

Model Following with Global Asymptotic Stability in Hybrid Systems with Periodic State Jumps

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Abstract: This work deals with model following by output feedback in hybrid systems subject to periodic state jumps. In particular, the hybrid systems addressed exhibit a free jump dynamics and a to-be-controlled flow dynamics. Moreover, they may present a direct algebraic link from the control input to the regulated output — such possible algebraic link is briefly called the control feedthrough. The problem of structural model following is investigated first. Namely, the structural aspect of model following consists in designing an output feedback hybrid compensator such that the output of the compensated system perfectly replicates that of the reference model for all the admissible input signals and all the admissible sequences of jump times, provided that the compensated system, the reference model, and the feedback compensator have zero initial conditions. A necessary and sufficient condition for the existence of a solution to the structural problem is proven. Then, the problem of also ensuring global asymptotic stability of the closed-loop compensated system, when the state is subject to periodic state jumps, is tackled and a necessary and sufficient condition to accomplish also this goal, under suitable assumption, is shown.

Key-Words: Hybrid systems, Periodic state jumps, Global asymptotic stability, Model following, Geometric approach.

1 Introduction

Model following is a classic and extensively investigated problem in system and control theory. At first, it was stated for linear time-invariant systems and a solution based on state feedback was presented [1]. Later, it was solved, still in the framework of linear time-invariant systems, by means of an output dynamic feedback control scheme [2, 3]. Meanwhile, the problem of model following was formulated and solved for several other classes of dynamical systems, such as nonlinear systems [4, 5], time-delay and uncertain systems [6–8], large-scale systems [9, 10], Markovian jump linear systems [11], switching systems [12–16]. It is also worth remarking that model following is interesting not only from a methodological point of view, but also from a practical point of view, since it is related to a large number of applications developed during the last decades [17–26].

In this work, the problem of model following is formulated and studied for a special class of hybrid dynamical systems — i.e., those featuring a continuous-time linear behavior, interrupted by abrupt state discontinuities occurring at precise time instants. The dynamics governing the continuous-time behavior is briefly referred to as the *flow dynamics*, while

the dynamics ruling the instantaneous changes of the state is called the *jump dynamics*. More precisely, this work considers hybrid systems whose jumps are equally spaced in time and satisfy the constraint that the number of jump times is finite in any finite time interval, so as to leave possible chattering phenomena out of consideration. Moreover, the hybrid systems addressed present a free jump dynamics, while the control input is applied to the continuous-time dynamics and may also directly affect the to-be-controlled output through an algebraic link, which is called the *control feedthrough*.

Hybrid systems with state jumps, in general, have recently attracted the attention and the research effort of the scientific community, mainly for their capability of modeling the main features of complex dynamical systems, such as colliding mechanical systems, multi-agent systems, electro-mechanical systems and many others — see, e.g., [27] for a thorough report on physical systems which can effectively be modelled as hybrid systems with state jumps. Hence, some control problems have already been formalized and studied for these dynamical systems in some previous papers — this is the case, for instance, of output regulation [28] and disturbance decoupling [29, 30].

Consequently, the contribution of this work consists in proposing a methodology for handling model following in hybrid systems with periodic state jumps, grounded on suitable extensions of well-settled previous results. More specifically, the possible presence of the control feedthrough requires further elaborating on recently-introduced geometric notions for hybrid systems. For instance, the notion of hybrid controlled invariance introduced in [28] needs to be extended to what will be called output-nulling hybrid controlled invariance, as will be shown in the paper. Moreover, the treatment of the issue of global asymptotic stability requires new notions to be introduced, like, e.g., that of minimum-phase hybrid system with periodic state jumps.

Indeed, the methods devised in this work in order to handle hybrid systems with state jumps ensue from the geometric approach to linear control theory [31, 32]. Although the geometric approach is a methodology established in the late sixties and originally aimed at linear time-invariant systems, it has recently proven to be very flexible and powerful in dealing with different kinds of hybrid dynamical systems. In particular, it has led to the solution of some basic control problems stated for switching systems, like signal decoupling [33, 34, 37–43] and output regulation [35, 36], in addition to the already mentioned model following [12–16].

On a last introductory note, it is worth mentioning that a typical feature of the geometric approach is distinguishing and separately treating the structural aspects and the stability aspects of any control problem, independently of the typology of the dynamical systems addressed. In the specific case dealt with herein, the structural aspects of model following are not affected by the special characteristics of the hybrid time domain. Namely, the fact that the jump times be equally spaced in time or not has no influence on solvability of the mere problem of making the difference between the output of the compensated system and that of the reference model to be equal to zero at any time (provided that the number of jump times be finite in any finite time interval). Instead, the fact that the jump times are equally spaced has a relevant impact on the stability issues.

For this reason, the structural model matching problem will be discussed and solved first, with reference to the more general case where the jump times are not necessarily uniformly spaced in time. Instead, the more exhaustive version of the model following problem, which also takes into account the requirement that the closed-loop compensated system be globally asymptotically stable, will be considered by focusing on the case of periodic jumps.

2 Notation and Preliminaries

The purpose of this section is twofold. The first aim is to introduce the notation that will be used throughout this work. The second goal is to review the basic geometric notions, referred to hybrid systems with state jumps, needed for the methodological developments presented in the paper.

The symbols \mathbb{R} , \mathbb{R}^+ , \mathbb{Z}_0^+ , and \mathbb{Z}^+ stand for the sets of real numbers, nonnegative real numbers, nonnegative integer numbers, and positive integer numbers, respectively. Matrices and linear maps are denoted by slanted upper-case letters, like A . The image and the kernel of A are denoted by $\text{Im } A$ and $\text{Ker } A$, respectively. The transpose of A is denoted by A^\top . The inverse of a nonsingular square matrix A is denoted by A^{-1} . Vector spaces and subspaces are denoted by calligraphic letters, like \mathcal{V} . The symbol I denotes an identity matrix of appropriate dimensions.

As will be shown in the following sections, a key role in the solution of the model following problem is played by the solution of a problem of disturbance decoupling for a suitably modified scheme. The wide literature available on decoupling and noninteraction shows that one of the most powerful tools to successfully master these problems is the geometric approach [31, 32]. For this reason, during the last decades, the fundamental concepts formerly established to deal with linear time-invariant systems have been extended to more complex dynamical systems.

More specifically, concerning hybrid systems with state jumps, some basic ideas, such as invariance and controlled invariance, have been generalized so as to adapt to this kind of dynamical systems in some earlier articles [28–30]. However, in this work, the considered hybrid systems may exhibit a direct algebraic link from the control input to the output. Hence, the notion of hybrid controlled invariance must be completed by the novel concept of *output-nulling* hybrid controlled invariance.

The definitions of hybrid invariant subspace, hybrid controlled invariant subspace and output-nulling hybrid controlled invariant subspace are given with reference to a hybrid system with state jumps that will be denoted by Σ . In order to formalize the mathematical description of Σ , the hybrid time domain must be first introduced through the following notation. The symbol \mathcal{T} denotes a finite or countably infinite ordered set $\{t_0, t_1, \dots\}$ of strictly increasing elements of \mathbb{R}^+ . The symbol t_f stands for the last element of \mathcal{T} when \mathcal{T} has a finite cardinality. The set \mathcal{T} is assumed to exhibit no accumulation points. The symbol \mathcal{S} denotes the set of all \mathcal{T} meeting the constraint mentioned above. The nonnegative real axis without the elements of \mathcal{T} is denoted by $\mathbb{R}^+ \setminus \mathcal{T}$.

Hence, the hybrid system with state jumps Σ is described by

$$\Sigma \equiv \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Hh(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x(t_k) = Gx(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = Ex(t) + Du(t), & t \in \mathbb{R}^+, \end{cases}$$

where $x \in \mathcal{X} = \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the control input, $h \in \mathbb{R}^q$ is the disturbance input, and $e \in \mathbb{R}^q$ is the to-be-controlled output.

The set of the admissible control input functions $u(t)$, with $t \in \mathbb{R}^+$, is defined as the set of all piecewise-continuous functions with values in \mathbb{R}^p . The set of the admissible disturbance input functions $h(t)$, with $t \in \mathbb{R}^+$, is defined as the set of all piecewise-continuous functions with values in \mathbb{R}^q . The so-called *flow dynamics* is ruled by the differential state equation. Meanwhile, the algebraic state equation governs the so-called *jump dynamics*. Thus, according to the hybrid structure of Σ , the state motion $x(t)$ in $[0, t_0)$ is the solution of the differential equation, with given initial state $x(0) = x_0$ and input functions $u(t)$ and $h(t)$, with $t \in [0, t_0)$. The state $x(t_k)$, with $t_k \in \mathcal{T}$, is the image through G of $x(t_k^-) = \lim_{\epsilon \rightarrow 0^+} x(t_k - \epsilon)$. The state motion $x(t)$ in $[t_k, t_{k+1})$, with $t_k, t_{k+1} \in \mathcal{T}$, is the solution of the differential equation, given the initial state $x(t_k)$ and the input functions $u(t)$ and $h(t)$, with $t \in [t_k, t_{k+1})$.

From now on, the symbol \mathcal{H} stands to qualify *hybrid invariance* or, respectively, *hybrid controlled invariance*. The symbol \mathcal{B} is the short notation for $\text{Im } B$, while \mathcal{H} stands for $\text{Im } H$. A subspace $\mathcal{W} \subseteq \mathcal{X}$ is said to be an \mathcal{H} -invariant subspace if $A\mathcal{W} \subseteq \mathcal{W}$ and $G\mathcal{W} \subseteq \mathcal{W}$. A subspace $\mathcal{W} \subseteq \mathcal{X}$ is said to be an \mathcal{H} -controlled invariant subspace if $A\mathcal{W} \subseteq \mathcal{W} + \mathcal{B}$ and $G\mathcal{W} \subseteq \mathcal{W}$. Furthermore, it can be shown that a subspace $\mathcal{W} \subseteq \mathcal{X}$, with a basis matrix W , is an \mathcal{H} -controlled invariant subspace if and only there exist matrices X_A , X_G , and U such that $AW = WX_A + BU$ and $GW = WX_G$. Hence, the definition of output-nulling \mathcal{H} -controlled invariant subspace is introduced as follows.

Definition 1 A subspace $\mathcal{W} \subseteq \mathcal{X}$, with a basis matrix W , is said to be an output-nulling \mathcal{H} -controlled invariant subspace if there exist matrices X_A , X_G , and U such that $AW = WX_A + BU$, $GW = WX_G$, and $EW = DU$.

A relevant characterization of the geometric concept of output-nulling \mathcal{H} -controlled invariant subspace is expressed by the following statement, whose proof directly ensues from the properties enjoyed by simultaneous invariant and output-nulling controlled invariant subspaces in the linear time-invariant case.

Proposition 2 A subspace $\mathcal{W} \subseteq \mathcal{X}$ is an output-nulling \mathcal{H} -controlled invariant subspace if and only if there exists a linear map K such that $(A + BK)\mathcal{W} \subseteq \mathcal{W}$ and $\mathcal{W} \subseteq \text{Ker}(E + DK)$ hold along with $G\mathcal{W} \subseteq \mathcal{W}$.

Any linear map K satisfying the conditions of Proposition 2 is said to be a *friend* of the output-nulling \mathcal{H} -controlled invariant subspace \mathcal{W} .

As can be shown by simple algebraic arguments, the set of all output-nulling \mathcal{H} -controlled invariant subspaces is an upper semilattice with respect to the sum and the inclusion of subspaces. The maximum of the set of all output-nulling \mathcal{H} -controlled invariant subspaces is henceforth denoted by $\mathcal{W}_{\mathcal{H}}^*$.

3 Structural Model Following by Output Feedback in Hybrid Systems: Problem Statement

The hybrid system with state jumps Σ_P is defined by

$$\Sigma_P \equiv \begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_P u(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_P(t_k) = G_P x_P(t_k^-), & t_k \in \mathcal{T}, \\ e_P(t) = E_P x_P(t) + D_P u(t), & t \in \mathbb{R}^+, \end{cases}$$

where $x_P \in \mathcal{X}_P = \mathbb{R}^{n_P}$ is the state, $u \in \mathbb{R}^p$ is the control input, and $e_P \in \mathbb{R}^q$ is the output, with $p, q \leq n_P$. A_P , B_P , G_P , E_P , and D_P are constant real matrices of appropriate dimensions. The algebraic link from the control input to the output, established by the matrix D_P , is referred to as the *control feedthrough*. The rank of the matrices

$$\begin{bmatrix} B_P \\ D_P \end{bmatrix}, \quad \begin{bmatrix} E_P & D_P \end{bmatrix},$$

is assumed to be full. The set of the admissible control input functions $u(t)$, with $t \in \mathbb{R}^+$, is defined as the set of all piecewise-continuous functions with values in \mathbb{R}^p . According to the hybrid structure of Σ_P , the state motion $x_P(t)$ in $[0, t_0)$ is the solution of the differential equation, with given initial state $x_P(0) = x_{P,0}$ and input function $u(t)$, with $t \in [0, t_0)$. The state $x_P(t_k)$, with $t_k \in \mathcal{T}$, is the image through G_P of $x_P(t_k^-) = \lim_{\epsilon \rightarrow 0^+} x_P(t_k - \epsilon)$. The state motion $x_P(t)$ in $[t_k, t_{k+1})$, with $t_k, t_{k+1} \in \mathcal{T}$, is the solution of the differential equation, given the initial state $x_P(t_k)$ and the input function $u(t)$, with $t \in [t_k, t_{k+1})$.

The hybrid reference model with state jumps Σ_R

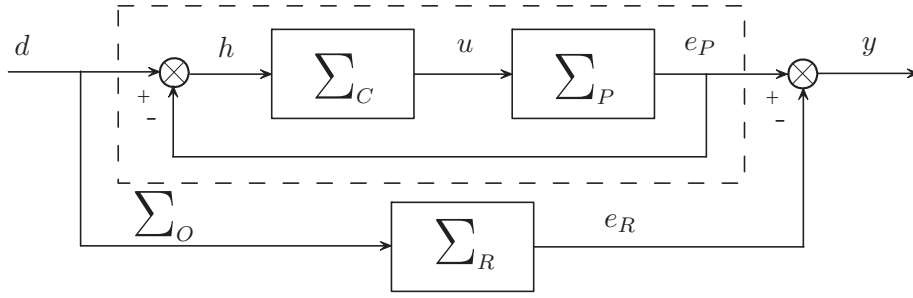


Figure 1: Model Following by Output Feedback

is defined by

$$\Sigma_R \equiv \begin{cases} \dot{x}_R(t) = A_R x_R(t) + B_R d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_R(t_k) = G_R x_R(t_k^-), & t_k \in \mathcal{T}, \\ e_R(t) = E_R x_R(t), & t \in \mathbb{R}^+, \end{cases}$$

where $x_R \in \mathbb{R}^{n_R}$ is the state, $d \in \mathbb{R}^q$ is the input, and $e_R \in \mathbb{R}^q$ is the output. The set of the admissible input functions $d(t)$, with $t \in \mathbb{R}^+$, is defined as the set of all piecewise-continuous functions with values in \mathbb{R}^q .

Hence, the problem of structural model following by output feedback in hybrid systems with state jumps is stated as follows.

Problem 3 (Structural Model Following by Output Feedback in Hybrid Systems with State Jumps) *Let the hybrid system with state jumps Σ_P and the hybrid reference model with state jumps Σ_R be given. Find a hybrid compensator with state jumps Σ_C , defined by*

$$\Sigma_C \equiv \begin{cases} \dot{x}_C(t) = A_C x_C(t) + B_C h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_C(t_k) = G_C x_C(t_k^-), & t_k \in \mathcal{T}, \\ u(t) = C_C x_C(t), & t \in \mathbb{R}^+, \end{cases}$$

where $h(t) = d(t) - e_P(t)$, such that the closed-loop hybrid system with state jumps Σ_O , defined by

$$\Sigma_O \equiv \begin{cases} \dot{x}_O(t) = A_O x_O(t) + D_O d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_O(t_k) = G_O x_O(t_k^-), & t_k \in \mathcal{T}, \\ e_P(t) = E_O x_O(t), & t \in \mathbb{R}^+, \end{cases}$$

where

$$A_O = \begin{bmatrix} A_P & B_P C_C \\ -B_C E_P & A_C - B_C D_P C_C \end{bmatrix},$$

$$D_O = \begin{bmatrix} 0 \\ B_C \end{bmatrix},$$

$$G_O = \begin{bmatrix} G_P & 0 \\ 0 & G_C \end{bmatrix},$$

$$E_O = [E_P \quad D_P C_C],$$

satisfies the requirement that the output $e_P(t)$ is equal to the reference model output $e_R(t)$, for all $t \in \mathbb{R}^+$, when the respective initial states are zero, for all the admissible input functions $d(t)$, with $t \in \mathbb{R}^+$, and all the admissible sequences of jump times $\mathcal{T} \in \mathcal{I}$.

A block diagram illustrating the system interconnection referred to in Problem 3 is presented in Fig. 1.

4 Structural Feedforward Disturbance Decoupling for the Hybrid Extended System: Problem Statement

As will be shown later on, the solution to the problem stated in Section 3 can be achieved by solving the problem which is the object of this section: namely, a problem of structural disturbance decoupling by dynamic feedforward, stated for a suitably-defined hybrid system with state jumps.

This newly-defined hybrid system, henceforth called the *extended* hybrid system with state jumps, is the output-difference connection between the given hybrid plant Σ_P and a modified hybrid reference model, henceforth denoted by Σ_R^+ . In particular, the hybrid reference model Σ_R^+ is derived from the original model Σ_R by closing a positive unit feedback of the output on the flow dynamics. Thus, Σ_R^+ is ruled by

$$\Sigma_R^+ \equiv \begin{cases} \dot{x}_R(t) = (A_R + B_R E_R) x_R(t) + B_R h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_R(t_k) = G_R x_R(t_k^-), & t_k \in \mathcal{T}, \\ e_R(t) = E_R x_R(t), & t \in \mathbb{R}^+. \end{cases}$$

The set of the admissible input functions to the modified reference model Σ_R^+ is defined as the set of all piecewise-continuous functions $h(t)$, with $t \in \mathbb{R}^+$, picking their values in \mathbb{R}^q .

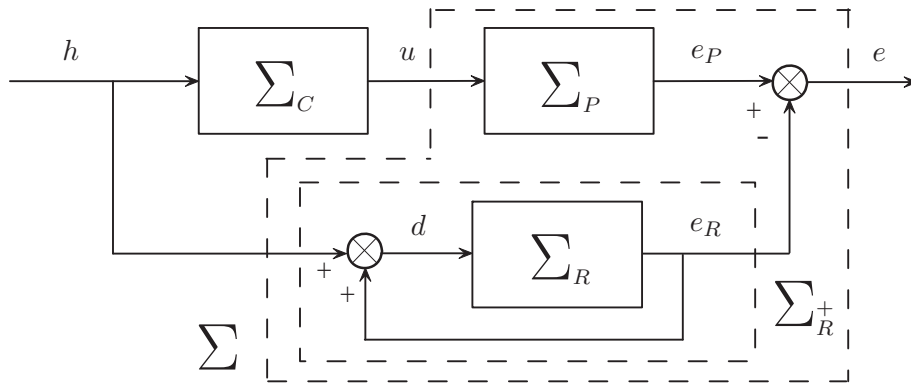


Figure 2: Structural Feedforward Disturbance Decoupling

Consequently, the hybrid extended system with state jumps — denoted by Σ — is defined as the connection of the given hybrid system Σ_P with the modified hybrid reference model Σ_R^+ , such that the control input, the disturbance input, and the output of Σ respectively are the control input of Σ_P , the input of Σ_R^+ , and the difference between the outputs of Σ_P and Σ_R^+ . Thus, Σ is described by

$$\Sigma \equiv \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Hh(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x(t_k) = Gx(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = Ex(t) + Du(t), & t \in \mathbb{R}^+, \end{cases}$$

where

$$\begin{aligned} A &= \begin{bmatrix} A_P & 0 \\ 0 & A_R + B_R E_R \end{bmatrix}, \\ B &= \begin{bmatrix} B_P \\ 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ B_R \end{bmatrix}, \\ G &= \begin{bmatrix} G_P & 0 \\ 0 & G_R \end{bmatrix}, \\ E &= [E_P \quad -E_R], \quad D = D_P. \end{aligned}$$

The state space of Σ will be denoted by \mathcal{X} : i.e., $\mathcal{X} = \mathbb{R}^n$, where $n = n_P + n_R$.

Hence, the structural disturbance decoupling problem by dynamic feedforward, for the hybrid extended system with state jumps Σ , can be stated as follows.

Problem 4 (Structural Feedforward Disturbance Decoupling for the Extended Hybrid System with State Jumps) *Let the hybrid extended system with state jumps Σ be given. Find a hybrid compensator with state jumps Σ_C such that the compensated hybrid system*

$$\bar{\Sigma} \equiv \begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{H}h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \bar{x}(t_k) = \bar{G}\bar{x}(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = \bar{C}\bar{x}(t), & t \in \mathbb{R}^+, \end{cases}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & BC_C \\ 0 & A_C \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H \\ B_C \end{bmatrix}, \\ \bar{G} &= \begin{bmatrix} G & 0 \\ 0 & G_C \end{bmatrix}, \\ \bar{C} &= [E \quad DC_C], \end{aligned}$$

satisfies the requirement that the output $e(t)$ is zero, for all $t \in \mathbb{R}^+$, when the initial state is zero, for all the admissible input functions $h(t)$, with $t \in \mathbb{R}^+$, and all the admissible sequences of jump times $\mathcal{T} \in \mathcal{I}$.

Figure 2 shows a block diagram of the system interconnection dealt with in Problem 4.

5 Structural Feedforward Disturbance Decoupling for the Hybrid Extended System: Problem Solution

Solvability of Problem 4 can be completely characterized by a necessary and sufficient condition exploiting the geometric notions introduced in Section 2. As will be shown in this section, such condition can be expressed in coordinate-free terms, since it amounts to an inclusion of subspaces. Nevertheless, since the proof of sufficiency is constructive — namely, it includes the synthesis of the compensator — some preliminary remarks are made with the purpose of expressing such condition with reference to suitably chosen coordinates.

To begin with, it is worth highlighting that the linear map $A + BK$, where K is a friend of the maximal output-nulling \mathcal{H} -controlled invariant subspace $\mathcal{W}_{\mathcal{H}}^*$ of the hybrid extended system Σ , is represented by a matrix with a typical upper block-triangular structure,

when a suitable similarity transformation is applied to the state space. In particular, let S be a change of basis defined by $S = [S_1 \ S_2]$, with $\text{Im } S_1 = \mathcal{W}_{\mathcal{H}}^*$. Then, in the new coordinates,

$$A' + B' K' = S^{-1} (A + B K) S = \begin{bmatrix} A'_{11} + B'_1 K'_1 & A'_{12} + B'_1 K'_2 \\ 0 & A'_{22} + B'_2 K'_2 \end{bmatrix}, \quad (1)$$

where the structural zero in the lower left corner — i.e.,

$$A'_{21} + B'_2 K'_1 = 0, \quad (2)$$

is due to $(A + B K)$ -invariance of $\mathcal{W}_{\mathcal{H}}^*$. Similarly, the linear map G is represented by

$$G' = S^{-1} G S = \begin{bmatrix} G'_{11} & G'_{12} \\ 0 & G'_{22} \end{bmatrix}, \quad (3)$$

in the same coordinates, where the structural zero in the lower left corner is due to G -invariance of $\mathcal{W}_{\mathcal{H}}^*$. Moreover, with respect to the same coordinates, the linear map $E + D K$, where K is the friend of $\mathcal{W}_{\mathcal{H}}^*$ considered, is represented by a matrix with a structural zero in the first block of columns: i.e.,

$$E' + D' K' = (E + D K) S = \begin{bmatrix} 0 & E'_2 + D' K'_2 \end{bmatrix}, \quad (4)$$

where the structural zero — i.e.,

$$E'_1 + D' K'_1 = 0, \quad (5)$$

is due to $\mathcal{W}_{\mathcal{H}}^* \subseteq \text{Ker } (E + D K)$.

Furthermore, the subspace inclusion that will be proven to be equivalent to solvability of Problem 4 can be conveniently recast in a coordinate-dependent form with reference to the basis considered above. This is to say that

$$\mathcal{H} \subseteq \mathcal{W}_{\mathcal{H}}^* \quad (6)$$

is equivalent to

$$H' = S^{-1} H = \begin{bmatrix} H'_1 \\ 0 \end{bmatrix}. \quad (7)$$

In fact, the structural zero in H' means that a basis matrix of \mathcal{H} is a linear combination of the column vectors of the basis matrix S_1 of $\mathcal{W}_{\mathcal{H}}^*$.

With these premises, the necessary and sufficient condition for Problem 4 to be solvable is formulated as in the following theorem.

Theorem 5 *Let the hybrid extended system with state jumps Σ be given. Problem 4 is solvable if and only if (6) holds.*

Proof: If. Let (6) hold. Let K be a friend of $\mathcal{W}_{\mathcal{H}}^*$. Hence, (1), (3), (4), and (7) hold with respect to the specified coordinates. Let

$$\begin{aligned} A'_C &= A'_{11} + B'_1 K'_1, \\ B'_C &= H'_1, \\ G'_C &= G'_{11}, \\ C'_C &= K'_1 \end{aligned}$$

be the matrices of the hybrid regulator Σ_C with respect to such coordinates. Then, it will be shown that Σ_C , with zero initial state, solves Problem 4. To this purpose, it is worth observing that the cascade, denoted by $\bar{\Sigma}$ in Problem 4, of the hybrid compensator Σ_C (thus determined) with the hybrid extended system Σ is ruled by (8), where the state of Σ , in the new coordinates, is partitioned as $[x_1^\top \ x_2^\top]^\top$ according to (1), (3), (4), and (7). By setting $\zeta(t) = x_1(t) - x_C(t)$, with $t \in \mathbb{R}^+$, the system $\bar{\Sigma}$ can be recast as in (9), where (2) and (5) have been taken into account. Hence, the assumption that the initial state is zero implies $\zeta(t) = 0$ and $x_2(t) = 0$, for all $t \in \mathbb{R}^+$, which also implies $e(t) = 0$, for all $t \in \mathbb{R}^+$, for all the admissible input functions $h(t)$, with $t \in \mathbb{R}^+$, and all the admissible jump time sequences $\mathcal{T} \in \mathcal{T}$.

Only if. If (6) does not hold, no other output-nulling \mathcal{H} -controlled invariant subspace containing \mathcal{H} exists, since the set of all output-nulling \mathcal{H} -controlled invariant subspaces is an upper semilattice and $\mathcal{W}_{\mathcal{H}}^*$ is the maximum. \square

6 Structural Model Following by Output Feedback in Hybrid Systems: Problem Solution

This section is aimed at showing that the problem of structural disturbance decoupling by dynamic feedforward respectively stated and solved for the hybrid extended system with state jumps in Sections 4 and 5 is equivalent to the problem of structural model following by output feedback stated in Section 3. This fact will be proven by demonstrating that a hybrid compensator with state jumps solves one of these problems if and only if it solves the other one. This result is formalized in the theorem below.

Theorem 6 *A hybrid compensator with state jumps Σ_C solves Problem 4 if and only if it solves Problem 3.*

Proof: If. Let the hybrid compensator Σ_C solve Problem 3. Consequently, the overall hybrid system with output feedback — from now called $\bar{\Sigma}'$ — is ruled by (10). It is worthwhile observing that, since

$$\bar{\Sigma} \equiv \begin{cases} \dot{x}_1(t) = A'_{11} x_1(t) + A'_{12} x_2(t) + B'_1 K'_1 x_C(t) + H'_1 h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_2(t) = A'_{21} x_1(t) + A'_{22} x_2(t) + B'_2 K'_1 x_C(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_C(t) = (A'_{11} + B'_1 K'_1) x_C(t) + H'_1 h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_1(t_k) = G'_{11} x_1(t_k^-) + G'_{12} x_2(t_k^-), & t_k \in \mathcal{T}, \\ x_2(t_k) = G'_{22} x_2(t_k^-), & t_k \in \mathcal{T}, \\ x_C(t_k) = G'_{11} x_C(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = E'_1 x_1(t) + E'_2 x_2(t) + D K'_1 x_C(t), & t \in \mathbb{R}^+, \end{cases} \quad (8)$$

$$\bar{\Sigma} \equiv \begin{cases} \dot{\zeta}(t) = A'_{11} \zeta(t) + A'_{12} x_2(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_2(t) = A'_{21} \zeta(t) + A'_{22} x_2(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_C(t) = (A'_{11} + B'_1 K'_1) x_C(t) + H'_1 h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \zeta(t_k) = G'_{11} \zeta(t_k^-) + G'_{12} x_2(t_k^-), & t_k \in \mathcal{T}, \\ x_2(t_k) = G'_{22} x_2(t_k^-), & t_k \in \mathcal{T}, \\ x_C(t_k) = G'_{11} x_C(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = E'_1 \zeta(t) + E'_2 x_2(t), & t \in \mathbb{R}^+, \end{cases} \quad (9)$$

Σ_C solves Problem 3, under the assumption that the initial state is the origin, the output of $\bar{\Sigma}'$ satisfies the condition that $e(t) = 0$, for all $t \in \mathbb{R}^+$, for all the admissible input functions $d(t)$, with $t \in \mathbb{R}^+$. Therefore, one can replace $e_P(t) = E_P x_P(t) + D_P C_C x_C(t)$ with $e_R(t) = E_R x_R(t)$ in the state equations of $\bar{\Sigma}'$. Consequently, the new system $\bar{\Sigma}''$ is described by (11). Further, since $e(t) = 0$ for all $t \in \mathbb{R}^+$, for all the admissible $d(t)$, with $t \in \mathbb{R}^+$, such condition holds when $d(t) = h(t) + E_R x_R(t)$, where $h(t)$, with $t \in \mathbb{R}^+$, stands for any admissible input function. Then, the system which turns out is the hybrid system $\bar{\Sigma}$ considered in Problem 4, as is proven by (12), which derive from $\bar{\Sigma}''$ with the abovementioned replacement. The equations of $\bar{\Sigma}$, which hold with $e(t) = 0$ for all $t \in \mathbb{R}^+$, for all the admissible $h(t)$, with $t \in \mathbb{R}^+$, show that the hybrid compensator Σ_C also solves Problem 4: i.e., the problem of decoupling the signal $h(t)$, with $t \in \mathbb{R}^+$, in the hybrid extended system Σ , including the modified hybrid reference model Σ_R^+ .

Only if. Let the hybrid compensator with state jumps Σ_C solve Problem 4. Therefore, to show that Σ_C also solves Problem 3, the reasoning presented in the if-part of the proof can be pursued backward — namely, starting from $\bar{\Sigma}$ and ending to $\bar{\Sigma}'$. \square

7 Model Following with Global Asymptotic Stability in Hybrid Systems with Periodic State Jumps

The discussion developed so far has been focused on structural model following: namely, the problem of finding a hybrid output feedback compensator such

that the respective forced responses of the closed-loop compensated hybrid system and of the hybrid reference model are equal for all the admissible input signals and all the admissible sequences of jump times.

The aim of this section is to investigate a more complete version of model following by output feedback in hybrid systems with state jumps: namely, a problem formulation where, under suitable assumptions on the given hybrid plant and the given hybrid reference model, the closed-loop hybrid compensated system is globally asymptotically stable for all the sequences of jump times belonging to a properly-defined set. In particular, the issue of global asymptotic stability of the closed-loop hybrid system is herein investigated by focusing on the case of hybrid systems with periodic state jumps. This is to say that the jump times of the sequence \mathcal{T} are multiple of a given positive real constant τ .

Hence, the plant Σ_P can be more conveniently described as

$$\Sigma_P \equiv \begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_P u(t), \\ \quad t \in [k\tau, (k+1)\tau), k \in \mathbb{Z}_0^+, \\ x_P(t) = G_P x_P(t^-), \\ \quad t = k\tau, k \in \mathbb{Z}^+, \\ e_P(t) = E_P x_P(t) + D_P u(t), & t \in \mathbb{R}^+. \end{cases}$$

Likewise, the hybrid reference model is described by

$$\Sigma_R \equiv \begin{cases} \dot{x}_R(t) = A_R x_R(t) + B_R d(t), \\ \quad t \in [k\tau, (k+1)\tau), k \in \mathbb{Z}_0^+, \\ x_R(t) = G_R x_R(t^-), \\ \quad t = k\tau, k \in \mathbb{Z}^+, \\ e_R(t) = E_R x_R(t), & t \in \mathbb{R}^+. \end{cases}$$

$$\bar{\Sigma}' \equiv \begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_P C_C x_C(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_C(t) = -B_C E_P x_P(t) + (A_C - B_C D_P C_C) x_C(t) + B_C d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_R(t) = A_R x_R(t) + B_R d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_P(t_k) = G_P x_P(t_k^-), & t_k \in \mathcal{T}, \\ x_C(t_k) = G_C x_C(t_k^-), & t_k \in \mathcal{T}, \\ x_R(t_k) = G_R x_R(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = E_P x_P(t) + D_P C_C x_C(t) - E_R x_R(t), & t \in \mathbb{R}^+. \end{cases} \quad (10)$$

$$\bar{\Sigma}'' \equiv \begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_P C_C x_C(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_C(t) = A_C x_C(t) - B_C E_R x_R(t) + B_C d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_R(t) = A_R x_R(t) + B_R d(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_P(t_k) = G_P x_P(t_k^-), & t_k \in \mathcal{T}, \\ x_C(t_k) = G_C x_C(t_k^-), & t_k \in \mathcal{T}, \\ x_R(t_k) = G_R x_R(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = E_P x_P(t) + D_P C_C x_C(t) - E_R x_R(t), & t \in \mathbb{R}^+. \end{cases} \quad (11)$$

In order to state the problem of model following with global asymptotic stability for hybrid systems with periodic state jumps, it is convenient to denote by \mathcal{T}_{τ_m} the set of all periodic sequences of jump times $\mathcal{T} = \{\tau, 2\tau, \dots\}$ such that the length of the time interval between two consecutive jump times is greater than or equal to τ_m , with τ_m denoting a given positive real constant: namely, $\tau \geq \tau_m$. Hence, it is assumed that both the hybrid plant Σ_P and the hybrid reference model Σ_R are globally asymptotically stable for all the jump time sequences belonging to \mathcal{T}_{τ_m} . This concept will be briefly referred to as *global asymptotic stability over \mathcal{T}_{τ_m}* of Σ_P and Σ_R , respectively. It is worth noting that there is no loss of generality in assuming that the hybrid plant Σ_P and the hybrid reference model Σ_R are globally asymptotically stable over the same set \mathcal{T}_{τ_m} of periodic sequences of jump times. Namely, if Σ_P and Σ_R are known to be globally asymptotically stable over $\mathcal{T}'_{\tau'_m}$ and $\mathcal{T}''_{\tau''_m}$, respectively, the constant τ_m can be assumed as the greater between τ'_m and τ''_m .

It is also worth noting that the hybrid plant Σ_P , subject to a periodic sequence of state jumps \mathcal{T} , with period τ , is globally asymptotically stable if and only if the state transition matrix over one period, i.e.,

$$\Phi_p(\tau) = G_P e^{A_P \tau},$$

is Schur stable, that is to say that all its eigenvalues lie inside the open unit disc of the complex plane. This statement, which is given with reference to Σ_P for the sake of immediacy, holds true for any hybrid system with periodic state jumps.

Hence, the problem of output feedback model following, with global asymptotic stability over \mathcal{T}_{τ} , in hybrid systems with state jumps can be stated as follows.

Problem 7 (Model Following by Output Feedback with Global Asymptotic Stability in Hybrid Systems with State Jumps) *Let the hybrid system with state jumps Σ_P and the hybrid reference model Σ_R be subject to periodic sequences of jump times with the same given period $\tau \geq \tau_m$, where τ_m denotes a given positive real constant. Let Σ_P and Σ_R be globally asymptotically stable over \mathcal{T}_{τ_m} . Find a hybrid compensator with periodic state jumps Σ_C , defined by*

$$\Sigma_C \equiv \begin{cases} \dot{x}_C(t) = A_C x_C(t) + B_C h(t), & t \in [k\tau, (k+1)\tau), k \in \mathbb{Z}_0^+, \\ x_C(t) = G_C x_C(t^-), & t = k\tau, k \in \mathbb{Z}^+, \\ u(t) = C_C x_C(t), & t \in \mathbb{R}^+, \end{cases}$$

where $h(t) = d(t) - e_P(t)$, such that the closed-loop hybrid system with state jumps Σ_O , defined by

$$\Sigma_O \equiv \begin{cases} \dot{x}_O(t) = A_O x_O(t) + D_O d(t), & t \in [k\tau, (k+1)\tau), k \in \mathbb{Z}_0^+, \\ x_O(t) = G_O x_O(t^-), & t = k\tau, k \in \mathbb{Z}^+, \\ e_P(t) = E_O x_O(t), & t \in \mathbb{R}^+, \end{cases}$$

where

$$\begin{aligned} A_O &= \begin{bmatrix} A_P & B_P C_C \\ -B_C E_P & A_C - B_C D_P C_C \end{bmatrix}, \\ D_O &= \begin{bmatrix} 0 \\ B_C \end{bmatrix}, \\ G_O &= \begin{bmatrix} G_P & 0 \\ 0 & G_C \end{bmatrix}, \\ E_O &= [E_P \quad D_P C_C], \end{aligned}$$

$$\bar{\Sigma} \equiv \begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_P C_C x_C(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_C(t) = A_C x_C(t) + B_C h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \dot{x}_R(t) = (A_R + B_R E_R) x_R(t) + B_R h(t), & t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_P(t_k) = G_P x_P(t_k^-), & t_k \in \mathcal{T}, \\ x_C(t_k) = G_C x_C(t_k^-), & t_k \in \mathcal{T}, \\ x_R(t_k) = G_R x_R(t_k^-), & t_k \in \mathcal{T}, \\ e(t) = E_P x_P(t) + D_P C_C x_C(t) - E_R x_R(t), & t \in \mathbb{R}^+. \end{cases} \quad (12)$$

satisfies the following requirements:

- $\mathcal{R}1.$ the output $e_P(t)$ is equal to the reference model output $e_R(t)$, for all $t \in \mathbb{R}^+$, when the respective initial states of the involved systems are zero, for all the admissible input functions $d(t)$, with $t \in \mathbb{R}^+$, and the given sequence of jump times $\mathcal{T} \in \mathcal{I}_{\tau_m}$;
- $\mathcal{R}2.$ the closed-loop hybrid system Σ_O is globally asymptotically stable for the given sequence of jump times $\mathcal{T} \in \mathcal{I}_{\tau_m}$.

In order to state a necessary and sufficient condition for Problem 7 to be solvable, the definition of minimum-phase hybrid system with periodic state jumps is introduced as follows. The hybrid plant Σ_P , subject to a periodic sequence of state jumps \mathcal{T} , with a given period τ , is said to be minimum-phase if its hybrid zero dynamics is globally asymptotically stable — namely, if the hybrid restricted dynamics $G_P e^{(A_P+B_P K_P)\tau}|_{\mathcal{W}_P^*}$, where \mathcal{W}_P^* denotes the maximal output nulling \mathcal{H} -controlled invariant subspace of Σ_P , is Schur stabilizable by a suitable choice of K_P .

Hence, the following necessary and sufficient condition for solvability of Problem 7 is established.

Theorem 8 *Let the hybrid plant Σ_P and the hybrid reference model Σ_R , subject to the sequence of periodic state jumps defined by \mathcal{T} be given. Let Σ_P and Σ_R be globally asymptotically stable over \mathcal{I}_τ , with the period τ given. Let the hybrid restricted dynamics $G_P e^{(A_P+B_P K_P)\tau}|_{\mathcal{W}_P^*}$, where \mathcal{W}_P^* denotes the maximal output nulling \mathcal{H} -controlled invariant subspace of Σ_P , be Schur stable for a suitable choice of K_P . Then, Problem 7 is solvable if and only if (6) holds, where the subspaces \mathcal{H} and $\mathcal{W}_{\mathcal{H}}^*$ refer to the hybrid extended system subject to periodic state jumps*

$$\Sigma \equiv \begin{cases} \dot{x}(t) = A x(t) + B u(t) + H h(t), \\ \quad \quad \quad t \in [k\tau, (k+1)\tau), k \in \mathbb{Z}_0^+, \\ x(t) = G x(t^-), \\ \quad \quad \quad t = k\tau, k \in \mathbb{Z}^+, \\ e(t) = E x(t) + D u(t), \quad t \in \mathbb{R}^+. \end{cases}$$

Proof: First, note that (6) is a necessary and sufficient condition to achieve structural model following by output feedback, by virtue of Theorem 6. Hence, a feedback compensator Σ_C , with periodic state jumps, achieving structural model following can be designed as shown in the if-part of the proof of Theorem 6. Then, in order to show that the same compensator Σ_C also accomplishes global asymptotic stability of the closed-loop system Σ_O for the given periodic sequence \mathcal{T} , it suffices to notice that global asymptotic stability of the reachable and observable modes is guaranteed by the assumption of global asymptotic stability of the hybrid reference model Σ_R and that inner unstable dynamics cancellations are prevented by the assumption that Σ_P is minimum-phase for the given sequence \mathcal{T} . \square

8 Conclusions

This work has been focused on model following by output feedback in hybrid systems with state jumps. First, a necessary and sufficient condition to achieve structural model following between a given plant and a given reference model, by means of an output feedback compensator, has been shown. In dealing with the structural aspects of the problem, no special assumptions have been made on the nature of the sequence of jump times, apart from that of considering a finite number of jump times in any finite time interval. Then, a necessary and sufficient condition for achieving model following with global asymptotic stability of the closed-loop hybrid system has been derived, by focusing on the case of hybrid systems subject to periodic state jumps.

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