functions φ_k, substitute them in expressions (16) for velocity potential and (17) for the free surface elevation. Then substitute the received relations in the boundary condition on the free surface that correspond to application of the dynamical condition [20,21].

$$\left.\frac{\partial\Phi}{\partial t} + g\zeta + a_s(t)x\right|_{S_0} = 0$$

As in the cylindrical system of coordinates there is $x = \rho\cos\theta$, there could be interest only in the first harmonica, i.e. in formula (17) it has been only considered α=1. Furthermore, there could be the following equation on the surface S_0

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + g\sum_{k=1}^{M} d_k \frac{\partial\varphi_k}{\partial n} + a_s(t)\rho = 0 .$$

Due to validity of relation (15) on the surface S_0 the equality given above takes the form

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + \sum_{k=1}^{M} \chi_k^2 d_k \varphi_k + a_s(t)\rho = 0 . \quad (21)$$

Accomplishing the dot product of equality (21) by $\varphi_l\left(l = \overline{1,M}\right)$ and having used orthogonality of own modes, it has been received the system of ordinary differential equations of the second order

$$\ddot{d}_k + \chi_k^2 d_k + a_s(t) F_k = 0;\quad F_k = \frac{(\rho,\varphi_k)}{(\varphi_k,\varphi_k)};\quad k = \overline{1,M} . \quad (22)$$

Suppose that before applying the horizontal loading, the tank was at the rest state. Then it must be solved system (22) under zero initial conditions.

The analogical procedure for the shell of revolution with ring baffles has been proposed and described in detail in authors' papers [16,18,21]. It should be noted that the system of second order differential equations has the form (22), but with other modes and frequencies.

According to [22-25] there have been assumed for the shell of revolution with two vertical baffles could be sought the basic function in the next form:

$$\varphi_k(r,z,\theta) = \varphi_k(r,z)\cos 2\alpha\theta,\quad \alpha = 0,1,\ldots,\quad k = 1,2,\ldots . \quad (23)$$

Using this correlation boundary conditions have been satisfied on the vertical baffles. The system of singular integral equations for $\varphi_k(r,z)$ acquires the form analogical to system (19). Let Γ be a generator of the surface S_1. Reducing integrals in this system to one-dimensional ones the system of one-dimensional integrals that has the same form as in [18] has been obtained. For one-dimensional integrals in (20) there have been gained formulas

$$\iint\limits_{S_1} \psi \frac{\partial}{\partial n}\left(\frac{1}{|P-P_0|}\right) dS_1 = \int\limits_{\Gamma} \psi(z)\Theta(z,z_0) r(z) d\Gamma ;$$

$$\iint\limits_{S_0} \psi \frac{1}{|P-P_0|} dS_0 = \int\limits_0^R \psi(\rho)\Phi(P,P_0)\rho d\rho .$$

Here kernels are differed from those obtained in [16,19] and are presented as follows:

$$\Theta(z,z_0) = \frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2 - r_0^2 + (z_0 - z)^2}{a-b} \mathsf{E}_\alpha(k) - \mathsf{F}_\alpha(k)\right] n_r + \frac{z_0 - z}{a-b} \mathsf{E}_\alpha(k) n_z\right\};$$

$$\Psi(P,P_0) = \frac{4}{\sqrt{a+b}} \mathsf{F}_\alpha(k);$$

$$\mathsf{E}_\alpha(k) = (-1)^\alpha (1 - 16\alpha^2) \int\limits_0^{\pi/2} \cos 4\alpha\psi \sqrt{1 - k^2 \sin^2\psi}\, d\psi ;$$

$$\mathsf{F}_\alpha(k) = (-1)^\alpha \int\limits_0^{\pi/2} \frac{\cos 4\alpha\psi\, d\psi}{\sqrt{1 - k^2 \sin^2\psi}} ;$$

$$a = r^2 + r_0^2 + (z - z_0)^2; \quad b = 2rr_0; \; k^2 = \frac{2b}{a+b}. \quad (24)$$

Note, that the system of second order differential equations has the form (22), but with other modes and frequencies, has been obtained by using the solution of the system of integral equation (19) but with kernels (24).

3.3 Liquid free vibrations

The prismatic reservoir $\{-a \le x \le a, \;\; -b \le y \le b, \;\; 0 \le z \le H\}$, filled with the ideal incompressible liquid has been considered. The own modes and frequencies of fluid vibrations in the prismatic reservoir have been determined by the Fourier method of separation of variables. As first, the solution of Laplace equation with non-penetration boundary conditions (first and second equalities in (9)) has been found

$$\Psi_{kl} = A_{kl} \operatorname{ch} \lambda_{kl} z \cos\frac{\pi k}{2a} x \cos\frac{\pi l}{2b} y, \quad \lambda_{kl} = \sqrt{\left(\frac{\pi k}{2a}\right)^2 + \left(\frac{\pi l}{2b}\right)^2}$$

Satisfying the condition on the free surface (third equation in (9)), there have been obtained

$$\lambda_{kl} \operatorname{sh} \lambda_{kl} H = \frac{\omega^2}{g} \operatorname{ch} \lambda_{kl} H .$$

From this the expression for the natural frequencies of the liquid in the prismatic tank have been found

$$\omega_{kl} = \sqrt{g\lambda_{kl} \tanh(\lambda_{kl} H)} .$$

The hereinafter the expressions for the first 8 own modes of fluid vibrations (they also are the system of basis functions for solving the problem of forced vibrations) have been presented

$$\Psi_{01}(x) = C_1 \cos(0 \cdot x) \sin\frac{\pi}{2b} y,$$

$$\Psi_{10}(x) = C_2 \sin\frac{\pi}{2a} x \cos(0 \cdot y), \; \Psi_{11}(x) = C_3 \sin\frac{\pi}{2a} x \sin\frac{\pi}{2b} y,$$

$$\Psi_{20}(x) = C_4 \cos\frac{\pi}{a} x \cos(0y), \; \Psi_{02}(x) = C_5 \cos 0x \cos(\frac{\pi}{b} y),$$

$$\Psi_{21}(x) = C_6 \cos\frac{\pi}{a} x \sin\frac{\pi}{2b} y,$$

$$\Psi_{12}(x) = C_7 \sin\frac{\pi}{2a} x \cos\frac{\pi}{b} y, \; \Psi_{22}(x) = C_8 \cos\frac{\pi}{a} x \cos\frac{\pi}{b} y \quad (25)$$

Table 1 shows the numerical values of frequencies ω_{ij} and the frequency parameter λ_{ij} for the prismatic reservoir in the form of a cube with $a = b = H = 1$m.

Table 1. Natural frequencies of fluid oscillations in the prismatic tank

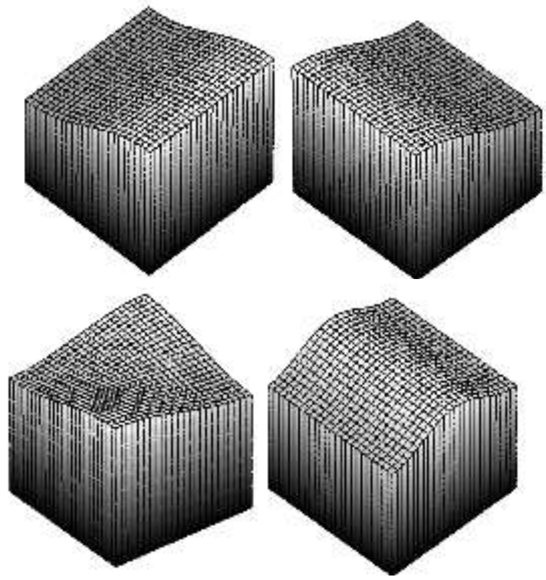
n	i	j	λ_{ij}	ω_{ij}
1	0	1	1.772453851	4.051164194
2	1	0	1.772453851	4.051164194
3	1	1	2.506628275	5.710012556
4	0	2	3.544907703	5.892165855
5	2	0	3.544907703	5.892165855
6	2	1	3.963327298	6.233151691
7	1	2	3.963327298	6.233151691
8	2	2	5.013256550	7.012538645

For receiving the expression for function ζ the following formula (15) for the velocity potential has been used. Dependence $n = n(i, j)$ has been shown in Table 1, the functions φ_n have been determined by the formulas

$$\varphi_n = \frac{1}{ab} \frac{\cosh(\lambda_{ij} z)}{\cosh(\lambda_{ij} H)} \Psi_{ij}(x, y); \quad n = n(i, j), (26)$$

where Ψ_{ij} has been found from (25). So the free surface elevation has been expressed by formula (16), where the basic function φ_n have been defined in (26)

The vibration modes of the free surface have been shown on Figure 2.



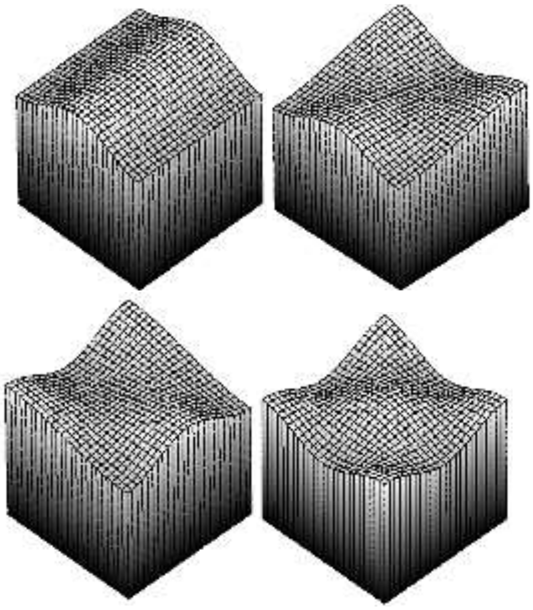


Fig.2. Modes of free surface vibrations in the prismatic tank

Thus, the basic system for the liquid forced vibration in rigid prismatic tanks has been built.

3.4 Forced vibrations of the liquid under harmonic loads

Suppose that at the initial time the liquid in the prismatic tank is at rest. The tank subjected to periodic loads cosωt, that applied in the horizontal direction (parallel to Ox axis, Fig. 1). The system of differential equations of the fluid motion has been obtained, starting from the next boundary condition on the free surface:

$$\left.\frac{\partial \Phi}{\partial t} + g\zeta(x, y) + x a_s(t)\right|_{z=H} = 0 . \qquad (27)$$

Substituting expressions (24), (25) into relation (27) leads to

$$\sum_{k=1}^{M} \ddot{d}_k(t)\varphi_k(x, y, H) + g\sum_{k=1}^{M} d_k(t)\left.\frac{\partial \varphi_k(x, y, z)}{\partial z}\right|_{z=H} + x\cos\omega t = 0 \qquad (28)$$

Then the following system of differential equations has been gained accomplishing dot product of correlation (28) by φ_n and using the orthogonality of sloshing modes:

$$\ddot{d}_1(t) + \omega_1^2 d_1(t) - \frac{8}{9\pi^2}\cos\omega t = 0 ;$$
$$\ddot{d}_2(t) + \omega_2^2 d_2(t) + \frac{8}{25\pi^2}\cos\omega t = 0 ;$$
$$\ddot{d}_3(t) + \omega_3^2 d_3(t) - \frac{8}{121\pi^2}\cos\omega t = 0 \qquad (29)$$

Solution of system (29) is following:

$$d_1(t) = \frac{8}{9\pi^2\left(\omega_1^2 - \omega^2\right)}\left(\cos\omega t - \cos\omega_1 t\right);$$
$$d_2(t) = \frac{8}{25\pi^2\left(\omega_2^2 - \omega^2\right)}\left(\cos\omega t - \cos\omega_2 t\right);$$
$$d_3(t) = \frac{8}{121\pi^2\left(\omega_3^2 - \omega^2\right)}\left(\cos\omega t - \cos\omega_3 t\right). \qquad (30)$$

Time history of the free surface elevation in the point ξ with coordinates $x = 1, y = 1, z = 1$ during 10 sec at ω =1.1Hz has been shown on Figure 3.

Figure 4 shows the influence of the load's frequency ω on the free surface level elevation.

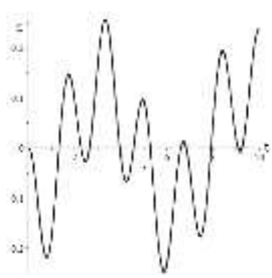


Figure 3. Time history of free surface via elevation

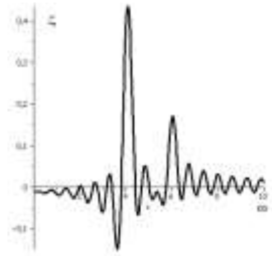


Figure 4. Free surface elevation loads frequencies

It would be noted that convergence here is achieved at M=3.

The peaks in the graph correspond to the frequencies ω_{10} and ω_{20} (table 1). These frequencies are the most dangerous ones, for example, during transportation of the tank under consideration. The function describing the free surface elevation of the liquid has been obtained as

$$\zeta = \sum_{k=1}^{M} d_k(t)\frac{\partial \varphi_k(x, y, H)}{\partial z},$$

where the coefficients $d_k(t)$ have been determined by formulas (30).

3.5 Forced vibrations of the liquid under impulse loads

It has been considered the rigid prismatic tank filled with the liquid. The tank parameters are the following: $H = b =$ 1m and a = 1m or a = 2m. The pressure p on tank walls from formula (7) has been determined. Here $a_s(t)$ is the function characterizing external influence (a horizontal seism or an impulse).

The load is suddenly applied to the lateral surface of the prismatic tank $a_s(t) = Q_0 a(t)$, where Q_0 = 1 MPa is the distributed pressure, and

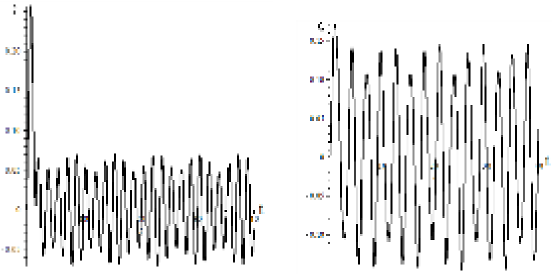
$$a(t) = \begin{cases} 1, & t < T, \\ 0, & t \geq T. \end{cases} \qquad (31)$$

It has been supposed before applying the horizontal impulse the tank was at the state of rest. Then there have been solved systems (30) under zero initial conditions. The operational method [20] has been applied here for receiving the solution of system (30).

The following values for coefficients $d_k(t),\quad k=\overline{1,M}$ have been obtained:

$$\frac{d_k(t)}{Q_0}=\begin{cases}\dfrac{1}{\chi_k^2}-\dfrac{1}{\chi_k^2}\cos(\chi_k t) & 0\le t\le T\\ \dfrac{1}{\chi_k^2}-\dfrac{1}{\chi_k^2}\cos(\chi_k t)-\dfrac{1}{\chi_k^2}+\dfrac{1}{\chi_k^2}\cos\chi_k(t-T) & t>T\end{cases}$$

On Figure 5 the free surface elevation in the point x=1, y=1, z=1 depending on time has been shown for T=1.5 sec.



a) b)

Figure 5. Time-histories of the free surface elevation

Figures 5 a) and b) correspond to a=1m and a=2m, respectively, and this is the side of the prismatic tank along which the load has been applied. Increasing this side leads to alignment and mitigation of sloshing amplitudes

4 Conclusion

The developed method allows us to estimate the level of the free surface elevation in prismatic tanks under suddenly enclosed loadings. The free and forced liquid vibrations in prismatic tanks of equal heights have been considered. The benchmark tests have been considered that validated the obtained results. The effects of the baffle installation and their influence on changing the elevation of the free surface have been taken into account. The elaborated approach allows us to carry out the numerical simulation of baffled tanks with baffles of different sizes and with different positions in the tank. This gives the possibility of governing the baffle radius and its position within the tank. It is very topical, because practically, the effect of baffles could be seen usually only after the baffle has been installed. The proposed method makes it possible to determine a suitable place with a proper height for installation of the baffles in tanks by using numerical simulation and thus shortening the expensive field experiments. It would be noted that only the ideal incompressible liquid is under consideration. The proposed approach could be applied to various environmentally hazardous liquids. The proposed approach will be easily generalized to elastic tanks with elastic baffles. Thus, the future research concerned with free and forced liquid vibrations in elastic tanks with elastic baffles. The geometry of the tank also could be easily changed, so the results will be obtained for conical, spherical and compound shells with and without baffles. It will allow giving recommendations about installation of protective elements (covers, partitions). This will increase the environmental safety level of the territories adjacent to stationary tanks with environmentally hazardous liquid. It will also be possible to prevent emergencies.

References:

[1] Charuaud L., Jarde E., Jaffrezic A., Thomas M.-F., Le Bot B., 2019. Veterinary pharmaceutical residues from natural water to tap water: Sales, occurrence and fate, Journal of Hazardous Materials. Vol. 361: 169-186. https://doi.org/10.1016/j.jhazmat.2018.08.075

[2] Gros M., Mas-Pla J., Boy-Roura M., Geli I., Domingo F., Petrović M., 2019. Veterinary pharmaceuticals and antibiotics in manure and slurry and their fate in amended agricultural soils: Findings from an experimental field site (Baix Empordà, NE Catalonia), Science of The Total Environment. Vol. 654: 1337-1349. https://doi.org/10.1016/j.scitotenv.2018.11.061

[3] Trávníček P., Kotek L., Junga P., Koutný T., Novotná J., Vítěz T., 2019. Prevention of accidents to storage tanks for liquid products used in agriculture. Process Safety and Environmental Protection. Vol. 128: 193-202.

[4] Tadros F. F., 2020. Environmental aspects of petroleum storage in above ground tank. The International Conference on Sustainable Futures: Environmental, Technological, Social and Economic Matters (ICSF 2020). E3S Web of Conferences. Vol. 166: 1-5. https://doi.org/10.1051/e3sconf/202016601006

[5] Dadashov I., Loboichenko V., Kireev A., 2018. Analysis of the ecological characteristics of environment friendly fire fighting chemicals used in extinguishing oil products. Pollution Research. V. 37, №1: 63-77.

[6] Khalmuradov B., Harbuz S., Ablieieva I., 2018. Analysis of the technogenic load on the environment during Forced ventilation of tanks. Technology audit and production reserves. #1/3 (39): 45-52. DOI: 10.15587/2312-8372.2018.124341

[7] Sierikova O., Strelnikova E., Degtyarev K., 2022. Seismic Loads Influence Treatment on the Liquid Hydrocarbon Storage Tanks Made of Nanocomposite Materials. WSEAS Transactions on Applied and Theoretical Mechanics, vol. 17: 62-70. DOI: 10.37394/232011.2022.17.9

[8] Sierikova O., Strelnikova E., Gnitko V., Tonkonozhenko A., Pisnia L., 2022. Nanocomposites Implementation for Oil Storage Systems Electrostatic Protection. Conf. Proc. of Integrated Computer

Technologies in Mechanical Engineering – ICTM-2021. Synergetic Engineering Springer Nature Switzerland AG 2022 M. Nechyporuk et al. (Eds.): ICTM 2021, LNNS 367: 573-585. https://doi.org/10.1007/978-3-030-94259-5_49

[9] Sierikova O., Strelnikova E., Gnitko V., K. Degtyarev, 2021. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. IEEE 2nd KhPI Week on Advanced Technology (KhPIWeek): 351-356, doi: 10.1109/KhPIWeek53812.2021.9570086.

[10] Sierikova O., Koloskov V., Degtyarev K., Strelnikova O., 2021. The Deformable and Strength Characteristics of Nanocomposites Improving. Materials Science Forum. Trans Tech Publications Ltd, Switzerland. Vol. 1038: 144-153.

[11] Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E., 2020. Flood risk management of Urban Territories. Ecology, Environment and Conservation 26 (3). India: 1068-1077.

[12] Zheng Jh., Xue M.A., Dou P., et al, 2021. A review on liquid sloshing hydrodynamics. J Hydrodyn, vol. 33: 1089–1104. DOI:10.1007/s42241-022-0111-7.

[13] Sun Y., Zhou D., Wang J., 2019. An equivalent mechanical model for fluid sloshing in a rigid cylindrical tank equipped with a rigid annular baffle. Applied Mathematical Modelling, vol. 72: 569-587.

[14] Raynovskyy I.A., Timokha A.N., 2020. Sloshing in Upright Circular Containers: Theory, Analytical Solutions, and Applications, CRC Press/Taylor & Francis Group: 170 p.

[15] Strelnikova E., Choudhary N., Kriutchenko D., Gnitko V., Tonkonozhenko A., 2020. Liquid vibrations in circular cylindrical tanks with and without baffles under horizontal and vertical excitations, Engineering Analysis with Boundary Elements, vol. 120: 13-27. DOI: 10.1016/j.enganabound.2020.07.02m

[16] Strelnikova E., Kriutchenko D., Gnitko V., Degtyarev K., 2020. Boundary element method in nonlinear sloshing analysis for shells of revolution under longitudinal excitations, Engineering Analysis with Boundary Elements, vol. 111: 78-87. DOI: 10.1016/j.enganabound.2019.10.008.

[17] Zang Q., Liu J., Lu L., Lin G., 2020. A NURBS-based isogeometric boundary element method for analysis of liquid sloshing in axisymmetric tanks with various porous baffles, European Journal of Mechanics - B/Fluids, Volume 81: 129-150. https://doi.org/10.1016/j.euromechflu.2020.01.010.

[18] Choudhary N., Narveen K., Strelnikova E., Gnitko V., Kriutchenko D., Degtyariov K., 2021. Liquid vibrations in cylindrical tanks with flexible membranes, Journal of King Saud University - Science, Vol. 33(8): 101589, https://doi.org/10.1016/j.jksus.2021.101589.

[19] Behshad A., Reza Shekari M., 2018. A Boundary Element Study for Evaluation of the Effects of the Rigid Baffles on Liquid Sloshing in Rigid Containers, International journal of maritime technology, Vol.10: 45 - 54, DOI: 10.29252/ijmt.10.45.

[20] Guo Y., Iyer S., 2021. Regularity and expansion for steady Prandtl equations, Comm. Math. Phys., 382(3): 1403-1447.

[21] Srivastava H. M., 2020. Integral Transformations, Operational Calculus and Their Applications, Symmetry 12(7): 1169 p. DOI: 10.3390/sym12071169

[22] Degtyariov K., Gnitko V., Kononenko Y., Kriutchenko D., Sierikova O., Strelnikova E. Fuzzy Methods for Modelling Earthquake Induced Sloshing in Rigid Reservoirs. 2022 IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek), 2022. P. 297-302. DOI: 10.1109/KhPIWeek57572.2022.9916466

[23] Sierikova O., Strelnikova E., Degtyarev K. Srength Characteristics of Liquid Storage Tanks with Nanocomposites as Reservoir Materials. 2022 IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek), 2022. P. 151-157. DOI: 10.1109/KhPIWeek57572.2022.9916369

[24] Sierikova O., Koloskov V., Degtyarev K., Strelnikova E. Improving the Mechanical Properties of Liquid Hydrocarbon Storage Tank Materials. Materials Science Forum. Trans Tech Publications Ltd, Switzerland. Vol. 1068, 2022. P. 223-229. doi:10.4028/p-888232

[25] Smetankina N., Merkulova A., Merkulov D., Postnyi O. Dynamic Response of Laminate Composite Shells with Complex Shape Under Low-Velocity Impact. In: Nechyporuk M., Pavlikov V., Kritskiy D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. ICTM 2020. *Lecture Notes in Networks and Systems*, vol. 188, 2021. Springer, Cham. https://doi.org/10.1007/978-3-030-66717-7_22

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Olena Sierikova: conceptualisation, data curation, formal analysis, methodology.
Elena Strelnikova carried out the simulation and the optimization.
Denys Kriutchenko: visualization, data curation.
Vasil Gnitko: carried out the simulation and the optimization.

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