ARC High Order Filters Suitable for Antialiasing and/or Reconstruction Filters

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Abstract – The discrete time signal processing circuit requires an anti-aliasing filter at the input and a reconstruction filter at the output, generally. In this paper, selected basic structures of some active filters are described and compared with a view to the degradation of the attenuation over the transient frequency of the operational amplifier. Firstly, the reasons for the degradation of the attenuation are explained theoretically. Secondly, these conclusions are verified by simulations. These simulations were performed by spice-like circuit simulator MicroCap version 11.

Keywords- low-pass filter structure; real operational amplifier; frequency response; decreasing of the attenuation at the high frequencies; transient frequency of the operational amplifier

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1. Introduction

THE main disadvantage of the all discrete-time signal processing circuits is the periodicity of its frequency response, as is depicted in Fig.1. Therefore, the main task of the antialiasing filter at the input is the attenuation of frequencies upper half of sampling frequency [1], [2], [3]. The realization of these filters can use active RC filters as well, where an active element can be an operational amplifier, i.e. filter in the voltage mode.

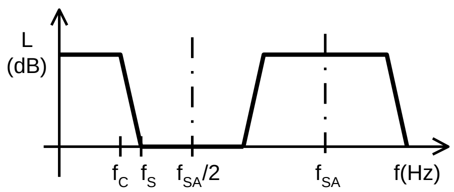

Figure 1. Digital filter attenuation.

In the following text, the use of the real operational amplifier in basic filter structures will be discussed.

Some biquad filters structures are characterized by a degradation of the attenuation at high frequencies [4], [5]. This degradation of the attenuation occurs only for some filters of the even orders, i.e. for the biquads, and is caused by the final value of the output resistance of the used operational amplifier [6] (see Fig. 2).

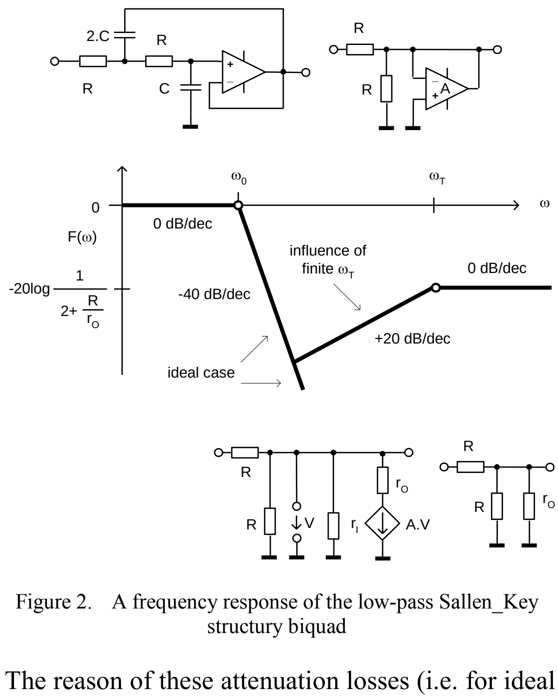

Figure 2. A frequency response of the low-pass Sallen_Key structury biquad

The reason of these attenuation losses (i.e. for ideal case) for a second-order low-pass filter is depicted in Fig. 2 as well. As is shown, the ideal course of the second-order low-pass filter frequency response is a monotonous decreasing of the magnitude value over the cutoff frequency ω_0 at whole frequency range with –40 dB/dec slope. But the finite value of the transient frequency ω_T of the used real operational amplifier leads to the break of this ideal slope (i.e. –40 dB/dec). The result is a loss of attenuation in the stopband for the frequencies around the transient frequency of the used operational amplifier ω_T.

Filter characteristics in the transition area between pass-band and stop-band frequencies are described in the generally available literature frequency [2], [3], [4] and many more.

For example, there are two variants possible, when Huelsman low-pass structure filter is designed by comparison with the general relation for low-pass transfer (1).

$$\frac{-Y_1 \cdot Y_3}{Y_1 \cdot Y_5 + Y_2 \cdot Y_5 + Y_3 \cdot Y_5 + Y_4 \cdot Y_5 + Y_3 \cdot Y_4} = \frac{\omega_O^2}{s^2 + s \cdot \frac{\omega_O}{Q} + \omega_O^2} \tag{1}$$

Behind the input resistor R, in the node 2 generally either a grounded capacitor C_2 and a feedback resistor R_3 or a grounded resistor and a feedback capacitor can be connected (where C_2 and R_3 are changed). However, the first variant is always chosen, with a grounded capacitor (see Fig.3).

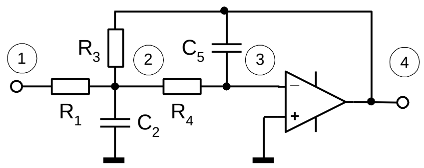

Figure 3. Structure of the LP filter in first variant

In this case the other part of this filter is thus excited by the zero voltage. Then even the output voltage at high frequencies must be zero, as is required for an ideal low pass filter. Then the input and grounded resistor create a voltage divider, which proceeds to the output of the biquad via the feedback capacitor. As a result, with a non-zero output resistance r_0 of the operational amplifier (see Fig.2), the output voltage for the high frequency from filter is non zero, as well.

2. LP-Filters Structure

In this paragraph, the basic low-pass filter type structures will be critically evaluated from the point of view described above. At the same time, the circuit diagram of the filter will always be replaced by the circuit valid for frequencies above the transit frequency of the operational amplifier.

2.1 Filter with distributed feedback loop

These filters using only one operational amplifier as a voltage follower exhibit the lowest known sensitivity Q to the passive elements. The general filter structure is described in [13], for 3rd order is shown in Fig. 4.

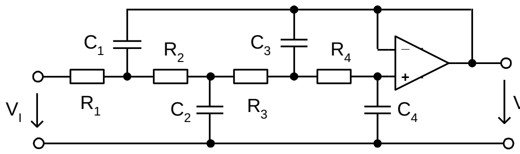

Figure 4. Structure of the LP filter with distributed feedback loop

Consider that the amplification of an operational amplifier is reduced to zero value at highest frequencies. The reactance of all capacitors is at highest frequencies equal to zero as well. In this case can be drawn model of this circuit, which is depicted in Fig.5.

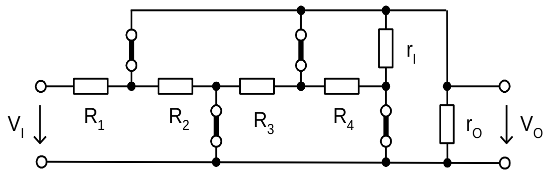

Figure 5. Model of the LP filter with distributed feedback loop in high frequencies

Consider for example $R_1 = R_2 = R_3 = R_4 = R$, thus if r_i is the input resistance and r_0 the output resistance of real operational amplifier, the output voltage V_O is determined as (2).

$$V_O = V_I \cdot \frac{\dfrac{\frac{R}{3} \cdot r_i \cdot r_O}{\frac{R}{3} \cdot r_i + \frac{R}{3} \cdot r_O + r_i \cdot r_O}}{R + \dfrac{\frac{R}{3} \cdot r_i \cdot r_O}{\frac{R}{3} \cdot r_i + \frac{R}{3} \cdot r_O + r_i \cdot r_O}} \overset{!}{>} 0 \tag{2}$$

Because normally r_i is in hundreds of kiloohms, R in kiloohms and r_o in ohms, therefore in all cases can be written $R >> r_O, r_i >> r_O$, thus eq.(2) can be simplified in following form (3)

$$\frac{V_O}{V_I} = \frac{r_O}{R + r_O} \tag{3}$$

Consider now commonly filter value R= 15 kOhm. For commonly used operational amplifier of type LM741, which has following typical main parameters $R_{OUTAC} = r_O = 50$ Ohm, A = 200 V/mV, GBW = 1 MHz, we see that

$$20 \cdot \log \frac{V_O}{V_I} = 20 \cdot \log \frac{r_O}{R + r_O} = 20 \cdot \log \frac{50}{15000 + 50} \approx$$

$$\approx 20 \cdot \log 0{,}0033 = 20 \cdot (-2{,}48) \approx -50\mathrm{dB} \qquad (4)$$

which is a typical value for this filter structure as well.

2.2 LP Sallen-Key Structure of Even Order

The Sallen-Key low-pass filter i.e. LP-SK [7] using an operational amplifier as a voltage follower exhibit the lowest sensitivity Q to the passive elements. Therefore, from the viewpoint the sensitivity of the filter, the composition from two (generally even) biquads LP-SK is the best. Commonly used high orders filter structure is shown in Fig.6.

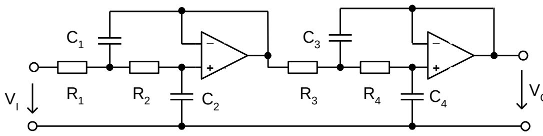

Figure 6. The LP filter 4th order SK structure

The main disadvantage of LP-SK biquad filters for odd orders, however, is a reduction of the attenuation in the stop-band. We know that the amplification factor A of an operational amplifier is reduced to zero at high frequencies. The reactance of capacitors nears to zero for frequencies close to infinite. The voltage ratio has a nonzero value in the zone over of transient frequency of the operational amplifier [9], this fact explains the degradation of the filter properties. Consider now $R_1 = R_2 = R_3 = R_4 = R$, thus output voltage V of the first biquad (see. Fig.7) derived from the model of the LP filter 4th order SK structure in high frequencies, which is depicted in Fig.6 and is given as Eq.(5)

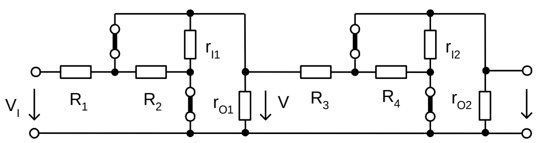

Figure 7. Model of the LP filter 4th order SK structure in high frequencies

$$V = V_I \cdot \frac{\dfrac{R.r_i.r_o}{R.r_i + R.r_o + r_i.r_o}}{R + \dfrac{R.r_i.r_o}{R.r_i + R.r_o + r_i.r_o}} \overset{!}{>} 0 \qquad (5)$$

Because in all typical cases (as is described above) is $R >> r_O$, $r_i >> r_O$, thus Eq.(5) can be simplified as the first biquad voltage transfer in form (6)

$$\frac{V}{V_I} = \frac{r_o}{R + r_o} \qquad (6)$$

Because the second one biquad voltage transfer is the same, thus the resulting transfer of the LP filter 4th order SK structure in high frequencies is given in Eq.(7)

$$\frac{V_O}{V_I} = \frac{r_o}{R + r_o} \cdot \frac{r_o}{R + r_o} = \frac{{r_o}^2}{(R + r_o)^2} > 0 \qquad (7)$$

Consider again commonly filter value R= 15 kOhm. For commonly used operational amplifier of type LM741 with following parameters $R_{OUTAC} = r_O =$ = 50 Ohm, A = 200 V/mV, GBW = 1 MHz, thus we see that

$$20 \cdot \log \frac{V_O}{V_I} = 20 \cdot \log \frac{{r_O}^2}{(R + r_O)^2} = 20 \cdot \log \frac{50^2}{15050^2} \approx$$

$$\approx 20 \cdot \log 1{,}1 \cdot 10^{-5} = 20 \cdot (-4{,}96) \approx -100\mathrm{dB} \qquad (8)$$

The approximately value -100 dB is in very well correlation with simulation result by program MicroCap version 11 which verifies this calculated solution as is depicted in Fig. 8, where is the resulting magnitude characteristics graph.

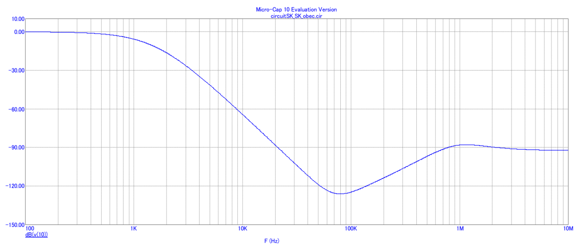

Figure 8. Magnitude characteristic of LP SK 4th order filer.

The result of the simulation is in full agreement with the theoretical analysis, as follows from the comparison of Fig. 8 and Fig.2.

2.3 Non-Cascade Structure

The non-cascading filter structure with galvanic connection between the input V_i and the output V_o voltage [10] is used when the transfer DC component is required. A resistor R_o is connected between input and output nodes, the frequency variable impedance is realized by a circuit containing one or more operational amplifiers, which are connected between the output node and ground. The schematic diagram of the 4th order non-cascading filter is depicted in following Fig. 9.

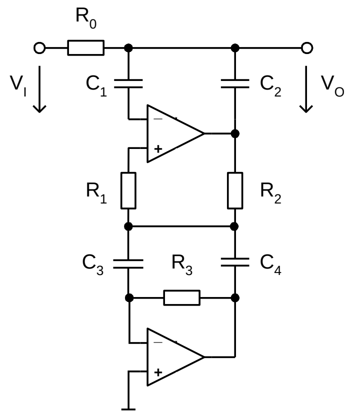

Figure 9. Schematic diagram of the LP filter 4th order non-cascade structure

Figure 10 is a simplified schematic diagram of this biquad at high frequencies (where all capacitors are substituted to a short circuit and the operational amplifier does not voltage amplify, in other word its voltage amplify A=0).

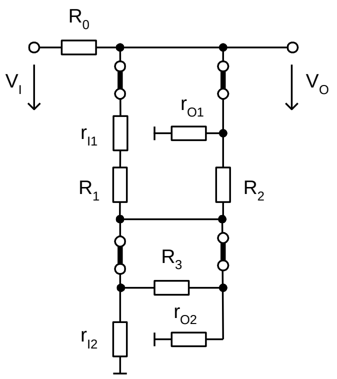

Figure 10. Model of the LP filter 4th order non-cascade structure in high frequencies

In all cases typically are $R >> r_O$, $r_i >> r_O$, thus output voltage V_O can be calculated in simplified form (9)

$$\frac{V_O}{V_I} = \frac{r_o}{R_o + r_o} \tag{9}$$

For commonly filter members value R = 15 kOhm and operational amplifier LM741 with following typical parameters $R_{OUTAC} = r_O = 50$ Ohm, A = 200, GBW = 1 MHz, we see that the damping above the transient frequency is approximately -50 dB again (see calculation of the same Eq.4 in the previous paragraph *B*).

2.4 Huelsmann Structure

Second one filter if the transfer DC component is required is 4th order filter Huelsmann Structure MFB. Its schematic diagram is depicted in Fig. 11, where we can see galvanic connection between the input and the output node (over the resistors R_1, R_3, R_4, R_6).

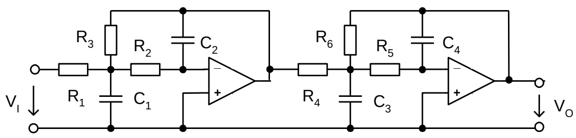

Figure 11. LP biquad 4th order Huelsmann (MFB) structure

Figure 12 is a simplified schematic diagram of the circuit from Fig.11 at higher frequencies (where all capacitors are substituted to a short circuit in the case of ideal passive elements).

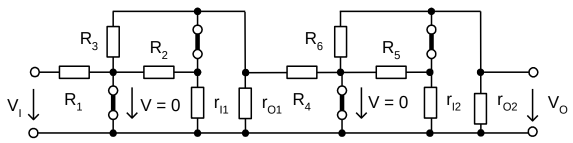

Figure 12. Model of the LP biquad 4th order Huelsmann (MFB) structure

The voltage V = 0 i.e. the other part of this filter is thus excited by the zero voltage. As a result, the output voltage V_o must be equal to zero as well. Therefore the magnitude of the voltage transfer ratio at the highest frequencies F_{inf} must be equal to zero, too (10)

$$F_{inf} = \lim_{\omega \to \infty} \frac{V_O}{V_I} = 0 \tag{10}$$

where: F_{inf}, V_I, V_O were specified above, thus the damping above the transient frequency is infinite in this case (11).

$$20 \cdot \log \frac{V_O}{V_I} = \lim_{\omega \to \infty} (\log \frac{V_O}{V_I}) \to -\infty \tag{11}$$

Note that this Huelsmann structure can be used when the transfer DC component of the signal is required.

2.5 2T Filter Structure

Now in this paragraph, we focus on the 2T LP filter [7], [8] at the highest frequencies, as well. Figure 13 is a schematic diagram of a 4th order filter, which is able transferred DC component as well, because there is a galvanic connection between the input and the output over all resistors R.

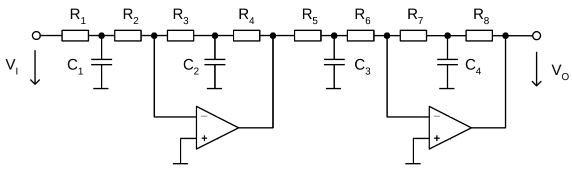

Figure 13. LP biquad 4th order in 2T structure.

Figure 14 depicted a simplified schematic diagram of the biquads from Fig.13 at higher frequencies (where all capacitors C are simply substituted to a short circuit).

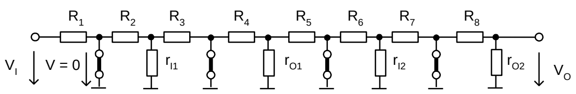

Figure 14. Model LP biquad 4th order 2T structure.

The operational amplifier loses its amplification factor, now. The voltage at the second, fourth, sixth and seventh node, i.e. in four nodes is equal to zero V=0, therefore the following parts of the filter is excited by zero voltage, thus output voltage V_o must be equal to zero, too. The magnitude of the voltage transfer ratio at the highest frequencies must be equal to zero as well (12)

$$F_{inf} = \lim_{\omega \to \infty} \frac{V_O}{V_I} = 0 \qquad (12)$$

where: F_{inf}, V_I, V_O were specified above, thus the damping above the transient frequency is infinite as well (see eq.(11)).

3. Discussion

Since the main function of both the anti-aliasing filter in discrete-time signal processing (see Fig. 15) is to suppress the frequency higher than half the sampling frequency [11], [12], it is possible, based on the theoretical analysis mentioned in paragraph II, select appropriate filter structures, taking into account the properties of real operational amplifier.

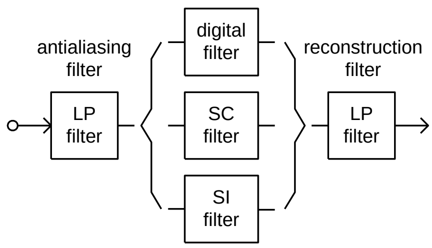

Figure 15. Antialiasing filter in signal processing.

In general, if the antialiasing filter is designed, it is necessary consider following basic values: the attenuations A_{corner}, A_{stop} and the frequencies f_{corner}, f_{stop} for the reference LP filter. But, as is described above, an important parameter is the attenuation over the transient frequency A_{over}, as well. The comparison of the attenuation is the worst cascade filter of the fourth order and Sallen-Key structure of even order filter, too. As is shown, approximatelly can be written (13)

$$A_{OVER} = N \cdot 50\text{dB} \qquad (13)$$

where: N is the number of the biquads. If this attenuation is over the quantize noise (see Fig.16), the degradation of magnitude LP filter is not relevanting, of course.

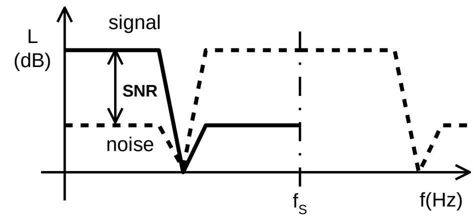

Figure 16. Low attenuation reduces the signal-to-noise ratio (SNR)

4. Conclusion

As we can see, only then H-LP and 2T-LP biquads have a monotone increasing attenuation in stop band. Another described structures, namely non-cascading filter structure and/or composition two (generally even) biquads LP-SK can be used as well. But it is necessary their attenuation must to be more, than quantize noise of digital filter.

Another possibility is to combine a filter structure, where a filter with a finite and small damping value above the transit frequency is replaced in one of its elements the structure with a theoretically infinite attenuation. Thus, one of the biquads of the SK-LP structure is replaced by one of the biquads of the H-LP and/or 2T structure. However, this requires converting the parameters of the SK structure to an H and/or 2T structure. Another way is to directly design n-1 biquads in the SK structure and a single biquad in the H and/or 2T structure for the n-th order filter, as wel.

For the above reasons, it seems appropriate to consider the facts described in the article when choosing a filter and its design.

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