

Comprehensive study on the role of the phase error distribution on the performances of the phased arrays systems based on a behavior mathematical model

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Abstract: - A study on the impact of the phase errors distribution on the performances of the phased array systems has been produced using a complete behavioral model for radiation-pattern characteristics. The main purpose of the study is to demonstrate that the rms phase error is a valuable figure of merit of phased array systems but it is not sufficient to completely describe the behavior of a real system. Indeed, even the way in which the phase errors are distributed among antenna elements actually affects the performances of the phased arrays antennas. The developed model has many input parameters and it has a lot of features, such as parameter simulations with results analysis, unconventional two-dimensional color graph representation capability in order to show more clearly the results. It is also a useful tool that can be used in system and design phase of a phase shifter. The results of the study demonstrating the previous considerations have been reported.

Key-Words: - Phase shifter, phased array, radiation pattern, behavioral model

1 Introduction

Phased array antennas play an important role in many applications. Their success is mainly due to high agility in reconfiguring pattern and quick steering capability. Many phased-array antennas are designed to use digital or digitalized phase shifters in which the phase shift varies in discrete steps rather than continuously. As a consequence of using digital phase shifter, the beam can be steered only in discrete steps and the granularity [1], defined as the finest realizable increment between adjacent beam positions, depends on the number of bits and the number of antennas. Furthermore, the phase shifts produced by the phase shifter are not ideal, because a phase shifter itself is affected by errors and non-idealities and this implies that the radiation pattern characteristics are further altered.

Usually [2]-[3], the phase accuracy is synthetically presented in terms of rms phase error defined as in

(1). Let's denote $\theta_{\Delta i}$ the i -th error phase shift, so the rms phase error can be defined as

$$\theta_{\Delta rms} = \sqrt{\frac{1}{N-1} \sum_{i=2}^N |\theta_{\Delta i}|^2} \quad (1)$$

where the phase of an arbitrary channel is chosen as the reference for the calculations of the errors in the other channels.

This figure of merit, even being a significant characteristic of the phase shifters, it is not satisfactory to describe consistently the behavior of real phased array systems. Indeed, an important role on determining the radiation-pattern characteristics is played by the phase errors distribution which can be defined as a vector with the length equal to the number of possible phase shifts and with the i -th

element equal to the phase error corresponding to the i -th ideal phase shift.

The phase shifter with a low rms phase error is a challenging building block, expressly at high frequencies [4]. It is easy to understand as the achievement presented in this study is an important key aspect to be taken into account designing a high performances phased array systems such as those used to achieve high directivity and high suppression of sidelobes for satellite, RADAR and wireless communications systems.

An advanced model, thus, must account for the non-idealities and the afore mentioned aspects related to phase errors distribution in the introduced phase shifts, besides of the classical parameters affecting the radiation-pattern characteristics, such as number of antennas, number of bits, and so on. In this chapter this complete and advanced behavioral model, developed in MATLAB® is presented and discussed. This tool is able to predict antenna array performance in terms of beam shape, directivity, side lobe levels, main lobe deviation etc. considering also the phase errors distribution.

Another important feature of the tool is the possibility to get the project specification of a phased array, starting from the radiation-pattern characteristics. In particular, the user can specify the minimum value of the side-lobe level, the maximum value of the lobe deviation and the maximum value of the half power beam width and the tool gives all those ideal values that satisfy the conditions imposed. For this type of simulation it was decided to limit the output to combinations of number of bit and number of antennas, fixing to 0.5λ the distance between the antennas. This choice is determined by the fact that it reduces the simulation time and by the fact that the modification of the distance between the elements in the array can be advantageous for some parameters but inconvenient for other parameters, as will be described below. For this last reason, a good compromise is setting the inter-element spacing equal to 0.5λ .

The paper is organized as follows: the model is described in Section 2; the designing tool is explained in the section 3; the analysis and results of rms effect on the phase shifter is presented in Section 4, while in the Section 5 the conclusions are given.

2 Model description

Matlab® furnishes a high-level technical computing language and an interactive environment for algorithm development, data visualization, data analysis, and numeric computation. For sake of

clarity, every detail of the complete model is discussed in the following paragraphs.

2.1 Array configuration

The tool takes into account for the array factors of linear arrays according to the basic antenna array theory. The array configuration is shown in Figure 1 where the elements, considered as point sources, are positioned on the horizontal axis.

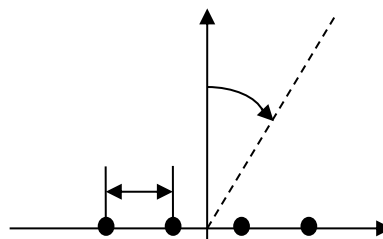


Fig. 1 Array configuration.

The angle θ represents the elevation angle which is the angle of the beam with respect to the z axis. The scan angle is the elevation angle imposed.

2.2 Result plot types

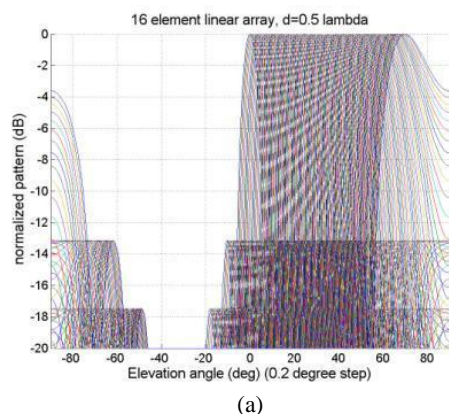
At the end of the simulations, the user can plot the obtained radiation-patterns in various formats: classical rectangular plot, classical polar plot or unconventional two-dimensional color graph representation in Cartesian coordinate system. Conventionally, radiation patterns are represented in two dimensions (2D), in Cartesian or polar coordinate systems. In the Cartesian coordinate system, the magnitude of the radiated field, usually in decibels (dB), is indicated on the Y axis, whereas the angular parameter, the elevation angle θ or the quantity $u = \sin(\theta)$, governs the X axis. In the polar coordinate system, the radius usually represents the magnitude of the radiated field while the angle represents the elevation angle. When a comparison is required or a parametric simulation is carried out, several curves are plotted on the same graph or, alternatively, another dimension is added. If we use another axis as a third dimension, the extrapolation of the data is not easily done, considering that the three dimensional representation is actually two dimensional on a planar surface [5]. An attractive representation adds the colors used for the graphs as the third dimension, which represents the variable or the parameter of user's interest. As a clarifying example, consider the case of a not parametric simulation in which the user aims to study the array factor of a sixteen element array with the inter-

element spacing equal to half a wavelength and exact phase at each element, over a range of scan angles. Assume, in our example, that the elevation start angle is -90° , the elevation stop angle 90° , the elevation increment angle 0.2° , the start scan angle 0° , the stop scan angle 70° and the increment scan angle 0.5° . Two different representations of the same results are shown in Figure 2, where Figure 2a is the classical representation with the overlap of all simulation in different color in rectangular and polar representation and Figure 2b is the proposed unconventional plot with the color scale in dB.

In this latter representation, the wanted elevation scan angle is the parameter on the Y axis, while on the X axis there is the elevation angle and finally the color indicates the magnitude in dB. Thus, each horizontal line corresponds to the array factor calculated for the relative wanted elevation scan angle. In such a representation it is easy to visualize the main lobe, sidelobes, nulls, and half-power-beamwidth (HPBW) variation when the scan angle changes and if there are some grating lobes. Particularly, with the help of the black contour line at the -3dB level, the user can notice the widening of the HPBW with the increase in the scan angle. Referring to Fig. 2 for comparison, it is almost impossible to extrapolate the results related to a single simulation.

It is clear the advantage in using the two dimensional color plot which gives more legibility to the graphical representation when, in general, a parametric or comparative simulation is required. This is only an example of the tool capabilities which depend on the characteristics the user wants to focus on.

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(a)

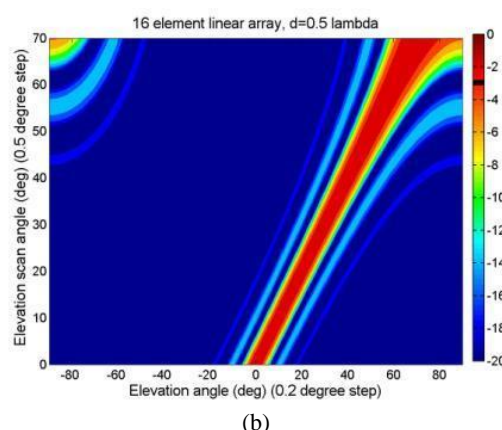


Fig. 2: (a) Overlapped simulations in a rectangular classical plot. (b) Unconventional color polar plot.

2.3 Round-off technique

The user can choose the preferred round-off technique among the following for each of the simulations described in the next section:

1. Exact phase at each element: this is the ideal situation represented by an ideal analog phase shifter. With this type of simulation the user can study the geometry of the array without worrying about the non-idealities of the phase shifter;
2. Ideal quantization: most phase shifters are digital devices or, at least, are digitally controlled. Therefore only discrete values of phase shift are allowed, and they may not be the precise values required. This simulation takes into account the errors due to an ideal quantization;
3. Real simulated quantization: the real digital phase shifts fed to each element are not ideal and they depend on the phase shifter adopted. A figure of merit is the rms error of the phase generation of the phase shifter. Starting from the ideal quantization, a casual and random error distribution, with the imposed rms, is added to the phase shifts in order to obtain the simulated allowable phase states;
4. Measured values: it is possible to insert the measurements taken by a prototype phase shifter in order to test the behavior of the real phase shifter and the resulting array pattern.

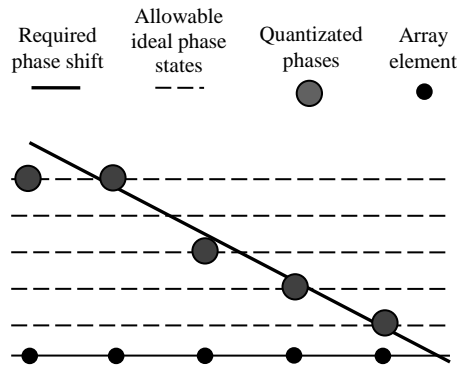


Fig. 3 Allowable phase state due to ideal quantization.

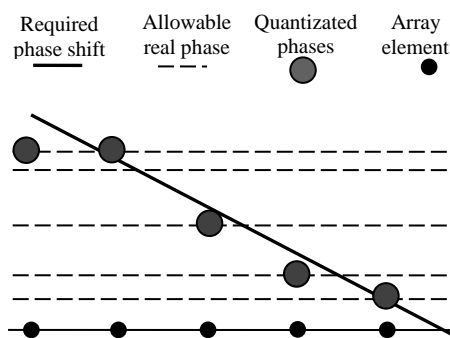


Fig. 4 Allowable phase state due to simulated real quantization.

2.4 Type of simulations

The model presented is very complete [6]-[8], in fact it accepts as input, according to the simulation type chosen, the range of elevation angles for pattern calculation, the number of antennas and the inter-element spacing between the antennas, the type of phase shifts fed at the antennas (ideal phase shift which simulates an ideal analog phase shifter, ideal quantization which simulates an ideal digital phase shifter, calculated real quantization which implements a simulated real digital phase shifter and finally real quantization obtained by inserting a vector of real digital phase shifter measurements and which implements a real measured phase shifter), the wanted elevation angle (scan angle) or range of scan angles.

The program is organized in menu; first of all the user can choose the type of simulation a subset of those is composed by:

1. Variable number of antennas: it is a parametric simulation which has as parameter the number of the antennas. The particular parameters required are the antenna start number, the antenna stop number and the antenna increment number. The other parameters that have to be inserted are the elevation

start angle, the elevation stop angle, the elevation increment angle, the elevation start scan or start steering angle, the stop scan angle, the increment scan angle, the antenna spacing.

2. Variable value of rms error: it is a parametric simulation which varies the rms error added to the ideal phase shifter in order to simulate a real phase shifter and to analyse the impact of the rms phase error introduced by a non-ideal phase shifter, on the radiation-pattern characteristic. This type of simulation, respecting the common practice, uses the rms phase error value as the only figure of merit of a phase shifter goodness while a more exhaustive study about this concept is achieved using the simulation option number 7 below described. In this case the parameters required are the number of simulation and the rms value to be used in those simulations. This is one of the most interesting feature of the program because it gives the opportunity to investigate if there is a particular relation between the rms phase error of the phase shifter and some parameter of the radiation-pattern [5]. The other parameters required are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle, the number of the antennas, the inter-element antenna spacing.

3. Variable number of bit: also in this case there is a parametric simulation which has as input the start number of bits, the stop number of bits and the increment. Such a simulation is the key point in those application where a great precision is required during the scan. In fact the more are the bits, the more is the scan precision till the ideal situation of analog phase shifter which could be assimilated to a phase shifter with infinite number of bits. So this type of simulation could be used as a dimension tool or as an instrument to study how effective is the increment of the number of bits and what is the impact on the radiation-pattern. The parameters required are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start, the stop scan angle, the increment scan angle, the number of antennas and the inter-element antenna spacing.

4. Rms behavior study: this type of simulation is very important in studying the impact of the effective phase error distribution on the beam shape. In fact for a generic phase shifter, it is often given the rms phase error as the only figure of merit, but this parameter does not represent the phase error distribution in an unambiguous way because there are infinite distribution with the same value of rms error, and different distributions can lead at significant different beam shapes. This simulation

has as peculiar inputs the phase error vector or distribution defined as the difference between the measured or simulated phase distribution and the ideal one; the simulation consists in combining the values of ideal phase shifts with the values of the phase errors in a different way for every simulation. The user can choose the type of this combination and in particular he can choose if the all permutations have to be used or a circular shift has to be used; the first option is only practical for situation where the number of phase is less than about 5 due to time and memory required by the simulation. As an example, let's consider a 6-bit phase shifter; the total phase shift are $2^6=64$ that corresponds to $1,2e89$ permutations ($64!=1,2e89$) and 64 circular shifts; in this case it has to be chosen the circular shift mode. The other parameter that have to be inserted are the number of antennas, the inter-spacing antenna, the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle.

3 Designing tool

The tool is, essentially, a simulation that examines all possible parameters of a phase shifter that meet certain requirements of the beam and stores the results for future simulations. The input parameters are the side lobe levels, the lobe deviations and the half power beam widths and the outputs are the combination of the number of antennas and the number of bits. Other inputs are the range of the number of antennas, the increment for the antenna variation, the range of number of bit, the increment for the bit variation, the bit-equivalent rms error to add iteratively at the simulations. The graphical representation shows immediately the combinations of number of antennas and number of bits that satisfies the beam requirements. As an example, consider the simulation parameters in Table 1.

In Table 2 are shown the best results for each simulation without considering the scan angle or number of antennas or input requirements. It is an initial screening and it gives an idea of the theoretical limit reachable.

Filtering the results by inserting the input values of lobe deviation, half power beam width and side lobe level we have a graphical representation as in Fig. 5 where the requested specifications are a lobe deviation less than 0.005° , a maximum half power beam width equal to 3° and a minimum side lobe level equal to -13dB . The triangle represents the requirement met for the lobe deviation, the circle represents the requirement met for the half power

beamwidth and the dot represents the requirement met for the side lobe level.

Table 1. Summary of the simulation setup

Parameter	Case study
Elevation start angle	-90
Elevation stop angle	90
Elevation increment angle	0.02
Start scan angle	-5
Stop scan angle	5
Increment scan angle	0.1
Number of antennas	Start: 4 Stop: 40 Increment: 1
Antenna inter-element spacing	0.5λ
Number of bit	Start: 4 Stop: 15 Increment: 1
Bit-equivalent rms error	4-6-8-10-12-14-15

Table 2. Summary of the simulation results

	Minimum lobe deviation (degree)	Minimum HPBW (degree)	Minimum SLL (dB)
4 bits rms error	0.0000	2.3400	-14.7619
6 bits rms error	0.0000	2.4600	-13.5384
8 bits rms error	0.0000	2.4600	-14.6136
10 bits rms error	0.0000	2.5400	-13.28.96
12 bits rms error	0.0000	2.5400	-13.2562
14 bits rms error	0.0000	2.5400	-13.2444
15 bits rms error	0.0000	2.5400	-13.2571

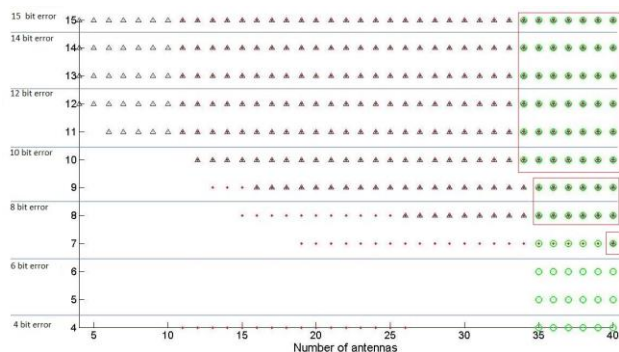


Fig. 6 Output results as function of bit-equivalent rms error for Lobe deviation $< 0.005^\circ$; HPBW $\le 3^\circ$; sll $\ge -13\text{dB}$. Compact view.

For lobe deviation $< 0.005^\circ$, HPBW $\le 3^\circ$ and sll $\ge -13\text{dB}$ the possible combinations are:

- (1) 34 antennas and 10÷15 bits
- (2) 35 antennas and 8÷15 bits
- (3) 36 antennas and 8÷15 bits
- (4) 37 antennas and 8÷15 bits
- (5) 38 antennas and 8÷15 bits
- (6) 39 antennas and 8÷15 bits
- (7) 40 antennas and 7÷15 bits

So probably the best choice could be the first one or the second one, depending on the design specifications.

4 RMS effect on the beam shape for a real phase shifter

This paragraph demonstrates the importance that the effective errors distribution has on the beam shape, through a practical example. To this purpose, the measurements made on a real 6-bit phase shifter will be used. The considered phase shifter is that described in [7], with a 0.223° rms phase error and a maximum and minimum phase error equal to about $\pm 0.5^\circ$ as shown in Figure 7. Different prototypes of the same architecture showed the same rms phase error, approximately, but certainly they don't have the same errors distribution that is, in this context, the vector containing the phase errors associated to vector of the possible phase shifts.

With the simulation number 4, called "Rms behavior study" explained in the previous paragraph (with the option of circular shift due to the great number of possible phase shifts), we are able to study the beam shape variations determined by a 64 different phase errors distribution with the same rms phase error. In this way, the i -th simulation uses the vector of the phase errors inserted with a circular shift of exactly i places.

As it is obvious, the rms phase error of the various distributions is the same for each simulation because

the values of the phase errors are the same and only the positions in the vector are changed. The hypothesis is that we have 16 identical phase shifters for the 16 antennas. The simulation setup is shown in Table 3, while Figure 8 report the results of the simulations.

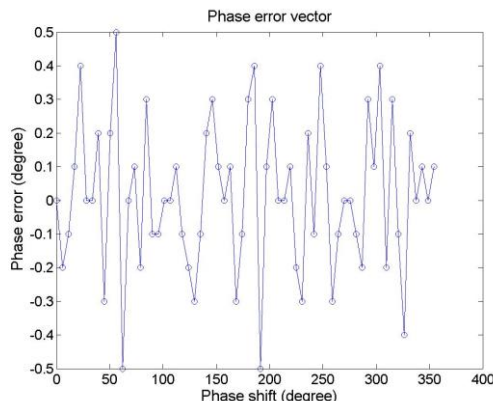


Fig. 7 Phase shifter error vector

An exhaustive and complete representation of the simulation results are reported in Figure 7a, Figure 7b and Figure 7c, where it has been used the unconventional color plot described in the previous section. In those type of plots all simulation results have been used without any significant loss of information.

Table 3. Summary of the simulation setup

Parameter	Variable error vector
Elevation start angle (degree)	-90
Elevation stop angle (degree)	90
Elevation increment angle (degree)	0.001
Scan start angle (degree)	0
Scan stop angle (degree)	70
Scan increment angle (degree)	0.1
Number of antennas	16
Inter-element distance	0.5
Simulation type	circular
Number of simulation	64

In Figure 8a it is reported the lobe deviation as a function of the number of simulations; the elevation scan angles in degrees are on the X axes, while on the Y axes there is the number of the simulations and the color bar represents the value of the main lobe deviation in degrees.

Thus, on each horizontal line are the lobe deviations related to the single simulation over the entire span of scan angles. On each vertical line the lobe deviations related to the single elevation scan angle over all the simulations are reported.

In Figure 8b it is represented the side lobe level as a function of the number of simulation; on the X axes there are the elevation scan angles in degrees, on the Y axes there is the number of the simulations, the color bar represents the value of the side lobe level in dB. Thus, on each horizontal line the side lobe level relates to each of the simulations over the entire span of scan angles and on each vertical line there are the side lobe level related to the single elevation scan angle over all the simulations.

In Figure 8c it is represented the lobe deviation as a function of the number of simulations: on the X axes there are the elevation scan angles in degrees, on the Y axes there is the number of the simulations, the color bar represents the value of the HPBW in degrees. So on each horizontal line there are the HPBW levels related to the single simulation over the entire scan angles span and on each vertical line there are the HPBW values related to the single elevation scan angle over all the simulations.

It is evident that the comparison among different simulations is very simple giving the opportunity to evaluate what are the critical angles.

The reported simulations demonstrate that the phase error rms value itself, even being an important figure of merit, it is not enough to evaluate antenna performances because the beam shape characteristics are strictly dependent on the actual phase errors vector.

Ideally, in absence of differences between the simulations, in Figure 8a, Figure 8b and Figure 8c would have shown perfectly matched vertical lines with the same colors; but the variations between the simulations are evident in the cited figures. For example, the lobe deviation and the SLL have significant variations for high values of the elevation angles, where there are red and blue zones for the same scan angle. The HPBW represented in Figure 8c seems to be not changed varying the simulations since the graph is composed by vertical line of almost the same color.

Actually, this is not true and it is determined by the great span of the HPBW represented, which doesn't give the opportunity to appreciate little changes.

As a consequence of these results we can say that for a phase shifter a little rms phase error value is not the only key issue but a good phase shifter should have the phase error vector as flat as possible, that means that each phase error should be very similar to another. The ideal condition is a constant phase error over all the possible phase shifts [7].

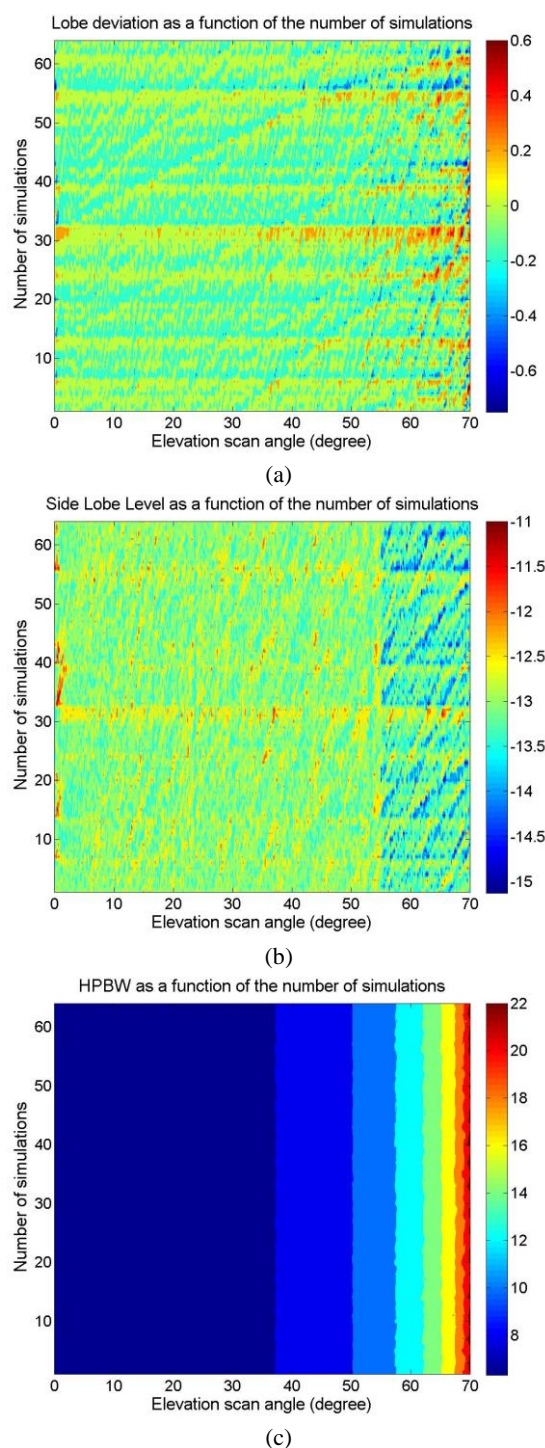


Fig. 8 Simulation results obtained changing the error phase vector. (a) Lobe deviation. (b) Side Lobe Level. (c) Half Power Beamwidth.

5 Conclusion

In this paper it has been presented a mathematical model very useful to investigate and simulate every aspect of a phase array and which is an optimum starting point for starting in design a phased array.

It has been demonstrated also that the beam shape of an array of antennas depends on the actual phase errors distribution and that a good phase shifter must exhibit phase errors with low rms values and as constant as possible.

Moreover the tool can be used for a yield analysis of sensitivity, simulating different implementation of the same architecture assuming that each phase shifter sample has a fixed rms phase error value but different phase errors vector.

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