TOPSIS Approach to Facility Location Selection Problem for Pythagorean Fuzzy Environment

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Abstract: - Pythagorean fuzzy sets (PFS) has much stronger ability than intuitionistic fuzzy set (IFS) to manage the uncertainty in real-world multi-criteria decision-making problems. Current research develops a Pythagorean fuzzy TOPSIS approach for formation and representing of expert's knowledge on the parameters of facility location planning in extreme environment. In this approach, we propose a score function based comparison method to identify the Pythagorean fuzzy positive ideal solution and the Pythagorean fuzzy negative ideal solution. Based on the constructed fuzzy TOPSIS aggregation a new objective function is formulated. Constructed criterion maximizes service centers' selection index. This criterion together with second criterion - minimization of number of selected centers creates the multi-objective facility location set covering problem. The approach is illustrated by the simulation example of emergency service facility location planning for a city in Georgia. More exactly, the example looks into the problem of planning fire stations locations to serve emergency situations in specific demand points – critical infrastructure objects.

Key-Words: - Emergency Service Facility Location planning, Pythagorean fizzy sets, fuzzy TOPSIS, critical infrastructure.

1 Introduction

Multi-criteria decision making (MCDM) problem is to find an optimal alternative that has the highest degree of satisfaction from a set of feasible alternatives characterized with multiple criteria, and these kinds of MCDM problems arise in many real-world situations. Considering the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, Bellman and Zadeh [1] introduced the theory of fuzzy sets in the MCDM problems. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) developed by Hwang and Yoon [2] in 1981 is one of the most useful distance measure based classical approaches to multi-criteria/multi-attribute decision making (MCDM/MADM) problems. It is a practical and useful technique for ranking and selecting of a number of externally determined alternatives through distance measures. The basic principle used in the TOPSIS is that the chosen alternative should have the shortest distance from positive-ideal solution (PIS) and farthest from the negative-ideal solution (NIS). There exists a large amount of literature involving TOPSIS theory and applications. In classical TOPSIS methods, crisp numerical values are used to express the performance rating and criteria weights. But for human judgment, preference values and criteria weights are often ambiguous and cannot be represented using crisp numerical value in real-life situation. To resolve the ambiguity frequently arising in information from human judgment and preference, the fuzzy set theory has been successfully used to handle imprecision and uncertainty in decision making problems. In this work a novel decision-making TOPSIS approach is developed to deal effectively with the interactive MCDM problems with Pythagorean fuzzy sets.

Intuitionistic fuzzy sets (IFS) was introduced by Atanassov [3], as a generalization of a Zadeh's fuzzy sets (FS). Because each element of IFS, as Intuitionistic fuzzy number (IFN) (μ,υ) is assigned a membership degree (μ), a non-membership degree (υ) and a hesitancy degree $(1-\mu-\upsilon)$, IFS is more powerful in dealing with uncertainty and imprecision than FS. IFS theory has been widely studied and applied to a variety of areas. But an IFN (μ,υ) has a significant restriction - the sum of the degrees of membership and the non-membership is equal or less than 1. In some cases, a decision maker (DM) may provide data for some attribute that the sum of two degrees is greater than $1\,(\mu+\upsilon>1)$. Yager in [4,5] presented the concept of the

Pythagorean fuzzy set (PFS) as extension of an IFS, where the pair of a Pythagorean fuzzy number (PFN) (μ,υ) has a less significant restriction - a square sum of the degrees of membership and the non-membership is equal or less than 1 $(\mu^2+\upsilon^2\le 1)$. In general, for practical problems, the PFSs can select significant decidions that IFSs cannot do. Therefore, PFSs are more able to process uncertain information and solve complex decision making problems.

Definition 1 [6,7]. Let be a fixed ordinary set. q-rung orthopair fuzzy set on is defined as membership grades:

$$A=\{\langle s,\mu_A(s),\nu_A(s)\rangle / s\in S\}, \tag{1}$$

where the functions $\mu_A(s)$ indicates support for membership of s in A and $\nu_A(s)$ indicates support against membership of s in A, where

$$q\ge 1,,\ 0\le\mu_A(s)\le 1,\ \ 0\le\nu_A(s)\le 1,$$
$$0\le(\mu_A(s))^q+(\nu_A(s))^q\le 1. \tag{2}$$

$Hes_q(s)=(1-((\mu_A(s))^q+(\nu_A(s))^q)^{1/q}$ is called a hesitancy associated with a q-rung orthopair membership grades $(q\ge 1)$ and $Str_q(s)=((\mu_A(s))^q+(\nu_A(s))^q)^{1/q}$ is called a strength of commitment viewed at rung q.

In [6] Yager showed that Attanassov's intuitionistic fuzzy sets [3] are $q=1$ -rung orthopair and Yager's Pythagorean fuzzy sets [5] are $q=2$ -rung orthopair fuzzy sets. For convenience, the authors for every $s\in S$ called $\alpha=\langle s,\mu_\alpha(s),\nu_\alpha(s)\rangle$ a q-rung orthothair fuzzy number (q-ROFN) denoted by $\alpha=(\mu_\alpha,\nu_\alpha)$. In future we will consider only Pythagorean fuzzy sets.

Definition 2 [5]. Suppose $\alpha=(\mu_\alpha,\nu_\alpha)$ be a PFN. a) A score function Sc of α is defined as

$$Sc(\alpha)=\mu_\alpha^2-\nu_\alpha^2; \tag{3}$$

b) An accuracy function Ac of α is defined as follows:

$$Ac(\alpha)=\mu_\alpha^2+\nu_\alpha^2. \tag{4}$$

Definition 3 [5]. Suppose $\alpha=(\mu_\alpha,\nu_\alpha)$ and $\beta=(\mu_\beta,\nu_\beta)$ are any two Pythagorean fuzzy numbers (PFN) and $Sc(\alpha),Sc(\beta)$ are the score functions and $Ac(\alpha),Ac(\beta)$ are the accuracy functions of α and β, respectively, then

a) If $Sc(\alpha)>Sc(\beta)$, then $\beta<\alpha$;
b) If $Sc(\alpha)=Sc(\beta)$, then
If $Ac(\alpha)>Ac(\beta)$, then $\beta<\alpha$;
If $Ac(\alpha)=Ac(\beta)$, then $\beta=\alpha$.

On the following basic operations can be defined:

Definition 4 [5]. Suppose for Pythagorean fuzzy numbers $\alpha=(\mu_\alpha,\nu_\alpha)$, α_1,α_2 we have:

$$\begin{aligned}
&1.\ \alpha^c=(\nu_\alpha,\mu_\alpha);\\
&2.\ \alpha_1\oplus\alpha_2=\left((\mu_{\alpha_1}^2+\mu_{\alpha_2}^2-\mu_{\alpha_1}^2\cdot\mu_{\alpha_2}^2)^{1/2},\nu_{\alpha_1}\cdot\nu_{\alpha_2}\right);\\
&3.\ \alpha_1\otimes\alpha_2=\left(\mu_{\alpha_1}\cdot\mu_{\alpha_2},(\nu_{\alpha_1}^2+\nu_{\alpha_2}^2-\nu_{\alpha_1}^2\cdot\nu_{\alpha_2}^2)^{1/2}\right);\\
&4.\ Min(\alpha_1,\alpha_2)=\left(\min(\mu_{\alpha_1},\mu_{\alpha_2}),\max(\nu_{\alpha_1},\nu_{\alpha_2})\right);\\
&5.\ Max(\alpha_1,\alpha_2)=\left(\max(\mu_{\alpha_1},\mu_{\alpha_2}),\min(\nu_{\alpha_1},\nu_{\alpha_2})\right);\\
&6.\ \lambda\cdot\alpha=\left((1-(1-\mu_\alpha^2)^\lambda)^{1/2},\nu_\alpha^\lambda\right),\ \ \lambda>0;\\
&7.\ \alpha^\lambda=\left(\mu_\alpha^\lambda,(1-(1-\nu_\alpha^2)^\lambda)^{1/2}\right),\ \ \lambda>0.
\end{aligned} \tag{6}$$

We define the distance between Pythagorean fuzzy numbers α_1,α_2 as:

$$\begin{aligned}&d(\alpha_1,\alpha_2)=\\&1/2\cdot(|(\mu_{\alpha_1})^2-(\mu_{\alpha_2})^2+|(\nu_{\alpha_1})^2-(\nu_{\alpha_2})^2|).\end{aligned} \tag{7}$$

It is not difficult to prove that this measure satisfies all properties of a distance function.

2 Description of TOPSIS Approach to Facility Location Selection Problem with Pythagorean Fuzzy Information

Location planning for candidate centers is vital in minimizing traffic congestion arising from facility movement in extreme environment. In recent years, transport activity has grown tremendously and this has undoubtedly affected the travel and living conditions in difficult and extreme urban areas. Considering the growth in the number of freight movements and their negative impacts on residents and the environment, municipal administrations are implementing sustainable freight regulations like restricted delivery timing, dedicated delivery zones, congestion charging etc. With the implementation of these regulations, the logistics operators are facing new challenges in location planning for service centers. For example, if service centers are located close to customer locations, then they increase traffic congestion in the urban areas. If they are located far from customer locations, then the service costs for the operators result to be very high. Under these circumstances, it is clear that the location planning for service centers in extreme environment is a complex decision that involves consideration of multiple attributes like maximum customer coverage, minimum service costs, least impacts on geographical points' residents and the environment, and conformance to freight regulations of these points.

Timely servicing from emergency service centers to the affected geographical areas (demand points as customers, for example critical infrastructure objects) is a key task of the emergency management system. Scientific research in this area focuses on distribution networks decision-making problems, which are known as a Facility Location Problem (FLP) [9]. FLP's models have to support the generation of optimal locations of service centers in complex and uncertain situations. There are several publications about application of fuzzy methods in the FLP. However, all of them have a common approach. They represent parameters as fuzzy values (triangular fuzzy numbers [8] and others) and develop methods for facility location problems called in this case Fuzzy Facility Location Problem (FFLP). Fuzzy TOPSIS approaches for facility location selection problem for different fuzzy environments are developed in [10-15]. In this work we consider a new model of FFLP based on the Pythagorean fuzzy TOPSIS approach for the optimal selection of facility location centers. This section first introduces the MCDM problem under Pythagorean fuzzy environment. Then, an effective decision-making approach is proposed to deal with such MCDM problems. At the end, an algorithm of the proposed method is also presented.

At first, we are focusing on a multi-attribute decision making approach for location planning for service centers under uncertain and extreme environment. We develop a fuzzy multi-attribute decision making approach for the service center location selection problem for which a fuzzy TOPSIS approach is used.

The formation of expert's input data for construction of attributes is an important task of the centers' selection problem. To decide on the location of service centers, it is assumed that a set of *candidate sites* (CSs) already exists. This set is denoted by $CS=\{cs_1,cs_2,...,cs_m\}$, where we can locate service centers and $S=\{s_1,s_2,...,s_n\}$ be the set of all attributes (transformed in benefit attributes) which define CS's selection. For example: ”access by public and special transport modes to the candidate site”, “security of the candidate site from accidents, theft and vandalism”, ”connectivity of the location with other modes of transport (highways, railways, seaports, airports, etc.)”, ”costs in vehicle resources, required products and etc. for the location of a candidate site”, “impact of the candidate site location on the environment, such as important objects of Critical Infrastructure, air pollution and others”, ”proximity of the candidate site location from the central locations”, ”proximity of the candidate site location from customers”, “availability of raw material and labor resources in the candidate site”, ”ability to conform to sustainable freight regulations imposed by managers for e.g. restricted delivery hours, special delivery zones”, ”ability to increase size to accommodate growing customers” and others.

Let $W=\{w_1,w_2,...,w_n\}$ be the weights of attributes. From invited group of experts (service dispatchers and so on) $E=\{e_1,e_2,...,e_t\}$, let α_{ij}^k be the fuzzy rating of the evaluation of expert e_k in PFNs for each candidate site cs_i, $(i=1,...,m)$, with respect to each attribute s_j, $(j=1,...,n)$. For the expert e_k we construct binary fuzzy relation $\mathrm{A}_k=\{\alpha_{ij}^k,\ i=1,...,m;\ j=1,...,n\}$ decision making matrix, elements of which are represented in PFNs. If some attribute s_j is cost type then we transform experts' evaluations and α_{ij}^k is changed by $(\alpha_{ij}^k)^c$. Experts' data must be aggregated in etalon decision making matrix - $\mathrm{A}=\{\alpha_{ij},\ i=1,...,m;\ j=1,...,n\}$. Our task is to build fuzzy TOPSIS approach, which for each candidate site cs_i, $(i=1,...,m)$, aggregates presented objective and subjective data into scalar values – *site's selection index*. This aggregation can be formally represented as a TOPSIS relative closeness of the alternative defined on α_{ij}, $j=1,...,n$:

$$\begin{aligned}\beta_i &= \text{relative closeness of the alternative}(\mathrm{cs_i})\\ &\equiv \text{TOPSIS aggregation}\,(\alpha_{i1},...,\alpha_{in}),\ i=1,...,m\end{aligned} \tag{8}$$

The proposed framework of location planning for candidate sites comprises the following steps:

Step 1: *Selection of location attributes*. Involves the selection of location attributes for evaluating potential locations for candidate sites. These attributes are obtained from discussion with experts and members of the city transportation group. We use *five attributes* $(n=5)$ defined above by short names: $s_1=$”Accessibility”, $s_2=$”Security”, $s_3=$“Connectivity to multimodal transport”, $s_4=$”Costs”, $s_5=$”Proximity to customers”. The fourth attribute is cost type and the others are benefit types. As mentioned above, cost type evaluation data must be transformed in the benefit forms.

Step 2: *Selection of candidate location sites*. Involves selection of potential locations for implementing service centers. The decision makers use their knowledge, prior experience in transportation or other aspects of the geographical area of extreme events and the presence of sustainable freight regulations to identify candidate locations for implementing service centers. For example, if certain areas are restricted for delivery by municipal administration, then these areas are barred from being considered as potential locations for implementing urban service centers. Ideally, the potential locations are those that cater for the interest of all city stakeholders, which are the city residents, logistics operators, municipal administrations, etc.

Step 3: Assignment of ratings to the attributes with respect to the candidate sites. Let $\mathrm{A}_k = \{\alpha_{ij}^k \in q-ROFNs,\ i=1,...,m;\ j=1,...,n\}$ be the performance ratings of each expert $e_k\,(k=1,2,..,t)$ for each candidate site $cs_i\,(i=1,2,..,m)$ with respect to attributes $s_j\,(j=1,2,..,n)$.

Step 4: Computation of the Pythagorean fuzzy decision matrix for the attributes and the candidate sites. Let the ratings of all experts be described by positive numbers $\omega_k, \omega_k > 0, k=1,...,t$. If ratings of the attributes evaluated by the k-th expert are α_{ij}^k then the aggregated fuzzy ratings (α_{ij}) of candidate sites with respect to each attribute are given by PFNs' weighted sum

$$\alpha_{ij} = \sum_{k=1}^{t} \alpha_{ij}^k \left(\omega_k / \sum_{l=1}^{t} \omega_l \right). \tag{9}$$

The fuzzy decision matrix $\{\alpha_{ij}\}$ for the *candidate sites* CS and the attributes S is constructed as follows:

$$\begin{array}{c c} & \begin{array}{cccc} s_1 & s_2 & & s_n \end{array} \\ \begin{array}{c} cs_1 \\ cs_2 \\ \\ cs_m \end{array} & \begin{bmatrix} \alpha_{11} & \alpha_{12} & ... & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & ... & \alpha_{2n} \\ ... & ... & ... & ... \\ \alpha_{m1} & \alpha_{m2} & ... & \alpha_{mn} \end{bmatrix} \end{array} \tag{10}$$

Construct the Pythagorean fuzzy decision matrix $\{\alpha_{ij}\}$ and calculate Sc and Ac functions values (Definition 2) of elements α_{ij}.

Step 5: *Identification of Pythagorean fuzzy PIS and NIS* . TOPSIS approach starts with the definition of the Pythagorean fuzzy PIS and the q-rung orthopair fuzzy NIS. Using formulas 6 of definition 4 the PIS is defined as a Pythagorean fuzzy set on attributes $S: cs^+ = \{s_j, \alpha_j^+ \equiv \underset{i}{Max}[(\alpha_{ij})] \mid j=1,2,..., n\}$ and the NIS is defined as a q-rung orthopair fuzzy set on attributes $S: cs^- = \{s_j, \alpha_j^- \equiv \underset{i}{Mix}[(\alpha_{ij})] \mid j=1,2,..., n\}$. In the real MCDM models PIS and NIS are usually not be feasible alternatives. They are extreme alternatives.

Step 6. Calculate the distances between the alternative candidate location site and the Pythagorean fuzzy PIS, as well as Pythagorean fuzzy NIS, respectively.

Then, we proceed to calculate the distances between each alternative and *Pythagorean* fuzzy PIS and NIS. Using equation (7), we define distances between the alternative cs_i and the *Pythagorean* fuzzy PIS and NIS, as a weighted sums of distances between extreme and evaluated PFNs:

$$\begin{aligned} & D(cs_i, sc^+) = \sum_{j=1}^{n} w_j d_q(\alpha_{ij}, \alpha_j^+) = 1/2 \cdot \\ & \sum_{j=1}^{n} w_j (|(\mu_{\alpha_{ij}})^2 - (\mu_{\alpha_j^+})^2| + |(\nu_{\alpha_{ij}})^2 - (\nu_{\alpha_j^+})^2|) \\ & D(cs_i, sc^-) = \sum_{j=1}^{n} w_j d_q(\alpha_{ij}, \alpha_j^-) = 1/2 \cdot \\ & \sum_{j=1}^{n} w_j (|(\mu_{\alpha_{ij}})^2 - (\mu_{\alpha_j^-})^2| + |(\nu_{\alpha_{ij}})^2 - (\nu_{\alpha_j^-})^2|) \end{aligned} \tag{11}$$

Step 7. Calculate the revised closeness or TOPSIS aggregation as a site's selection index for every alternative.

In general the bigger $D(cs_i, sc^-)$ and the smaller $D(cs_i, sc^+)$ the better the alternative cs_i. In the classical TOPSIS method , *authors usually need to calculate the relative closeness (RC) of the alternative* cs_i. *We define candidate site's selection index as RC with respect to Pythagorean PIS* sc^+ as bellow:

$$\beta_i \equiv RC(cs_i) = \frac{D(cs_i, cs^+)}{D(cs_i, cs^+) + D(cs_i, cs^-)}, \quad i=1,...,m. \tag{12}$$

3 Multi-Objective Optimization Model of Facility Location Set Covering Problem

The location set covering problem (LSCP) was proposed by C. Toregas and C. Revell in 1972, which seeks a solution for locating the least number of facilities to cover all demand points within the service distance. In some of our works we are focusing on the multi-objective fuzzy set covering problems [16,17] for extreme conditions. In this work we construct new fuzzy LSCP model for emergency service facility location planning.

As we discussed in previous section, constructed Fuzzy TOPSIS technology forms center's selection rational index. The center's index reflects expert evaluations with respect to the center, considering all actual attributes. If $x = \{x_1, x_2, ..., x_m\}$ is Boolean decision vector, which defines some selection from candidate centers $CS = \{cs_1, cs_2, ..., cs_m\}$ for facility location, we can build centers' selection index as linear sum of $\beta_j x_j$ values: As a result, new objective function – *centers' selection index* $\sum_{j=1}^{m} \beta_j x_j$ is constructed. Maximizing it will select group of centers with the best total ranking from admissible covering selections. Classical facility location set covering problem tries to *minimize the number of centers*, where service facilities can be located - $\sum_{j=1}^{m} x_j$. The problem aims to locate service facilities in minimal travel time from candidate centers. Let customers covered by service centers in distribution networks be denoted by $A = \{a_1, ..., a_k\}$. The problem aims to locate service facilities in minimal travel time from candidate sites. Let experts evaluated movement fuzzy times (evaluated in triangular fuzzy numbers (TFNs) [8]) between customer and candidate sites - $\tilde{t}_{ij}$, $a_i \in A$; $cs_j \in CS$.

In extreme environment for emergency planning a radius of service center is defined not based on distance but based on maximum allowed time T for movement, since the rapid help and servicing is crucial for customers in such situations. Respectively, a set of candidate sites N_i, covering customer $a_i \in A$, is defined as $N_i = \{cs_j, \ cs_j \in CS / E(\tilde{t}_{ij}) \le T\}$, $i = 1, ..., m$, where

$$E(\tilde{t}_{ij}) = \tilde{t}_{ij}^2 + (\tilde{t}_{ij}^3 - 2\tilde{t}_{ij}^2 + \tilde{t}_{ij}^1)/4, \qquad (13)$$

is an expected value of a TFN $\tilde{t}_{ij} \equiv (\tilde{t}_{ij}^1, \tilde{t}_{ij}^2, \tilde{t}_{ij}^3)$ [8]. Then we can state bi-objective facility location set covering problem:

$$\min\ z_1 = \sum_{j=1}^{m} x_j \quad (1), \quad \max\ z_2 = \sum_{j=1}^{m} \beta_j x_j \qquad (14)$$

$$\sum_{s_j \in N_i} x_j \ge 1 \ (i = 1,2,...,k) \ ; \ x_j \in \{0,1\} \quad j = 1,2,...,m .$$

4 Numerical Simulation of Emergency Service Facility Location Model

We illustrate the effectiveness of the constructed optimization model by the numerical example. Let us consider an emergency management administration of a city in Georgia that wishes to locate some fire stations with respect to timely servicing of critical infrastructure objects. Assume that there are 6 demand points as customers (critical infrastructure objects) and 5 candidate facility centers (fire stations) in the urban area. Let us have 4 experts from Emergency Management Agency (EMA) of Georgia for the evaluation of the travel times and the ranking of candidate facility centers. The travel times between demand points and candidate centers are evaluated in triangular fuzzy numbers (see Table 1). According to the standards of EMA (Georgia), the principle of location of fire stations is that the fire station can reach the area edge within 5 minutes after receiving the dispatched instruction. Therefore, we set covering radius $T = 5$ minutes.

Table 1. Fuzzy Travel times $\tilde{t}_{ij}$ from fire stations to critical infrastructure objects (in minutes).

	a_1	a_2	a_3	a_4	a_5	a_6
cs_1	(3,5,7)	(2,4,6)	(4,6,7)	(4,7,9)	(1,3,5)	(1,3,4)
cs_2	(6,10,14)	(4,9,14)	(2,4,6)	(5,7,10)	(1,4,8)	(1,4,5)
cs_3	(4,8,12)	(4,7,11)	(4,6,9)	(2,4,7)	(4,7,10)	(4,6,8)
cs_4	(4,7,10)	(7,11,15)	(6,9,13)	(4,6,8)	(2,4,6)	(1,3,5)
cs_5	(1,3,5)	(2,4,6)	(1,3,6)	(2,4,7)	(4,6,8)	(5,9,12)

Covering sets of candidate sites N_i are defined (omitted here). Let experts generated the attributes weights as values of overall importance based on the consensus:

$$w_1 = 0.25\ ; \quad w_2 = 0.15\ ; \quad w_3 = 0.25\ ; \quad w_4 = 0.20\ ; \quad w_5 = 0.15\ .$$

Each expert $e_k \left(k=1,2,3\right)$ presented the ratings r_{ij}^k for each candidate center s_i, $(i=1,...,5)$, with respect to each attribute s_j, $(j=1,...,5)$.

Table 2. Appraisal matrix A_1 by expert-1

	s_1	s_2	s_3	s_4	s_5
cs_1	(0.7, 0.4)	(0.7, 0.3)	(0.7, 0.4)	(0.7, 0.4)	(0.8, 0.3)
cs_2	(0.6, 0.5)	(0.7, 0.4)	(0.4, 0.6)	(0.8, 0.3)	(0.7, 0.4)
cs_3	(0.7, 0.4)	(0.9, 0.3)	(0.6, 0.5)	(0.7, 0.4)	(0.8, 0.3)
cs_4	(0.6, 0.5)	(0.8, 0.3)	(0.8, 0.3)	(0.9, 0.3)	(0.8, 0.4)
cs_5	(0.8, 0.3)	(0.6, 0.4)	(0.9, 0.3)	(0.7, 0.4)	(0.8, 0.3)

Table 3. Appraisal matrix A_2 by expert-2

	s_1	s_2	s_3	s_4	s_5
cs_1	(0.7, 0.3)	(0.8, 0.3)	(0.6, 0.3)	(0.6, 0.3)	(0.7, 0.4)
cs_2	(0.6, 0.5)	(0.7, 0.3)	(0.7, 0.4)	(0.9, 0.3)	(0.8, 0.3)
cs_3	(0.8, 0.4)	(0.9, 0.3)	(0.6, 0.4)	(0.8, 0.1)	(0.6, 0.2)
cs_4	(0.6, 0.4)	(0.8, 0.3)	(0.9, 0.3)	(0.7, 0.3)	(0.6, 0.2)
cs_5	(0.8, 0.5)	(0.7, 0.4)	(0.8, 0.4)	(0.7, 0.2)	(0.9, 0.2)

Table 4. Appraisal matrix A_3 by expert-3

	s_1	s_2	s_3	s_4	s_5
cs_1	(0.7, 0.4)	(0.8, 0.3)	(0.7, 0.5)	(0.7, 0.4)	(0.9, 0.4)
cs_2	(0.6, 0.5)	(0.7, 0.4)	(0.5, 0.3)	(0.7, 0.2)	(0.6, 0.3)
cs_3	(0.6, 0.2)	(0.9, 0.4)	(0.7, 0.5)	(0.7, 0.3)	(0.6, 0.3)
cs_4	(0.8, 0.4)	(0.9, 0.4)	(0.8, 0.5)	(0.8, 0.5)	(0.8, 0.3)
cs_5	(0.9, 0.2)	(0.6, 0.3)	(0.8, 0.5)	(0.9, 0.4)	(0.7, 0.4)

Let experts have equal ratings $\{\omega_j = 1/3\}$. Using formula (9) experts' evaluations are aggregated in Pythagorean fuzzy decision making matrix $\{\alpha_{ij}\}$ (Table 5).

Table 5. Accumulated Pythagorean fuzzy decision matrix $\{\alpha_{ij}\}$

	s_1	s_2	s_3	s_4	s_5
cs_1	(0.7,0.36)	(0.77,0.3)	(0.67,0.39)	(0.67,0.36)	(0.82,0.36)
cs_2	(0.6,0.5)	(0.7,0.36)	(0.56,0.42)	(0.82,0.26)	(0.71,0.33)
cs_3	(0.71,0.32)	(0.9,0.33)	(0.64,0.46)	(0.74,0.23)	(0.69,0.26)
cs_4	(0.69,0.43)	(0.84,0.33)	(0.84,0.36)	(0.82,0.36)	(0.75,0.29)
cs_5	(0.84,0.31)	(0.64,0.36)	(0.84,0.39)	(0.8,0.32)	(0.82,0.29)

Using the algorithm from Section 2 of new fuzzy TOPSIS we calculated values of candidate centers' selection indexes:

$$\beta_1 = 0.428,\ \beta_2 = 0.914,\ \beta_3 = 0.524,\ \beta_4 = 0.397,\ \beta_5 = 0.287.$$

After these calculations a Combinatorial Programming Problem (14) has been constructed:

$$\begin{cases} f_1 = x_1 + x_2 + x_3 + x_4 + x_5 \Rightarrow \min, \\ f_2 = 0.428x_1 + 0.914x_2 + 0.524x_3 \\ \qquad + 0.397x_4 + 0.287x_5 \Rightarrow \max \\ x_1 + x_5 \geq 1, \\ x_2 + x_5 \geq 1, \\ x_3 + x_5 \geq 1, \\ x_1 + x_2 + x_4 \geq 1, \\ x_i \in \{0,1\},\ i = 1,2,3,4,5. \end{cases} \tag{15}$$

Based on the developed software for the problem (15) Pareto solutions [18] are founded. There are:

$a)$ $\{cs_1, cs_5\}$, $f_1 = 2;\ f_2 = 1.201,$

$b)$ $\{cs_1, cs_2, cs_3\}$, $f_1 = 3;\ f_2 = 1.866,$

$c)$ $\{cs_1, cs_2, cs_3, cs_4\}$, $f_1 = 4;\ f_2 = 2.263,$

$d)$ $\{cs_1, cs_2, cs_3, cs_4, cs_5\}$, $f_1 = 5;\ f_2 = 2.55.$

It is clear that, increasing of fire stations numbers in Pareto solutions gives us better level of the second objective function - *fire stations' selection index*. But the decision on the choice of the fire stations as service centers depends on the decision making person's preferences with respect to risks of administrative actions.

5 Conclusion

The paper presents a new approach for facility location problem for selection of the locations of service centers in extreme and uncertain situations. The approach utilizes experts' knowledge represented by Pythagorean fuzzy numbers and considers the suitability of a location (i.e. affordability, security, etc.) using constructed new fuzzy TOPSIS approach. From the other hand model also considers the necessity to reach all critical infrastructure points and time that is required to reach them, presented by triangular fuzzy numbers. As a results bi-objective set covering problem is obtained. The constructed approach is illustrated by a numerical example for locating fire stations servicing critical infrastructure points in a city in Georgia. For the constructed problem Pareto solutions are obtained. For the large dimension cases of the problem the epsilon-constraint approach for the Pareto front obtaining is constructed.

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