

Complete Electrical Equivalent Circuit Based Modeling and Analysis of Permanent Magnet Direct Current (DC) Motors

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Abstract: The modeling and analysis of permanent-magnet DC motor, whose attractiveness has increased with the availability of high-energy rare earth permanent magnet materials such as SmCo and NdFeB, is very important in recent years. The analysis of the established model is very important for researchers and engineers before siteworks. The most important features of the established model are that the equations are simple to obtain, the model can be set up using a simple platform, and ease of model analysis. Generally, for DC motor models, only the armature circuit model is given and only the equations of the mechanical system are given. In the armature circuit model, equations are obtained by switching to the s-domain, and analyzes are made on highly advanced platforms by using these equations. In this study, electrical equivalents of mechanical equations were obtained and both armature circuit and mechanical parameters of a DC motor were expressed as electrical circuit elements. Although the motor model includes mechanical parameters and variables, the whole model is expressed only with electrical elements and variables. Thus, a complete electrical equivalent circuit is proposed for the dynamic model of the permanent-magnet DC motor, in which both the armature and the mechanical part can be modeled as an electrical circuit. With the analyzes, the performance of the permanent magnet DC motor model was examined and it was seen that the system dynamics responses were compatible.

Keywords: Permanent Magnet DC Motor, Complete Electrical Equivalent Circuit, Analysis, Modeling

Received: July 21, 2021. Revised: June 25, 2022. Accepted: July 15, 2022. Published: August 2, 2022.

1. Introduction

DC motors are widely used in industry today due to their low cost, higher efficiency than AC machines, less complex control structure and wide speed and torque range. General structures, properties, analysis, dynamical equations and control of DC motors are given in many textbooks [1–3] and some articles. Permanent magnet DC motors do not need field excitation regulation and consume less input power. Due to these advantages, permanent magnet DC motors have a wide range of applications where constant speed will be maintained at varying loads or different speeds will be obtained at constant load [4,5]. In permanent magnet machines, there is only one armature circuit and the flux produced by the magnets is constant. Compared with conventional DC electric machines, permanent magnet machines stand out with their higher efficiency and simpler structures [6-10]. Due to the absence of excitation windings and therefore no excitation current, permanent-magnet machines offer high efficiency, simple and robust structure and high power-to-weight ratio in operation. The attractiveness of permanent-magnet machines is further enhanced by the availability of high-energy rare-earth permanent magnet materials such as SmCo and NdFeB [11]. Besides, since there is no excitation circuit, it is not possible to control the speed of a permanent-magnet DC motor by changing the excitation current. However, since it is only armature circuit, speed and torque control is done by controlling this circuit and this simplifies the controller design. With all these features, permanent magnet DC motors stand out with their simpler structures and easy controllability when compared to other DC motors.

In system analysis, there are two basic methods for obtaining circuit equations: Modified Nodal Approach (MNA) and State Variables Method (SVM) [12]. These methods have advantages and disadvantages over each other. The most important advantage of the equations of state method is that the number of unknown variables is minimal. However, it is very difficult to obtain the equations of the circuits with this method. In the classical node method, all kinds of circuit elements (such as voltage source, controlled sources) could not be included in the system equations. To overcome these drawbacks, the Modified Nodal Approach has been proposed [13]. While the large number of unknowns of the generalized node method is the disadvantage of the method, the convenience in obtaining the equations is the most important advantage of the method. [14-15]. The structure of Modified Nodal Approach in the s-domain and t-domain are given below in equation (1) and equation (2), respectively.

$$(G + s.C).X(s) = B.U(s) \quad (1)$$

$$G.x(t) + C.\frac{dx(t)}{dt} = B.u(t) \quad (2)$$

Here G, C, B are coefficient matrices. The frequency-independent values resulting from obtaining all conductivity and node equations form the G matrix, and the capacitance and inductance values related to the frequency variable form the C matrix. The U matrix is the vector containing the initial conditions of independent current and voltage sources and non-zero capacitances and inductances. Its unknown vector $x(t)$ contains both current and voltage variables. Considering

the variable types, the unknown vector $x(t)$ can be split into $x1(t)$ and $x2(t)$ vectors as shown in equation (3).

$$x(t) = \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} \quad (3)$$

Here, the $x1(t)$ vector represents the node voltage variables, and the $x2(t)$ vector represents the current variables. Considering the unknown vector decomposed by eq.(3), the structure of generalized nodal equations in the time domain can be rearranged with equation (4) in the following form.

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & 0 \end{bmatrix} \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} = B \begin{bmatrix} u1(t) \\ u2(t) \end{bmatrix} \quad (4)$$

In the studies examined in the literature [4-11], while DC motor models are given, only the electrical circuit model of the armature circuit is given. On the other hand, only the equations of the mechanical system are used by expressing them. No equivalent circuit models of these equations have been produced. However, similar approaches were used for the full electrical equivalent circuit model of the free-excited DC motor in the study [16] and for the full electrical equivalent circuit model of the PMSM in the study [17]. In this study, analyzes will be made using the full electrical equivalent circuit model and the MNA method. The most important contribution of the model is the mechanical equations and obtaining the electrical exact equivalent circuit model of the position equation.

In this study, first of all, the electrical full equivalent circuit of a permanent-magnet DC motor will be established. Then, the model of the permanent-magnet DC motor will be constructed using the generalized node equations structure in the time domain given by equation (4). Numerical analyzes based on the Backward-Euler method of the engine with the MNA model will be made.

In Chapter II, armature, torque and motion equations of permanent-magnet DC motor will be subtracted and dynamic equations will be given. In Chapter III, the electrical exact equivalent circuit of the permanent-magnet DC motor will be constructed. In Chapter IV, the MNA model of the permanent magnet DC motor with its full electrical equivalent circuit will be given, and numerical analyzes based on the Backward-Euler method regarding the established model will be carried out for a sample motor. Transient stability analysis time constants related to the system dynamics of the created MNA model will be found, the duration of the transient state will be determined, and the convergence step length will be determined for numerical analysis. The compatibility between the system dynamics outputs of the model and the calculated values will be examined. At the same time, a performance review of the established model will be made.

2. Equations of Permanent Magnet DC Motors

Due to the absence of excitation windings and therefore the excitation current, permanent-magnet machines, which is preferred because of their high efficiency in operation, their low volume and weight, their high moment densities and torque/weight ratios, have the armature equivalent circuit shown in Figure 1.

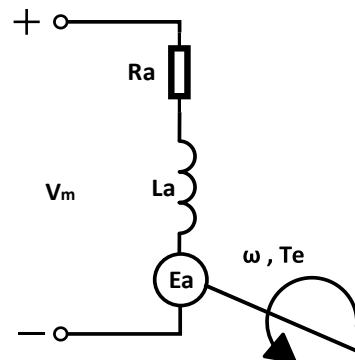


Figure 1 Armature Equivalent Circuit Diagram of Permanent-Magnet DC Motor

Equation of armature circuit,

$$V_m = R_a i_a + L_a \frac{di_a}{dt} + E_a \quad (5)$$

$$E_a = K_b \omega \quad (6)$$

Equation (6) is substituted in equation(5).

$$V_m = R_a i_a + L_a \frac{di_a}{dt} + K_b \omega \quad (7)$$

Moment equation:

$$T_e = K_b i_a \quad (8)$$

$$T_a = T_e - T_L = K_b i_a - T_L \quad (9)$$

Motion equation:

$$T_a = B_1 \omega + J \frac{d\omega}{dt} \quad (10)$$

Equation (9) and Equation (10) taken together.

$$\begin{aligned} B_1 \omega + J \frac{d\omega}{dt} &= K_b i_a - T_L \\ J \frac{d\omega}{dt} + B_1 \omega - K_b i_a + T_L &= 0 \end{aligned} \quad (11)$$

Position-velocity equation,

$$\frac{d\theta}{dt} = \omega \quad (12)$$

V_m : Motor terminal voltage (V)

L_a : Armature winding inductance (H)

R_a : Armature winding resistance (Ω)

i_a : Armature current (A)

E_a : Motor EMF Voltage (V)

K_b : EMF constant - Motor design constant (V.s/rad)

T_e : Motor air gap torque (Nm)

T_L : Load torque (Nm)
 T_a : Motor shaft torque – Acceleration torque (Nm)
 ω : Angular speed of motor rotor (rad/s)
 B_1 : Motor friction constant (Nm.s/rad)
 J : Rotor inertia ($kg.m^2/s^2$)
 θ : Rotor position (rad)

By using equation (7), (11) and (12), the dynamic equations of permanent magnet DC motor can be expressed as given in equation (13).

$$\frac{d}{dt} \begin{bmatrix} i_a \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_b}{J} & -\frac{B_1}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_m \\ T_L \end{bmatrix} \quad (13)$$

3. Complete Electrical Equivalent Circuit Model of Permanent Magnet DC Motor

When we look at the permanent magnet DC motor mechanical equations, it is seen that the motor motion equation can be modeled as a series RL circuit with the following inferences.

- Equation (7), which represents the armature circuit, Series RL elements correspond to an electrical circuit containing the dependent source and supply voltage. This circuit equivalent is a very common notation.
- An electrical circuit belonging to the equation of motion given in Equation (10) will be obtained. For this purpose, when equation (10) is examined carefully, it can be seen that this equation can be handled like the series RL circuit given in equation (14) due to its structure.

$$V_a = R_1 i + L_j \frac{di}{dt} \quad (14)$$

- An electrical circuit belonging to the mechanical equation given in equation (11) will be obtained. For this purpose, when equation (11) is examined carefully, it can be seen that this equation can be handled like the equivalent circuit in Figure 2 due to its structure. It can be seen here that T_e (Motor air gap torque) value can be modeled as a dependent voltage source and T_L (Acceleration torque) value can be modeled as a constant voltage source.

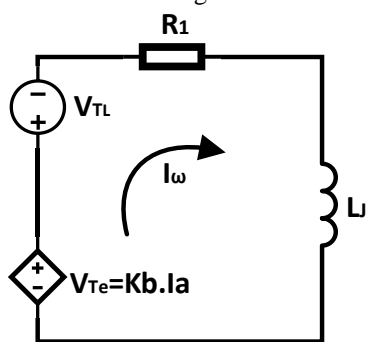


Figure 2 Electrical Equivalent Circuit Diagram of Moment and Motion Equations of Permanent-Magnet DC Motor

- In these equations, it can be clearly seen that the following transformations are made between the electrical circuit and the mechanical system.

- $T_e (Nm) \rightarrow V_{Te} (V)$
- $T_L (Nm) \rightarrow V_{TL} (V)$
- $B_1 (Nm.s/rad) \rightarrow R_1 (ohm)$
- $J (kg - m^2/s^2) \rightarrow L_j (H)$
- $\omega (rad/s) \rightarrow I_\omega (A)$

Likewise, the position-velocity equation from the motor mechanical equations can also be modeled as an L-circuit, as can be seen in Figure 3.

- An electrical circuit belonging to the position-velocity equation given in equation (12) will be obtained. For this purpose, when equation (12) is examined carefully, it is understood that this equation is similar to the terminal equation of an inductance as in equation (15). However, the terminal voltage will be equivalent to a dependent voltage source as in equation (16). In these equations, if $k_\theta = 1, L_\theta = 1 H$ is taken, the value of ω corresponds to the current I_ω and the value of θ to the current of the L circuit (I_θ). As a result, the position-velocity equation is modeled with the equivalent circuit in Figure 3.

$$\frac{dI_\theta}{dt} = \frac{V_\theta}{L_\theta} \quad (15)$$

$$V_\theta = k_\theta I_\omega \quad (16)$$

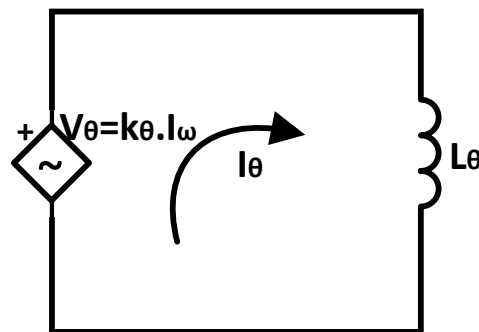


Figure 3 Electrical Equivalent Circuit Diagram of Position-Speed Equation of Permanent-Magnet DC Motor

- $k_\theta = 1$
- $L_\theta = 1H$
- $\theta (rad) \rightarrow I_\theta (A)$

After these inferences, the proposed full electrical equivalent circuit of permanent-magnet DC motor is given in Figure 4. The first block of the fully electrical equivalent circuit in Figure 4 expresses the armature circuit, the second block expresses the moment and motion equations, and the

third block expresses the position-velocity equation. In the fully equivalent circuit obtained, the variables that the dependent sources depend on are also shown on the figure.

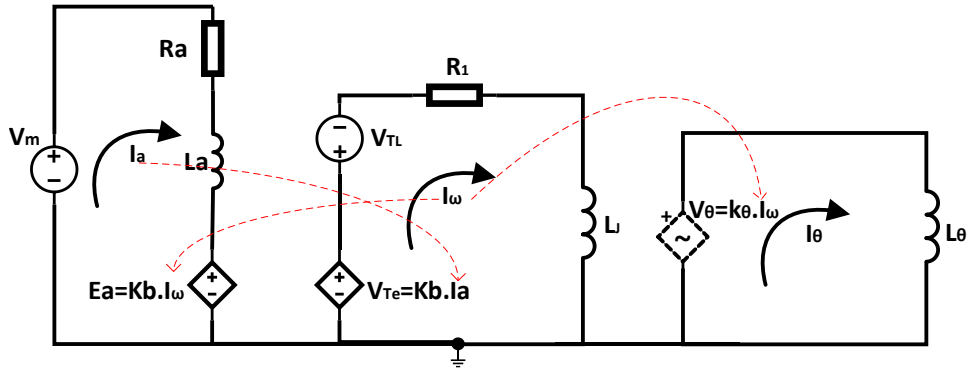


Figure 4 Complete Electrical Equivalent Circuit Diagram of Permanent-Magnet DC Motor

4. Mna Model of Permanent Magnet DC Motor

In this section, the equations for the GDD method of the full electrical equivalent circuit given in Figure 4 will be

given. Nodes and currents for the MNA model will be used as shown in Figure 5.

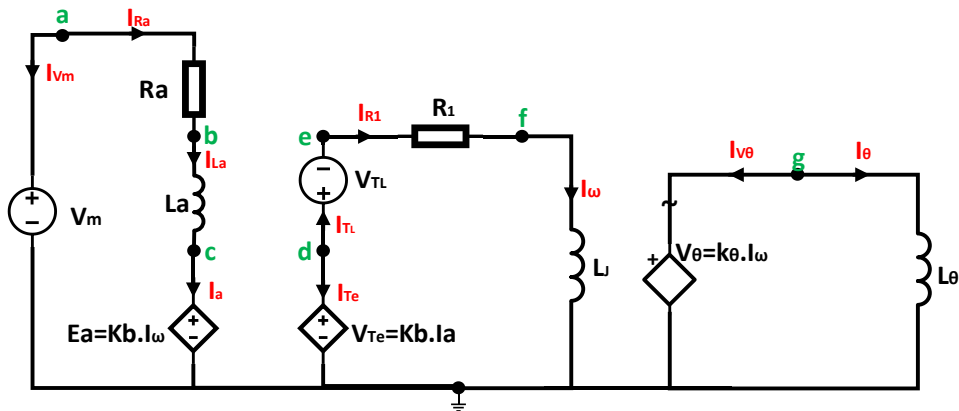


Figure 5 Nodes and Node Currents for the MNA Model

Main Equations;

$$\begin{aligned}
 a &\rightarrow I_{V_m} + I_{R_a} = G_{R_a} \cdot U_a - G_{R_a} \cdot U_b + I_{V_m} = 0 \\
 b &\rightarrow -I_{R_a} + I_{L_a} = -G_{R_a} \cdot U_a + G_{R_a} \cdot U_b + I_{L_a} = 0 \\
 c &\rightarrow -I_{L_a} + I_a = 0 \\
 d &\rightarrow I_{T_e} + I_{T_L} = 0 \\
 e &\rightarrow -I_{T_L} + I_{R_1} = G_{R_1} \cdot U_e - G_{R_1} \cdot U_f - I_{T_L} = 0 \\
 f &\rightarrow -I_{R_1} + I_\omega = -G_{R_1} \cdot U_e + G_{R_1} \cdot U_f + I_\omega = 0 \\
 g &\rightarrow I_{V_\theta} + I_\theta = 0
 \end{aligned}$$

Additional equations;

$$\begin{aligned}
 V_m &\rightarrow U_a = V_m \\
 L_a &\rightarrow U_b - U_c - sL_a I_{L_a} = 0 \\
 E_a &\rightarrow U_c - K_b I_\omega = 0 \\
 V_{T_e} &\rightarrow U_d - K_b I_a = 0 \\
 V_{T_L} &\rightarrow U_d - U_e = V_{T_L} \\
 I_\omega &\rightarrow U_f - sL_f I_\omega = 0
 \end{aligned}$$

$$V_\theta \rightarrow U_g - k_\theta \cdot I_\omega = 0$$

$$L_\theta \rightarrow U_g - sL_\theta I_\theta = 0$$

The equations obtained above, according to the system in equation (2), the matrix \mathbf{G} , \mathbf{C} , $\mathbf{X}(\mathbf{t})$, \mathbf{B} and $\mathbf{U}(\mathbf{t})$ are given in equation (17), (18), (19), and (20), respectively.

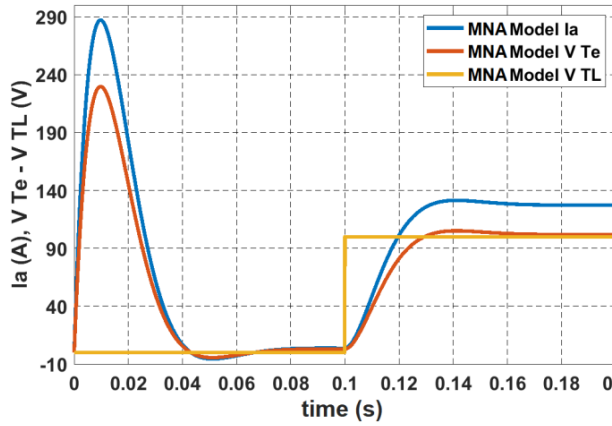


Figure 7 $I_a(t), V_{Te}(t)$ and $V_{TL}(t)$ Graphics for The MNA Model

Figure 8 shows the I_ω graph that emerged as a result of the simulation study. For this study, the value of V_{TL} was ensured to be 200 V throughout the entire simulation process. The transient region of $I_\omega(t)$ value is examined and as expected, $I_\omega(t)$ value, which symbolizes motor speed, takes negative values until V_{Te} value, which is dependent on I_a value, is greater than V_{TL} value.

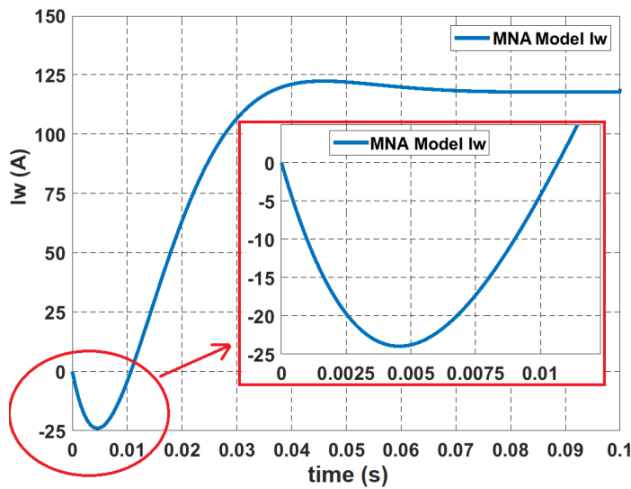


Figure 8 Transient Graphic for The MNA Model $I_\omega(t)$ for $V_{TL}(t) = 200 V$

5. Conclusion

In the study carried out, a very simple electrical equivalent circuit of the permanent-magnet DC motor was extracted and this equivalent circuit was modeled with MNA. In this study, all variables and components in the mechanical equations of permanent magnet DC motors are expressed in terms of electrical variables and components. Thus, the complete electrical equivalent circuit for the entire electromechanical system of the DC motor is obtained. In the numerical example, it is shown that various analysis and dynamic properties of DC motor can be obtained with this electrical equivalent circuit. At the same time, a temporary situation analysis of the MNA model was made to the board and it was seen that the dynamic results here were exactly as expected. The most important features of the developed model are that the equations are simple to obtain, the model can be set up using a simple platform, and model analysis is quite easy. It is possible to analyze a motor with a mechanical structure with programs that solve electrical circuits.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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