An optimization algorithm for the calculation of the electrostatic discharge current equations' parameters

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Abstract: - The international standard IEC 61000-4-2 for the electrostatic discharge does not provide an equation for the electrostatic discharge current, but it just defines a waveform for it. In the current paper a new optimization algorithm is presented that has been developed in order to calculate the parameters of equations proposed in the technical literature to describe the current during an electrostatic discharge. Electrostatic discharge current data obtained from real measurements are also used in this work in order to compare the results produced using the optimization algorithm in an effort to prove the efficiency and accuracy of the proposed algorithm and to identify the equation that describes better the electrostatic discharge current.

Key-Words: - Discharge current; Electrostatic discharge; Electrostatic discharge generators; Optimization algorithm.

1. Introduction

Electrostatic Discharge (ESD) is common phenomenon that is very crucial for electronic devices such as integrated circuits or fast complementary metal oxide semiconductor systems. The IEC 61000-4-2 [1] describes the test procedure for electronic equipment under electrostatic discharges and defines the shape of the discharge current that the ESD generators must produce, however it does not provide an equation for the electrostatic discharge current. An accurate equation is an indispensable requirement for the description of the ESD generators in simulation programs.

Many studies have been conducted worldwide to study the ESD current waveforms. In [2] it has been concluded that the amplitudes and the rise times depend on the charging voltages, the approach speeds, the electrode types, the relative arc length and the humidity. The parameters that characterize the discharge current waveforms of ESD testers have been studied in [3]. Murota in [4] presented the variations that appear on the discharge current, when various conditions change during the test using the simulation program PSpice. An improved

circuit for the ESD generators with a reference waveform close to the one defined by the Standard and an equation describing the reference waveform have been proposed [5]. Another proposed equation [6] for the reference waveform has been developed in order to study the ESD phenomenon in coaxial cable shields. Finally the downhill simplex optimization method in [7] and genetic algorithms in [8] have been used in order to determine an accurate electrostatic discharge current equation.

The current paper aims to calculate the electrostatic discharge current equations' parameters in an effort to obtain an accurate discharge current equation. For this purpose a new optimization algorithm is proposed and the obtained results are compared with electrostatic discharge current data obtained from real measurements proving the efficiency of the algorithm.

2. ESD generators' discharge current

The ESD generators are used for testing the robustness of electronics towards electrostatic

discharges. Electrostatic discharges can occur either as contact discharges or as air discharges. The test level voltages for the contact discharges range between 2 and 8 kV and for the air discharges between 2 and 15 kV. According to the Standard for the verification of ESD generators there are 4 parameters whose values must be confined by certain limits. These parameters are: the rise time (t_r), the maximum discharge current (I_{max}), and the current at 30 ns (I_{30}) and 60 ns (I_{60}). A typical waveform of the output current of an ESD generator is shown in Fig. 1 and the limits of the 4 parameters (valid for contact discharges only) are shown in Table 1 [9, 10].
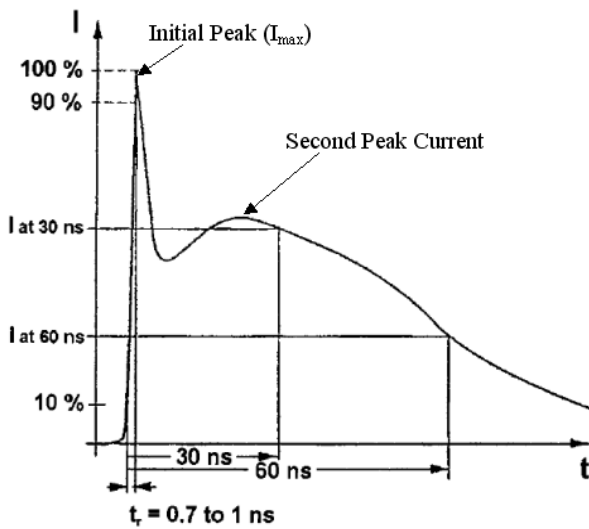

Fig. 1 Typical waveform of the output current of the ESD generator [1]

Table 1 Waveform parameters

Voltage (kV)	I_{max} (A)	t_r (ns)	I_{30} (A)	I_{60} (A)
2	6.75-8.25	0.7-1	2.8-5.2	1.4-2.6
4	13.50-16.50	0.7-1	5.6-10.4	2.8-5.2
6	20.25-24.75	0.7-1	8.4-15.6	4.2-7.8
8	27.00-33.00	0.7-1	11.2-20.8	5.6-10.4

An equation that does not correspond to the discharge current, but will be used in the further analysis is the lightning current equation [11]:

$$i(t) = i_0 \cdot \left(e^{-\frac{t}{t_1}} - e^{-\frac{t}{t_2}} \right) \tag{1}$$

An approximate equation of the discharge current for commercial simulators was firstly introduced by [12] using a double exponential function:

$$i(t) = i_1 \cdot e^{-\frac{t}{t_1}} - i_2 \cdot e^{-\frac{t}{t_2}} \tag{2}$$

The reference waveform for the discharge current according to [6] is:

$$i(t) = A \cdot e^{-\left(\frac{t-t_1}{\sigma_1}\right)^2} + B \cdot t \cdot e^{-\left(\frac{t-t_2}{\sigma_2}\right)^2} \tag{3}$$

The waveform of Fig. 1 may be viewed as the sum of two Gaussians in the time domain, one narrow and the other broad. Equation (3) is closer to this observation, since the factors $A \cdot e^{-\left(\frac{t-t_1}{\sigma_1}\right)^2}$ and $B \cdot t \cdot e^{-\left(\frac{t-t_2}{\sigma_2}\right)}$ represent the narrow and broad Gaussian respectively.

In [5] based on the equation of the lightning current of Heidler [13], the referred waveform is given by:

$$i(t) = \frac{i_1}{k_1} \cdot \frac{\left(\frac{t}{\tau_1}\right)^n}{1+\left(\frac{t}{\tau_1}\right)^n} \cdot e^{-\frac{t}{\tau_2}} + \frac{i_2}{k_2} \cdot \frac{\left(\frac{t}{\tau_3}\right)^n}{1+\left(\frac{t}{\tau_3}\right)^n} \cdot e^{-\frac{t}{\tau_4}} \tag{4}$$

where:

$$k_1 = e^{-\frac{\tau_1}{\tau_2}\left(\frac{n\tau_2}{\tau_1}\right)^{\frac{1}{n}}} \text{ and } k_2 = e^{-\frac{\tau_3}{\tau_4}\left(\frac{n\tau_4}{\tau_3}\right)^{\frac{1}{n}}},$$

i_1, i_2 are currents in A,
τ_1, τ_2, τ_3, τ_4 are time constants in ns and
n signifies how many times the equation can be differentiated with respect to time.

In order to analytically describe the measured electrostatic discharge current, the unknown parameters of the equations (1)-(4) must be optimized.

3. The optimization algorithm

The waveform of the discharge current that occurs for the optimized parameter data of the four possible equations, must be as much closer to the measured discharge current. Considering that the accuracy of the computed discharge current depends on the model parameters, these can be determined by minimizing the function:

$$e = \frac{I_c(t,x) - I_m(t)}{I_m(t)} \cdot 100 \qquad (5)$$

where:

$I_c(t, x)$ is the computed discharge current and
$I_m(t)$ is the measured discharge current.

The equations (1)-(4) that can describe the discharge current, are consisted of different parameters whose values may have various values. Equations (1), (2), (3) and (4) have 3, 4, 6 and 7 parameters respectively. The objective is the minimization of the absolute error of (5), which is a function of 3, 4, 6 or 7 variables depending on the model. The parameters of each equation form a column vector x:

$$X = \begin{bmatrix} x_1, x_2, x_3 \end{bmatrix}^T = \begin{bmatrix} i_0, t_1, t_2 \end{bmatrix}^T \qquad (6)$$

$$X = \begin{bmatrix} x_1, x_2, x_3, x_4 \end{bmatrix}^T = \begin{bmatrix} i_1, i_2, t_1, t_2 \end{bmatrix}^T \qquad (7)$$

$$X = \begin{bmatrix} x_1, x_2, x_3, x_4, x_5, x_6 \end{bmatrix}^T =$$
$$= \begin{bmatrix} A, B, t_1, t_2, \sigma_1, \sigma_2 \end{bmatrix}^T \qquad (8)$$

$$X = \begin{bmatrix} x_1, x_2, x_3, x_4, x_5, x_6, x_7 \end{bmatrix}^T =$$
$$= \begin{bmatrix} i_1, i_2, \tau_1, \tau_2, \tau_3, \tau_4, n \end{bmatrix}^T \qquad (9)$$

The application of an optimization algorithm will determine the optimal values x_i, with its goal the minimization of the relative error, which is a function of several variables. $e(x)$ is the relative error, where x is the column vector presented in (6)-(9). The optimal solution can be found after the iteration of a formula of the form:

$$x_{n+1} = x_n - \lambda_n \cdot colM_n \qquad (10)$$

where:

X_n is the value of the design characteristic vector at the nth iteration,
λ_n is the coefficient vector, which accelerates the convergence,
$colM_n$ is the column vector formed from the Jacobian matrix M_n and
M_n is the Jacobian matrix, defined as:

$$M_n(x_1, x_2, ..., x_N) = \begin{bmatrix} \frac{\partial e_1}{\partial x_1} & \cdots & \frac{\partial e_1}{\partial x_N} \\ \cdots & \cdots & \cdots \\ \frac{\partial e_N}{\partial x_1} & \cdots & \frac{\partial e_N}{\partial x_N} \end{bmatrix} \qquad (11)$$

where:

N equals to 3, 4, 6 or 7 depending on the equation.

This method computes the first partial derivatives of the objective function in reference to the dependent variables [14]. It is internally made, by writing a suitable approximation of the objective function up to a desired degree.

The following algorithm based on quasi-Newton method has been implemented.

Step 1: Set initial values for the i_0, i_1, i_2, t_1, t_2, τ_1, τ_2, τ_3, τ_4, A, B, σ_1, σ_2, n of each equation (6)-(9).

Step 2: Calculate e_i from (5), M_n from (11) and x_{n+1} from (10).

Step 3: As long as: $\| x_{n+1} - x_n \| < \varepsilon$, repeat Step 2, where ε is a positive parameter, which defines the desired convergence precision.

Step 4: Display converged values X_n.

4. Results

Tables 2-5 present the computed parameters for each one current equation described in section 2, as well as the optimum parameter values obtained using the proposed optimization algorithm.

Table 2 Parameters for equation (1)

Parameters	Computed values	Optimized values
i_0[A]	31.20	34.24
t_1[ns]	33.20	35.87
t_2[ns]	20.71	22.46

Table 3 Parameters for equation (2)

| Parameters	Computed values	Optimized values
i_1[A]	16.50	19.07
i_2[A]	13.40	16.99
t_1[ns]	45.20	54.12
t_2[ns]	13.20	16.21

Fig. 2 presents in a single figure the comparison between the discharge current waveform plotted using experimental data that have been obtained through real measurements [15, 16] and the discharge current waveforms plotted using the obtained optimized parameter values for the equations (1)-(4).

Table 4 Parameters for equation (3)

Parameters	Computed values	Optimized values
A[A]	6.30	5.07
B[A/ns]	0.20	0.29
t_1[ns]	5.50	6.28
t_2[ns]	9.10	8.74
σ_1[ns]	4.00	4.46
σ_2[ns]	50.00	54.01

Table 5 Parameters for equation (4)

Parameters	Computed values	Optimized values
i_1[A]	10.00	15.13
i_2[A]	6.10	8.95
τ_1[ns]	3.15	2.12
τ_2[ns]	1.67	2.22
τ_3[ns]	7.50	10.42
τ_4[ns]	35.00	37.36
n	1.70	1.91
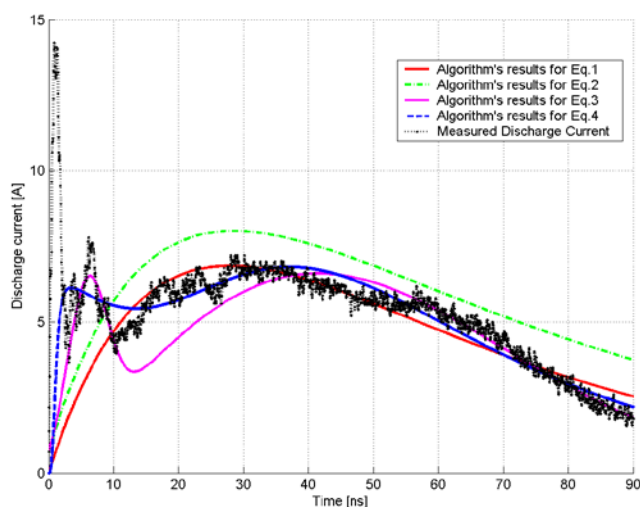
Fig. 2 Comparison between the experimental data of the discharge current and the discharge current for the optimized parameter values of (1)-(4) (charging voltage = +4kV).

Comparing the curves of Fig. 2 it is obvious that the equation, that has the best fitting to the experimental data, is equation (4). This is the most suitable of all the examined equations for the electrostatic discharge current description, since it simulates the discharge current in the best way. The second most suitable equation is (3), since it has a shape similar to the experimental results. The third more suitable equation is (2) that although it cannot simulate the first peak of the electrostatic discharge current, it can calculate accurately the parameters of the double exponential function.

5. Conclusion

The international standard IEC 61000-4-2 for the electrostatic discharge does not provide an equation for the electrostatic discharge current. In this paper an optimization algorithm was developed in order the optimum equation for the description of the electrostatic discharge current to be determined. The obtained results have been compared with real electrostatic discharge current data obtained through experimental measurements in an effort to justify the efficiency and accuracy of the proposed methodology. The proposed optimization algorithm can be proved very valuable in the achievement of more accurate and reliable results in the study of the ESD phenomenon.

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