Modeling and Analysis of DC-DC CUK Converter with Coupled Inductors

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Abstract: In this article, the analysis of the inductive coupled CUK topology, which is a DC-DC converter, which is of great importance for power electronics and used in structures such as electric vehicles and PV systems, with Modified Nodal Analysis (MNA) and the modeling of the elements in the circuit structure will be explained. The numerical values of the semiconductors and passive circuit elements to be modeled will be given the voltage, and current at the output will be calculated. In addition, the graphs of the parameters analyzed and analyzed with MNA will be created with the code system written in the MATLAB environment and will be explained in this article.

Keywords: Modified Nodal Analysis (MNA), Cuk Converter, DC-DC Converter
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1. Introduction

Nowadays, with the widespread use of electric vehicles and the increase in the need for energy, there are many DC-DC converter topologies in active use. The main reason for the increase in these structures is due to the fact that the structures used in energy storage systems are DC. Additionally, solar panels and electric vehicle charging stations, which are becoming more and more common today, can be given as examples for these. The DC-DC converter structure is used to convert a constant DC current to an adjustable DC according to needs. If the input voltage is lower than the output voltage, it is called a boost converter, and if the input voltage is higher than the output voltage, it is called a buck converter [1-5]. The boost DC-DC converter is a popular power electronics device with a simple low-component structure with a continuous input current, a switch and a diode. However, this DC-DC converter is not capable of providing a high voltage gain ratio. The buck DC-DC converter structure is a converter structure in which similar circuit elements are used, just like the boost DC-DC converter structure. The main difference in this circuit topology is the positioning of the semiconductor and passive circuit element used in the switching. The buck DC-DC converter is similarly incapable of providing a high voltage gain ratio. Providing high voltage gain ratio varies depending on the duty cycle applied to the switch. According to the ideal voltage gain relationship of the mentioned DC-DC converter circuit topologies, the voltage occupancy ratio should reach infinity when the occupancy ratio reaches infinity. However, when the duty cycle reaches infinity, it does not result in a high value of voltage gain due to parasitic effects in the voltage gain equation. The Cuk DC-DC converter structure is basically similar to other topologies, but it is a topology created by eliminating some of the disadvantages of the amplifier topology [6]. The continuous input current in the Amplifier and Cuk topologies helps to reduce the capacitance value by reducing the input current stress on the capacitance. Moreover, the Cuk topology, unlike the amplifier topology, ensures that the output current is also continuous. Another advantage provided by the Cuk converter structure allows it to be used in three different operating modes; reducer, riser and pass through. It decides in which operating mode it will work according to the selected duty cycle. In this article, in the first part, the DC-DC converter Cuk structure will be explained, and the mathematical equations used in Cuk converter will be expressed. In the second part, the modeling of the circuit elements used in the Cuk converter structure with modified nodal analysis (MNA) and the solution methods will be explained. In the fourth chapter, the modeling of the circuit structure with the package program and the comparison of the obtained results with the modified nodal analysis (MNA) results made in the third chapter will be made. In the final part, conclusions and suggestions for future work will be made.

2. The Explanation of DC- DC Cuk Converter with Coupled Inductors Topology

The CUK converter is a converter topology that is created by using the above-mentioned buck and boost converter topologies. Unlike these converter types, 2 coupled inductors and 2 capacitors are used in this topology structure. In addition, there are 2 semiconductor elements in the circuit structure, one fully controlled and the other uncontrolled. Here, it was important to take the values of the inductor acting in conjunction with each other, which creates the magnetomotive force that does not work like a transformer [7]. It is known by the name of its finder, “Slobodan Cuk”. The Cuk converter structure is actively involved in many areas today as a field of use. These areas include various telecommunications applications, including power supplies, spacecraft power systems, and laptop computers. The Cuk converter structure works differently from other PWM converter structures and the main motivation behind the topology is to reduce the switching losses. Figure 1 shows the general CUK converter topology. As can be seen in the figure, there are two modes of the circuit structure controlled by the S1 controlled switch. These modes are called mode-1 when the S1 switch is in the on position, and as mode-2 when the S1 switch is in the off position.

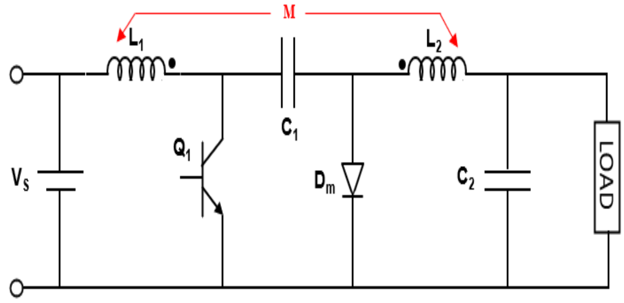

Fig. 1. CUK Converter Topology

Unlike many other DC-DC converter structures, it is the use of capacity instead of inductor in energy storage. The capacitor C1 in the circuit structure mediates the energy transfer from the source to the load. [8]. The mode-1 structure of the CUK converter is explained in figure 2.

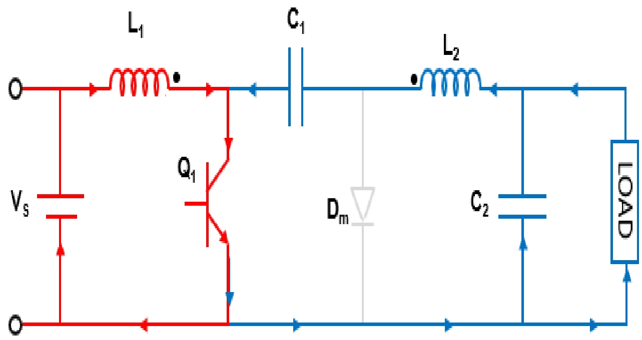

Fig. 2. CUK Converter Mode-1

In the Mode-1 circuit structure, the Q1 switch is turned on. The energy of inductor L1 increases in period T. Meanwhile, the C1 capacitance both charges the C2 capacitance and increases the energy of the L2 inductor via the Q1 switch. In addition to these, these passive circuit elements also feed the load.

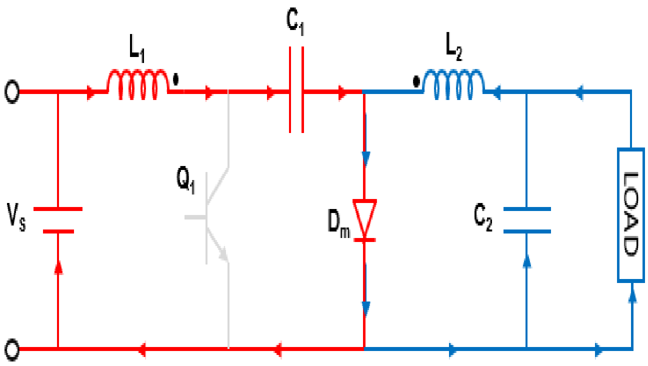

Fig. 3. CUK Converter Mode-2

In the Mode-2 circuit structure, the Q1 switch is cut off. The diode, which is an uncontrolled switch, turns on. Source Vs is increasing the energy of inductor L1 while charging the capacitance C1. The L2 inductor feeds the load and its energy decreases. This structure is explained in figure 3.

2.1. General Equations

Due to the fact that DC-DC converters, which are frequently used today, provide convenience in practical calculations in general, their equations have been created depending on certain parameters. The expression of the duty cycle of the switches in the power electronics circuits is given in this circuit structure. [9-11]

Also, equations such as ripple current on the input inductor, output voltage ripple, duty cycle will be given in this article.

Duty Cycle:

$$\frac{V_a}{V_s} = \frac{k}{1-k} \quad (1)$$

Output Voltage Ripple:

$$\Delta v_{c2} = \frac{kV_s}{8C_2L_2f^2} \quad (2)$$

Input Inductor Ripple Current:

$$\Delta I_1 = \frac{kV_s}{L_1 f} \quad (3)$$

3. System Equations and Swicth Modelling

The state space model and modified nodal analysis (MNA) are two often utilized methodologies for system modelling and equations. The State space method, which is based on the graph theory, is used in the analysis of simple circuits, it facilitates the solution, yet requires very difficult and intensive operations in obtaining the equations. Despite the fact that the modified nodal analysis model has a large number of operations and unknowns, the equations are very simple to derive.[12] In this study, the solution of DC to DC Cuk topology with modified nodal analysis (MNA) will be given. Modified Nodal Analysis (MNA) can be expressed in the time and Laplace domain as follows:

$$G_x(t) + C\frac{dx(t)}{dt} = Bu(t) \quad (4)$$

$$(G + sC)X(s) = BU(S) \quad (5)$$

In the specified coefficient matrices, the frequency-independent elements are represented in the G matrix. The C matrix describe the circuit elements associated with frequency, while the voltage and current sources connected to the circuit are expressed in the B matrix.

In this study, MOSFET and diode semiconductors, one of which is a fully controlled and the other an uncontrolled switch, in a power electronics circuit will be modeled with a bivalent resistor element.

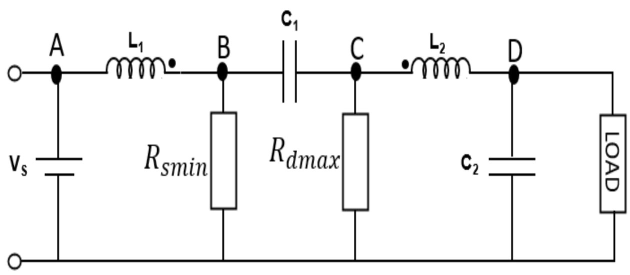

Fig. 4. The modeling of the Mod-1

The modeling of the Q_1 switch with a resistor is shown in Figure 4, and the R_{dmax}, uncontrolled switch, is in cutoff. Q_1switch is expressed as R_{smin}.

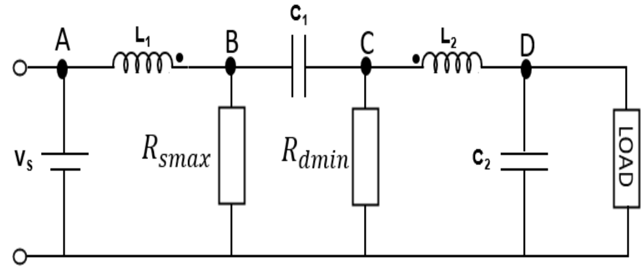

Fig. 5. The modeling of the Mod-2

The modeling of the D_m, uncontrolled switch, with a resistor is shown in Figure 5, and the R_{smax} switch is in cutoff. D_m key is denoted as R_{dmin}.

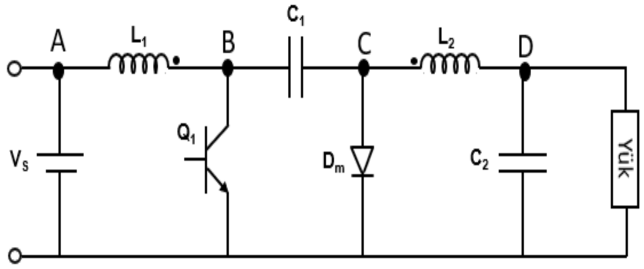

Fig. 6. Presantation of Nodal Voltage

DC-DC Cuk converter circuit structure can basically be examined in $2^2 = 4$ cases [1]. However, due to topology, only the two modes Q_1 conduct D_m diode cut-off and D_m diode conduct Q_1 switch cut-off status, which are two modes in which the switches work opposite each other, will be examined. The equation structures formed according to these situations are given below.

When the Q_1 switch is in the conduction state:

$$A \to I_{L_1} + I_e = 0 \tag{6}$$

$$B \to -I_{L_1} + I_{R_{amin}} - I_{C_1} = 0 \\ \to -I_{L_1} + I_{R_{amin}} - I_{C_1} \tag{7}$$

$$B \to -I_{L1} + I_{Ramin} - sC1(Vc - vb) = 0 \tag{8}$$

$$C \to -I_{L2} + I_{Rdmax} + I_{C1} = 0 \\ \to -I_{L2} + sC1(Vb - Vc) \\ + I_{Rdmax} \tag{9}$$

$$D \to I_{L2} - I_{C2} - I_{RL} = 0 \to I_{L2} - sC_2Vd - G_LVd \tag{10}$$

$$I_{L1} \to V_A - V_B - sL_1I_{L1} + sML_2 = 0 \tag{11}$$

$$I_{L2} \to V_C - V_D - sL_2I_{L2} + sML_1 = 0 \tag{12}$$

Given equation 11 and 12 is valid for all cases. Using these equations MNA matrix can be written as follows:

$$G1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & \frac{1}{Ramin} & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1/Rdmax & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -GL & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$\text{CA1}= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -C1 & C1 & 0 & 0 & 0 & 0 \\ 0 & C1 & -C1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L1 & M & 0 \\ 0 & 0 & 0 & 0 & M & L2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$B1 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] \tag{15}$$

When the D_m diode is in the conduction state:

$$A \to I_{L1} + I_e = 0 \tag{16}$$

$$B \to -I_{L1} + I_{Ramax} + I_{C1} = 0 \\ \to -I_{L1} + G_{Ramax} * Vb \\ + I_{C1} \tag{17}$$

$$B \to -I_{L1} + I_{Ramax} + sC1(Vb - Vc) = 0 \tag{18}$$

$$C \to -I_{L2} + I_{Rdmin} - I_{C1} = 0 \\ \to -I_{L2} + sC1(Vb - Vc) \\ + G_{Rdmin} * Vc \tag{19}$$

$$D \to I_{L2} - I_{C2} - I_{RL} = 0 \\ \to I_{L2} - sC_2Vd - G_LVd \tag{20}$$

MNA matrix can be specified as below based on these equations:

$$G1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & \frac{1}{Ramax} & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1/Rdmin & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -GL & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

$$\mathrm{CA1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C1 & -C1 & 0 & 0 & 0 & 0 \\ 0 & -C1 & C1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L1 & M & 0 \\ 0 & 0 & 0 & 0 & M & L2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$B1 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] \quad (23)$$

$$X = [V_A \quad V_B \quad V_C \quad V_D \quad I_{L1} \quad I_{L2} \quad I_E] \quad (24)$$

4. Applying the B.euler Method to Modified Nodal Analysis

The B.Euler method is applied in order to apply the numerical solution of the DC-DC Cuk Converter whose equations are obtained above. The general expression of the B.Euler method, h being the step interval;

$$\frac{dx}{dt} = -\frac{G}{C}X_{(t)} + \frac{B}{C}U_{(t)} \quad (25)$$

$$X_{(n+1)} = X_{(n)} + h[-\frac{G}{C}X_{(n+1)} + \frac{B}{C}U_{(n+1)}] \quad (26)$$

Equation (19) is obtained by substituting the structure formed in equation (17) in equation (18).

$$X_{(n+1)} = K_{(A)}.X_{(n)} + K_{(B)}.U_{(n+1)} \quad (19)$$

Numerical solution was obtained with the values given in Table-1 below, and the code generated on MATLAB. It is also supported by the graphics of the numerical solution.

Table 1 Design Parameter

Type	Values	Unit
L1	180	µH
L2	150	µH
M	1	µH
C1	200	µF
C2	220	µF
E	12	V
R_L	4	Ω
$Q_1 - R_{smax}$	$1 \times e^6$	Ω
$Q_1 - R_{smin}$	$1 \times e^{-6}$	Ω
$D_m - R_{dmax}$	$1 \times e^6$	Ω
$D_m - R_{dmin}$	$1 \times e^{-6}$	Ω

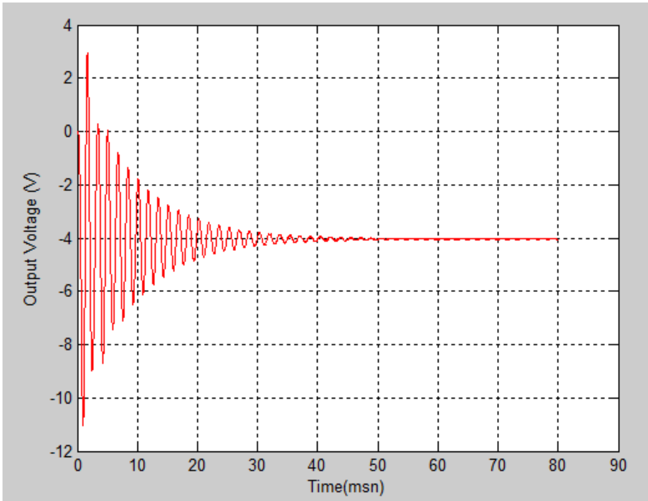

Fig. 7. Output Voltage

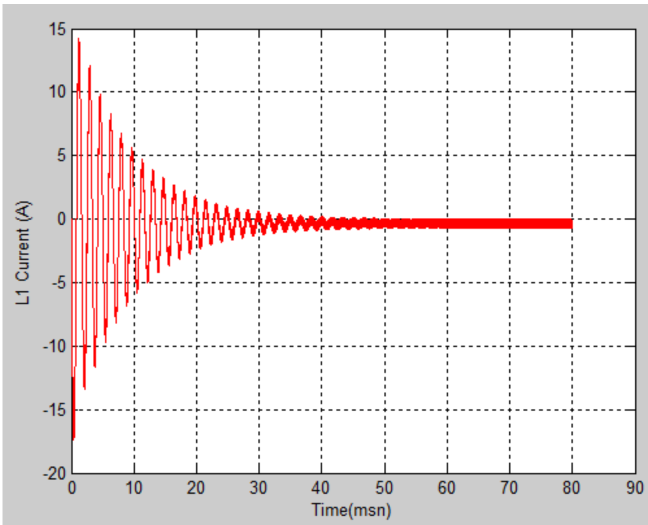

Fig. 8. L_1 Inductor Current

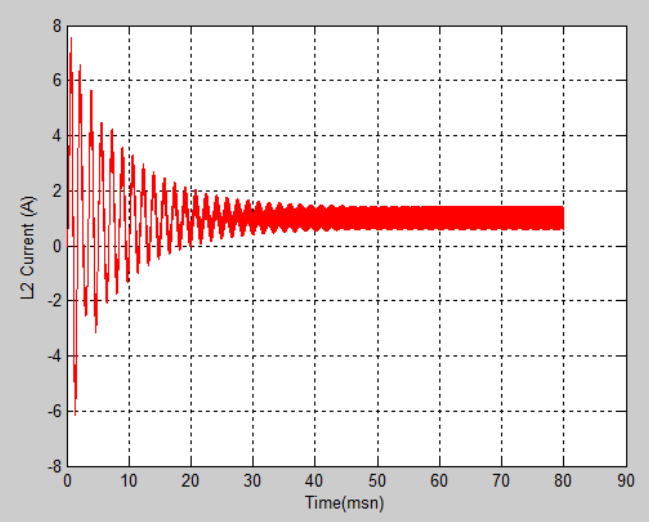

Fig. 9. L_2 Inductor Current

5. Conclusion and Future Works

In this research, DC-DC CUK converter analysis of Modified Nodal Analysis (MNA) was performed. The stages of obtaining the equations illustrates the superiority and ease of the method. The switches in the converter are modeled with the concept of the bivalent resistor element. The B. Euler method, which is one of the numerical solution methods, was applied to the system equations in the MATLAB and the results of the analysis were presented with graphics.

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